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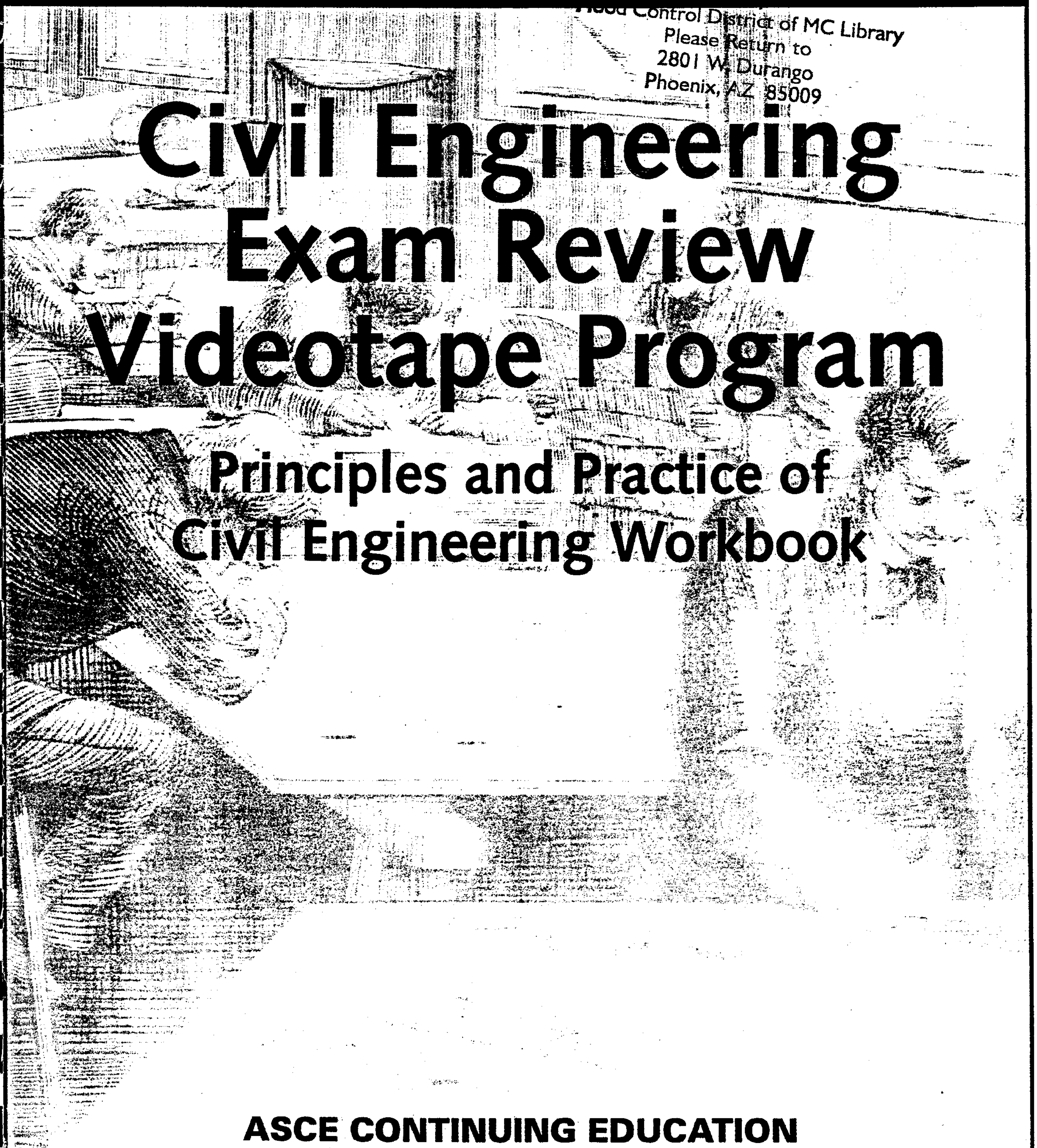

Civil Engineering Exam Review Videotape Program

Principles and Practice of Civil Engineering Workbook

ASCE CONTINUING EDUCATION

Civil Engineering Exam Review Videotape Program

Principles and Practice of Civil Engineering Workbook

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Instructor: Harry Parker

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ASCE CIVIL ENGINEERING EXAM REVIEW VIDEOTAPE PROGRAM

INTRODUCTION

Pursuing an engineering license is one of the most challenging efforts you will undertake in your professional life. Passing the Principles and Practice of Engineering Examination (P.E. Exam) and earning your license will be one of the most important accomplishments in your professional development - it will help you throughout your career.

The ASCE P.E. Exam Review videotape program can help you achieve this goal . The program covers all the topics you are expected to know for the P.E. exam. It includes problems similar to those that have appeared P.E. exams. The presentations are simple and straight-forward with emphasis on understanding the concepts. Review of the reference manual will assist you in understanding the mechanics of the problem-solving techniques covered in the videotapes. Most of the pages in the manual are also shown on the videotapes.

This program was videotaped in cooperation with the ASCE Boston Society of Civil Engineers exclusively for ASCE Continuing Education and the material covered is generally applicable nationwide.

LICENSE PROCEDURE

After passing the "Fundamentals of Engineering (F.E.) exam which covers all of the technical, mathematical and engineering subjects covered in an undergraduate degree program, the next step, after adequate experience, to become licensed is to pass the eight-hour "Principles and Practice of Civil Engineering" (P.E.) Exam. The P.E. Exam contains practical civil engineering problems encountered in the civil engineering field.

The time factor is significant for both the F.E. and the P.E. exams. Most states require four years or more of experience (after graduation and the F.E. exam) before you can take the P.E. exam. Both exams are typically given in mid-April and late October. Exam application deadlines may be several months in advance of the exam. Each state charges different fees, requires different qualifications, and uses different forms. Contact your state board of registration to request an examination application, exam date details and confirm which reference books will be allowed into the exam.

All states use the National Council of Examiners for Engineering and Surveying (NCEES) P.E. examination. If you pass the P.E. Exam in one state, your certificate will be honored by other states using the same NCEES examination (Uniform Examination). Each state may chose its own minimum passing score or add special questions, and, therefore, reciprocity is not automatically ensured.

A license from one state will not permit you to practice engineering in another state. You must have a professional engineering license from each state in which you are practicing.

PREPARATION

Study the list of examination subjects and concentrate on the subjects in which you have extensive professional experience. There will be a good probability of finding enough problems in your area of expertise that can be solved.

The exam is very fast paced. You must be able to recall procedures, formulas, and important data quickly. You will not have time in the exam to derive solution methods. You may want to prepare a one-page summary of all important formulas and information in each subject area. You can use these summaries during the exam instead of searching for the correct page in a book. Remember, preparation must be done in advance.

EXAM OVERVIEW

The P.E. Exam consists of two four-hour sessions separated by a one-hour lunch. Both sessions usually contain 12 questions each. Session one of the exam, given in the morning, is usually in essay format and includes 12 essay questions, of which any four must be answered. Session two of the exam, given in the afternoon, is usually in multiple choice format. Once again, 12 multiple choice questions are presented, of which any four must be answered. Each multiple choice question may consist of 10 parts. Multiple choice questions are either right or wrong. Essay questions can receive partial credit.

Thoroughly explain how you arrived at the answer to each question chosen. If only part of your answer is correct, you may still be eligible to receive partial credit for the answer given. It is important to show all your work for each solution/answer so that you can be given the maximum amount of credit. All solutions are recorded in an official solution booklet.

Some states have a "combined" exam which contains problems for civil, mechanical, electrical and chemical engineers. Other states, such as California will only allow you to work on problems from the civil part of the booklet and Massachusetts has a "discipline" exam. You must choose only the questions that apply to your particular discipline within civil engineering.

The following 16 subjects areas include all the important activities in which engineers engage, and since the examination structure is not rigid, it is not possible to give the exact number of problems that will appear in each subject area. There is no guarantee that any single subject will appear.

1. Design and analysis of traffic systems
2. Operations of traffic systems
3. Design and analysis of transportation facilities
4. Construction of transportation facilities
5. Design and analysis of buildings and special structures
6. Design and analysis of bridges and special structures
7. Design and analysis of foundation and retaining structures
8. Design and analysis of drainage and flood control
9. Design and analysis of natural water systems
10. Design and analysis of water supply systems
11. Design and analysis of waste water systems
12. Waste water treatment system operations
13. Design and analysis of solid/hazardous waste systems
14. Design and analysis of geotechnical soil projects
15. Construction of geotechnical soil projects
16. Construction material testing

Most states allow solution aids into the exam, including nomographs, and silent calculators without printers. Such aids should be used only to check your work. Very few points will be earned if a pre-programmed calculator is used to solve a surveying problem without showing interim steps.

Most states allow you to bring "bound" reference material into the exam. Some states ban only collections of solved problems and a few prohibit all review books. Loose papers and writing pads are usually not allowed. However, a three-ring binder with loose reference papers may be allowed. Books, calculators or any other items cannot be shared with other examinees. Keep in mind there is insufficient time to use books with which you are not thoroughly familiar.

The following are techniques to use during the exam:

- Classify the problems and then do them in order of increasing difficulty: easy problems, hard problems, problems for which you can get partial credit, and problems you cannot do.
- Do not rewrite the problem statement or unnecessarily redraw figures.
- Print all text and draw a box around each answer.
- Use one page per problem and write on one side of the page only.
- Check for mathematical errors.

Full credit is achieved by correctly working eight problems. NCEES has recommended the passing standard be established as a minimum raw score of 48 and a new "Criterion-Referenced Method" which requires specific elements you must include in your solution to receive credit for your solution. Getting ten points requires solving the problem correctly to completion and making no mathematical errors in the solution. For mathematical errors, a point or two would be lost. Each state is free to specify passing requirements.

The results of your examination will be received by mail about 12-14 weeks after the exam. Your score may not be revealed. You may retake the exam without having to reapply each time. Some people do not pass on the first try. You are allowed to review, with most state board, your exam results, to identify areas that you need to work on. Contact your state board to make these arrangements. Should you feel an appeal is necessary, after reviewing your exam results with the state board, it is up to you to provide a valid argument as to why you feel your exam was not scored properly.

NATIONAL COUNCIL OF EXAMINERS FOR ENGINEERING AND SURVEYING

P.O. BOX 1686
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Listed below are the phone numbers for each state's board of engineering registration.

Alabama	(205) 242-5568	Montana	(406) 444-4285
Alaska	(907) 465-2540	Nebraska	(402) 471-2021
Arizona	(602) 255-4053	Nevada	(702) 688-1231
Arkansas	(501) 324-9085	New Hampshire	(603) 271-2219
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Florida	(904) 488-9912	Ohio	(614) 466-3650
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Indiana	(317) 232-2980	South Carolina	(803) 734-9166
Iowa	(515) 281-5602	South Dakota	(605) 394-2510
Kansas	(913) 296-3053	Tennessee	(615) 741-3221
Kentucky	(502) 564-2680	Texas	(512) 440-7723
Louisiana	(504) 568-8450	Utah	(801) 530-6628
Maine	(207) 289-3236	Vermont	(802) 828-2837
Maryland	(410) 333-6322	Virginia	(804) 367-8514
Massachusetts	(617) 727-9956	Virgin Islands	(809) 774-3130
Michigan	(517) 335-1669	Washington	(206) 753-2548
Minnesota	(612) 296-2388	West Virginia	(304) 348-3554
Mississippi	(601) 359-6160	Wisconsin	(608) 266-1397
Missouri	(314) 751-0047	Wyoming	(307) 777-6155

II.
Hydrology and Hydraulics

Instructor: Gary Mercer

Tapes

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PROFESSIONAL ENGINEERING REVIEW

HYDRAULIC REVIEW SECTION

APPROACH

This section reviews the basic of hydraulics and hydrology for the Professional Engineer Examination. The audience is intended to be civil engineers that have had hydraulics and hydrology courses. The topics covered include: fluid properties, hydrostatics, equations of continuity, momentum, and energy of pipe and open channel flow. Various headloss equations, including Darcy-Weisbach, Hazen-Williams, and Mannings are presented, discussed, and compared. In addition, applications of: pumps and turbines in reservoir systems, Rational formula, water distribution analysis by Hardy-Cross method, and hydraulic jump are presented. The material provided includes: review notes, class notes, and example problems. The focus of the notes and class room work is on concepts and problem solving.

CONTENTS

The hydraulics review section contains three parts:

1. Review Notes

2. Videotape with Class Notes

3. Example Problems

The material should be reviewed in the above order for greatest benefit. A brief explanation of each part follows.

Review Notes

The review notes will refresh the reviewer's knowledge of the terms, methods, formulas, and applications used in hydraulics and hydrology. The units and symbols used in the material are introduced and defined in the review notes. Also an emphasis is made on introducing and understanding the fundamentals. To assist the reviewer some of the common reference information is included.

Videotape and Class Notes

The hydraulics review videotape consists of four - 50 minute classroom sessions. The sessions compliment the review notes and should be viewed sequentially. The class notes are included which form the basis of the classroom presentation. The approach for the classroom sessions is to introduce a concept and do several example problems applying the concept.

Example Problems

To compliment the review notes and the classroom notes are a set of example problems. The problems are similar to problems that have been on past examines. Full solutions are provided for each problem.

PROFESSIONAL ENGINEERING REVIEW

HYDRAULIC REVIEW SECTION

REVIEW NOTES

HYDRAULICS
REVIEW NOTES

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HYDRAULICS
REVIEW NOTES

The review notes contain the basic concepts and formulas with example problems worked and discussed for the Principal and Practice of Engineering Professional Engineering Review. These notes are supplemented with class notes and example problems.

1. Hydrostatics (Pressure-Density-Height Relationships)

Basic

The hydrostatic pressure difference, $P_1 - P_2$, between two elevations Z_1 and Z_2, is proportional to the unit weight of fluid and to the elevation difference:

$$P_1 - P_2 = \gamma (Z_2 - Z_1) = \rho g(Z_2 - Z_1) \tag{1-1}$$

where: P_1, P_2 = Pressure @ 1 or 2 (units = force per area, psi)
Z_1, Z_2 = Elevation @ 1 or 2 (units = distance, ft)
γ = Unit weight (units = force per volume, lb/ft^3)
γ_{WATER} = 62.4 lb/ft^3 or 1000 kg/m^3
ρ = Density (units = mass per volume, slug/ft^3)
ρ_{WATER} = 1.94 slugs/ft^3
g = Acceleration due to gravity = 32.2 ft/sec^2 = 9.8 m/sec^2

also: $\gamma = \rho g$

In Figure 1-1, at a depth h beneath the water surface in the tank, the gauge pressure is just γh. The absolute pressure is gauge pressure plus atmospheric pressure.

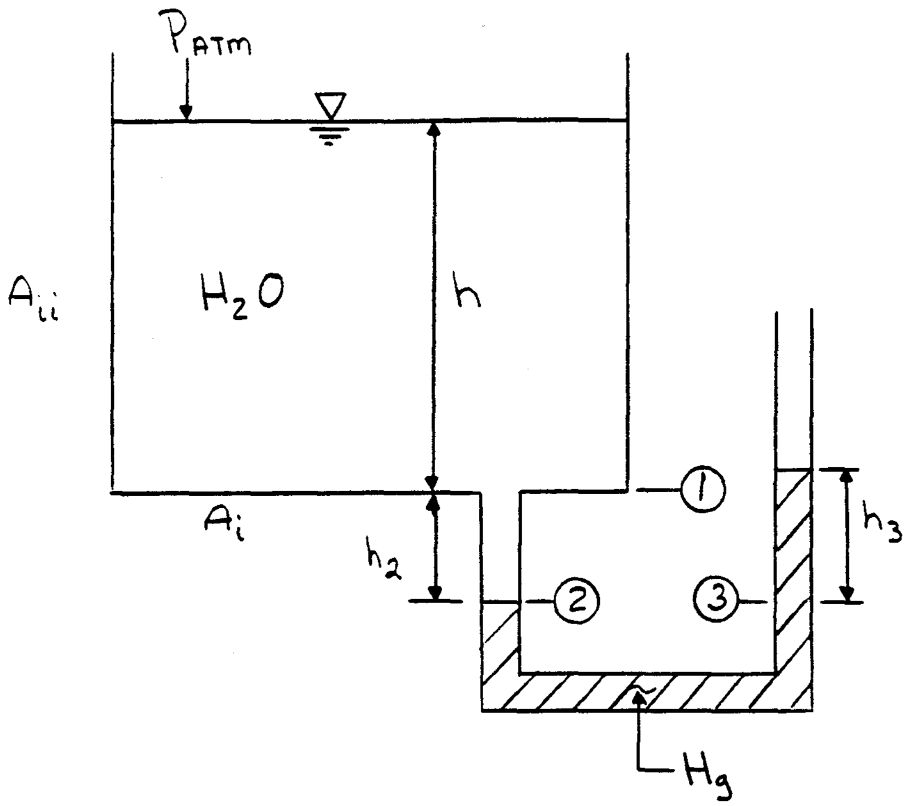


Figure 1-

At sea level, atmospheric pressure is about 14.7 psi, or 2116.8 psf; in terms of head of water, 34 ft or 10.4 m; in terms of head of mercury, 30 in. or about 760 mm.

Again in Figure 1-1, the pressure at (2) is $P_1 + \gamma h_2$, or $\gamma(h_1 + h_2)$. Since (2) and (3) are at the same elevation, $P_2 = P_3$. P_3 also equals $p_{ATM} + \gamma s h_3 m$ where s is the specific gravity of mercury, 13.6.

Knowing s, h_2 and h_3, one may deduce h_1, the depth of water in the tank. Conversely, knowing h_1, h_2 and h_3 one may deduce s.

Forces on Submerged Planes

The hydrostatic force on a plane area is equal to the hydrostatic pressure that exists at the centroid of the area, multiplied by the area; and the force acts in a direction normal to the plane. The force acts at the center of pressure. In Figure 1-1, the force on the bottom of the tank is $(P_{ATM} + \gamma h)A_i$, and acts downward. The force on the left wall of the tank is $(P_{ATM} + \gamma h/2)A_{ii}$ and acts horizontally to the left.

2. Continuity (Inflow = Outflow)

Unless there is a storage accumulation or drawdown in a conduit, the mass flow rate through a conduit is constant from point to point along the conduit.

In Figure 2-1, the mass flow rate at Station 1 is $\rho_1 A_1 V_1$, the product of density, cross-sectional area, and velocity. At Station 2 it is $\rho_2 A_2 V_2$.

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

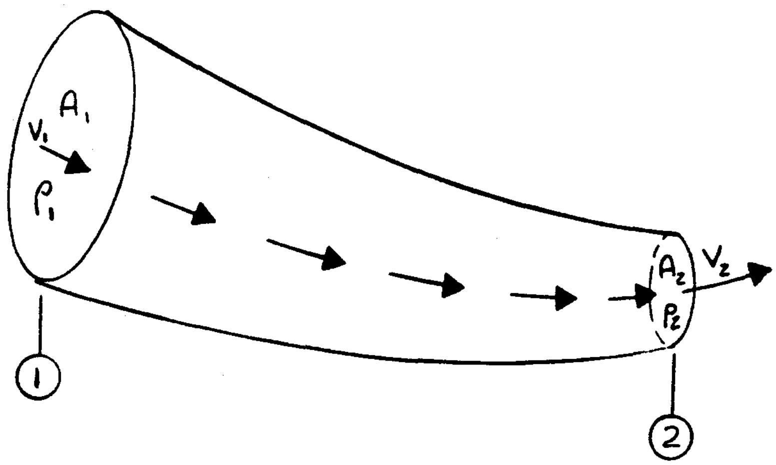


Figure 2-

For many hydraulics problems where fluid compressibility is negligible, ρ_1 and ρ_2 may be considered equal, and

$$A_1V_1 = A_2V_2$$

Example: Hydraulic jump in a flow of water in a channel of width, b_1, b_2. (See Figure 3-1). The density, ρ, may be considered constant, so

$$A_1V_1 = A_2V_2$$

Now $A_1 = b_1y_1$ and $A_2 = b_2y_2$, where **b** = width of channel and **y** = depth of channel

$$b_1y_1V_1 = b_2y_2V_2$$

If channel width is constant, $b_1=b_2$, and $y_1V_1 = y_2V_2$.

3. Momentum

Force = mass x acceleration, Newton's Second Law of Motion

Force, momentum, distance, velocity, and acceleration, are vector quantities, in that one must specify their direction as well as their magnitude. Temperature, pressure, and energy are scalar quantities, with magnitude only. Pressure (a scalar) times area (a vector whose "direction" is perpendicular to the plane of the area) = a force (a vector).

Force equals mass times acceleration. Acceleration includes a time rate of change in velocity in unsteady flow at one station, and a change in velocity in a steady flow as fluid moves from Station (1) to Station (2).

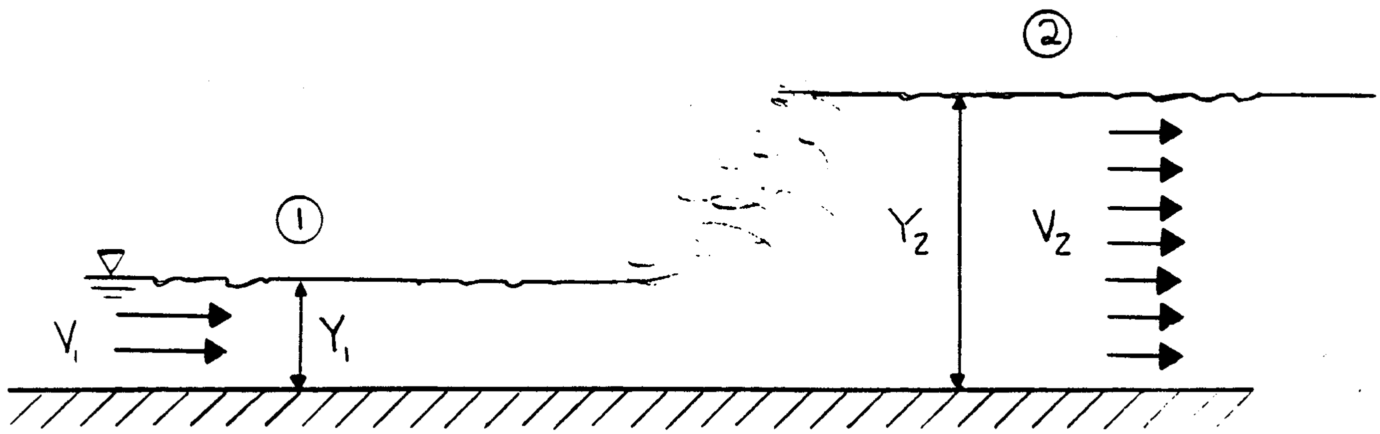


Figure 3-1

The basic momentum equations for steady flow in three space dimensions x, y, and z are:

$$\Sigma F_x = \rho Q\,(V_{x2} - V_{x1})$$
$$\Sigma F_y = \rho Q\,(V_{y2} - V_{y1})$$
$$\Sigma F_z = \rho Q\,(V_{z2} - V_{z1})$$

remember:

ρ = density $\Rightarrow$ mass per volume

Q = flowrate $\Rightarrow$ volume per time

F = P x A

Figure 3-2 presents examples of applying these equations.

As a third example, consider a hydraulic jump in a rectangular channel of constant width, with zero slope, and negligible friction (see Figure 3-1).

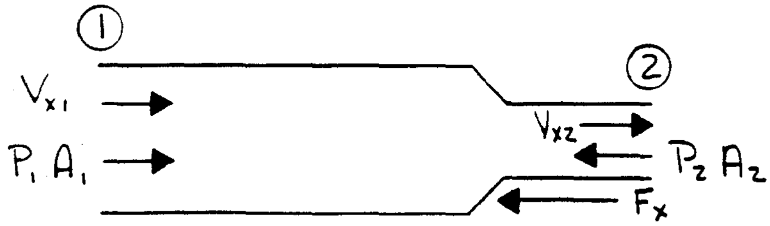
There are no bends, so we only need to consider forces acting upstream and downstream (on the x-axis).

At (1), the force on the fluid body is the hydrostatic pressure integrated over the flow depth:

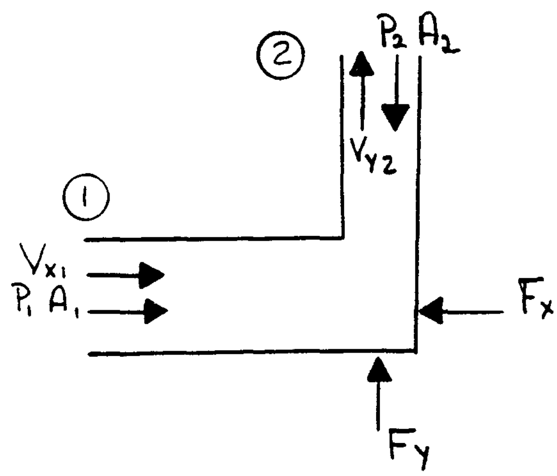
$$F_1 = b\int_0^{y_1} P_1\,(y)\,dy, \qquad P_1 = \rho g\,(y_1 - y)$$

$$F = \rho g b \int_0^{y_1} y_1 y - \frac{y^2}{2} \quad = \quad \frac{\rho g b y_1^2}{2}$$

The force acts to the right, inward on the body; at (2), the force is $F_2 = \rho g b y^2/2$. The force acts to the left, inward on the body.



$$F_x + P_1 A_1 - P_2 A_2 = \rho Q (V_{x2} - V_{x1})$$



$$P_1 A_1 - F_x = \rho Q (-V_{x1})$$
$$F_y - P_2 A_2 = \rho Q (V_{y2})$$

Figure

The rate of change of momentum is

$$F_x = F_1 - F_2 = \rho Q (V_2 - V_1)$$

$$\rho g b \left(\frac{y_1^2}{2} - \frac{y_2^2}{2}\right) = \rho Q (V_2 - V_1)$$

If one is given Q, which equals $V_1 b_{y1}$, and $V_2 b_{y2}$; and if one is given **b** and, say, y_1, one can now solve for V_1, V_2 and y_2.

4. Energy

The energy equation, or Bernoulli equation, is a means of keeping track of the energy budget in a flow system: potential energy, kinetic energy, energy lost in gradual or sudden dissipation, energy provided by a pump, energy extracted by a turbine.

In one-dimensional hydraulics analysis, energy is measured in terms of pressure head, units of length (feet, meters). "Head" multiplied by unit weight, ρg, is "pressure", which is "energy per unit volume." Recall that the potential energy of a body of mass **m** situated at a height **h** above a reference datum is **mgh**, and that its kinetic energy is **1/2 mv²**, and that its total energy is the sum of the two: **m (gh + 1/2v²)**. Divide by the volume of the body to get ρ(gh + 1/2v²), where ρ = m/volume. Divide by ρg to express total energy in terms of head: **h + v²/(2g)**. This is often called the specific energy.

The Bernoulli equation may be written as:

$$Z_1 + y_1 + \frac{v_1^2}{2g} = Z_2 + y_2 + \frac{v_2^2}{2g} + h_f + h_t - h_p \qquad (4\text{-}1)$$

The left-hand side, $A_1 + y_1 + v_1^2/2g$, is the total energy head at a station (1), upstream. The first term, A_1, is the elevation of the floor of the channel or the invert of the pipe. The second term, y_1, is the piezometric elevation. Together, $Z_1 + y_1$, indicate the potential energy at (1). The velocity head, $(v_1^2)/2g$, denotes the kinetic energy.

The piezometric elevation, y, in an open channel can usually be taken as depth. In a closed conduit flowing full, it is the "artesian pressure head", the height to which fluid would rise in a piezometer tube, a test bore piercing the crown of the pipe. (Figure 4-1)

The right-hand side of Equation 4-1 includes the total head at a second station, $Z_2 + y_2 + (v_2^2)/2g$, and all terms responsible for change in total head between stations (1) and (2): friction and form loss, h_f; turbine head, h_t; and pumping head, h_p.

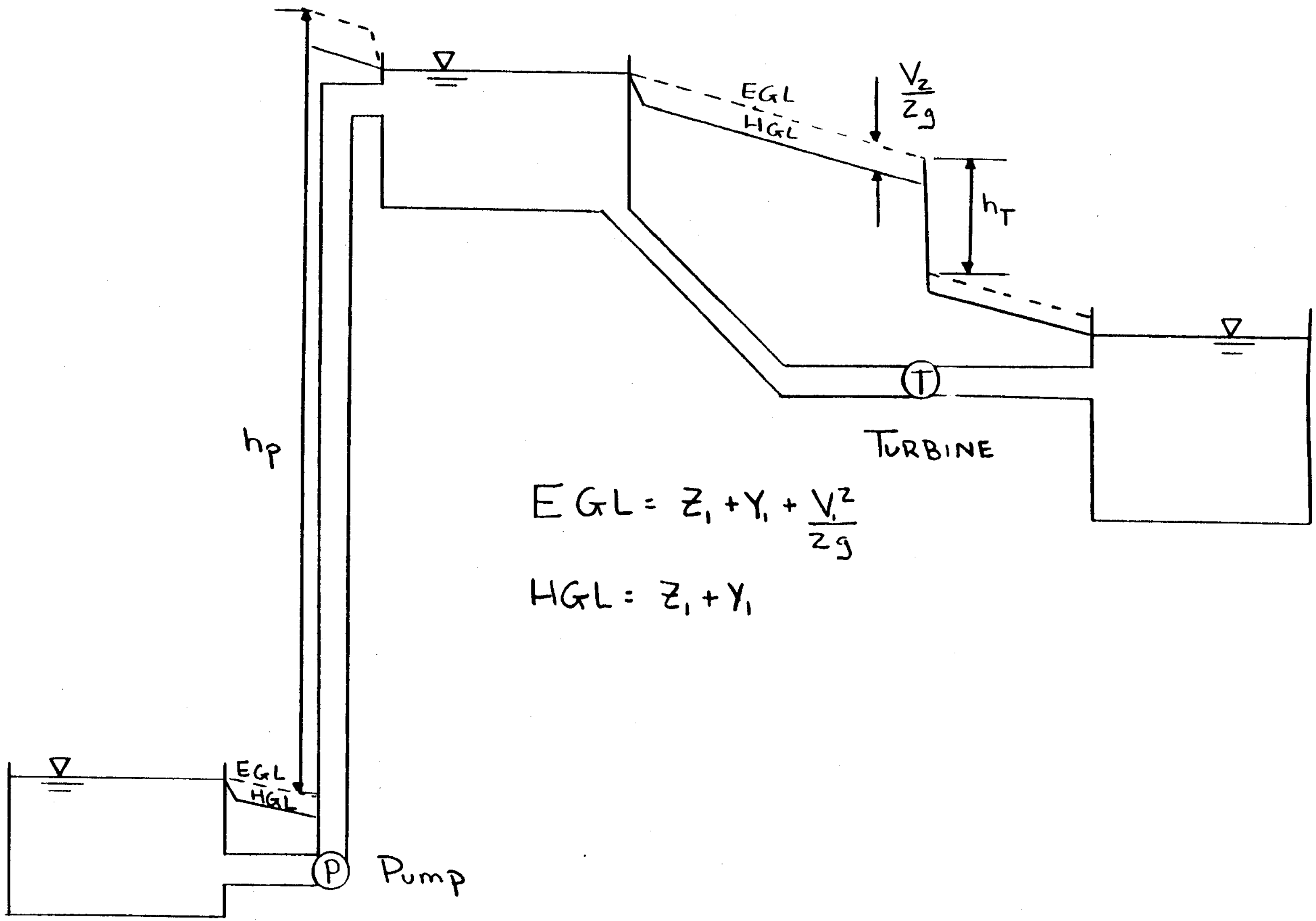
In Figure 4-2, consider a Pitot tube, used for measuring current speed. A Pitot tube actually contains two tubes, one bent so that its end aperture faces directly into the oncoming flow (A), the other's opening facing across the flow (B). Write the Bernoulli equation, neglecting any friction losses:

$$Z_A + Y_A + \frac{V_A^2}{2g} = Z_B + Y_B + \frac{V_B^2}{2g}$$

The elevations Z_A and Z_B are the same. The velocity of flow past B, is the velocity of the flow being measured. However, (A) is designed to be at a stagnation point, where the velocity directly ahead of the tube is brought to a stop, so that $V_A = 0$. Thus:

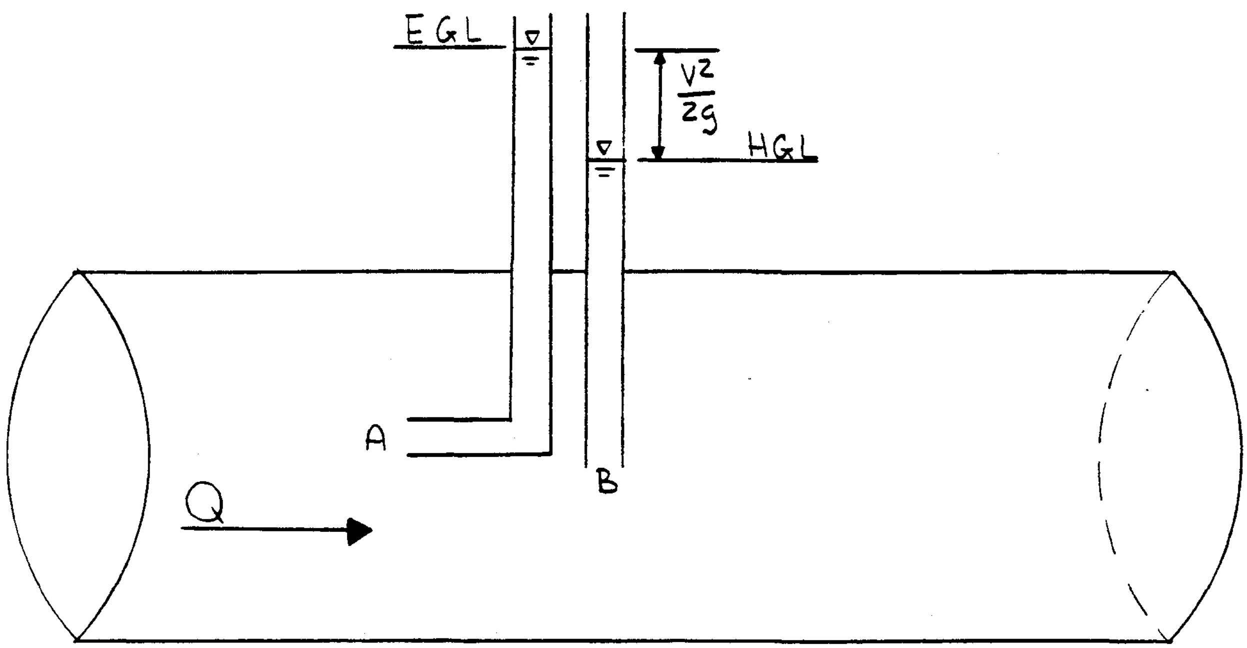
$$Y_A = Y_B + \frac{V_B^2}{2g}$$

Tube A measures total head and is called a <u>total head tube</u>. Tube B is another example of a piezometer tube. By measuring the difference in water levels in the two tubes, $Y_A - Y_B$, we can deduce the flow velocity, V_B.



EGL
HGL
$\frac{V_2}{2g}$
h_T
h_p
T
TURBINE
P
Pump
EGL
HGL
$EGL = Z_1 + Y_1 + \frac{V_1^2}{2g}$
$HGL = Z_1 + Y_1$

Figure 4-1



PITOT TUBE

Figure 4-2

5. Pressure Losses, due to Form and to Friction

As fluid moves through a channel or conduit, it dissipates kinetic energy, and loses pressure, or "head".

See the definition for "total head" in Section 4. Total head represents the sum of the potential and kinetic mechanical energy in the flow. Head is lost when kinetic energy is dissipated, due to wall friction, and due to inefficient flow patterns at bends.

Kinetic energy is proportional to V^2 and Q^2 (remember, K.E. = 1/2 mV^2 from elementary mechanics). Head loss is also proportional to V^2 and Q^2 via a "loss factor" or a "friction factor." These are dependent on the Reynolds number, $\mathbf{R}$. For sufficiently large flows, as found in medium to large conduits, the loss factors are independent of $\mathbf{R}$ and hence of velocity:

$$h = K\frac{v^2}{2g} \qquad (5\text{-}1)$$

in which K depends on conduit geometry, but not on V. Another example is the Manning friction formula:

$$v^2 = \left(\frac{1}{n^2}\right) R^{\frac{4}{3}} \frac{h}{L} \quad \rightarrow \quad v = \left(\frac{1}{n}\right) R^{\frac{2}{3}} S^{\frac{1}{2}} \qquad (5\text{-}2)$$

in which R is the hydraulic radius (compare Equation 5-2 with Equations 5-5 and 5-6).

For very small Reynolds number, such as for flow through sand or clay aquifer or through capillary tubes, the friction factor, **f**, is inversely proportional to **R**, hence V, so that

h is proportional to **fV²**, and f is proportional to **1/v**, then

h is proportional to **(1/V)V²**, so that

h is proportional to **V**.

Examples of equations for this condition are the Darcy law for groundwater flow and the Kozeny and Fair Hatch equations for flow through porous filter media.

Friction Loss Formulas

The Hazen-Williams formula describes the headloss relationship for intermediate values of **R**, Reynolds number:

$$h = (10.6)\, L\left(\frac{Q}{CD^{2.63}}\right)^{1.85} \qquad (5\text{-}3)$$

in which head loss **h**, diameter **D**, and length **L** are in feet, and **Q** is in mgd. The friction factor **C** is about 130 for smooth pipe, 100 for average pipe, and about 80 for rough-walled pipe.

This formula, with the exponent of 1.85 less than 2 and more than 1, is intended for small-diameter pipe systems. It is not theoretically elegant, but has been and continues to be used widely, rendering adequate results when used as intended.

The Darcy-Weisbach formula is the most elegant and general of the friction formulas, in that it covers the full range of **R**, and may be used with any liquid or gas:

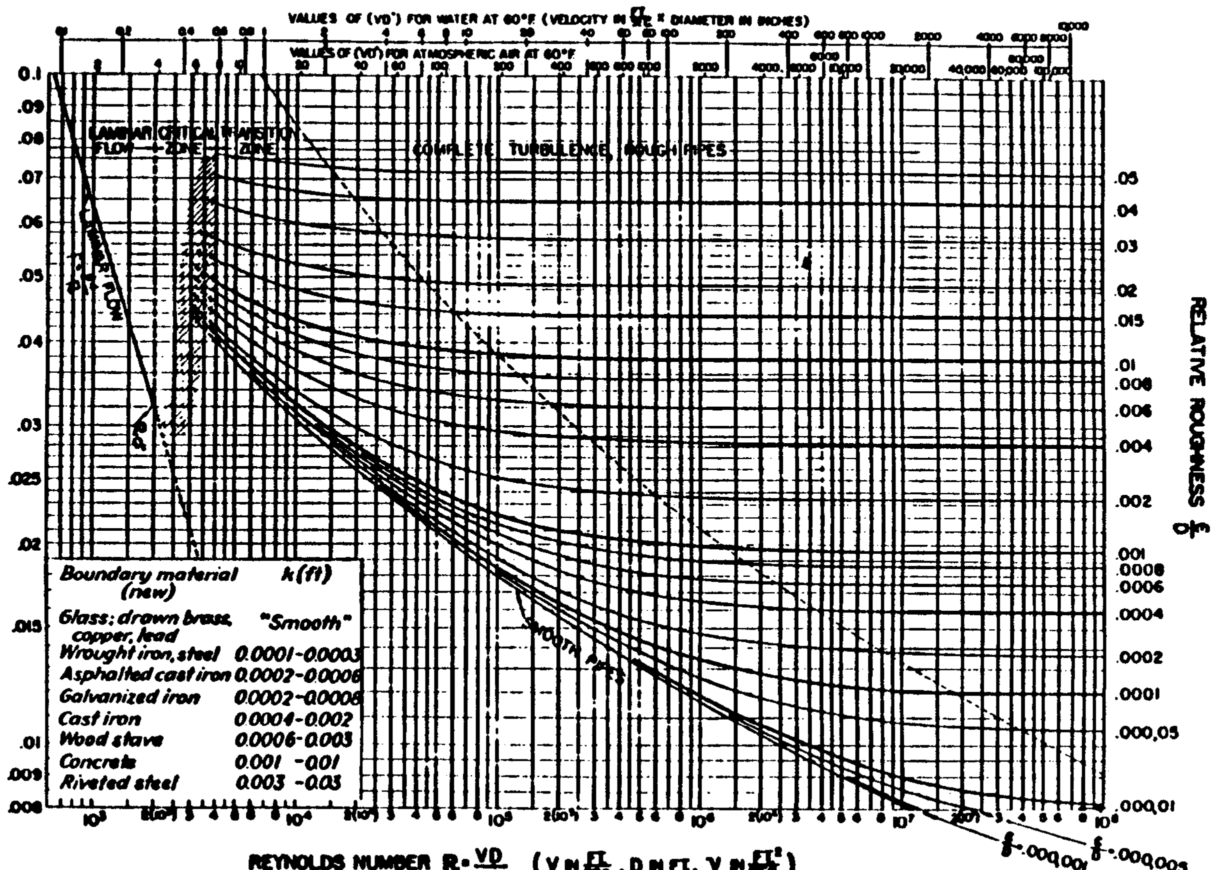
$$S = \frac{h}{L} = \frac{f}{D}\left(\frac{V^2}{2g}\right) \quad \rightarrow \quad h = \frac{fL}{D}\left(\frac{V^2}{2g}\right) \qquad (5\text{-}4)$$

In this formula the friction slope, S = h/L, is simply related to the velocity head and the pipe diameter by the coefficient f. This coefficient f is a function of Reynolds number, **R** = VD/υ, and the relative roughness of the pipe or channel wall, ε/D, as shown on the friction factor, or Moody, diagram in Figure 5-1.

The figure shows that for small **R**, **f = 64/R**, so that **s = h/L = 64υV/2gD²** , i.e., s and h are proportional to V, as in Darcy's Law for groundwater flow.

For large **R**, the figure shows horizontal lines of constant f, indicating that f is independent of **R**, and depends only on ε/D. This represents completely turbulent flow very often encountered in practical large-pipe or open-channel flow problems. The roughness dimension, ε, is the typical height of roughness elements or "bumps" on the pipe wall. Ranges of values of ε characteristic of several pipewall materials are given in the inset table of Figure 5-1.

For intermediate values of **R**, f is a function of both **R** and ε/D. The functional relationship in this zone has been expressed in terms of empirical formulas, such as those by Colebrook and White; for review purposes it is adequate to simply derive the relationship from the curves on Figure 5-1.



SOURCE: AMERICAN SOCIETY OF MECHANICAL ENGINEERS, NEW YORK, N.Y. TRANSACTIONS - ASME, VOL. 66 (1944) L. F. MOODY.

Figure 5-1

The Manning formula, applicable to large **R** flows in which friction is independent of **R**, is convenient and widely used for many civil engineering flow applications. In SI units:

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}} \qquad (5\text{-}5)$$

in which:

Q, m^3/sec, is the discharge

A, m^2 is the cross-sectional area

R, m, is the hydraulic radius

S, dimensionless, is the friction slope

In the foot-lb-second system, in which **Q** is in cfs, **A** is in ft^2, and **R** is in feet:

$$Q = \frac{1.49}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}} \qquad (5\text{-}6)$$

The friction factor, **n**, increases with increasing wall roughness. Suggested values are given in Table 5-1.

Form Loss

"Form loss" occurs whenever there is a separation of flow at a boundary, creating a zone of eddying and turbulent dissipation of energy. Such flow separation occurs at nearly any sudden change in channel wall direction, as at sharp bends or sudden contractions or expansions. The form headloss proportionality constant K in Equation 5-1 is a function of local pipe geometry and Reynolds number; at sufficiently high Reynolds numbers, the case for many practical applications, K is a function of geometry only. Values of K for use in Equation 5-1 are listed in Table 5-2.

Table 5-1

VALUES OF n FOR USE IN MANNING OR KUTTER FORMULA *

0.009 and 0.010	Very smooth and true surfaces, without projections. Clean new glass, pyralin, or brass, with straight alignment.
0.011 and 0.012	Smoothest clean wood, metal, or concrete surfaces, without projections, and with straight alignment.
0.013	Smooth wood, metal, or concrete surfaces without projections, free from algae or insect growth, and with reasonably straight alignment.
0.014	Good wood, metal, or concrete surfaces with very small projections, with some curvature, with slight insect or algae growth, or with slight gravel deposition. Shot concrete surfaced with troweled mortar.
0.015	Wood with algae and moss growth, concrete with smooth sides but roughly troweled or shot bottom, metal with shallow projections. Same with smoother surface but excessive curvature.
0.016	Metal flumes with large projections into the section. Wood or concrete with heavy algae or moss growth.
0.017	Shot concrete, not troweled, but fairly uniform.
0.018 to 0.025	Metal flumes with large projections into the section and excessive curvature, growths, or accumulated debris.
0.016 to 0.017	Smoothest natural earth channels, free from growths, with straight alignment.
0.020	Smooth natural earth, free from growths, little curvature. Very large canals in good condition.
0.022	Average, well constructed, moderate-sized earth canal in good condition.
0.025	Very small earth canals or ditches in good condition, or larger canals with some growth on banks or scattered cobbles in bed.
0.030	Canals with considerable aquatic growth. Rock cuts, based on average actual section. Natural streams with good alignment, fairly constant section. Large floodway channels, well maintained.
0.035	Canals half choked with moss growth. Cleared but not continuously maintained floodways.
0.040 to 0.050	Mountain streams in clean loose cobbles. Rivers with variable section and some vegetation growing in banks. Canals with very heavy aquatic growths.
0.050 to 0.150	Natural streams of varying roughness and alignment. The highest values for extremely bad alignment, deep pools, and vegetation, or for floodways with heavy stand of timber and underbrush.

*From Rouse, Engineering Hydraulics, Wiley, 1950.

TABLE 5-2

VALUES OF HEADLOSS COEFFICIENT FOR SOME FITTINGS, BENDS, AND SECTION CHANGES

ITEM		K
Square elbow, single-mitre bend		1.0
Sudden enlargement (use upstream velocity for V):	$d_1/d_2 = 3/4$	0.19
	$d_1/d_2 = 1/2$	0.56
	$d_1/d_2 = 1/4$	0.92
ordinary outlet:	$d_1/d_2 = 0$	1.00
Sudden contraction (use downstream velocity for V):	$d_1/d_2 = 3/4$	0.25
	$d_1/d_2 = 1/2$	0.43
	$d_1/d_2 = 1/4$	0.49
ordinary inlet:	$d_1/d_2 = 0$	0.50
Rounded, or bell-mouth, inlet:		0.001
Borda inlet:		0.75
Gate valve, fully open:		0.25

6. Critical Depth in Open Channel Flow

In rectangular open channels, the flow is called critical when the Froude number, F, is one:

$$F = \frac{v}{\sqrt{g\,y}} = 1 \qquad (6\text{-}1)$$

where **v** is the average flow velocity, **g** is the gravitational acceleration, and **y** is the water depth.

Subcritical flow, where **F** is less than 1, is the relatively quiescent flow upstream of weirs, or found in long channels with mild slope, and downstream of full hydraulic jumps. Supercritical flow, in which **F** is greater than 1, is the "shooting flow" often seen downstream of weirs and sluice gates, often characterized by oblique (or "diamond-pattern") fixed waves.

In rectangular channels, critical depth and critical velocity for a given flow, Q, are easily computed from Equation (6-1):

$$Q = vby \;; \qquad v = \sqrt{gy_c} \quad for \quad F = 1 \qquad (6\text{-}2)$$

then:

$$\frac{Q}{b} = v_c y_c = y_c\sqrt{gy_c} = g^{\frac{1}{2}}\,y_c^{\frac{3}{2}} \qquad (6\text{-}3)$$

$$y_c = g^{-\frac{1}{3}}\left(\frac{Q}{b}\right)^{\frac{2}{3}} \qquad \text{or} \qquad v_c = \left(g\,\frac{Q}{b}\right)^{\frac{1}{3}} \qquad (6\text{-}4)$$

However, for a channel that does not have a constant depth like a rectangular channel, the definition of critical flow conditions is a little more complex.

Recall from Section 4 that the specific energy, E, is the sum of potential and kinetic energy:

$$E = y + \frac{v^2}{2g} \qquad (6\text{-}6)$$

Express velocity, **v**, as flow per unit of cross-sectional area: **v = Q/A**:

$$E = y + \frac{(Q/A)^2}{2g} \qquad (6\text{-}7)$$

Since **A** is a function of depth, **y**, the plot of **E** against **y** appears as in Example 3. For all conditions but critical, there can be two values for **y** for each value of **E**; subcritical and supercritical. The critical condition is found when **E(y)** is a minimum, i.e., when
dE/dy = 0:

$$\frac{dE}{dy} = 1 + \left(\frac{Q^2}{2g}\right)\left(\frac{-2}{A^3}\right)\frac{dA}{dy} \qquad (6\text{-}8)$$

$$= 0 \quad when \quad \left(\frac{Q^2}{gA^3}\right)\left(\frac{dA}{dy}\right) = 1 \qquad (6\text{-}9)$$

Now, **dA/dy** is simply the width of the channel at the water surface; call it "**b**":

For critical conditions: $$\frac{Q^2 b}{gA^3} = 1 \qquad (6\text{-}10)$$

Equation (6-10) is then the general equation for critical flow conditions in a channel of any cross-section. To check its agreement with Equations (6-1) and (6-3) for rectangular channels, set **y = A/b** and **Q/A = v**.

7. Free-Surface Weirs

Weirs regulate the rate at which flow passes over them, and provide a convenient means of flow measurement. For many cases of interest, the flow passes through a critical flow condition. Therefore, weir formulas are all related to Equation 6-10 in one way or another.

Rectangular weirs:

$$Q = C_d \frac{2}{3} \sqrt{2g} \, L h^{\frac{3}{2}}$$

or

$$Q = C L h^{\frac{3}{2}} \qquad where \qquad C = C_d \frac{2}{3} \sqrt{2g}$$

Triangular weirs (90° only), in f-p-s units:

$$Q = 2.5 \, H^{2.5}$$

8. Some Basic Elements of Hydrology

The Rational Formula. To quote Williams[1]:

> "In basic concept the rational method presumes that the maximum rate of runoff from a small drainage basin occurs when the entire basin is contributing, and that this rate of runoff equals a percentage 'C' of the average rate of rainfall. In equation form,
>
> $$Q = CIA \tag{8-1}$$

---

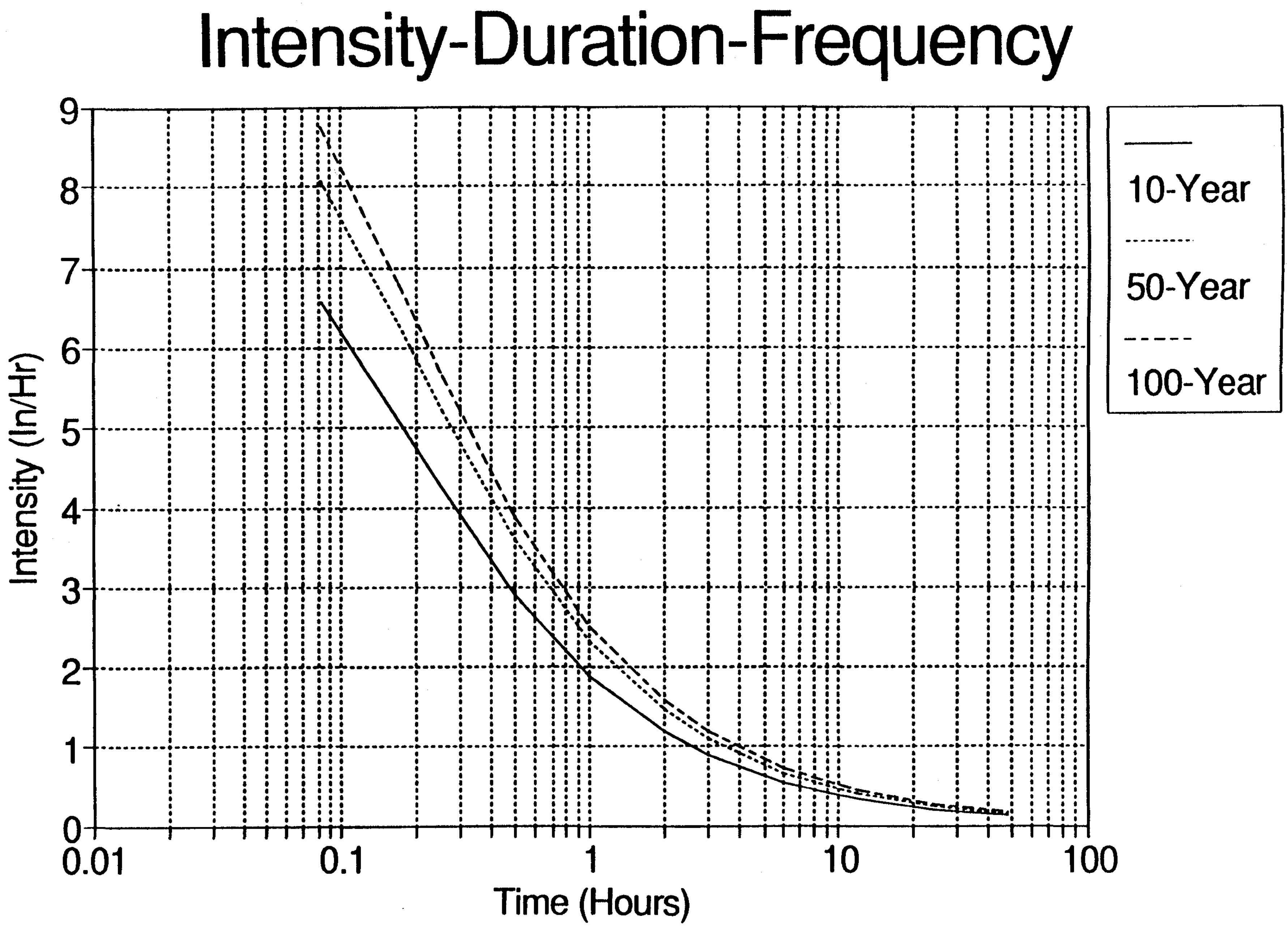
[1] Williams, G.R., "Chapter IV: Hydrology", in Engineering Hydraulics, H. Rouse, ed., Wiley, 1950.

> where 'Q' is the rate of runoff in acre-inch per hour = 1.008 cubic feet per second, which may be taken as one for all practical purposes, 'C' is the ratio of peak runoff to average rainfall, I is the average rainfall intensity in inches per hour, and 'A' is the drainage area in acres. Implicit in the determination of I and in the stipulation "when the entire basin is contributing: is the determination of what is usually called the time of concentration, designated by T_c. As visualized by earlier investigators, T_c is the time of travel of a water particle from the most remote point to the outlet of the drainage basin."

Equation (8-1) is widely used in hydrologic analysis of small areas, simply as it stands and also as a core algorithm of some hydrologic computer programs that lag and route the Q's from many sub-areas through a large area, and account for such things as depression storage and initial infiltration. The review herein will be restricted to the straightforward use of Equation (8-1).

The value of 'C' is a function of land use and soil type. Paved parking lots have a high value; level, grassed lawns have a low value. See Table 8-1 for a range of conditions.

The value of design average rainfall intensity, 'I', is a function of local climate, the time of concentration, T_c defined in the above quotation, and the frequency of the design event. Figure 8-1 shows one form in which intensity-frequency-duration curves may be plotted. (Usually, in design applications of the rational formula, the rainfall duration of interest is the time of concentration of the drainage area under study.)



II-24

Figure 8-1

TABLE 8-1

VALUES OF C FOR USE IN THE RATIONAL FORMULA

Land Use or Condition	C
Lawns, flat, sandy	.05 - .10
Lawns, steep, heavy soil	.25 - .35
Business: Downtown	.70 - .95
Neighborhood	.50 - .70
Residential Single family house	.30 - .50
Apartment house	.50 - .70
Industrial Light	.50 - .80
Heavy	.60 - .90

Reservoir Storage. A common water resources problem is: Given a seasonally varying water supply, and a relatively constant demand that exceeds the supply in the dry season, what active reservoir storage volume is needed to ensure that demand is met?

First of all, annual demand must not exceed annual supply. Secondly, reservoir storage will have a net draft every day (or week, or month) of the dry season, starting from when supply roughly equals demand continuing throughout the months when demand exceeds supply, until supply (river inflow, perhaps) once again equals demand. The required storage volume is the net draft (volume out less volume in) in each time period (day, week, or month), summed over all time periods in the dry season.

9. Pumps and Turbines

Pumps and turbines can share much of the same discussion. Consider pumps first.

In a pump, mechanical energy is provided to turn an impeller at speed N. Depending on the pump size and design, rotation at speed N will overcome an imposed head differential, H, across the pump, and supply a flow at rate Q. For a given N, the relationship between H and Q may be plotted as shown by the N_1 or N_2 curve on Figure 9-1.

A pump may perform tolerably over a range of H, Q, and N, but is found to have its greatest efficiency at a single point on the diagram. This point, commonly denoted by a "hard corner" symbol, defines the rated conditions of head, speed, and flow.

An increase of adverse head, H, forces a decrease in output, Q, for a given N. The head at which Q is reduced to zero, or "shut off", is called the "shutoff head" for that N.

Power is the product of pressure and flow rate, HQ. The horsepower delivered by a pump is

$$\text{HP delivered} = \frac{wHQ}{550} \tag{9-1}$$

where **w** is the unit weight of fluid in lb/ft^3, wH is the pressure in lb/ft^2, and Q is the flow rate in cfs. The power delivered is less than the power applied, by the efficiency factor:

$$\text{Power delivered} = (e) \times (\text{Power applied}) \tag{9-2}$$

where **e** is the fractional efficiency, so that

$$\text{Mechanical horsepower applied} = \frac{wHQ}{550e} \tag{9-3}$$

Pump Curve
Head vs. Flow

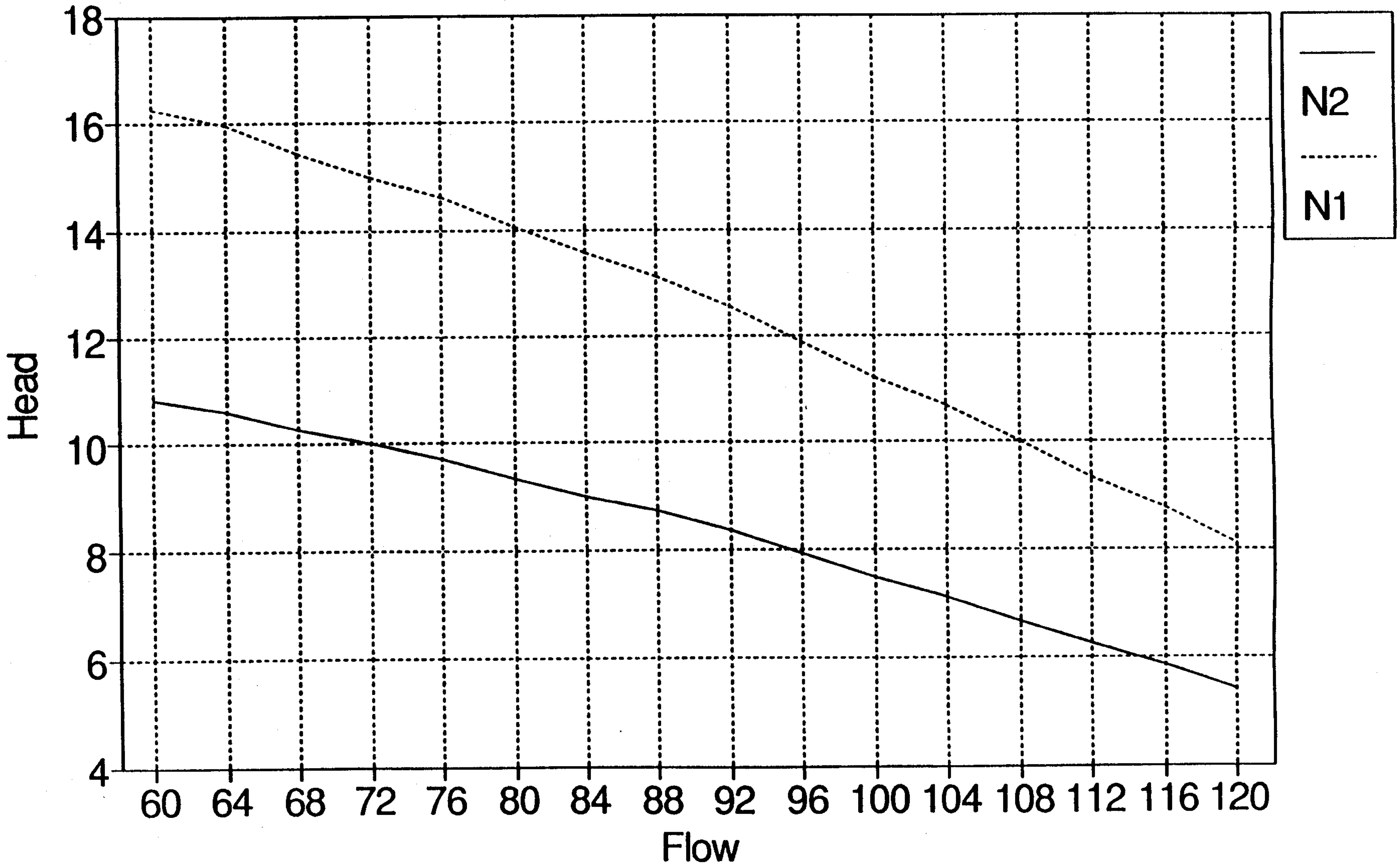


Figure 9-1

For turbines, the horse power delivered is the hydraulic power applied, wHQ, reduced by the efficiency factor:

$$\text{Mechanical horsepower delivered} = \frac{wHQe}{550} \tag{9-4}$$

In metric units, consider one cubic meter per sec being raised one meter. The unit weight of water is 1000 kg-wt/m^3.

$$Power = 1000 \frac{kg\ wt}{m^3} \times 1\ m \times 1 \frac{m^3}{\text{sec}}$$

$$= 9800 \frac{newtons}{m^3} \times 1\ m \times 1 \frac{m^3}{\text{sec}}$$

$$= 9800 \frac{joules}{\text{sec}} = 9800\ watts = 9.8\ kw$$

$$9.8\ kw = 62.4 \frac{lb}{ft^3} \times 3.28\ m \times \frac{35.3\ cfs}{(550 \frac{ft.lb}{sec})} = 13.12\ horsepower$$

Thus 1 hp = 0.746 kilowatts.

Referring again to Figure 9-1, consider the ratio, N_R, of two rotor speeds N_1 and N_2:

$$N_R = \frac{N_1}{N_2}$$

$$Q_R = \frac{Q_1}{Q_2} = N_R \qquad\qquad H_R = \frac{H_1}{H_2} = N_R^2$$

$$\therefore Typically,\ hp_R = Q_R\, H_R = N_R^3$$

PROFESSIONAL ENGINEERING REVIEW

HYDRAULIC REVIEW SECTION

CLASS NOTES

PROFESSIONAL ENGINEER REFRESHER COURSE

HYDRAULICS REVIEW

OUTLINE

A. GENERAL
   1. Hydrostatics
   2. Pressure Measurements

B. CLOSED CONDUITS
   1. Continuity
   2. Head
   3. Bernoulli
   4. Friction Equations
      - Darcy Weisbach
      - Hazen-Williams
      - Manning's
   5. Minor Losses
   6. Momentum

C. HYDROLOGY
   1. Rational Formula

D. OPEN CHANNEL
   1. Specific Energy
   2. Manning's Equation

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APPROACH TO REVIEW COURSE

- Class complements Notes
- Focus on doing review problems
- Problems are either different ways to solve same problem or slightly different approach than in notes.
- Course is intended to refresh your knowledge of hydraulics, covers a great deal of material.

1.0 HYDROSTATICS

Definition: Hydrostatics deals with pressures and forces in a liquid (water) when there is no movement

Concepts: **a) Weight**

W = Density * Gravity (F = M A)

$$\gamma = \rho * g$$

$$\rho\ (h_2 0) = 1.94\ \frac{slugs}{ft^3} \qquad slugs = \frac{lbs\ sec^2}{ft^4}$$

b) Pressure

P = Weight * Height = W * h

$$P = \rho * g * h \qquad [lb/ft^2 \text{ or Force/Area}]$$

c) Atmospheric Pressure

$$P_{atm} = \rho * \text{thickness of atmosphere} * g$$

$$P_{atm} = 14.7 \text{ lb/in}^2 = 2116.8 \text{ lb/ft}^2 \quad \text{at sea level}$$

d) At Sea Level

$$P_A = \text{atmospheric pressure} = 14.7 \text{ lb/in}^2$$

$$P_o = \text{gage pressure} = 0 \text{ lb/in}^2$$

This is simply a datum difference

Absolute Pressure = Pressure on a gage + Atmospheric pressure

PROBLEM 1a

A rectangular panel, 4 ft wide x 8 ft long is held in a tank at a depth of 9 feet and parallel with the surface.

a) What is the absolute pressure at the centroid of the panel?
b) What is the absolute force on the panel?

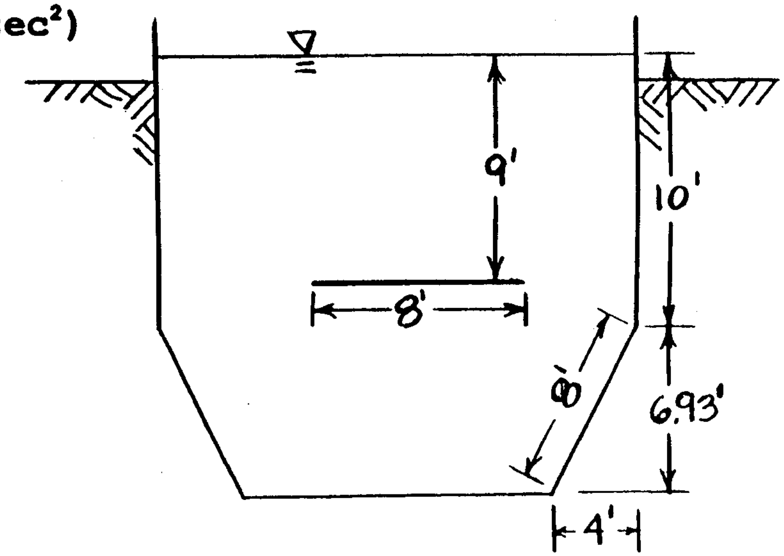
i) determine gage pressure

$$P_g = \rho\ g\ h$$

$$\rho\ g = 62.4\ \text{lb/ft}^3$$
$$= (1.94\ \text{slugs/ft}^3) * (32.2\ \text{ft/sec}^2)$$

$$h = 9\ \text{ft.}$$

$$P_g = (62.4)(9) = 561.6\ \text{lb/ft}^2$$



ii) determine absolute pressure

$$P_{abs} = P_{atm} + P_g \quad = \quad 2116.8 \text{ lb/ft}^2 \; + \; 561.6 \text{ lb/ft}^2$$

$$= 2678.4 \text{ lb/ft}^2$$

iii) Force

$$\text{Force} = \text{pressure} * \text{area} \quad = \quad (\text{lb/ft}^2)(\text{ft}^2) \quad = \quad \text{lb}$$

$$F = 2678.4 \text{ lb/ft}^2 * (8 \text{ ft} * 4 \text{ ft}) = \; 85{,}709 \text{ lb}$$

PROBLEM 1b

What is the force on a one foot length (into the page) of an inclined plane at 30°? Where is the resultant located on the inclined section?

P_1 = Pressure at top = $\rho\ g\ h_1 = (62.4)(10) = 624\ \mathrm{lb/ft^2}$

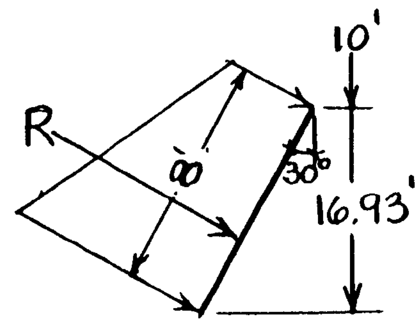
P_2 = Pressure at bottom = $\rho\ g\ h_2 = (62.4)(16.93) = 1056\ \mathrm{lb/ft^2}$

$$P_{AVG} = \frac{P_1 + P_2}{2} = \frac{1056 + 624}{2} = 840\ \mathrm{lb/ft_2}$$

$$F = (P_{AVG})(A) = (840\ \mathrm{lb/ft^2})*(8\ \mathrm{ft})*(1\ \mathrm{ft}) = 6720\ \mathrm{lb}$$

$$h_R = \frac{2}{3}\left[h_1 + h_2 - \left(\frac{h_1 h_2}{h_1 + h_2}\right)\right]$$

$$= \frac{2}{3}\left[10 + 16.93 - \left(\frac{(10)\ (16.93)}{10 + 16.93}\right)\right] = 13.76 ft$$



2.0 PRESSURE MEASUREMENT

- The term "head" is a common way of expressing pressure. "Head" is the height of a fluid column under a given pressure.

If: $P = \rho\, g\, h$

$$h = \frac{P}{\rho\, g} \qquad \text{("pressure" head)}$$

What is atmospheric pressure in terms of "head" of water?

$$h = \frac{P}{\rho\, g} = \frac{2116.8\ \text{lb/ft}^2}{62.4\ \text{lb/ft}^3} = 33.9 \approx 34 \text{ feet of } H_2O$$

What is atmospheric pressure in terms of "head" of mercury? The specific gravity of mercury is 13.6.

$$S.G. = \frac{\rho_{Hg}}{\rho_{H_2O}} \qquad \rho_{Hg} = (\rho_{H_2O})(13.6) = 26.38 \text{ slugs/ft}^3$$

$$h = \frac{P}{\rho\ g} = \frac{2116.8 \text{ lb/ft}^2}{(26.38)(32.2)} = 2.49 \text{ ft}_{Hg} \cong 30\text{" Hg}$$

Therefore, atmospheric pressure = 34 ft H_2O = 30" Hg

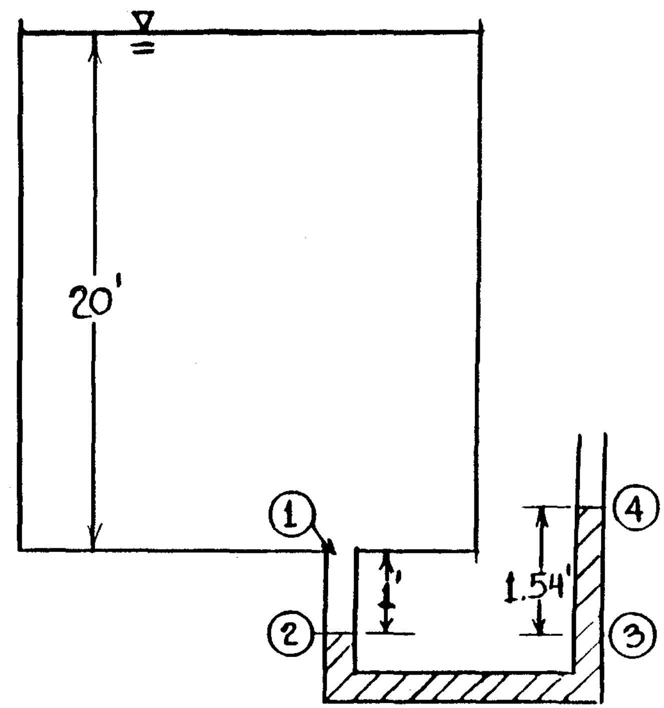
PROBLEM 2a   Pressure Measurement

The tank is 20 feet deep. An open air U-shaped manometer is tapped at the bottom of the tank, as shown below. What is the measuring (cross-hatched) fluid in the manometer?

$P_3 = \rho g * h = 1.54\ \rho g$

$P_2 = \rho g * h = 62.4\ \mathrm{lb/ft^3} * 21\ \mathrm{ft}$

$= 1310.4\ \mathrm{lb/ft^2}$



$P_2 = P_3 \qquad 1.54\ \rho g = 1310.4\ \mathrm{lb/ft^2}$

$\rho g = 850.9\ \mathrm{lb/ft^3}$

$$\rho = \frac{850.9}{g} = \frac{850.9}{32.2} = 26.4\ \mathrm{slugs/ft^3}$$

$$\mathrm{S.G.} = \frac{P_?}{P_W} = \frac{26.4}{1.94} = 13.6$$

Therefore, measuring fluid is Hg

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B. CLOSED CONDUITS

1.0 CONTINUITY

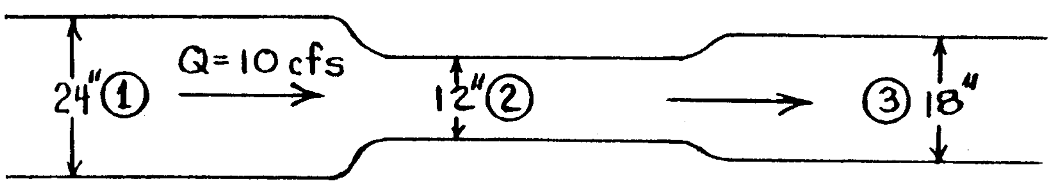
If you assume an incompressible fluid,

$Q_{in} = Q_{out} \pm \Delta$ Storage   If no Δ Storage, then $Q_{in} = Q_{out}$

$Q = Q_1 = A_1V_1 = A_2V_2 = Q_2$

PROBLEM 1a

10 cfs flows through a pipe of changing dimension, as shown. What is the velocity at each section?



(1) $Q_1 = AV, \quad V_1 = \dfrac{Q}{A_1} = \dfrac{10}{\pi(2)^2/4} = \dfrac{10}{3.14} = 3.18$ ft/s

(2) $Q_2 = AV, \quad V_2 = \dfrac{Q}{A_2} = \dfrac{10}{\pi(1)^2/4} = \dfrac{10}{.785} = 12.7$ ft/s

(3) $Q_3 = AV, \quad V_3 = \dfrac{Q}{A_3} = \dfrac{10}{\pi(1.5)^2/4} = \dfrac{10}{1.77} = 5.6$ ft/s

2.0 OTHER TYPES OF HEAD

Pressure head $= h_p = \dfrac{P}{\rho \; g}$

Velocity head $= h_v = \dfrac{V^2}{2 \; g}$

HGL = P

EGL = P + V

Specific Energy $= h_p + h_v = \dfrac{P}{\rho \; g} + \dfrac{V^2}{2 \; g}$

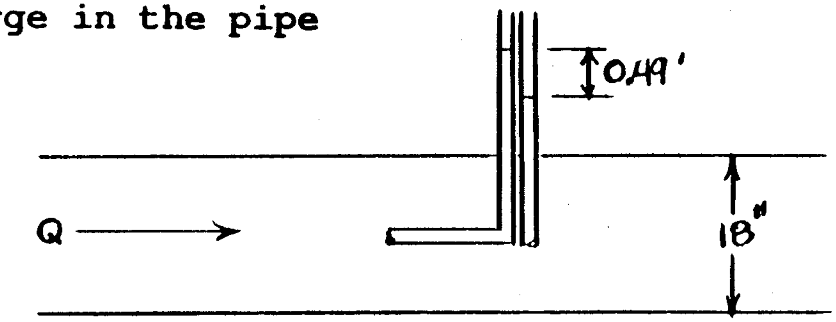
Elevation Head $= h_z =$ elevation above a datum

Velocity head is a <u>kinetic</u> energy term

Pressure head and elevation head are <u>potential</u> energy terms

PROBLEM 2a

Determine the discharge in the pipe



Tube A measures Specific Energy = pressure head and velocity head
Tube B measures pressure head

The elevation in Tube A is known as the Energy G.L.
The elevation in Tube B is known as the Hydraulic G.L.

$H_A = h_p + h_v$ $\qquad\qquad$ $H_B = h_p$

Therefore, $H_A - H_B = h_p + h_v - h_p = h_v$

$$h_v = \frac{v^2}{2g} = .49\ ft$$

$$v = \sqrt{.49(2)(32.2)} = 5.6\ ft/s$$

$$Q = vA = (5.6)\left(\frac{\pi(1.5)^2}{4}\right) = (5.6)(1.77) = 10\ cfs$$

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3.0 BERNOULLI EQUATION

Keeps track of the energy budget of a system

$E_1 = E_2 +$ [loss of energy from 1 to 2]

$$Z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + [\, h_f + h_t - h_p]$$

h_p = head added by a pump

h_t = head subtracted by a turbine

$$\text{H.P.} = \frac{\rho g \ h_p \ Q}{550 \ e}$$

$$\text{H.P.} = \frac{\rho g \ h_t \ Q \ e}{550}$$

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PROBLEM 3a

A pump delivers 2.5 cfs of 70° F H_2O from the reservoir at El 50 to the reservoir at El 150. There is 1000 ft of 8" steel suction pipe from the first reservoir to the pump. There is 2000 ft. of 6" steel discharge pipe from the pump to the second reservoir. Neglect friction and form losses. What is the horsepower of the pump? Pump efficiency is 0.80.

$$Z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_f - h_p$$

$$Z_1 = Z_2 - h_p$$

1 = surface of 1st reservoir   2 = surface of 2nd reservoir

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$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} = 0 = \textit{atmospheric pressure}$$

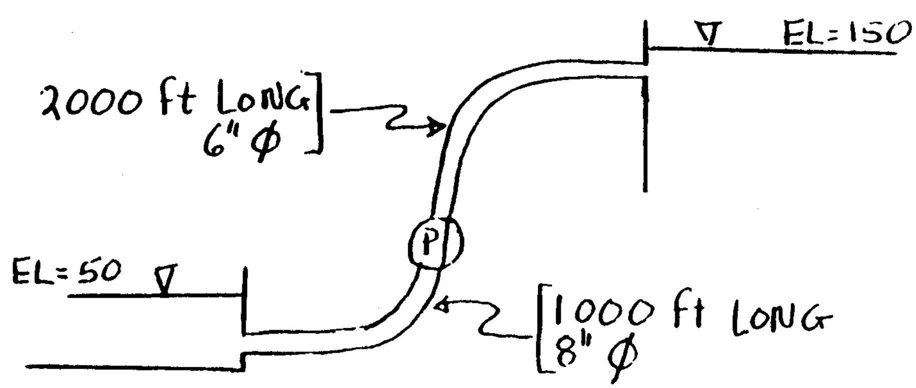
$$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} = 0 = \textit{water has no velocity}$$

$$h_f = 0$$

$$Z_1 = Z_2 - h_p$$

$$h_p = Z_2 - Z_1 = 150 - 50 = 100\ ft = h_p$$

$$H.P. = \frac{(62.4)(100)(2.5)}{550(0.8)} = 35.4$$



4.0 FRICTION

- Probably most important cause of loss of energy
- Many ways to compute it, from elegant to empirical
- We will cover three ways:

  Darcy

  Hazen Williams

  Manning's

DARCY

$$\mathbf{R} = \frac{v\ D}{\upsilon}$$

$\mathbf{R}$ = Reynolds #

v = velocity

D = inside diameter (or depth)

υ = kinematic viscosity (1×10^{-5} ft^2/sec)

For small $\mathbf{R}$ (<2000), laminar flow:

friction inversely related to $\mathbf{R}$

$f = 64/\mathbf{R}$ where f = friction factor

For very large $\mathbf{R}$ ($> 1 \times 10^6$), turbulent flow:

friction is independent of $\mathbf{R}$

friction dependent on roughness (ε/D) only

For intermediate values of $\mathbf{R}$:

friction is function of $\mathbf{R}$ and roughness (ε/D)

Moody Diagram demostrates this:

Line on left for laminar flow

Area between laminar & dotted line is intermediate

To the right of dotted line, complete turbulance, f is independent of $\mathbf{R}$

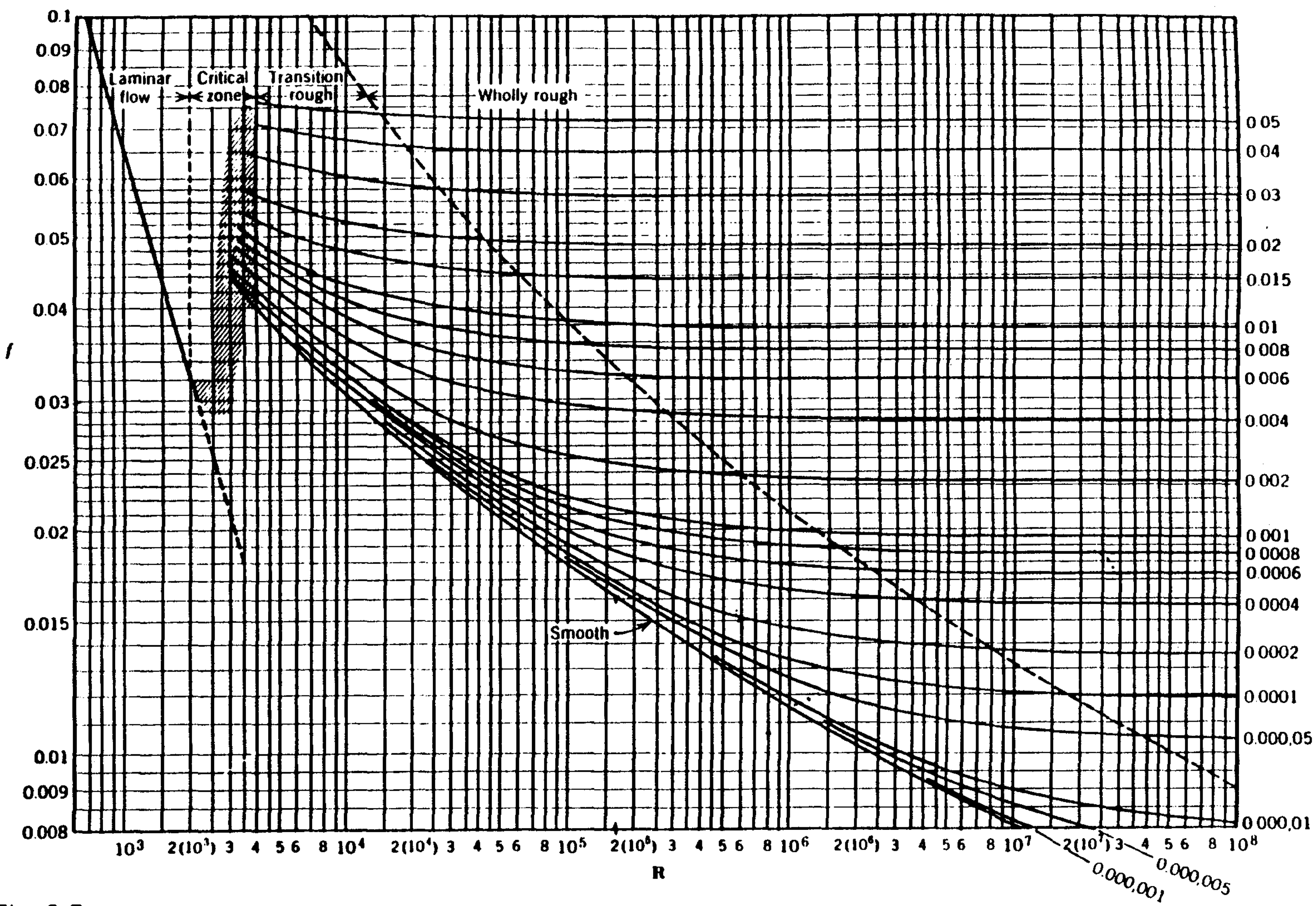


Fig. 9.5
Relation of friction factor, Reynolds number, and roughness for commercial pipes (see footnote 9).

PROBLEM 4a

Calculate the $\mathbb{R}$ in the 2 pipes in previous problem.

υ at $70°F$ H_2O = 1.059×10^{-5} ft^2/sec

$$\mathbb{R}_{8''} = \frac{vD}{\upsilon} \qquad v_{8''} = \frac{Q}{A} = \frac{2.5\,ft^3/s}{\frac{\pi}{4}(.67)^2} = 7.16\,ft/s$$

$$\mathbb{R}_{8''} = \frac{7.16(.67)}{1.059x10^{-5}} = 4.5x10^5$$

$$\mathbb{R}_{6''} = \frac{vD}{\upsilon} \qquad v_{6''} = \frac{Q}{A} = \frac{2.5\,ft^3/s}{\frac{\pi}{4}(.5)^2} = 12.72\,ft/s$$

$$\mathbb{R}_{6''} = \frac{12.72(.5)}{1.059x10^{-5}} = 6x10^5$$

<u>Darcy Weisbach Equation</u>

for all ranges of $\mathbb{R}$

$$h_f = \left(\frac{fL}{D}\right)\left(\frac{V^2}{2g}\right)$$

Physical Properties of Water

English Units

TEMPERATURE, °F	SPECIFIC WEIGHT,[a] γ, lb/ft³	DENSITY,[a] ρ, slug/ft³	MODULUS[b] OF ELASTICITY,[c] $E/10^3$, psi	VISCOSITY,[a] $\mu \times 10^5$, lb-sec/ft²	KINEMATIC VISCOSITY,[a] $\nu \times 10^5$, ft²/sec	SURFACE[a] TENSION,[d] σ, lb/ft	VAPOR PRESSURE,[e] p_v, psia
32	62.42	1.940	287	3.746	1.931	0.00518	0.09
40	62.43	1.940	296	3.229	1.664	0.00614	0.12
50	62.41	1.940	305	2.735	1.410	0.00509	0.18
60	62.37	1.938	313	2.359	1.217	0.00504	0.26
70	62.30	1.936	319	2.050	1.059	0.00498	0.36
80	62.22	1.934	324	1.799	0.930	0.00492	0.51
90	62.11	1.931	328	1.595	0.826	0.00486	0.70
100	62.00	1.927	331	1.424	0.739	0.00480	0.95
110	61.86	1.923	332	1.284	0.667	0.00473	1.27
120	61.71	1.918	332	1.168	0.609	0.00467	1.69
130	61.55	1.913	331	1.069	0.558	0.00460	2.22
140	61.38	1.908	330	0.981	0.514	0.00454	2.89
150	61.20	1.902	328	0.905	0.476	0.00447	3.72
160	61.00	1.896	326	0.838	0.442	0.00441	4.74
170	60.80	1.890	322	0.780	0.413	0.00434	5.99
180	60.58	1.883	318	0.726	0.385	0.00427	7.51
190	60.36	1.876	313	0.678	0.362	0.00420	9.34
200	60.12	1.868	308	0.637	0.341	0.00413	11.52
212	59.83	1.860	300	0.593	0.319	0.00404	14.70

PROBLEM 4b

Assuming the pipes are steel, what is the head loss due to friction, h_f, if friction is not neglected in Problem 3a? What would the horsepower of the pump really have to be if h_f were included in the calculations? Use Darcy's equation to compute h_f.

ε of steel pipe = 0.0002 feet (design)

8" pipe:

$$\frac{\varepsilon}{D} = \frac{0.0002}{0.67} = 0.0003$$

$$\mathbb{R} = 4.5x10^5$$

$$f = 0.0165$$

$$v = 7.16\ ft/sec$$

$$\frac{v^2}{2g} = \frac{7.16^2}{64.4} = 0.793ft$$

$$L = 1000\ ft$$

$$h_f = \left(\frac{fL}{D}\right)\left(\frac{v^2}{2g}\right)$$

$$= \frac{(.0165)(1000)}{0.67}(.793) = 19.5ft$$

6" pipe:

$$\frac{\varepsilon}{D} = \frac{0.0002}{0.5} = 0.0004$$

$$\mathbb{R} = 6x10^5$$

$$f = 0.017$$

$$v = 12.72\ ft/sec$$

$$\frac{v^2}{2g} = \frac{12.72^2}{64.4} = 2.5\ ft$$

$$L = 2000\ ft$$

$$h_f = \left(\frac{fL}{D}\right)\left(\frac{v^2}{2g}\right)$$

$$= \frac{(.017)(2000)}{0.5}(2.5) = 170ft$$

Specific Roughness and Hazen-Williams Constants for Various Pipe Materials

Type of pipe or surface	ε (ft) Range	ε (ft) Design	C Range	C Clean	C Desig
STEEL					
welded and seamless	.0001-.0003	.0002	150-80	140	100
interior riveted, no projecting rivets				139	100
projecting girth rivets				130	100
projecting girth and horizontal rivets				115	100
vitrified, spiral-riveted, flow with lap				110	100
vitrified, sprial-riveted, flow against lap				100	90
corrugated				60	60
MINERAL					
concrete	.001-.01	.004	152-85	120	100
cement-asbestos			160-140	150	140
vitrified clays					110
brick sewer					100
IRON					
cast, plain	.0004-.002	.0008	150-80	130	100
cast, tar (asphalt) coated	.0002-.0006	.0004	145-50	130	100
cast, cement lined	.000008	.000008		150	140
cast, bituminous lined	.00008	.00008	160-130	148	140
cast, centrifugally spun	.00001	.00001			
galvanized, plain	.0002-.0008	.0005			
wrought, plain	.0001-.0003	.0002	150-80	130	100
MISCELLANEOUS					
fiber				150	140
copper and brass	.000005	.000005	150-120	140	130
wood stave	.0006-.003	.002	145-110	120	110
transite	.000008	.000008			
lead, tin, glass		.000005	150-120	140	130
plastic		.000005	150-120	140	130

(Problem 4b, cont.)

Total friction head loss = 19.5 + 170 = 189.5 ft

Bernoulli's equation:

$$h_p = (z_2 - z_1) + h_f$$

$$h_p = (150 - 50) + 189.5 = 289.5 \text{ ft}$$

Determine horsepower:

$$\text{H.P.} = \frac{(62.4)(289.5)(2.5)}{550\ (0.8)} = 103$$

[compare with 35.4 HP before when we didn't consider friction]

Note: The only difference between this problem and one shown in the notes, is that for that problem ε was given as 0.001 ft. This is 5 times as "rough" as the value determined in this problem (0.0002). Note the difference in answers: 103 HP vs. 130 HP.

Hazen-Williams Equation

- Common in water distribution problems
- For intermediate values of $\mathbf{R}$

$$Q = 0.432\ C_1\ D^{2.63}\ S^{0.54}$$

$$v = 0.55\ C\ D^{0.63}\ S^{0.54}$$

$$S = \frac{h_f}{L}$$

$$h_f = \frac{3.012\ v^{1.85}\ L}{C^{1.85}\ D^{1.165}}$$

v in ft/sec

L in ft

D in ft

C = H - W (coefficient)

Q in cfs

II-53

PROBLEM 4c

For Problem 4b, we computed total friction head loss to be 189.5 ft, using Darcy's equation. What would our answer be if we use Hazen-Williams?

C = Hazen Williams coefficient = 100 for steel pipe

$$h = \frac{3.012 \; V^{1.85} \; L}{C^{1.85} \; D^{1.165}}$$

$$h_8 = \frac{3.012 \; (7.16)^{1.85} \; (1000)}{(100)^{1.85} \; (0.67)^{1.165}} = 36.6 \text{ ft}$$

$$h_6 = \frac{3.012 \; (12.72)^{1.85} \; (2000)}{(100)^{1.85} \; (0.5)^{1.165}} = 297.8 \text{ ft}$$

$H_f = h_8 + h_6 = 334$ ft

Manning's Equation

- Common in open channel flow problems

- for fully turbulent flow

$$v = \frac{1.49}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \qquad S = \frac{h_f}{L}$$

$$h_f = \frac{v^2 n^2 L}{2.22 R^{\frac{4}{3}}} \qquad n = \text{Manning's } n$$

$$R = \frac{A}{P_w} = \frac{\left(\frac{\pi D^2}{4}\right)}{\pi D} = \frac{D}{4}$$

II-55

PROBLEM 4d

What is the friction head loss if we use Manning's equation?

n = Manning's "n" value = 0.012 for steel pipes

$$h_8 = \frac{(7.16)^2(0.012)^2(1000)}{(2.22)\left(\frac{2/3}{4}\right)^{\frac{4}{3}}} = 36.2\ ft$$

$$h_6 = \frac{(12.72)^2(0.012)^2(1000)}{(2.22)\left(\frac{1/2}{4}\right)^{\frac{4}{3}}} = 335\ ft$$

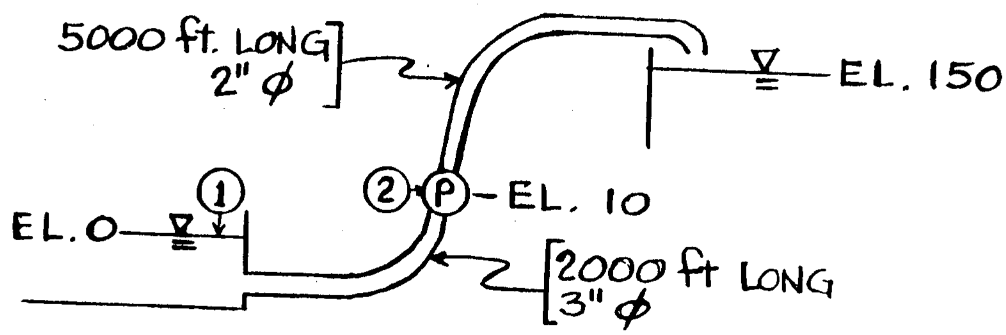
$$h_T = 372\ ft$$

VALUES OF n FOR USE IN MANNING OR KUTTER FORMULA *

0.009 and 0.010	Very smooth and true surfaces, without projections. Clean new glass, pyralin, or brass, with straight alignment.
0.011 and 0.012	Smoothest clean wood, metal, or concrete surfaces, without projections, and with straight alignment.
0.013	Smooth wood, metal, or concrete surfaces without projections, free from algae or insect growth, and with reasonably straight alignment.
0.014	Good wood, metal, or concrete surfaces with very small projections, with some curvature, with slight insect or algae growth, or with slight gravel deposition. Shot concrete surfaced with troweled mortar.
0.015	Wood with algae and moss growth, concrete with smooth sides but roughly troweled or shot bottom, metal with shallow projections. Same with smoother surface but excessive curvature.
0.016	Metal flumes with large projections into the section. Wood or concrete with heavy algae or moss growth.
0.017	Shot concrete, not troweled, but fairly uniform.
0.018 to 0.025	Metal flumes with large projections into the section and excessive curvature, growths, or accumulated debris.
0.016 to 0.017	Smoothest natural earth channels, free from growths, with straight alignment.
0.020	Smooth natural earth, free from growths, little curvature. Very large canals in good condition.
0.022	Average, well constructed, moderate-sized earth canal in good condition.
0.025	Very small earth canals or ditches in good condition, or larger canals with some growth on banks or scattered cobbles in bed.
0.030	Canals with considerable aquatic growth. Rock cuts, based on average actual section. Natural streams with good alignment, fairly constant section. Large floodway channels, well maintained.
0.035	Canals half choked with moss growth. Cleared but not continuously maintained floodways.
0.040 to 0.050	Mountain streams in clean loose cobbles. Rivers with variable section and some vegetation growing in banks. [illegible]

PROBLEM 4e

For the pumping system shown, what is the maximum possible flow rate? Use f = 0.020. The pump will not function if absolute pressure, P_a, is less than zero.



$$P_a = P_{atm} - \rho g\,(10) - \rho g\left(\frac{f\,L}{D}\frac{v^2}{2\,g}\right)$$

P_{atm} = 14.7 $lb/inch^2 = \rho g\,(34\ ft)$ *at sea level*

$$P_a = 0 = \rho g\left[34 - 10 - \left(\frac{(0.02)\,(2000')}{0.25}\right)\left(\frac{v^2}{(2)\,(32.2)}\right)\right]$$

v = 3.11 *ft/sec*

$$Q = \left(\frac{\pi}{4}D^2\right)v = 0.153\ cfs$$

Note: Actually, cavitation effects would begin to degrade pump performance at a lesser Q, before P_a is reduced to zero.

11-58

5.0 MINOR LOSSES (FORM LOSSES)

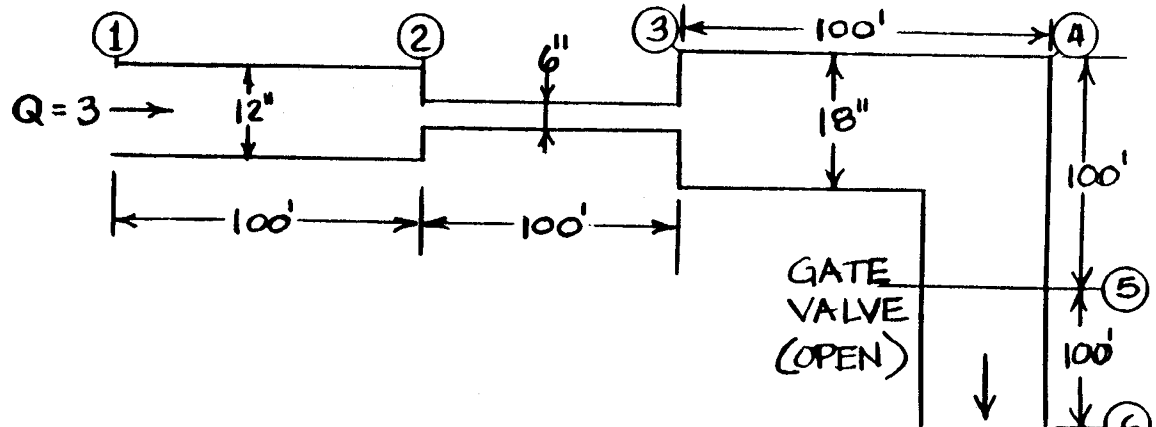
Losses of kinetic energy because of:

- expansions in the flow line (6" pipe to 12" pipe)
- contractions in the flow line (12" pipe to 6" pipe)
- bends, gates and valves
- any disruptions in the flow field

Remembering that these are kinetic energy losses, and remembering that $v^2/2g$ is a kinetic energy term, determine form loss by multiplying $v^2/2g$ by a coefficient.

PROBLEM 5a

Determine the friction and form losses in the following pipe network. Use Darcy's friction formula, assume $f = 0.02$.



use D/S velocity: 1 - ordinary inlet, $K = 0.5$

use D/S velocity: 2 - sudden contraction, $d_2 / d_1 = 6/12 = 0.5$, $K=0.43$

use U/S velocity: 3 - sudden enlargement, $d_1 / d_2 = 6/18$, $K = 0.85$

4 - square elbow, $K = 1.0$

5 - gate valve, $K = 0.25$

use U/S velocity: 6 - ordinary outlet, $K = 1.0$

11-60

VALUES OF HEADLOSS COEFFICIENT FOR SOME FITTINGS, BENDS, AND SECTION CHANGES

Item		K
Square elbow, single-mitre bend		1.0
Sudden enlarge (use upstream velocity for V):	$d_1/d_2 = 3/4$	0.19
	$d_1/d_2 = 1/2$	0.56
	$d_1/d_2 = 1/4$	0.92
ordinary outlet:	$d_1/d_2 = 0$	1.00
Sudden contraction (use downstream velocity for V):	$d_2/d_1 = 3/4$	0.25
	$d_2/d_1 = 1/2$	0.43
	$d_2/d_1 = 1/4$	0.49
ordinary inlet:	$d_2/d_1 = 0$	0.50
Rounded, or bell-mouth, inlet:		0.001
Borda inlet:		0.75
Gate valve, fully open:		0.25

(problem 5a, cont.)

Friction Losses: h_f

$$h_f = \left(\frac{f\,L}{D}\right)\left(\frac{v^2}{2\,g}\right)$$

$$h_{f\,1\text{-}2} \Rightarrow \; v = \frac{Q}{A} = \frac{3}{\left(\frac{\pi\,(1)^2}{4}\right)} = 3.82 \qquad h_f = \left(\frac{(0.02)\,(100)}{2}\right)\left(\frac{3.82^2}{(2)\,(32.2)}\right) = 0.45\ ft$$

$h_{f\,2\text{-}3} \Rightarrow \; v = 15.38 \qquad h_f = 14.69\ ft$

$h_{f\,3\text{-}4} \Rightarrow \; v = 1.7 \qquad h_f = 0.06\ ft$

$h_{f\,4\text{-}5} \Rightarrow \; v = 1.7 \qquad h_f = 0.06\ ft$

$h_{f\,5\text{-}6} \Rightarrow \; v = 1.7 \qquad h_f = 0.06\ ft$

$\therefore\; h_f = 15.02\ ft$

Form Losses: h_m

$$h_m = K\left(\frac{v^2}{2g}\right)$$

$$h_{m1} = 0.5\left(\frac{v^2}{2g}\right) \qquad v_{1-2} = 3.82, \qquad h_{m1} = 0.11\ ft$$

$$h_{m2} = 0.43\left(\frac{v^2}{2g}\right) \qquad v_{2-3} = 15.38, \qquad h_{m2} = 1.58\ ft$$

$$h_{m3} = 0.85\left(\frac{v^2}{2g}\right) \qquad v_{2-3} = 15.38, \qquad h_{m3} = 3.12\ ft$$

$$h_{m4} = 1.0\left(\frac{v^2}{2g}\right) \qquad v_{3-4} = 1.7, \qquad h_{m4} = 0.04\ ft$$

$$h_{m5} = 0.25\left(\frac{v^2}{2g}\right) \qquad v_{4-5} = 1.7, \qquad h_{m5} = 0.01\ ft$$

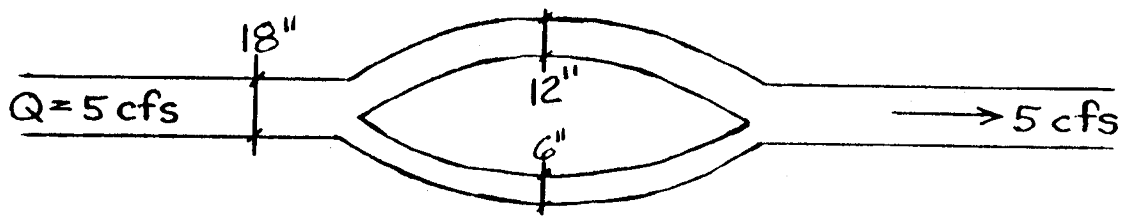
$$h_{m6} = 1.0\left(\frac{v^2}{2g}\right) \qquad v_{5-6} = 1.7, \qquad h_{m6} = 0.04\ ft$$

h_m = 4.9 ft

$h = h_f + h_m = 15.02 + 4.90 = 19.92$ ft

PROBLEM 5b Parallel Pipes

When two pipes flow parallel to one another, the flow will split so that the friction loss is the same in both pipes. The flow in the 18" pipe is 5 cfs. What is the flow in the 12" and 6" pipes? The 6" pipe is 100 ft long and the 12" pipe is 150ft long. Assume f=.02 in Darcy formula.



$$Q_{12} + Q_6 = 5 \qquad Q = V * A$$

$$h_{12} = h_6 \qquad h_f = \left(\frac{f\,L}{D}\right)\left(\frac{v^2}{2\,g}\right)$$

$$\frac{(0.02)\,(150)\,(v_{12}^2)}{(2)\,(1)\,(32.2)} = \frac{(0.02)\,(100)\,(v_6^2)}{(2)\,(.5)\,(32.2)}$$

$$v_{12}^2 = 1.35\,v_6^2 \quad \rightarrow \quad v_{12} = 1.15\,v_6$$

$$Q_6 + Q_{12} = 5 \quad \rightarrow \quad 1.15\,v_6\,A_{12} + v_6\,A_6 = 5$$

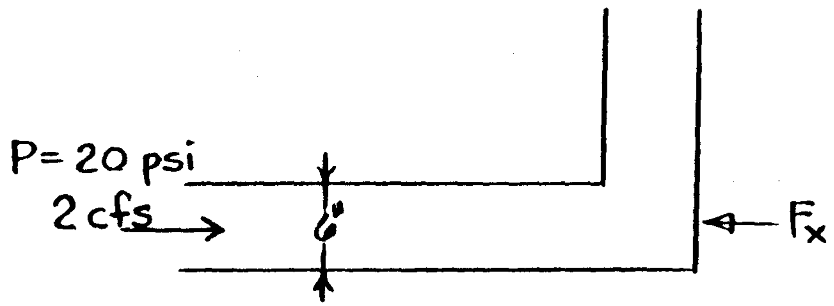
$$A_{12} = \frac{\pi\,D^2}{4} = 0.785 \qquad A_6 = 0.196$$

$$0.785\,(1.15\ \ v_6) + 0.196\,v_6 = 5 \quad \rightarrow \quad v_6 = 4.54\ ft/s$$

$$Q_6 = 0.89\ cfs \quad \rightarrow \quad Q_{12} = 5 - 0.89 = 4.11\ cfs$$

6.0 MOMENTUM - VECTOR

$$\sum F_x = \rho\ Q\ (V_{x2} - V_{x1}) = 0 \qquad (F = M\ A)$$



$$\sum F_x = 0 = P\ A + \rho\ Q\ (V_{x1} - V_{x2}) - F_x$$

$$0 = [(20)(144)][\pi(0.5)^2/4] + 1.94(2)\left|\frac{2}{\pi(0.5)^2/4}\right| - F_x$$

$$= (2880\ \text{lbs/ft}^2)\ (0.2) + 1.94(2)(10) - F_x$$

$$= 576 + 38.8 - F_x$$

$$F_x = 614.8\ \text{lbs}$$

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

ρ = slugs/ft^3

Q = ft^3/s

ρQ = mass/rate

11-65

C. HYDROLOGY

1. RATIONAL FORMULA

- Simple method to determine a design Q
- For use on small areas

$$Q = C\ I\ A$$

C = runoff coefficient = runoff/rainfall; dimensionless < 1

I = rainfall intensity for a given design event and a given time of concentration, made from a site specific I-D-F curve; units = inches/hr

A = area in acres

- Many ways to determine T_c. One method is Kirpich's equation, where:

$$T_c = \frac{0.0078\,L^{0.77}}{S^{0.385}} \qquad T = \frac{L}{V}$$

T_c in minutes

L is length in feet

S is slope in feet/foot

PROBLEM C1a

Determine design Q for these 4 conditions. Drainage area is 436 ft by 1000 ft. Top of parking lot is 10 ft higher than outlet. Longest flow length along the diagonal.

1) Drainage area is park, 10 yr design
2) Drainage area is park, 100 yr design
3) Drainage area is parking lot, 10 yr design
4) Drainage area is parking lot, 100 yr design

$$t_c = \frac{0.0078(1091)^{0.77}}{\left(\frac{10}{1091}\right)^{0.385}} \qquad t_c = 10.4 \text{ min} \approx 10 \text{ min}$$

	C
Park	0.15
Parking lot	0.9

	I in/hr
10 yr	4.9
100 yr	6.1

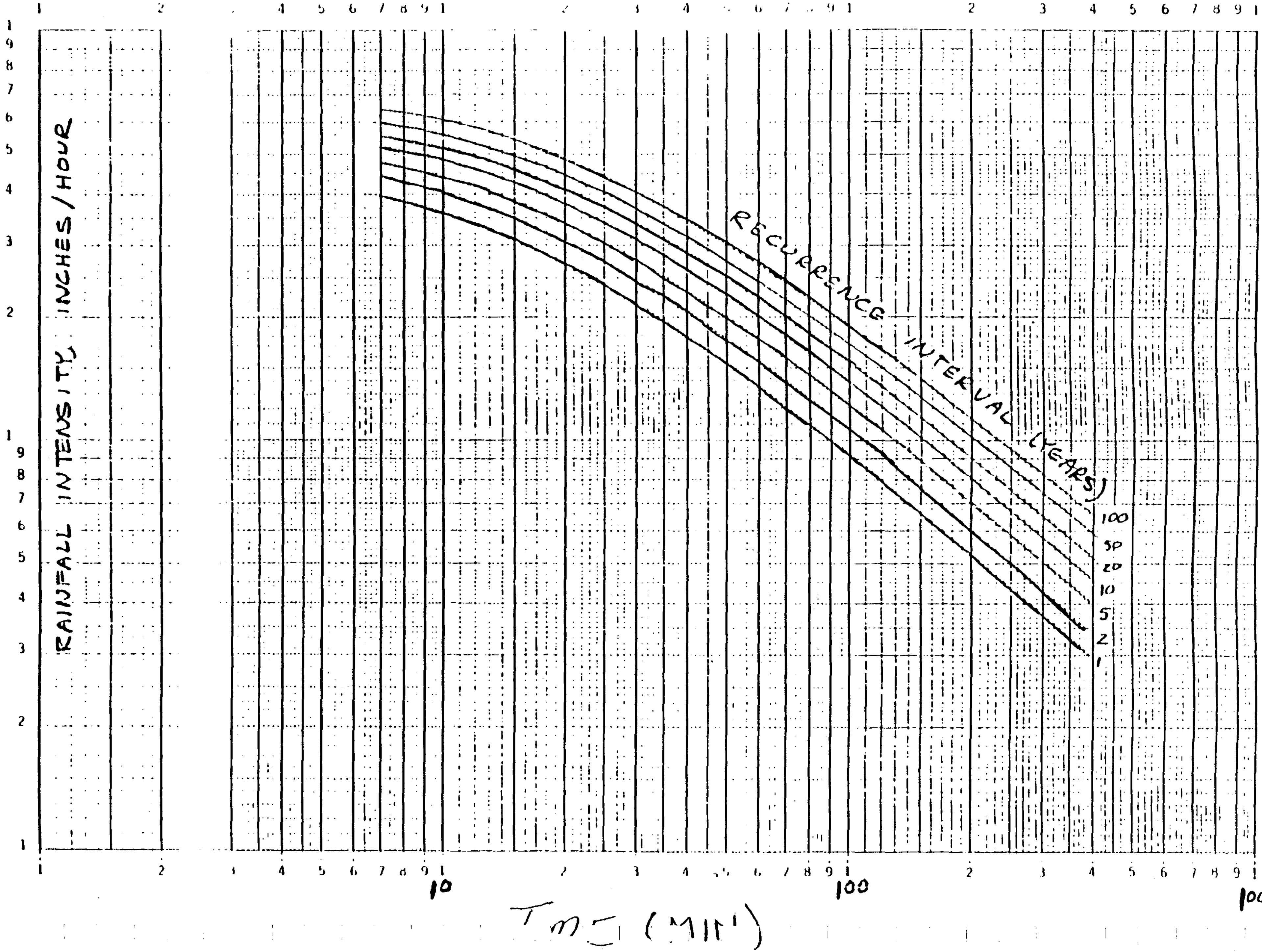
Rational Method Runoff Coefficients

Categorized by Surface

Forested	.05 - .2
Asphalt	.7 - .95
Brick	.7 - .85
Concrete	.8 - .95
Shingle roof	.75 - .95
Lawns, well drained (sandy soil)	
Up to 2% slope	.05 - .1
2% to 7% slope	.10 - .15
Over 7% slope	.15 - .2
Lawns, poor drainage (clay soil)	
Up to 2% slope	.13 - .17
2% to 7% slope	.18 - .22
Over 7% slope	.25 - .35
Driveways, walkways	.75 - .85

Categorized by Use

Farmland	.05 - .3
Pasture	.05 - .3
Unimproved	.1 - .3
Parks	.1 - .25
Cemetaries	.1 - .25
Railroad yard	.2 - .40
Playgrounds (except asphalt or concrete)	.2 - .35
Business districts	
neighborhood	.5 - .7
city (downtown)	.7 - .95
Residential	
single family	.3 - .5
multi-plexes, detached	.4 - .6
multi-plexes, attached	.6 - .75
suburban	.25 - .4
apartments, condominiums	.5 - .7
Industrial	
light	.5 - .8
heavy	.6 - .9


RAINFALL INTENSITY, INCHES/HOUR
RECURRENCE INTERVAL (YEARS)
100
50
20
10
5
2
1
10
100
1000
TIME (MIN)

$$Q_1 = C\ I\ A = (0.15)(4.9)(10) = 7\ \text{cfs}$$

$$Q_2 = C\ I\ A = (0.15)(6.1)(10) = 9\ \text{cfs}$$

$$Q_3 = C\ I\ A = (0.9)(4.9)(10) = 44\ \text{cfs}$$

$$Q_4 = C\ I\ A = (0.9)(6.1)(10) = 55\ \text{cfs}$$

	10 yr	100 yr
Park	7	9
Parking lot	44	55

D. OPEN CHANNEL FLOW

1. SPECIFIC ENERGY

Critical Depth, Subcritical Depth, Supercritical Depth

- Specific Energy = Pressure Head + Velocity Head = E
- In open channels, Pressure Head = Water Surface
  $\therefore$ Specific Energy = Water Surface Elev. + Velocity Head
- If conditions are "quite", velocities low, then depth must be high $\Rightarrow$ subcritical flow
- If flow is "shooting" down over a dam, and velocities are high $\Rightarrow$ supercritical flow
- For rectangular channels, Froude # defines type of flow

$$F = \frac{v}{(gy)^{1/2}} = 1, \quad \text{critical depth}$$

$$F < 1, \quad \text{subcritical}$$

$$F > 1, \quad \text{supercritical}$$

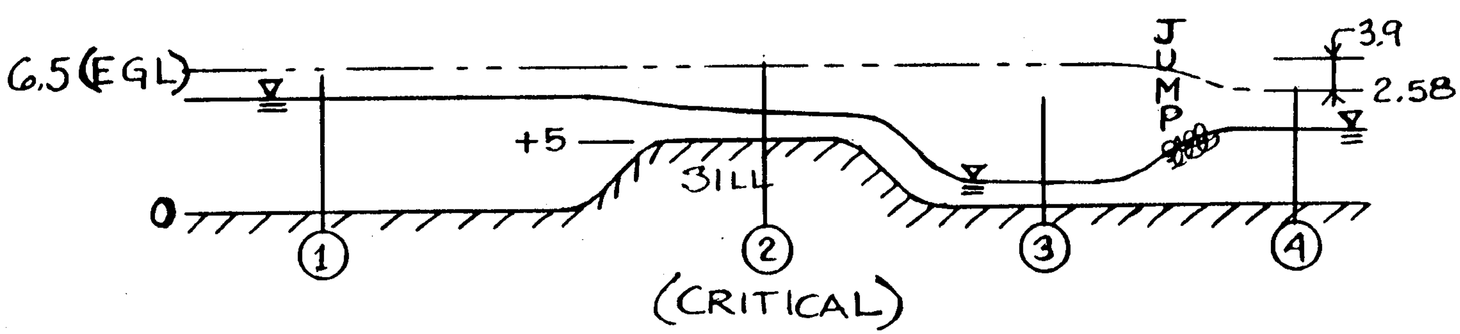
PROBLEM D1a.

In a 1 ft wide channel approaching a sill, the floor elevation is zero. The sill elevation is +5. The 1 ft wide channel downstream of the sill also has floor elevation zero.
Flow passing over the sill has a critical depth of 1 ft.

a) What is the flow rate?
b) What is the water depth at (1) ?
c) What is the water depth in supercritical flow at (3) ?
d) What is the water depth at (4) , downstream of a stationary hydraulic jump?

Neglect friction losses other than energy loss in the jump.

11-72



a:

$Q = bVy$; $b = 1\ ft$; at (2), critical conditions exist

$$F = \frac{V_c}{(g\,y_c)^{1/2}} = 1$$

$V_c = (g\,y_c)^{1/2}$; $y_c = y_2 = 1\ ft$; $Q = AV$; $A = y\,b$

$Q = (1)(32.2*1)^{1/2}(1) = 5.67\ cfs$; $V_c = ((32.2)(1))^{1/2}$; $V_c = 5.67$ 11-73 b: The specific energy (E) $\qquad E = y + \dfrac{V^2}{2g}$

$$@\ (2),\quad E_2 = 1 + \frac{(5.67)^2}{2g} = 1.5\ ; \qquad y = 1\ ; \qquad v_c = \sqrt{(32.2)(1)} = 5.67$$

$$so\ \ EGL_2 = E_2 + Z = 1.5 + 5 = 6.5\,ft\ ; \qquad EGL_1 = EGL_2$$

$$EGL_2 = Z_1 + Y_1 + \frac{V_1^2}{2g} = 6.5$$

$$V_1 = Q/bY\ ; \qquad Q = 5.67\ ; \qquad b = 1$$

$$Y_1 + \frac{(Q/(bY_1))^2}{2g} = 6.5 \quad \Rightarrow \quad Y_1 + \frac{(5.67/(1)Y_1)^2}{64.4} = 6.5$$

Trial & Error $\Rightarrow$ $Y_1 = 6.49$ Subcritical $F < 1$

11-74

c. $Y_3 = ?$ $6.5 = EGL_1 = EGL_2 = EGL_3 \neq EGL_4$ Supercritical F>1

$$Y_3 + \left(\frac{Q/bY_3}{2g}\right)^2 = 6.5 \qquad T/E \quad \Rightarrow \quad Y_3 = 0.284$$

To determine Y_4, apply the momentum equation

d. $Y_4 = ?$ $EGL_3 \neq EGL_4$ Energy loss across a hydraulic jump

However, momentum is conserved & Continuity

11-75

$$\sum F_x = 0 = \rho g b\left(\frac{Y_4^2}{2} - \frac{Y_3^2}{2}\right) + \rho Q(V_4 - V_3)$$

Pull apart:

$$\rho g b\left(\frac{Y_4^2}{2}\right) + \rho Q V_4 = \rho g b\left(\frac{Y_3^2}{2}\right) + \rho Q V_3$$

$$Q_3 = 5.67 = Q_4 \qquad\qquad Y_3 = 0.284$$

Substitute V = Q/by

$$gb\frac{Y_4^2}{2} + \frac{Q^2}{bY_4} = gb\frac{Y_3^2}{2} + \frac{Q^2}{bY_3}$$

$$16.1\,Y_4^2 + \frac{32.2}{Y_4} = 1.30 + 113.2 = 114.5$$

Trial & error $\Rightarrow$ $Y_4 = 2.51$ ft

How much energy is lost in jump?

11-76

$Y_4 = 2.51$ $A_4 = (2.51)(1) = 2.51$ $Q = 5.67$

$$v = \frac{Q}{A} = 2.259 \qquad \left(\frac{v^2}{2g}\right) = 0.079$$

$$E = Y + \frac{v^2}{2g} = 2.51 + 0.079 = 2.59$$

Energy lost in jump = 6.5 − 2.59 = 3.91 ft

PROBLEM D2a. Manning's Equation

If 10 cfs are flowing down a rectangular channel with a slope of 1/1000, what is the depth of flow for a top width of 5 feet? The channel is a "natural" stream with good alignment.

Manning's Equation:

$$Q = \frac{1.49}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}}$$

n = .03 (reference)

Q = 10

S = 0.001

$$R = \frac{A}{P} = \frac{bY}{b + 2Y} = \frac{5Y}{5 + 2Y}$$

$$10 = \frac{1.49}{0.03} (5Y) \left[\frac{5Y}{5 + 2Y}\right]^{\frac{2}{3}} (0.001)^{\frac{1}{2}}$$

$$6.29 = (5Y) \left[\frac{5Y}{5 + 2Y}\right]^{\frac{2}{3}}$$

$$\Rightarrow \quad Y \cong 1.35$$

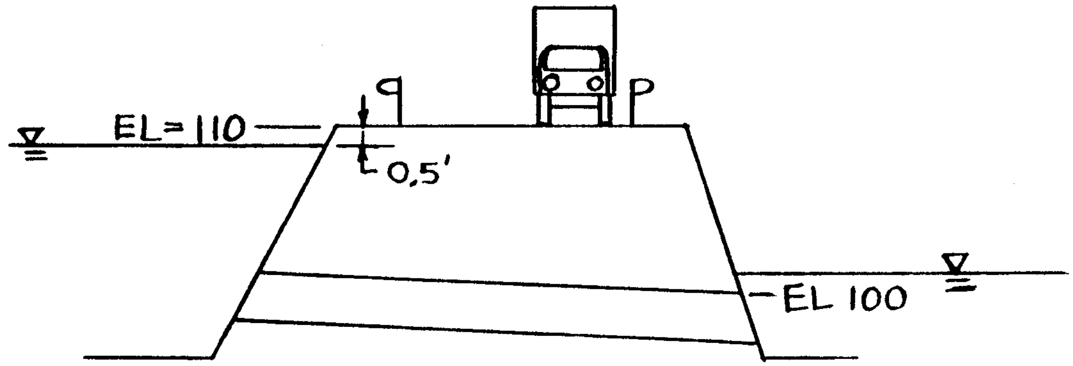
Trial and Error

Y	[5Y...
1	4
2	10.7
1.5	7.2
1.3	5.4
1.4	6.5
1.35	6.18

VALUES OF n FOR USE IN MANNING OR KUTTER FORMULA *

0.009 and 0.010	Very smooth and true surfaces, without projections. Clean new glass, pyralin, or brass, with straight alignment.
0.011 and 0.012	Smoothest clean wood, metal, or concrete surfaces, without projections, and with straight alignment.
0.013	Smooth wood, metal, or concrete surfaces without projections, free from algae or insect growth, and with reasonably straight alignment.
0.014	Good wood, metal, or concrete surfaces with very small projections, with some curvature, with slight insect or algae growth, or with slight gravel deposition. Shot concrete surfaced with troweled mortar.
0.015	Wood with algae and moss growth, concrete with smooth sides but roughly troweled or shot bottom, metal with shallow projections. Same with smoother surface but excessive curvature.
0.016	Metal flumes with large projections into the section. Wood or concrete with heavy algae or moss growth.
0.017	Shot concrete, not troweled, but fairly uniform.
0.013 to 0.025	Metal flumes with large projections into the section and excessive curvature, growths, or accumulated debris.
0.016 to 0.017	Smoothest natural earth channels, free from growths, with straight alignment.
0.020	Smooth natural earth, free from growths, little curvature. Very large canals in good condition.
0.022	Average, well constructed, moderate-sized earth canal in good condition.
0.025	Very small earth canals or ditches in good condition, or larger canals with some growth on banks or scattered cobbles in bed.
0.030	Canals with considerable aquatic growth. Rock cuts, based on average actual section. Natural streams with good alignment, fairly constant section. Large floodway channels, well maintained.
0.035	Canals half choked with moss growth. Cleared but not continuously maintained floodways.

PROBLEM D2b. Culvert Problem



Q_P = 24 cfs

L = 50

Freeboard of 0.5 ft

W.S. EL. U/S = 110 - 0.5

= 109.5

h_L = 109.5 - 100 = 9.5 ft

$h_L = h_I + h_F + h_E$

n = 0.013

K_I = 0.75 K_E = 1.00

$$V = \frac{1.49}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \qquad Q = \frac{1.49}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$S = \frac{h_F}{L} \quad \Rightarrow \quad h_F = \frac{V^2 n^2 L}{2.22 R^{\frac{4}{3}}}$$

$$R = \frac{A}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} \qquad V = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}}$$

$$h_{E,I} = \frac{K v^2}{2g} \qquad h_L = h_I + h_F + h_E$$

$$9.5 = K_I \frac{V^2}{2g} + \left[\frac{V^2 n^2 L}{2.22\,(R)^{\frac{4}{3}}}\right] + K_E \frac{V^2}{2g}$$

$$9.5 = (0.75)\frac{\left(\frac{24}{\pi D^2/4}\right)^2}{2g} + \frac{\left(\frac{24}{\pi D^2/4}\right)^2 (0.013)^2 (50)}{2.22\left(\frac{D}{4}\right)^{\frac{4}{3}}} + \frac{(1.00)\left(\frac{24}{\pi D^2/4}\right)^2}{2g}$$

11-80

$$9.5 = \frac{250}{(\pi D^2)^2} + \frac{35}{(\pi D^2)^2}(D/4)^{\frac{4}{3}}$$

Trial & Error:

choose 18"

D	right side
1	25.8
2	1.64
1.5 = 18"	5.2
1.25 = 15"	10.7

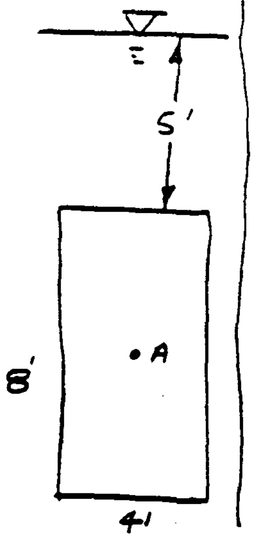
PROFESSIONAL ENGINEERING REVIEW

HYDRAULIC REVIEW SECTION

EXAMPLE PROBLEMS

EXAMPLES

1. A rectangular panel, 4' wide x 8' tall, in a vertical wall, is submerged 5 ft below the water surface as shown in the diagram. What is the gauge pressure at the center of the door, Point A? What is the hydrostatic force on the door?



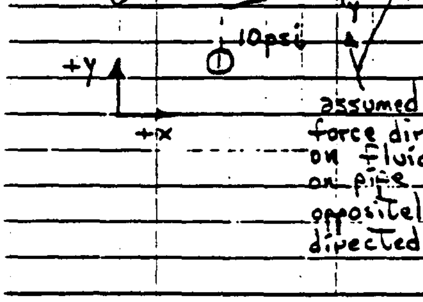
-- Point A is $5 + \frac{8}{2} = 9$ ft below the surface. $P = \rho g h = 62.4 \frac{lb}{ft^3} \times 9 ft =$

$\underline{561.6\ lb/ft^2}$

--- Force is pressure at the centroid x panel area:

$561.6\ lb/ft^2 \times 32\ ft^2 = \underline{17971.2\ lb}$

2. Find force on horizontal pipe bend shown below. Neglect friction loss.



CONTINUITY

$V_1 = \frac{10}{\frac{\pi}{4}(1)^2} = 12.72\ fps$ $\quad V_1^2/2g = 2.51\ ft$ $\quad A_1 = \frac{\pi}{4} \quad = \cancel{113\ in^2}$

$V_2 = \frac{10}{\frac{\pi}{4}(.67)^2} = 28.6\ fps$ $\quad V_2^2/2g = 12.70\ ft$ $\quad A_2 = \frac{\pi}{4} \quad = 50.2\ in^2$

BERNOULLI

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + 0 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + 0$$

$$2.51 + \frac{10(144)}{62.4} = 12.70 + \frac{p_2(144)}{62.4} \quad \therefore \quad p_2 = 5.57\ psi$$

MOMENTUM

$$\Sigma F_x = \rho Q (V_{out} - V_{in})_x$$

$$p_1A_1 + p_2A_2 \sin 30° - P_x = \rho Q(-V_2 \sin 30° - V_1)$$

$$10(113) + 5.57(50.2)(0.5) - P_x = 1.94(10)[-28.6(0.5) - 12.72]$$

$$P_x = {}^{+}1790 \leftarrow$$

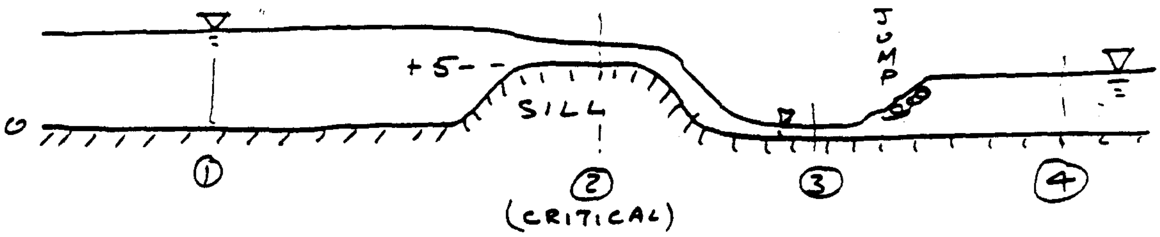
$$\Sigma F_y = \rho Q (V_{out} - V_{in})_y$$

$$P_y - p_2A_2 \cos 30° = \rho Q(V_2 \cos 30°)$$

$$P_y - 5.57(50.2)(.866) = 1.94(10)(28.6)(0.866)$$

$$P_y = {}^{+}724 \uparrow$$

3. In a 1-ft wide channel approaching a sill, the floor elevation is zero. The sill elevation is +5. The 1-ft wide channel downstream of the sill also has floor elevation zero.



Flow passing over the sill has a critical depth of 1 ft.
a. What is the flow rate?
b. What is the water depth at ①?
c. What is the water depth in supercritical flow at ③?
d. What is the water depth at ④, downstream of a stationary hydraulic jump?

Neglect friction losses other than energy loss in the jump.

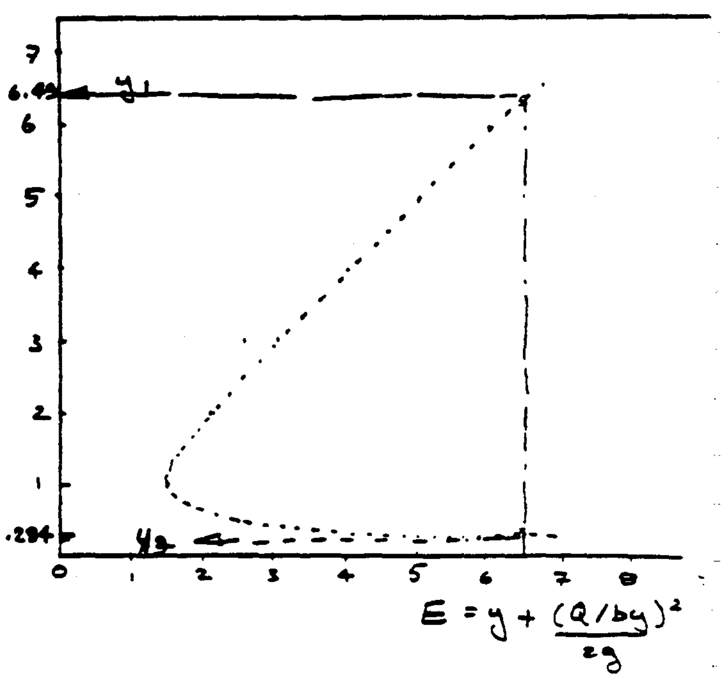
∴ a. $Q = bVy$; $b = 1\,ft$; at ② critical conditions exist :
$V_c = \sqrt{g y_c}$; $y_c = y_2 = 1\,ft$; $Q = bV_c y_c$; $Q = 1 \times \sqrt{32.2 \times 1} \times 1 = \underline{\underline{5.67\ cfs}}$

b. The specific energy is $E = y + \frac{V^2}{2g} = y + \frac{[Q/(by)]^2}{2g}$

Construct the specific energy curve y vs E, for this Q and b.

At ②, y = 1, so E = 1.5 ft.

The energy grade line is thus at elevation 1.5 + 5 ft. Since there is no friction loss between ① and ②, The EGL is at 6.5 ft at ① and ③ as well as ②.



3a (cont.)

Since the floor at ① and ③ is at Elevation zero,
The specific energy at ① and ③ is 6.5 ft.
From the curve, deduce two values of y for
E = 6.5: the greater, for subcritical flow, is $y_1 = 6.49 ft$
The lesser, for supercritical flow, is $y_3 = .284$.

To determine y_4, apply the momentum equation:

$$\rho g \frac{y_4^2}{2} + \rho Q V_4 = \rho g \frac{y_3^2}{2} + \rho Q V_3$$

$$\frac{g}{2} y_4^2 + \frac{Q^2}{b y_4} = \frac{g}{2} y_3^2 + \frac{Q^2}{b y_3}$$

$g = 32.2\ ft/sec^2$; $Q = 5.67$ cfs; $b = 1 ft$; $y_3 = 0.284 ft$

$$\frac{32.2}{2} y_4^2 + \frac{(5.67)^2}{1 \times y_4} = \frac{32.2}{2}(.284)^2 + \frac{(5.67)^2}{1 \times .284}$$

$$(16.1) y_4^2 + \frac{(5.67)^2}{y_4} = 1.30 + 113.2 = 114.5$$

By trial and error:

Try y_4:	$\frac{32.2}{2} y_4^2 + \frac{(5.67)^2}{y_4} = 114.5?$
5	408.9
3	155.6
2	80.5
2.5	113.5
2.51	114.2 → use this:

$y_4 = 2.51 ft$

4. What size must a V.C. sewer be to carry 4 cfs, assuming a grade necessary to produce $V_{MIN} = 2$ fps when flowing full?

$Q = AV = 4 \text{ cfs} = (\frac{\pi}{4} D^2) \times 2$; $\therefore D = 1.60 \text{ ft} = 19.2 \text{ in}$

(Use 18 inch diameter for $V > 2$ fps)

What is the slope for this pipe if the Manning $n = 0.013$?

If $D = 18$ inches $= 1.5$ ft, then $A = \frac{\pi}{4}(1.5)^2 = 1.77 \text{ ft}^2$

$V = 4 \text{ cfs} / 1.77 \text{ ft}^2 = 2.25$ fps ; $V = \frac{1.49}{n} R^{2/3} S^{1/2}$

$R = (1.5/4)$;

$S = \left[\frac{V^2 n^2}{1.49^2 \; R^{4/3}}\right] = 0.00142$

5. Two 24-inch dia. reinforced concrete drains, at a slope of 0.0025 discharge into a long rectangular open channel of width 5 ft and slope of 0.004. What is the normal depth of flow when the drains are flowing full without surcharge? Use $n = .013$.

a. $Q = \overset{\text{2 drains}}{2} \times \left[\frac{1.49}{n} A R^{2/3} S^{1/2}\right] = 2 \times \left[\frac{1.49}{.013} \times \frac{\pi}{4}(2')^2 (2'/4)^{2/3} \sqrt{.0025}\right]$

$= 22.7$ cfs

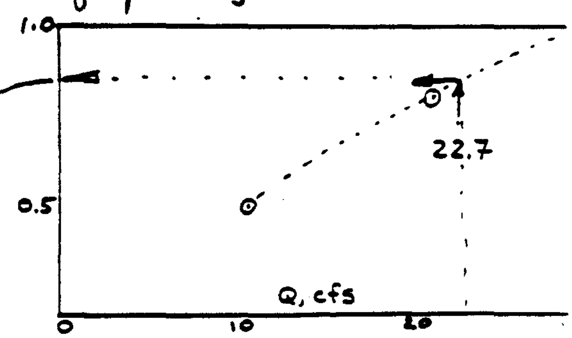
b. Find normal depth, y, in open channel.

$A = \overset{by}{5y}$; $R = A/P = 5y/(5+2y)$; $Q = \frac{1.49}{n} A R^{2/3} S^{1/2}$

Solve by trial-and-error, and graphically:

Trial y	Resulting $AR^{2/3}$	Resulting Q
0.5 ft	1.395	10.11 cfs
1.0	3.995	28.96
0.8	2.865	20.77
0.85	3.138	22.74 cfs - confirmed

confirm



6. Compute the rectangular "best hydraulic section" to convey a flow of 17 m^3/sec. The wall material is smooth concrete, and the slope is 0.08m per 1000 m.

A "best hydraulic section" has the least wetted perimeter for a given cross-sectional area. For a rectangular channel, the BHS is one in which the flow depth, y, is half of the width.

Use the Manning formula for SI units:

$$Q = \frac{1}{n} AR^{2/3} S^{1/2}$$

For smooth concrete, use n = 0.013

$S = 8 \times 10^{-5}$

$Q = 17\ m^3/sec$

$A = 2y^2$

$R = A/P = \qquad = y/2$

$R = \frac{A}{P} = \frac{by}{b+2y}$

$b = 2y$

Then $\frac{2y(y)}{2y+2y} = \frac{2y^2}{4y}$

$= y/2$

$AR^{2/3} = 1.26y^{2.667}$ $2y^2(y/2)^{2/3} =$

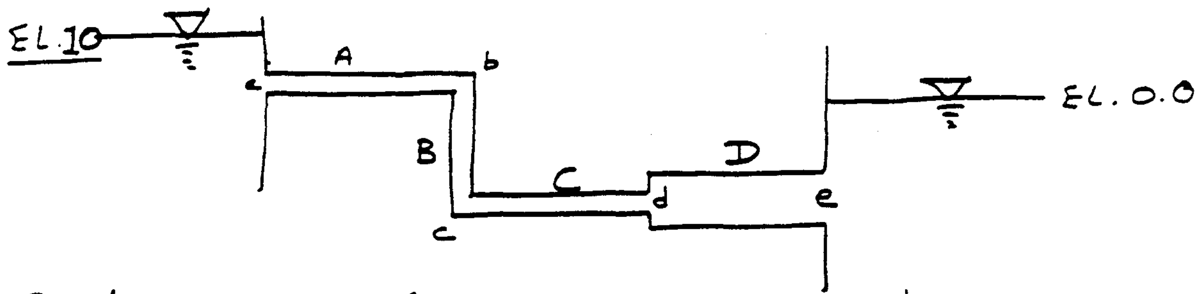
But $AR^{2/3} = Qn/S^{1/2} = 24.7$ $(17)(0.013)/(0.00008)^{1/2}$

$y = (24.7/1.26)^{\frac{1}{2.667}} = 3.05$ m, the depth

The width is 2y, or **6.1m**

$24.7 = 1.26y^{2.667}$

7. A pipeline connecting two reservoirs consists of four sections, each 1000 m long.



Sections A, B, and C have 0.1 m diameter; Section D has 0.2 m diameter. For Manning $n = 0.013$, what Q in the system results from a 10 m head difference?

__Friction__: $Q = \frac{1}{n} A R^{2/3} S^{1/2}$, or

$$h = \frac{L Q^2 n^2}{A^2 R^{4/3}}$$

$L = 1000$ m, $n = .013$. For A, B, C,

$A = \frac{\pi}{4}(.1)^2$, $R = 0.025$

$h_A = h_B = h_C = 374789 Q^2$

For D, $A = \frac{\pi}{4}(.2)^2$, $R = .05$

$h_D = 9296 Q^2$

Total friction loss $= h_A + h_B + h_C + h_D$

$= 1{,}133{,}663 Q^2$

__Form__: $h = K \frac{Q^2}{A^2 2g}$; $g = e$

At a: $K = 0.5$, $A = \frac{\pi}{4}(.1)^2$; $h_a = 41$

b, c: $K = 1.0$, $A = \frac{\pi}{4}(.1)^2$; $h_b = h_c$

$h_b = h_c = 827$

d: $K = 0.56$, $A = \frac{\pi}{4}(.1)^2$;

$h_d = 463$

e: $K = 1.0$, $A = \frac{\pi}{4}(.2)^2$;

$h_e = 52$

Total form loss $=$

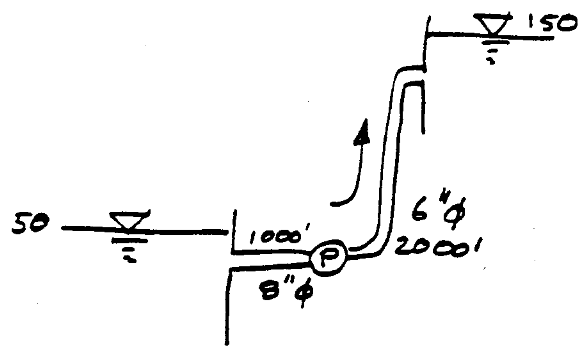
$h_a + h_b + h_c + h_d + h_e =$

$2583 Q^2$

Form loss + friction loss $= 1{,}136{,}246 Q^2 = 10$ m

$Q = 0.00297$ cms $=$ __2.97 litres/sec.__

8. A pump delivers 2.5 cfs from the reservoir at elev. 50 to a reservoir at elev. 150. There is 1000 ft of 8" suction pipe and 2000 ft of 6" of discharge pipe. Deduce the horsepower of the pump. Neglect form losses. Water temperature is 70 °F. The roughness is $\varepsilon = 0.001$ ft, and the pump efficiency is 0.8



Bernoulli equation applied between two reservoirs:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_{f_{8''}} + h_{f_{6''}} - h_{pump}$$

$$0 + 0 + 50 = 0 + 0 + 150 + h_{f_{8''}} + h_{f_{6''}} - h_{pump}$$

In 8" pipe:

$$V_8 = \frac{2.5}{\frac{\pi}{4}(.67)^2} = 7.16 \text{ fps} \qquad \frac{V_{8''}^2}{2g} = 0.793 \text{ ft}$$

$$R = \frac{VD}{\nu} = \frac{7.16 \times 0.67}{1.095 \times 10^{-5}} = 4.5 \times 10^5; \quad \frac{\varepsilon}{D} = \frac{.001}{.67} = .0015$$

From Figure 5-1, $f_{8''} = 0.022$

$$h_{f_{8''}} = \frac{f_8 \cdot L_8}{D_{8''}} \frac{V_{8''}^2}{2g} = \frac{0.022 \times 1000}{0.67} \times 0.793 = 26 \text{ ft}$$

In 6" pipe:

$$V_6 = \frac{2.5}{\frac{\pi}{4}(.5)^2} = 12.72 \text{ fps} \qquad \frac{V_{6''}^2}{2g} = 2.50 \text{ ft}$$

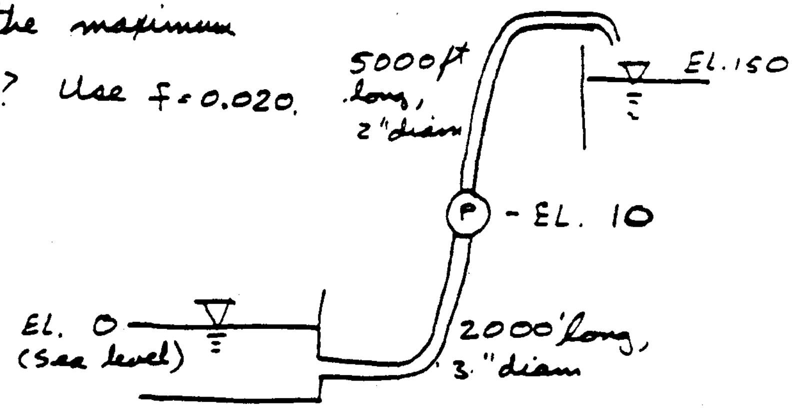
$$R = \frac{VD}{\nu} = \frac{12.72 \times 0.5}{1.059 \times 10^{5}} = 6 \times 10^5; \quad \frac{\varepsilon}{D} = \frac{.001}{0.2} = .002$$

From Figure 5-1, $f_{6''} = 0.024$

$$h_{f_{6''}} = \frac{f_6 L_6}{D_6} \frac{V_6^2}{2g} = \frac{0.024 \times 2000}{0.5} \times 2.5 = 240 \text{ ft}$$

$$h_{pump} = (150 - 50) + 26 + 240 = 366 \text{ ft} \qquad hp = \frac{\rho g Q h_{pump}}{550 e} = 130 \text{ horse-power}$$

9. In the pumping system shown, what is the maximum possible flow rate? Use $f = 0.020$.



(In the past, some students have declared that there is insufficient information given. Resist jumping to that conclusion!)

This example differs most from No. 10 in that the pump is located <u>above</u> the suction reservoir level. The pump will at function if absolute pressure, P_a, is less than zero:

$10 = \frac{fL}{D}\frac{V^2}{2g}$

$$P_a = P_{atm} - \rho g(10) - \rho g\left(\frac{fL}{D}\frac{V^2}{2g}\right)$$

At sea level, $P_{atm} = 14.7$ psi $= \rho g(34\,ft)$

$$P_a = 0 = \rho g\left[34 - 10 - \frac{(0.02)(2000')}{0.25'}\frac{V^2}{2(32.2)}\right]$$

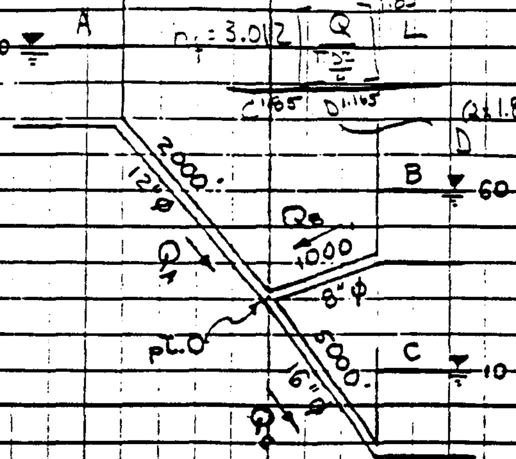
$V = 3.$ ft/sec; $Q = \frac{\pi}{4} D^2 \times V = 0.153$ cfs

(Actually, cavitation effects would begin to degrade pump performance at a lessor Q, before P_a is reduced to zero.)

10. Given: Three reservoir problem: El. A = 100, L_A = 2000', D_A = 12"; El. B = 60, L_B = 1000'
D_B = 8"; El. C = 10, L_C = 5000', D_C = 16". Hazen & William's C = 130

Find: Q_A, Q_B, Q_C, pressure elevation at junction

$$h_f = 3.012 \left(\frac{Q}{\frac{\pi D^2}{4}}\right)^{1.85} \frac{L}{C^{1.85} D^{1.165}}$$



Assume flows as shown ∴ $Q_A + Q_B = Q_C$

Heads must balance ∴

$100 - h_{AO} = 60 - h_{BO}$; $60 - h_{BO} - h_{OC} = 10$

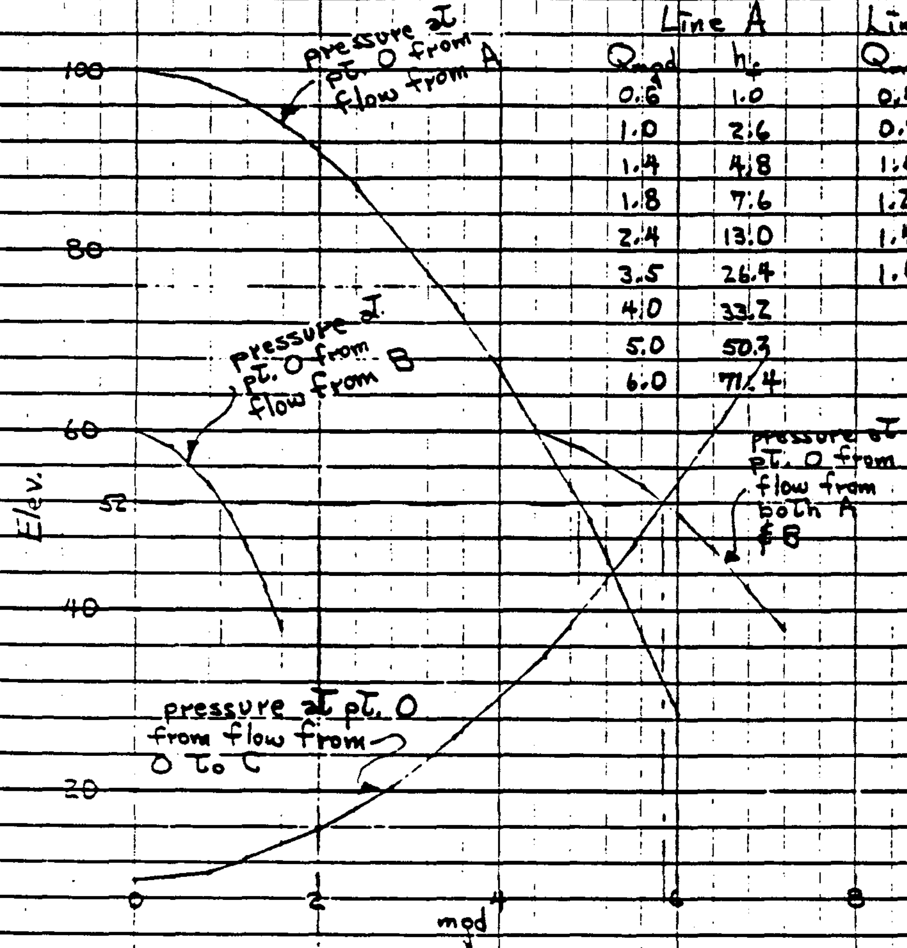
Solve graphically: $h_f = \frac{3.012\, V^{1.85} L}{C^{1.85} D^{1.165}}$

$h_f = K Q_{mgd}^{1.85}$ $\quad K = \frac{10.6\, L_{ft}}{C^{1.85} D_{ft}^{4.87}}$ $\quad V = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}}$

$K_A = \frac{10.6(2000)}{(130)^{1.85}(1)^{4.87}} = 2.58$ $\quad K_B = \frac{10.6(1000)}{(130)^{1.85}(.67)^{4.87}} = 9.3$

$K_C = \frac{10.6(5000)}{(130)^{1.85}(1.33)^{4.87}} = 1.59$

Line A Q_{mgd}	Line A h_f	Line B Q_{mgd}	Line B h_f	Line C Q_{mgd}	Line C h_f
0.6	1.0	0.4	1.7	0.8	1.1
1.0	2.6	0.8	6.1	1.2	2.2
1.4	4.8	1.0	9.3	2.0	5.8
1.8	7.6	1.2	13.0	2.8	10.3
2.4	13.0	1.4	17.3	3.5	16.1
3.5	26.4	1.6	22.0	4.5	26.0
4.0	33.2			5.5	37.5
5.0	50.3			7.0	58.5
6.0	71.4				



Ans:

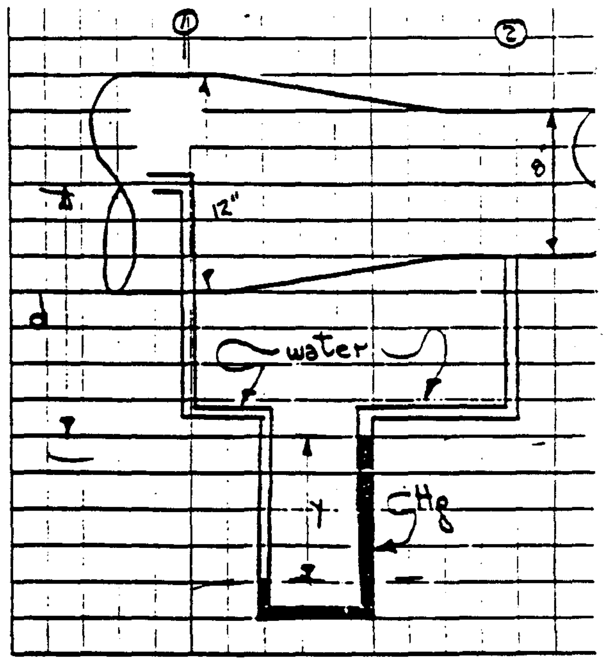
Q_B : 0.9 mgd

Q_A : 4.9 mgd

Q_C = 5.8 mgd

pressure at O: 52

11. Given the tubing configuration shown below, find the deflection, y, for a flow of 10 cfs. Assume that the centerline velocity is 1.5 times the average velocity. Neglect headloss in the pipe contraction. The specific gravity of mercury is 13.6.



... The tube at (1) measures total head at the pipe centerline. The tube at (2) measures piezometric head.

Bernoulli Eqn

$$\frac{V_{1avg}^2}{2g} + \frac{P_1}{\rho g} + z_1 = \frac{V_{2avg}^2}{2g} + \frac{P_2}{\rho g} + z_2$$

$$z_1 = z_2$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_{2avg}^2}{2g} - \frac{V_{1avg}^2}{2g}$$

$$V_{1avg} = \frac{10}{\pi(1^2)} = 12.72 \text{ fps} ; \quad \frac{V_{1avg}^2}{2g} = 2.51 \text{ ft}$$

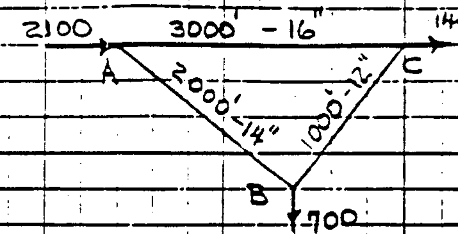
$$V_{2avg} = \frac{10}{\pi(.67)^2} = 28.6 \text{ fps}; \quad \frac{V_{2avg}^2}{2g} = 12.70 \text{ ft}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 12.70 - 2.51 = 10.19 \text{ ft of water}$$

The total head at (1) is $\frac{P_1}{\rho g} + \frac{(1.5 V_{1avg})^2}{2g}$; the piezometric head at (2) is $\frac{P_2}{\rho g}$. The difference is $10.19 + (1.5)^2 2.51 = 15.84$ feet of water. As for y, $15.84 \text{ ft}_{H_2O} + 1 \times y \text{ ft}_{H_2O} = 13.6 y$,

$$15.84 = 12.6 y ; \quad \underline{y = 1.26 \text{ ft}}$$

12. Given the level section of a city water distribution system and the flows in gallons per minute entering or leaving the pipe network at points A, B and C shown, find the amount and direction of flow in each pipe, disregarding any differences in elevation of the pipes. Use Hazen-Williams $C = 100$.



$Q_A = 2100$ gpm $= 3.03$ mgd — supply

$Q_C = 1400$ gpm $= 2.02$ mgd — Take-off

$Q_B = 700$ gpm 1.01 mgd — Take-off

$$h_f = \frac{4.78\,L\,Q}{C^{1.85}\,D^{4.87}}$$

$$K_{AC} = \frac{10.6(3000)}{(100)^{1.85}(1.33)^{4.87}} = 1.56$$

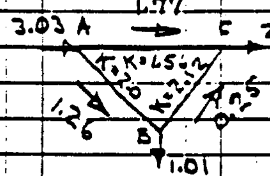
$$K_{AB} = \frac{10.6(2000)}{(100)^{1.85}(1.167)^{4.87}} = 2.00 \qquad h_f = K Q_{mgd}^{1.85}$$

Solving by Hardy-Cross method:

1. Assume initial flows in pipes $\qquad K_{BC} = \dfrac{10.6(1000)}{(100)^{1.85}(1)^{4.87}} = 2.12$
2. Loop correction,

$$\Delta Q_r = \Sigma h_f \Big/ \Sigma\left|1.85 K Q_r^{0.85}\right|$$

ΔQ_r is in direction tending to ma[ke] heads along either path more nearly [equal]



	Q_r	h_f	$\Sigma\lvert 1.85KQ_r^{0.85}\rvert$
AC	1.77	$1.56\times(1.77)^{1.85} = 4.53$	$1.56(1.77)^{0.85} = 2.54$
AB	1.26	$2.00\times(1.26)^{1.85} = 3.06$	$2.00(1.26)^{0.85} = 2.43$
BC	0.25	$2.12\times(0.25)^{1.85} = .16$	$2.12(0.25)^{0.85} = 0.65$
		$\Sigma h_f = 1.31$	$1.85 \times 5.62 = 10.4$

$\Delta Q_r = 1.31/10.40 = 0.1267 \cong 0.13$

AC	1.64	$1.56(1.64)^{1.85} = 3.90$	$1.56(1.64)^{0.85} = 2.38$
AB	1.39	$2.00(1.39)^{1.85} = 3.68$	$2.00(1.39)^{0.85} = 2.65$
BC	0.38	$2.12(0.38)^{1.85} = .35$	$2.12(0.38)^{0.85} = 0.93$
		0.13	$1.85 \times 5.96 = 11.0$

$\Delta Q_r = 0.13/11.02 = 0.0118 \cong 0.01$

AC	1.65
AB	1.38
BC	0.37

Discussion: First, assume a plausible distribution of flow that satisfies the inflow and outflow conditions. In general this trial distribution will not satisfy headloss conditions, e.g. h_{AC} will not equal $h_{AB} + h_{BC}$, which it must for the correct solution.

The trial solution will differ from the correct distribution by an amount equal to an imaginary circulation, ΔQ, around the loop. In each leg of the loop:

$$h_f = K Q^{1.85} \qquad (14\text{-}1)$$

The difference between the true headloss and the headloss associated with the trial flow is $h_{true} - h_{trial} = \Delta h_f$. Differentiate Eq. 14-1 to get ΔQ: $\quad \Delta h_f = 1.85 K^{0.85} \Delta Q$. Over all legs in the loop: $\Sigma h_f = \Delta h_1 + \Delta h_2 + \Delta h_3 = 1.85\left[K_1 Q_1^{.85} + K_2 Q_2^{.85} + K_3 Q_3^{.85}\right]\Delta Q$. The first trial's mismatch of headloss is Σh_f: i.e. $(h_{AC}) - (h_{AB} + h_{BC}) = \Sigma h_f$. Using trial Q_1, Q_2 and Q_3, solve for ΔQ.

13. An essentially plane parking lot, asphalt surfaced, 1000 ft × 436 ft, slopes to one corner for drainage. The diagonally opposite corner is 10 ft higher than the drainage corner. The rainfall statistics for the area are described by the curves in Figure 8-1.

The drainage culvert is to be of concrete pipe, whose alignment has a slope of 0.0025. What is an appropriate pipe diameter to handle a 5-year design storm?

$Q = cia$ Use $c = 0.9$ $a = 436000 \text{ ft}^2 \cong 10$ acre

To determine i, use Figure 8-1; determine T_c.

Slope of parking lot $= 10' / \sqrt{1000^2 + 436^2} = 0.0092$

$T_c = V \times \sqrt{1000^2 + 436^2} = 1091/V$ sec

$V = \frac{1.49}{n} R^{2/3} S^{1/2}$ What to use for R? Use $n = .015$

Try $R = 0.05$ ft; then $V = 1.29$ ft/sec, and $T_c = 14$ min

From Figure 8-1, for 5 years and $T_c = 14$ min, $\underline{i = 4 \text{ in/hr}}$

(Check: 4 in/hr × 14 minutes = 0.93 inches = 0.078 ft.
Use this for R; then $V = 1.74$ ft/sec, and $T_c = 10.5$ min.
From Figure 8-1, $\underline{i = 4.3 \text{ in/hr}}$. Use this second trial.)

$Q = cia = 38.7$ acre·in/hr $\cong 38.7$ cfs

Drain: $Q = \frac{1.49}{n} A R^{2/3} S^{1/2}$ use $n = .013$; $S = 0.0025$

$AR^{2/3} = \frac{Qn}{1.49 S^{1/2}} = 6.753$

But $AR^{2/3} = \frac{\pi}{4} D^2 \left(\frac{D}{4}\right)^{2/3}$ $D^{2^{2/3}} = 21.67$; $D = \underline{3.169 \text{ ft}}$

$= \underline{38 \text{ inches}}$

(Use standard size, 42 inches

14. Compute the storage requirements for a reservoir to meet a constant draft of 300 MG/year/mi^2, given the following critical or design period:

Month:	F	M	A	M	J	J	A	S	O	N	D	J
Monthly Inflow: (MG/Year)	30	60	90	10	5	10	5	30	40	20	15	85

Solve this graphically. This is a problem involving volume ("What is the required storage of the reservoir?"), and rate of change of volume with respect to time ("300 MG/year per mi^2" and the monthly inflow volumes).

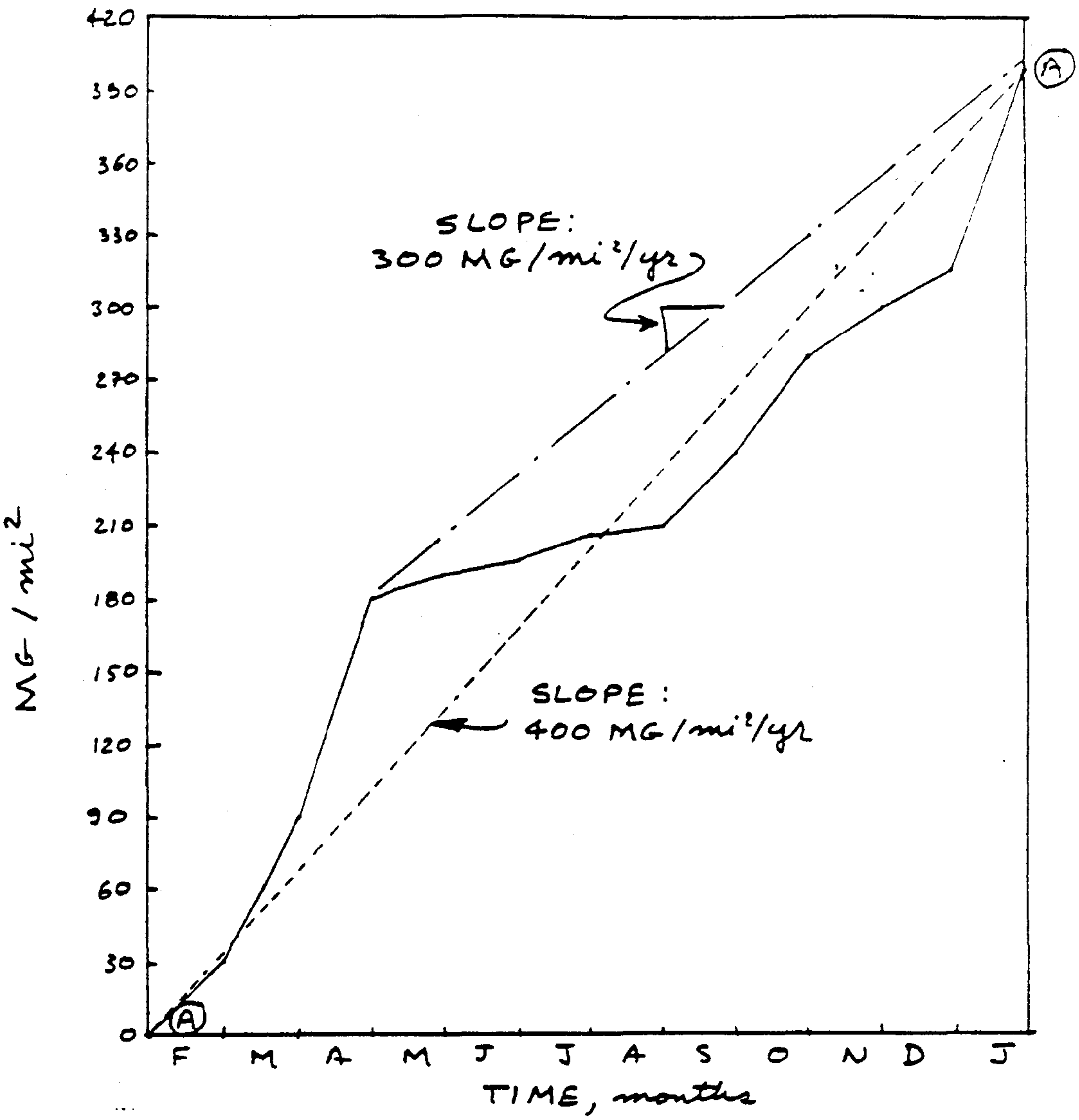
Construct a graph whose vertical axis is in units of volume (million gallons, MG, per squre mile of watershed area) and whose horizontal axis is in units of time (months). The slope of a curve on this graph will have units of discharge (MG/month/mi^2).

Plot the given monthly inflow in a cumulative distribution, as shown in Figure 16-1. The dashed line A - A connecting the two end points (beginning of February to end of January) has a slope representing the average inflow:

400 MG/mi^2/year, or 33.33 MG/mi^2/mo. The safe yield cannot exceed this average inflow rate, no matter what the reservoir size.

The desired safe yield is 300 MG/mi^2/year. Draw a line having a slope of 300 MG/mi^2/year, and interesecting the inflow curve at the "knee" (end of April, Point A). This represents a condition of a full reservoir, to which there will continue to be inflow according to monthly inflow rates, and from which there will be draft at 300 MG/mi^2/year. At any given time, the vertical distance between the two curves in the reservoir, which reaches 70 MG/mi^2 at the end of August. This is a maximum, and this is the required storage volume.

Non-graphical check: At the end of April, the reservoir drawdown is zero. The yearly draft is 300 MG/mi^2, so the monthly draft is 25 MG/mi^2. During May, June, July, and August, the net draft is 4x25 MG/mi^2 less (10+5+10+5) M for a net draft of 70 MG/mi^2.



420
390
360
330
300
270
240
210
180
150
120
90
60
30
0
A
A
SLOPE:
300 MG/mi²/yr
SLOPE:
400 MG/mi²/yr
MG / mi²
F M A M J J A S O N D J
TIME, months

FIGURE 16-1.

15. City Water Department billing records indicate the following service history:

Year	Population Served
1915	12,500
1940	54,000
1965	125,000

If in 1982 there is treated-water storage of 400 acre-ft, and water consumption averages 115 gallons/capita/day, how long would storage last if the water treatment plant breaks down?

a. Estimate the 1982 population served by extrapolation of the billing data (Figure 17): 200,000 is the estimate using a French curve.

b. This served population x 115 gallons capita/day is 23 million gallons/day.

c. $400 \text{ acre-ft} \times \left(\frac{43{,}560 \text{ ft}^2}{\text{acre}}\right) \times \left(\frac{7.48 \text{ gall}}{\text{ft}^3}\right) = 130 \text{ million gallons}$

d. $130 \text{ million gallons} \times \left(\frac{1 \text{ day}}{23 \text{ million gallons}}\right) = \underline{5.67} \text{ days}.$

16. The cross-section of a channel, together with its adjacent floodplain, is shown in Figure 18. The average longitudinal slope of the channel and its valley is S = 0.00031. During a flood in which the water level rises to El. 910 ft, what is the total discharge and average velocity?

Apply Manning's formula separately to each region of flow: overbank (cornfield), main channel, and overbank (brush). The formula in foot - pound - second system units is:

$$Q = \frac{1.49}{n} AR^{2/3} S^{1/2}$$

No side-slope information is given, but in cases where the flow width is much greater than the depth, there is little error introduced by neglecting the channel sides. Furthermore, the hydraulic radius, R, is approximately equal to the flow depth for wide, shallow channels.

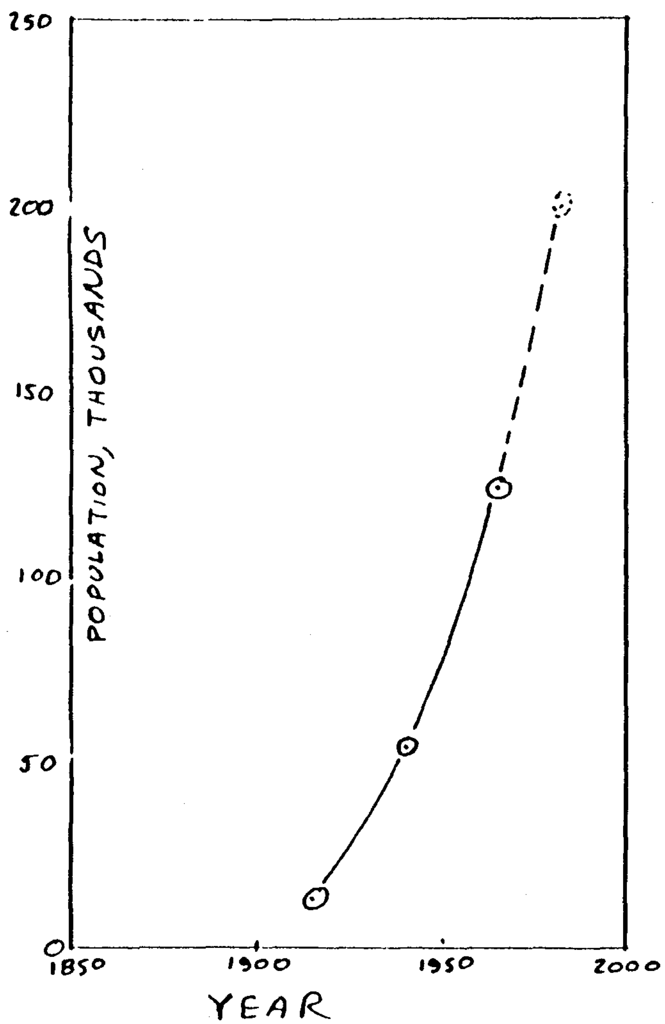


FIGURE 17.

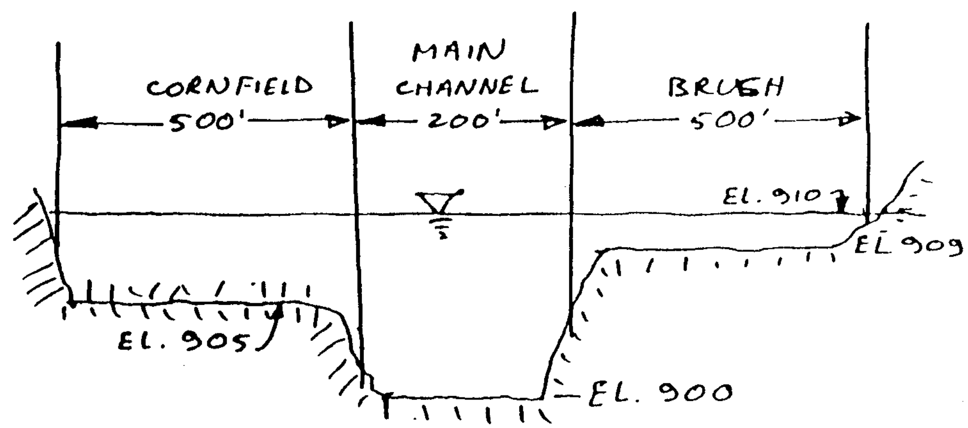


FIGURE 18.

Thus, for the cornfield:

n = 0.06, depth = 910-905 = f ft, A = 500x5 = 2500 ft,^{2}

R = 5 ft;

$$Q = \frac{1.49}{0.06} \times 2500 \times 5^{2/3} \times (.00031)^{0.5}$$

<u>Q = 3196 ft^3.sec (cornfield)</u>.

For the main channel:

n = 0.015, depth = 910-900 = 10 ft, A = 200x10 = 2000 ft^2,

R = 10 ft;

$$Q = \frac{1.49}{.015} \times 2000 \times 10^{2/3} \times 0.00031^{0.5}$$

<u>Q = 16236 ft^3/sec (main channel)</u>.

For the brush-covered area:

n = 0.12, depth = 910-909 = 1 ft, A = 500x1 = 500 ft^2,

R = 1 ft;

$$Q = \frac{1.49}{0.12} \times 500 \times 1^{2/3} \times (.00031)^{0.5}$$

<u>Q = 109 ft^3/sec (brush)</u>.

The sum of these three discharges is the <u>total discharge: 19-41 ft^3.sec.</u> The sum of the three areas is 5000 ft^2; the average velocity is the total discharge divided by the total area, or <u>3.9 ft/sec</u>.

(NOTE: even in cases where one explicitly considers the rigid banks of the flow, the vertical interface between two sections of the flow, e.g., between the flow covering the cornfield and the flow in the main channel, should <u>not</u> be considered a frictional surface. The cornfield flow may exert a slowing drag on the main channel flow; but the main channel flow exerts an equal and opposite impulsive drag, or boost, to the cornfield flow, for zero net effect.)

HYDRAULICS GLOSSARY

Uniform flow has the same speed and direction at all points within the flow boundaries. Uniform flow need not be steady flow.

Steady flow has speed and direction that may vary from point to point, but which nowhere changes speed or direction with respect to time.

Rapidly varied flow is open-channel flow that changes speed and/or direction suddenly, due to channel geometry such as a bend or a sill, or due to a hydraulic jump.

Gradually varied flow is open-channel flow that changes depth and speed in adjustment to wall friction force on the fluid. Examples are backwater and drawdown.

Piezometric head, representing the amount of potential energy in a flow, is the free surface elevation in open-channel flow. In a full closed conduit, the piezometric head is the elevation to which fluid would rise in a test bore piercing the conduit wall.

Hydraulic Grade Line (HGL) is the locus of piezometric head all along a flow conduit.

Velocity Head, $v^2/2g$, is a measure of kinetic energy in the flow.

Total Head is the sum of piezometric head and velocity head.

Specific Energy is the elevation of total head above the channel floor at any point.

The Energy Grade Line (EGL) is the locus of total head all along a flow conduit.

Froude Number, $F = V/\sqrt{gy}$. In critical flow, F=1. In supercritical flow, F is greater than 1. In subcritical flow, F is less than 1.

Critical Depth: For a given channel size and shape and a given flow rate, the depth at which flow is critical. See Section 6.

Normal depth in open-channel flow is achieved in a long straight channel of uniform wall roughness and channel geometry. In normal-depth flow, energy dissipation by side-wall fricition is equal to the decrease in potential energy as fluid proceeds down a channel. Put another way, friction slope, S, calculated from the Manning formula from Q, n, R, and A, just equals the longitudinal channel slope.

Hydraulic Radius: the ratio of flow cross-section area to wetted perimeter: R = A/P. For a very wide, shallow rectangular channel, R is approximately equal to the channel depth. For a circular pipe flowing full, R = 1/4 times the diameter.

Best Hydraulic Section: For a given flow area, the channel section that has the least wetted perimeter, hence the greatest conveyance. For a rectangular channel, the BHS is half of a square (width equal to twice the depth).

III.
Structural Analysis and Design

Instructor: Peter Branagan

Tapes

4 5

Introduction:

The subject matter that is presented as a review of structural analysis and design has been selected and prepared with the Professional Engineers Exam as the criterion. A comprehensive review of the extremely broad field of both areas is neither reasonable nor feasible. It is felt that as a review, the material will serve to prompt one's memory and direct the review process along the general paths covered in both classrooms and office practice.

The structural section of the PE refresher course consists of two sections. The first section reviews various "hand" methods of analysis that may be useful in determining the design parameters of a problem. The second section reviews the design of a structure's members using materials that are commonly encountered: concrete, steel, wood, and masonry.

The first section includes analysis methods for simple structures. It considers the Cable theorem, and the determination of deflections by both the Moment Area Method and the Law of Virtual Work. Analysis of Indeterminate structures by the Flexibility Method and by a Moment Distribution Method are reviewed. Frame sidesway by the moment distribution method is also considered. Finally the rigidity of masonry walls with openings is analyzed for determining the distribution of lateral forces to the individual piers.

The second section reviews structural design for imposed forces and moments . Structural steel members are reviewed to resist axial, flexural, and combined forces per, AISC, Manual of Steel Construction, 9th ed. Reinforced concrete design by the ACI 318 Building Code is reviewed for beams, shear, and column design. The Specification for Wood Construction, NFPA,is used to review wood members for axial, flexural, and shear forces. Masonry members are designed by ACI 531 Building Code Requirements for Concrete Masonry Structures to resist axial and shear forces. Composite construction of wood beams with steel side plates, and concrete slabs on steel sections are included. Finally, seismic code requirements are reviewed.

The format is primarily a problem solving presentation and attempts to address areas which have appeared in past exams. It is hoped that the material will accomplish its goal of stimulating the memory banks and supplying a measure of confidence to the applicant's approach to the coming exam.

PREFACE

The subject matter that is presented as a review of structural analysis and design has been selected and prepared with the professional engineer examination as a criterion. A comprehensive review of the extremely broad field of both areas is neither reasonable nor feasible. It is felt that as a review, the material will serve to prompt one's memory and direct the review process along the general paths covered in both classrooms and office practice.

No attempt has been made to discuss derivation of formulas, or development of analytical or design processes, since these must be presumed as prerequisites for the examination.

The format is a problem solving presentation and attempts to address the areas which have appeared in past exams. It is hoped that the material will accomplish its goal of stimulating the memory banks and supplying a measure of confidence to the applicant's approach to the coming exam.

The writer expresses thanks for their help to the following: Professors Kenneth Leet and Leroy Cahoon of the Department of Civil Engineering, Northeastern University, for permission to use material from their publications; to Doctor Ronald Sharpin for his kind encouragement, and for giving me the opportunity to present this material; and to my wife, Theresa, who patiently supported the project and expertly typed much of the material.

James Regan

S T R U C T U R A L

A N A L Y S I S

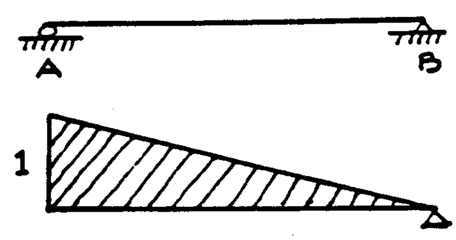
INFLUENCE LINES

An influence line is a curve, the ordinates of which give the values of some particular function (shear, moment, reaction, bar force, etc.) in a fixed element (member section, support, bar in a truss, etc.) due to a unit load acting at the point corresponding to the particular ordinate being considered.

Influence lines for statically determinate structures are straight lines. To draw such diagrams it is only necessary to compute the ordinates at a few control points and connect their terminals with straight lines.

For statically indeterminate structures the influence lines are curved and their construction involves an analysis which may require considerable time. The Meuller-Breslau principle provides a simple method to draw approximate shapes of these influence lines. This principle may be stated as follows:

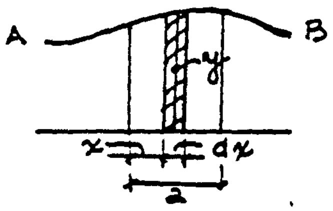
> The ordinates of the influence line for any stress element of any structure are proportional to those of the deflection curve which is obtained by removing the restraint corresponding to the stress resultant and introducing in its place a corresponding deflection.



The reaction restraint at A is removed and an upward deflection (corresponds to an upward reaction) is introduced. The area enclosed between the original and the final positions of the beam is the influence diagram for the reaction at A.

USE OF INFLUENCE LINES

1. Influence lines show where to place a load on a structure to maximize the value of the function for which the influence line is drawn.
2. The value of the function due to a concentrated load equals the product of the magnitude of the load and the ordinate of the influence line at the point of application of the load.
3. For maximum uniform loads place the load over those portions of the curve for which the ordinates have the same sign.
4. The value of a function due to a uniformly distributed load equals the product of the intensity of the load and the net area under that portion of the influence line which is under consideration.



AB is the influence line for a given function (shear, reaction, moment, etc.) on a beam which has a uniform load of w pounds per foot.

The portion of load in distance dx is treated as a concentrated load and is equal to wdx.

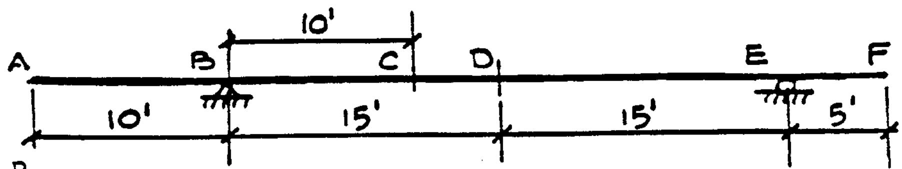
By Rule 4, the value of function F due to this differential load is dF = wdx y.

$$\therefore\ F=\int_0^a dF = \int_0^a wdx\ y = w\int_0^a ydx$$

which is the same as saying, w(the uniform load per foot) multiplied by the area under the influence curve corresponding to the portion of the structure being considered, in this case, distance a.

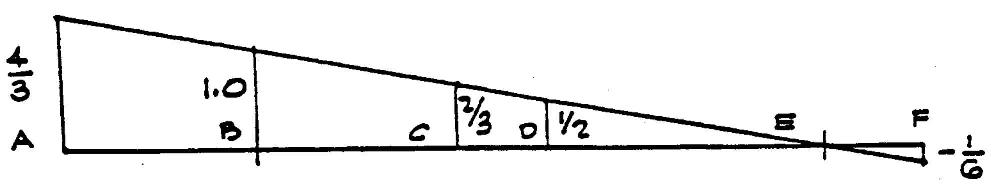
EXAMPLE

1. Draw the influence lines for the following functions: Reaction at B, Reaction at E, Shear at D, Moment at B, Moment at C.
2. Use the influence line to determine the reaction at point B due to a uniform dead load of 200 lb per ft.
3. What is the maximum positive and maximum negative moment produced at point C by a concentrated load of 50 kips?



I.L. R_B

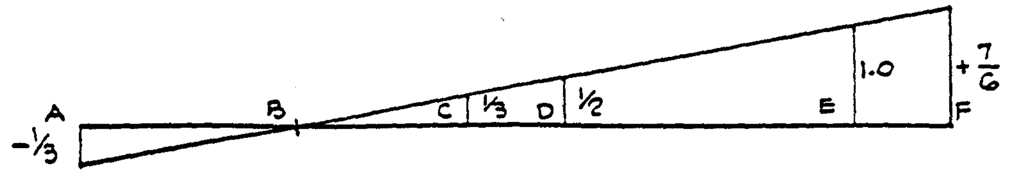
Place a unit load at B, the reaction at B is 1; place a unit load at E, the reaction at B is 0. We now have two points on the influence line, draw a straight line through these points and extend this line to intersect perpendiculars from A and F. This sloping line is the influence line for the reaction at B.



By proportion, when the unit load is at A, the reaction at B is 4/3; and when the unit load is at F, the reaction at B is -1/6.

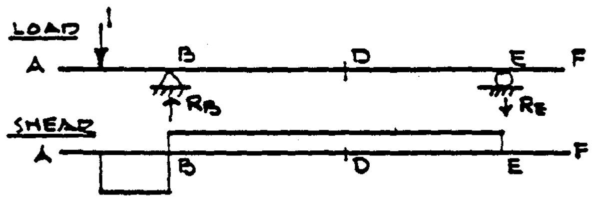
I.L. R_E

Place a unit load at E, the reaction at E is 1, place a unit load at B, the reaction at E is 0. Again, we have two points on a straight line, and connecting these points in the same way as for the influence line for the reaction at B we can easily draw the influence line for the reaction at E.

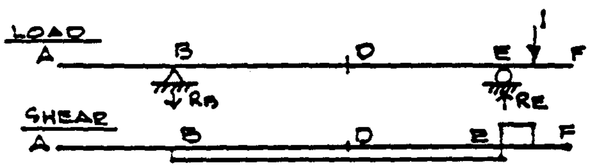


From the diagram, when the unit load is at A, the reaction at E is -1/3; and when the unit load is at F, the reaction at E is +7/6.

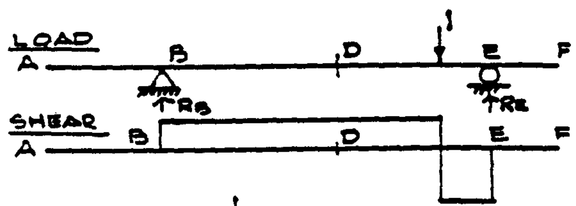
I.L. Shear at D.



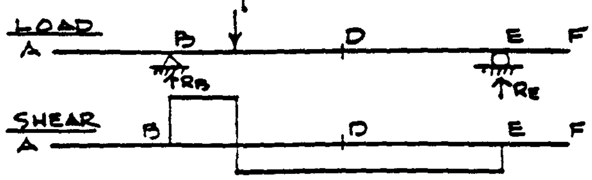
A unit load at any point between A & B will induce a shear at D equal to the negative of the reaction at E.



Placing a unit load at any point between E & F will cause a shear at D equal to the reaction at B.

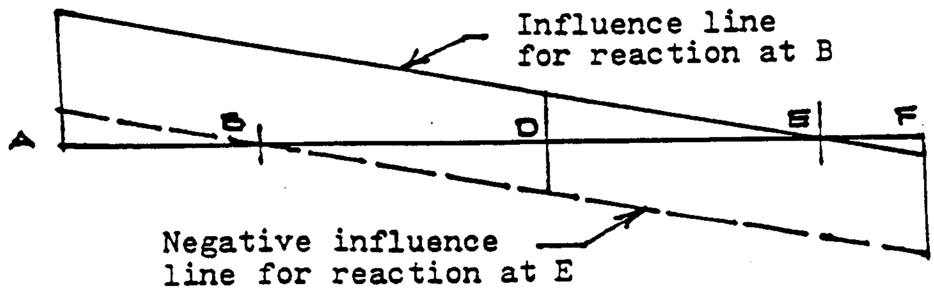


For a unit load at any point between D and E the shear at D is equal to the reaction at B.

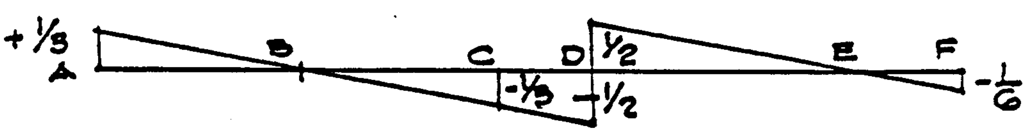


If the unit load is placed at any point between B and D the shear at D is equal to the negative of the reaction at E.

So, to draw the influence line for the shear at D, draw the influence line for the reaction at B and add to it the negative influence line for the reaction at E. Then draw at D a perpendicular line which intersects both of the influence lines.



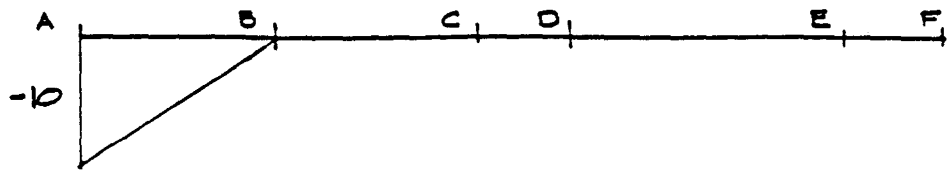
The final influence line for the shear at D is the left-hand portion of the lower influence line and the right-hand portion of the upper influence line, thusly



I.L. for the Moment at B

Place the unit load at A and take moments about point B; 1 x 10 counter-clockwise is negative, and $M_B = -10$.

For a unit load at B the moment at B = 0. At any other point to the right of B the application of the unit load does not induce any moment at B. Then the influence line for the moment at B is



I.L. for the Moment at C

If the unit load is at A the reaction at B, from the influence line for the reaction at B, is equal to 4/3 and taking moments about C we have

$$(+)\quad -1 \times 20 + 4/3 \times 10 = -20/3$$

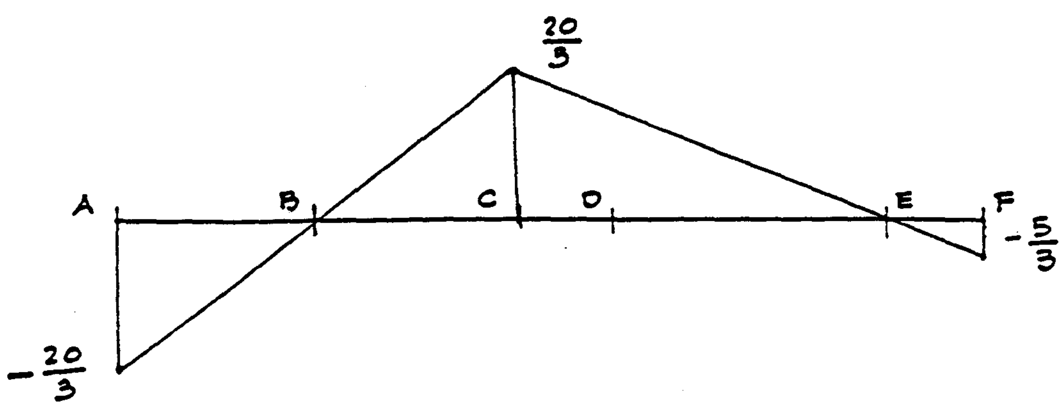
When the unit load is at C the reaction at B is 2/3 and taking moments about C

$$(+)\quad 2/3 \times 10 = +20/3$$

Placing the unit load at F the reaction at E = 7/6 and summing moments about C

$$(+)\quad -1 \times 25 + 7/6 \times 20 = -5/3$$

When the unit load is at B or E there is no moment induced at C. The final influence line is drawn by connecting the control points with straight lines.



The reaction at B due to a uniform load of 200 lbs/ft by Rule 4 is

200 x Area enclosed by the influence line.

$$= 200 \left[\tfrac{1}{2}\left(4/3 \times 40 - 1/6 \times 5\right)\right]$$

$$= 15750/3 = 5250^{\#}$$

The maximum reaction at B occurs when the positive portion of the influence line is loaded.

$$200 \times 1/2 \times 4/3 \times 40 = 16000/3 = 5333^{\#}$$

The maximum positive moment at C produced by a concentrated load of 50^k is obtained by placing the 50^k at the point of maximum positive value for the influence line for moment at C, this is at point C itself and the ordinate value is + 20/3. Therefore, the maximum positive moment is

$$20/3 \times 50 = 333.3^{ft\text{-}k}$$

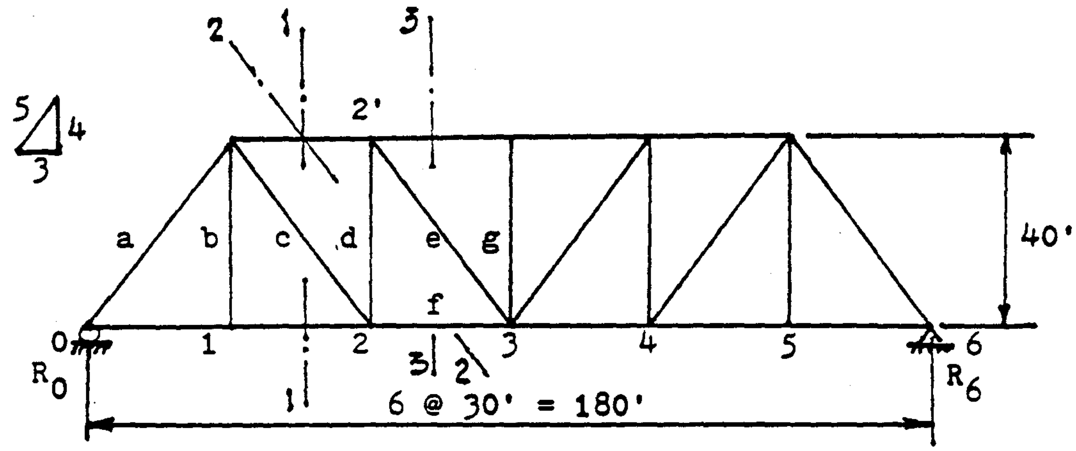
The maximum negative moment at C is produced by placing the 50^k load at the point of maximum negative value of the influence line for moment at C. This location is at point A, and the maximum negative moment is

$$50 \times (-20/3) = -333.3^{ft\text{-}k}$$

Influence lines for bar forces in a truss are drawn for a unit load moving along the truss, the load is usually placed on the bottom chord. Since the truss is determinate, the influence lines are straight lines between the panel points.
The evaluation is done by either the method of sections or the method of joints. In using the method of sections it is best to use the free body diagram which does not contain the unit load.

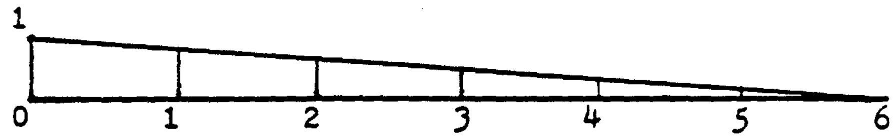
<u>EXAMPLE</u>

Given a truss having parallel top and bottom chords.
Draw the influence lines for the forces in bars a thru g.

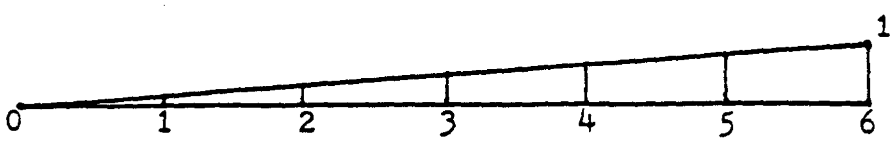


Draw the influence lines for the reactions R_0 and R_6.

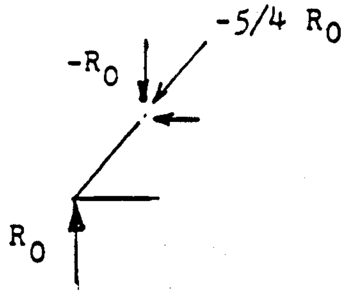
I.L. for the reaction R_0



I.L. for the reaction R_6



I.L. for the force in bar a.



Using the method of joints and isolating joint 0 as a free body, by inspection; when the unit load is at point 0 the bar force in a is equal to zero. If the unit load is at any point from 1 to 6 then

$$S_a = -5/4\ R_0$$

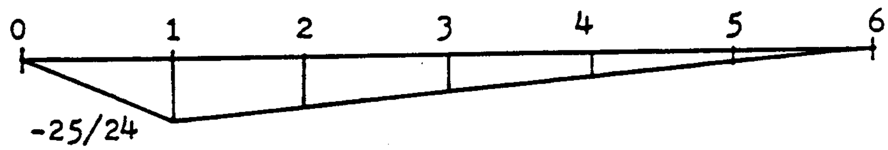
and is minus because the bar is in compression.

When the unit load is at point 6 the force in bar a is = 0.

A unit load at point 1 will induce a force in bar a =

$$S_a = -5/4 \times 5/6 = -25/24$$

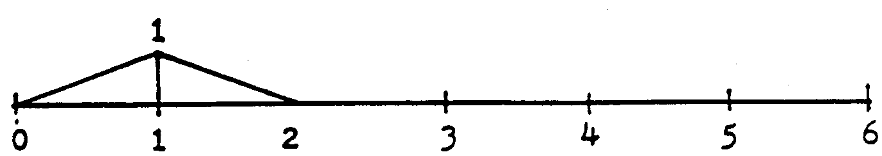
∴ The influence line for the force in bar a is



I.L. for the force in bar b.

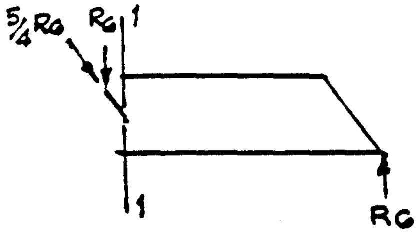
When the unit load is at 0, $S_b = 0$; when the unit load is at point 1, $S_b = 1$; and when the unit load is at points 2 thru 6 the force in bar b is = 0.

The influence line for the force in bar b is then



<u>NOTE</u> In general, in the free body diagram above, and in those to follow, only the forces related to the pertinent discussion are shown.

I.L. for the force in bar c.

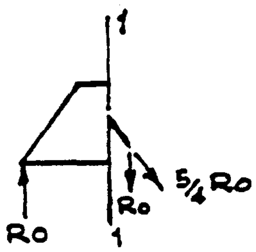


Cutting section at 1-1 and using the right-hand free body, if the unit load moves from point 0 to point 1, the force in bar c is

$$S_c = -5/4\ R_6 \text{ compression.}$$

If the load is at point 1 then

$$S_c = -5/4 \times 1/6 = -\ 5/24$$

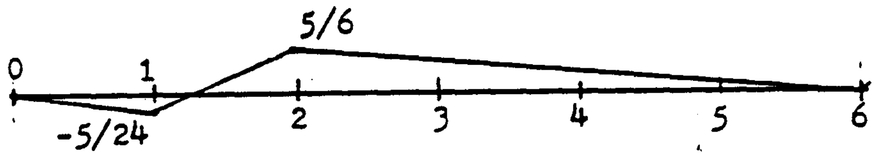


Using the left-hand free body, as the load moves from point 2 to point 6, the force in bar c is $5/4\ R_0$ tension.

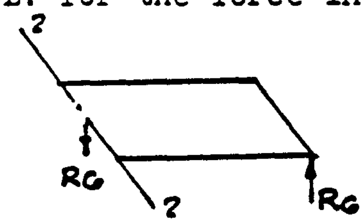
When the unit load is at point 2

$$S_c = 5/4 \times 2/3 = 10/12 = 5/6$$

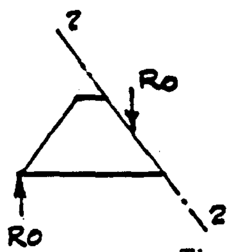
∴ The influence line for the force in bar c is



I.L. for the force in bar d.



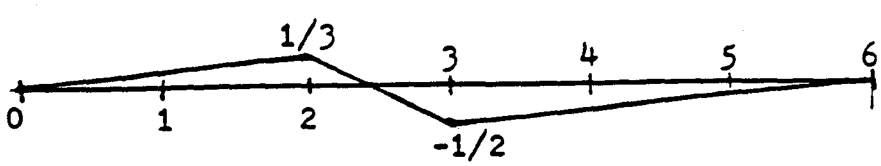
Cutting section at 2-2 and using the right-hand free body, as the unit load moves from 0 to 2 the bar force in d is $+\ R_6$ tension. With the unit load at 2 the force in bar d is +1/3.



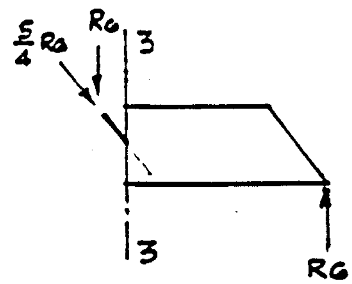
Using the left-hand free body, the bar force in d is equal to $-R_0$ compression. If the unit load is at 3,

$$-R_0 = -1/2$$

The influence line for the bar force in d is



I.L. for the force in bar e.

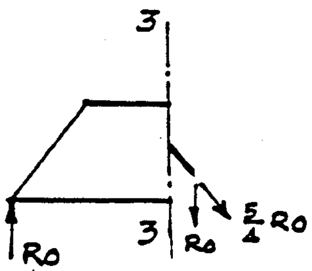


Cutting section 3-3 and using the right-hand free body, if the unit load moves from point 0 to point 2, the force in bar e is

$$S_e = -5/4\ R_6$$

If the unit load is at point 2 then

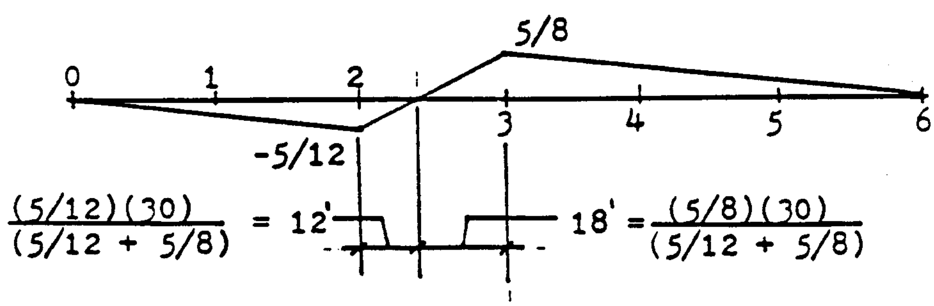
$$S_e = -5/4 \times 1/3 = -5/12$$



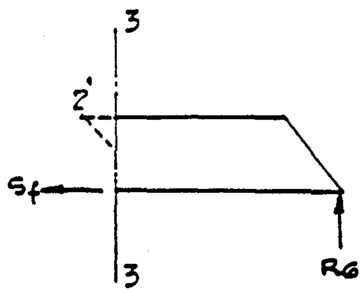
Using the left-hand free body, as the load moves from point 3 to point 6, the force in bar e is 5/4 R_0 tension. When the unit load is at point 3

$$S_e = 5/4 \times 1/2 = +5/8$$

∴ The influence line for the force in bar e is



I.L. for the force in bar f.

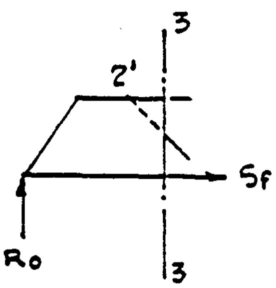


Cutting section 3-3 and using the right-hand free body, take moments about point 2' and consider the unit load as moving from point 2 to point 6.

$$+\quad -120\,R_6 + 40\,S_f = 0$$

$$S_f = 3\,R_6 \text{ tension}$$

When the unit load is at point 2,
$R_6 = 1/3$ and $S_f = 3 \times 1/3 = 1$



Using the left-hand free body, take moments about 2' again, but now consider the unit load to be moving from point 0 to point 2.

$$+\quad 60\,R_0 - 40\,S_f = 0$$

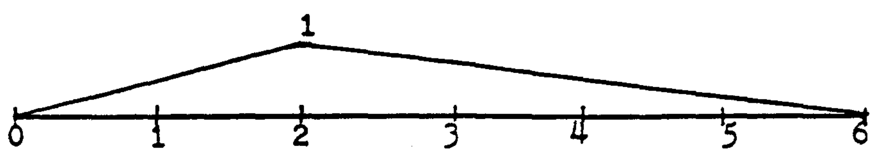
$$S_f = 3/2\,R_0 \text{ tension}$$

When the unit load is at point 2,
$R_0 = 2/3$ and $S_f = 3/2 \times 2/3 = 1$

If the unit load is at points 0 or 6

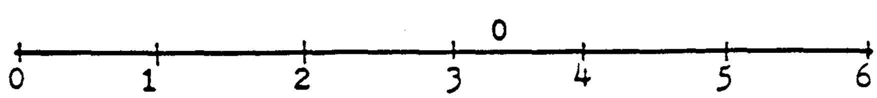
$$S_f = 0$$

∴ The influence line for the force in bar f is



I.L. for the force in bar g.

When the unit load is placed anywhere along the bottom chord of the truss there is no stress induced in bar g; therefore, the influence line for the force in bar g is



EXAMPLE Use of truss influence lines.

a. Determine the force in bar e due to dead weight of the truss of 0.6^k per foot.

b. What is the maximum tension in this bar if a uniform live load of 1^k per foot and a concentrated load of 8^k moves along the bottom chord of the truss?

c. What is the maximum compression induced in bar e by these live loads?

d. Find the total tensile and compressive forces in bar e.

Solution;

a. Since the dead load applies throughout the length of the truss, by rule 4, the force in bar e due to the dead load is

$$S_e = 1/2(-5/12)(72)(0.6) + 1/2(5/8)(108)(0.6)$$
$$S_e = -9.00 + 20.25 = 11.25 \text{ tension}$$

b. For the live load tension in bar e, use the positive portion of the influence line.

$$S_e = 1/2(5/8)(108)(1) + (5/8)(8)$$
$$S_e = 33.75 + 5.00 = 38.75$$

c. For the live load compression in bar e, use the negative portion of the influence line.

$$S_e = 1/2 \times (-5/12)(72)(1) -(5/12)(8)$$
$$S_e = -15.00 -3.33 = -18.33$$

d. The maximum tension in bar e is

$$38.75 + 11.25 = +50^k$$

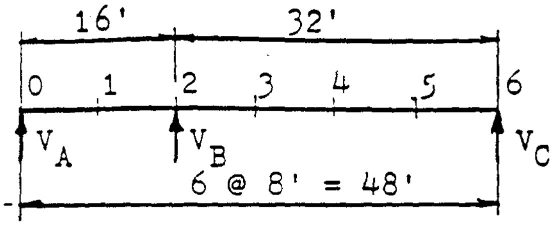
The maximum compression in bar e is

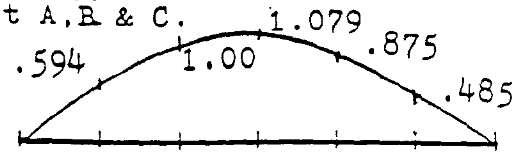
$$-18.33 + 11.25 = -7.08^k$$

It is apparent from these calculations, that a reversal of stress occurs in bar e as the live load moves across the truss. This is important in the design of the member and its connections because stress reversals affect metal fa[illegible] could become a critical consideration.

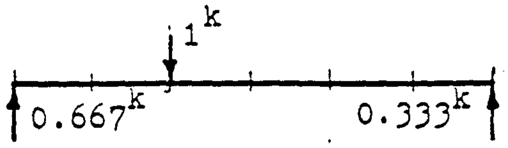
Influence Lines for Indeterminate Beam

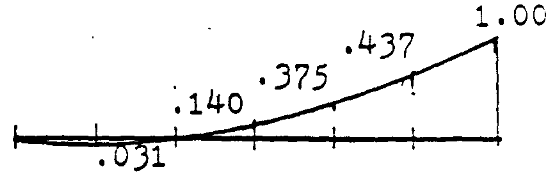
Find the I.L. for the reactions at A, B & C.



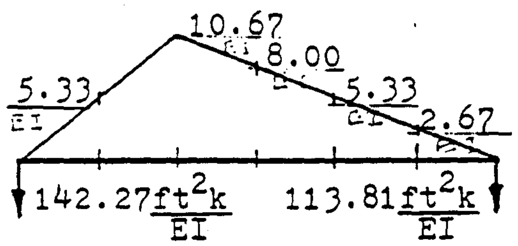


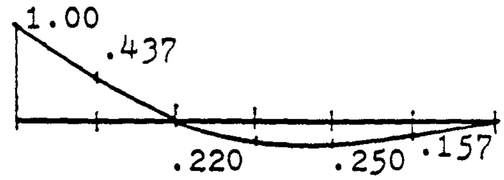
I.L. for V_B





I.L. for V_C





I.L. for V_A

Conjugate Beam loaded with M/EI diagram.

Solution:

Remove V_B and place a unit load at B (point 2), compute the deflections at points 1 through 5.

Using the conjugate beam method, cut sections at points 1 through 5 and compute the moment on the conjugate beam at the respective points.

$$\delta_1=(142.27)(8)-\tfrac{1}{2}(8)(5.33)(2.67)=1081$$
$$\delta_2=(142.27)(16)-\tfrac{1}{2}(16)(10.67)(5.33)=1820$$
$$\delta_3=(113.81)(24)-\tfrac{1}{2}(24)(8.00)(8.00)=1963$$
$$\delta_4=(113.81)(16)-\tfrac{1}{2}(16)(5.33)(5.33)=1593$$
$$\delta_5=(113.81)(8)-\tfrac{1}{2}(8)(2.67)(2.67)=882$$

$$V_B=\frac{\delta_B}{\delta_{bb}}$$

Since δ_{bb} (the deflection at B due to a unit load at B) is the same as δ_2, dividing each of the point deflections above by δ_2 will give the value of the ordinate of the influence line at the corresponding point.

The ordinate at point 1 is:

$$\frac{1081}{1820} = 0.594$$

at point 2:

$$\frac{1820}{1820} = 1.00$$

at point 3:

$$\frac{1963}{1820} = 1.079 \quad \text{etc.}$$

To find the influence line for the reaction at C, use the influence line for the reaction at B to obtain values for V_B at the various points along the beam. Applying a specific value of V_B at B and a unit load at the related point (determined from the IL for V_B), the reaction at C is determined by statics. Then using the influence lines for the reactions at B and C, the reaction at A is readily found.

e.g. For point 1, place a unit load at point 1, the reaction at B, from the influence line for V_B, is 0.594. Apply this at B and take moments about point 0 and solve for V_C.

For the reaction at A, simply sum the forces in vertical direction.

Repeat this process for each of the points along the beam.

Load point 1 and sum moments about 0.

$$\circlearrowright{+} \quad 1 \times 8 - 0.594 \times 16 + 48V_C = 0$$

$$V_C = 0.031$$

Summing forces in the vertical direction:

$$+\uparrow \quad -1.00 + 0.0594 - 0.031 + V_A = 0$$

$$V_A = 0.437$$

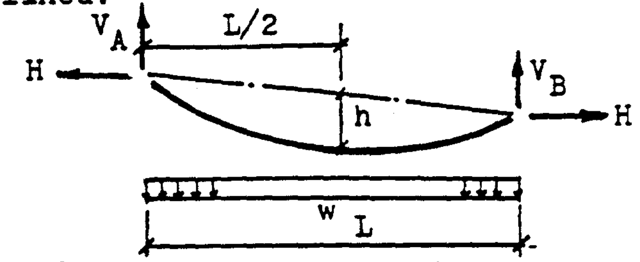
Load point 2.

$$\circlearrowright{+} \quad 1.00 \times 16 - 1.00 \times 16 + 48V_C = 0$$

$$V_C = 0 \quad \text{etc.}$$

CABLES

General Cable Theorem: At any point on a cable acted upon by vertical loads, the product of the horizontal component of cable tension and the vertical distance from that point to the cable chord equals the bending moment which would occur at that section if the loads carried by the cable were acting on an end-supported beam of the same span. The cable theorem holds whether the cable chord is horizontal or inclined.



Applying the cable theorem to the uniformly loaded cable, at mid-span

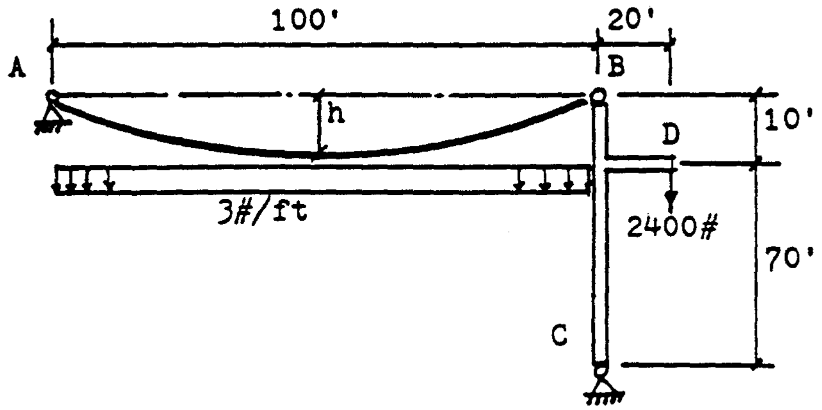
$$H \cdot h = wL^2/8$$

EXAMPLE

For the given structure:

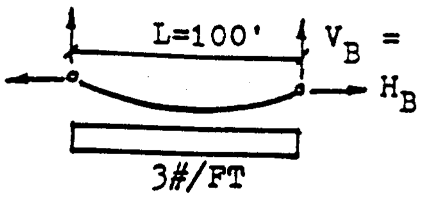
a. Find h at mid-span.

b. Determine the maximum cable tension.

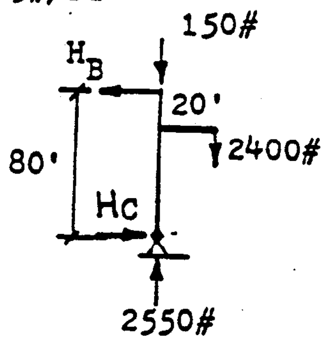


Solution:

a.



$V_B = (100 \times 3)/2 = 150\#$



$$M_C = 0 \quad (+)$$

$$(-H_B \times 80)+(20 \times 2400)=0$$

$$48000/80 = H_B = 600\#$$

$$\therefore\ h = wL^2/8H$$
$$= (3 \times 100^2)/(8 \times 600)$$
$$= 30000/(8 \times 600)$$
$$= 50/8 = 6.25' \text{ Ans.}$$

b. $$T_{max.} = \sqrt{V_B^2 + H_B^2} = \sqrt{(150)^2 + (600)^2}$$
$$= \sqrt{382500} = 618.5\# \text{ Ans.}$$

Also, using the expression for T_{max} developed in strength of materials texts,

$$T_{max.} = H\left[1+16n^2\right]^{1/2}$$

where $n = h/L = 6.25/100 = .0625$

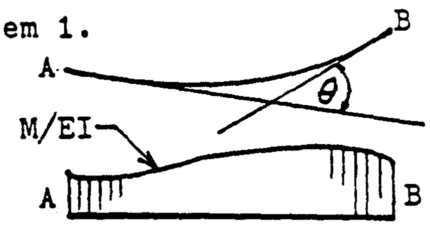
$$T_{max.} = 600\left[1+16(.0625)^2\right]^{1/2} = 618.5\#$$

DEFLECTIONS

MOMENT AREA METHOD: (Based on the following two theorems)

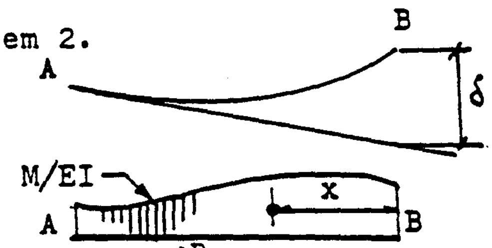
1. The change in slope of the tangents to the elastic curve between two points, A and B on the elastic curve, is equal to the area under the M/EI curve between these two points.
2. The deflection of a point B on the elastic curve from the tangent to this curve at point A is equal to the static moment about an axis through point B of the area under the M/EI curve between the points A and B.

Theorem 1.



$$\theta = \int_A^B Mdx/EI$$

Theorem 2.

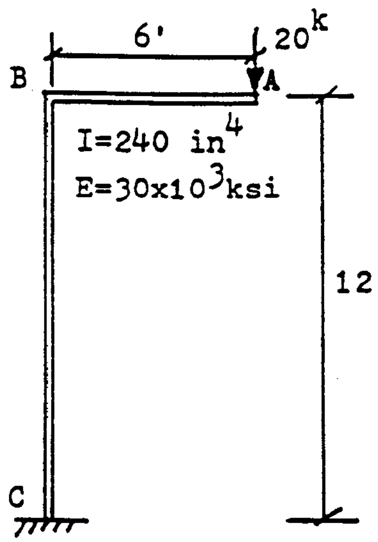


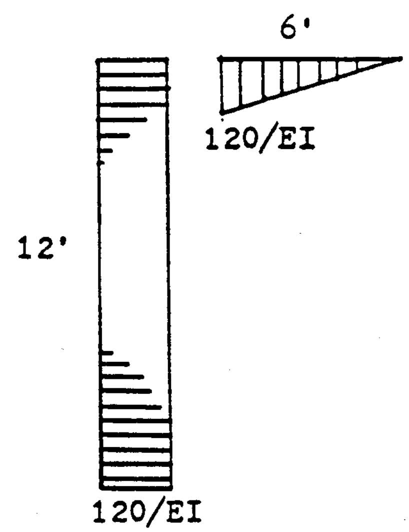
$$\delta = \int_A^B Mxdx/EI$$

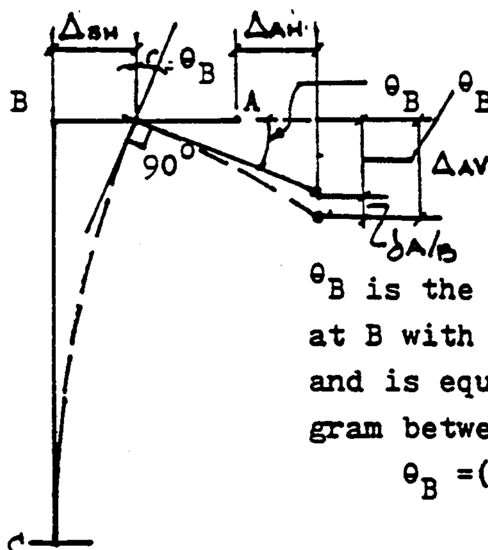
The areas and the moments of the areas of the bending moment diagrams may be found by integration, but in most cases the moment diagram may be broken up into a series of relativly simple geometric shapes. Knowing the areas and centroids of these shapes, the moments of the areas equal the centroidal distances from the axis of moments multiplied by the respective areas.

EXAMPLE

Find the vertical and horizontal deflection of point A due to bending strains. (Use the moment-area method)







θ_B is the change in slope of the tangent at B with respect to the tangent at C and is equal to the area of the M/EI diagram between B and C.

$$\theta_B = (120'^k x 12')/EI = \frac{(1440 ft^2k \; x 144 in^2)}{EI \; ft^2}$$

$$\delta_{A/B} = 1/2 \; x \; \frac{120'^k}{EI} x \; 6' \; x \; 2/3 \; x \; 6' x \frac{1728 in^3}{ft^3}$$

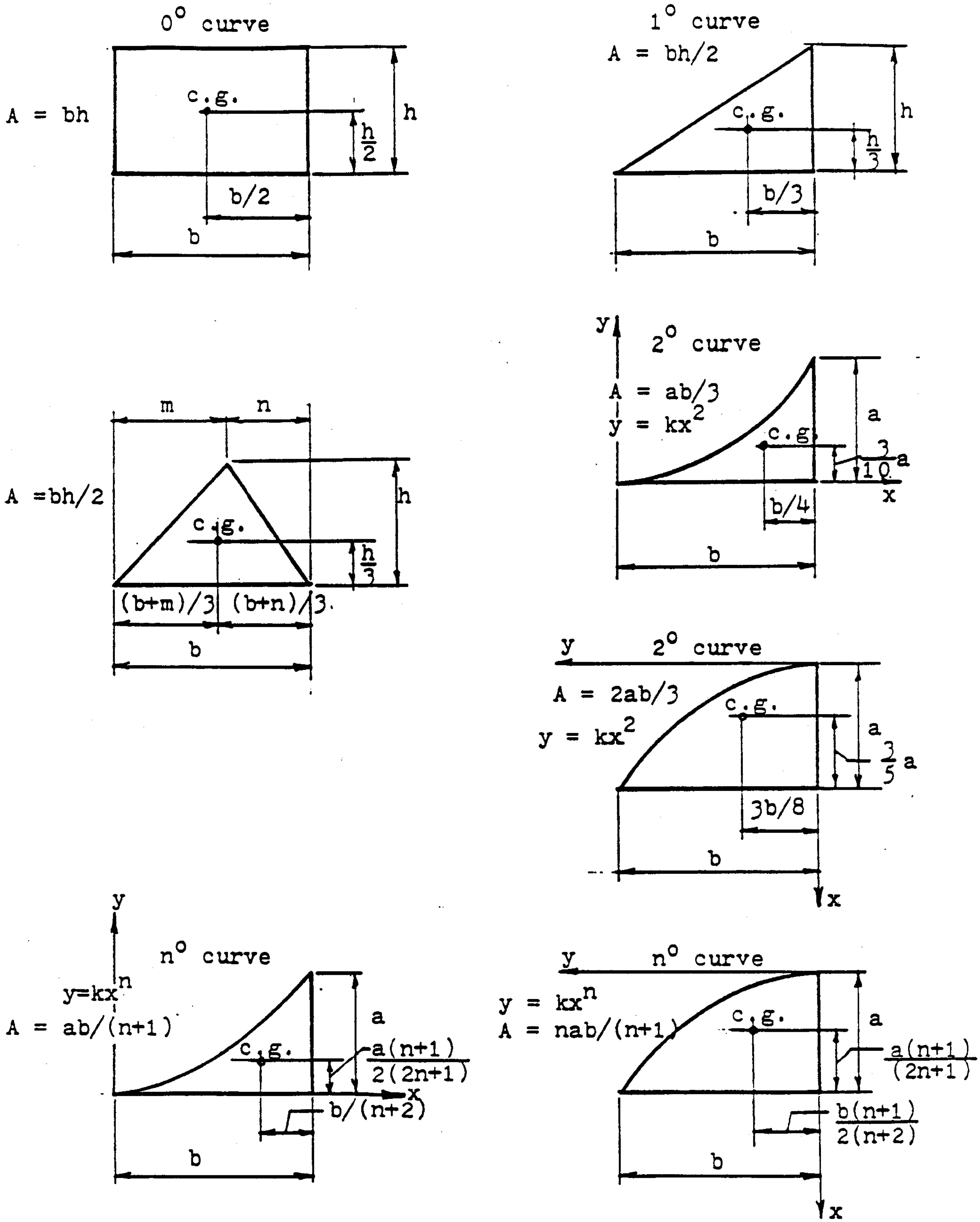
$$= \frac{1440 \; ft^3k}{EI} \; x \; \frac{1728 \; in^3}{ft^3}$$

$$\Delta_{AV} = (\theta_B \times 6') + \delta_{A/B}$$

$$\Delta_{AV} = \frac{1440}{EI} \times 144 \times 6' \times \frac{12"}{ft} + \frac{1440}{EI} \times 1728 = 2.4" \text{ Ans.}$$

$$\Delta_{BH} = \Delta_{AH} = \delta_{B/C}$$

$$\delta_{B/C} = \frac{120^{ft\ k}}{EI} \times 12' \times 6' \times \frac{1728^{in^3}}{ft^3} = 2.07" \text{ Ans.}$$



GEOMETRIC DATA FOR VARIOU

DEFLECTIONS (cont.)

LAW OF VIRTUAL WORK:

If a structure is in equilibrium under a set of forces and if the structure is given a virtual displacement consistent with the constraints of the structure, then the external virtual work done is equal to the internal virtual work done.

$$W_{(external)} = W_{(internal)}$$

The general equation is:

$$\begin{matrix} 1 \times \Delta \\ 1 \times \alpha \end{matrix} + W_R = \sum \frac{F_Q F_P L}{AE} + \int_0^L M_Q M_P \frac{ds}{EI}$$

$1 \times \Delta$ is used for deflections.

$1 \times \alpha$ is used for rotations.

W_R is the work done by the supports, this is due to Q forces only.

The first term on the right side of the equation is the work done by bar forces; that is, by axial forces.

The second term on the right is the work done by flexural forces.

Note: We have neglected the work done by shear and torsion.

F_Q = Forces in frame due to a unit load Q.

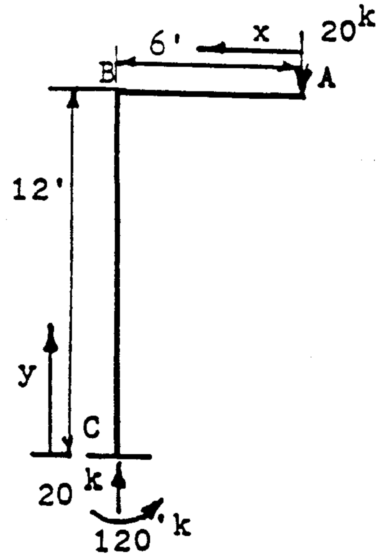
M_Q = Moment in beam due to a unit load Q.

F_P = Forces in frame due to actual real loads.

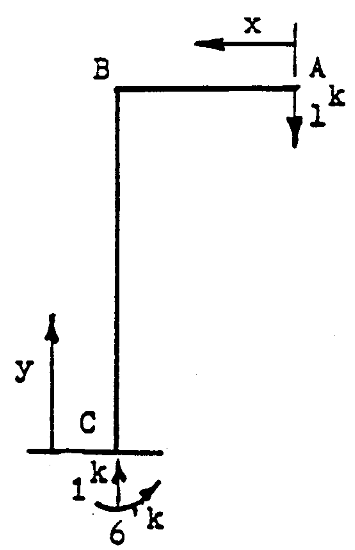
M_P = Moment in beam due to actual real loads.

EXAMPLE

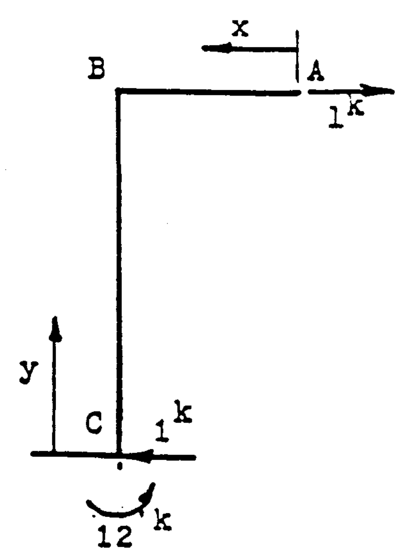
Find the vertical and horizontal deflection of point A due to bending strains. (Use the virtual work method)



P Forces



Q_V Forces



Q_H Forces

Member		M_P	M_{Q_V}	M_{Q_H}
AB	x increases A to B $0 < x < 6$	$20x$	x	0
BC	y increases C to B $0 < y < 12$	-120	-6	$-12+y$

$$Q\Delta_{AV} = \int_0^L M_Q M_P \frac{ds}{EI}$$

$$EI(1^k)\Delta_{AV} = \int_0^6 (x)(20x)dx + \int_0^{12}(-6)(-120)dy$$

$$EI\Delta_{AV} = \frac{20x^3}{3}\Big|_0^6 + 720y\Big|_0^{12}$$

$$EI\Delta_{AV} = 1440 + 8640$$

$$\Delta_{AV} = \frac{1440 + 8640}{30000 \times 240} \times 1728 = 2.4\text{" Ans.}$$

Example (Cont.)

$$Q\,\Delta_{AH} = \int_0^L M_Q M_P \frac{ds}{EI}$$

$$EI(1^k)\,\Delta_{AH} = \int_0^6 \cancelto{0}{(20x)(0)dx} + \int_0^{12} (-12+y)(-120)dy$$

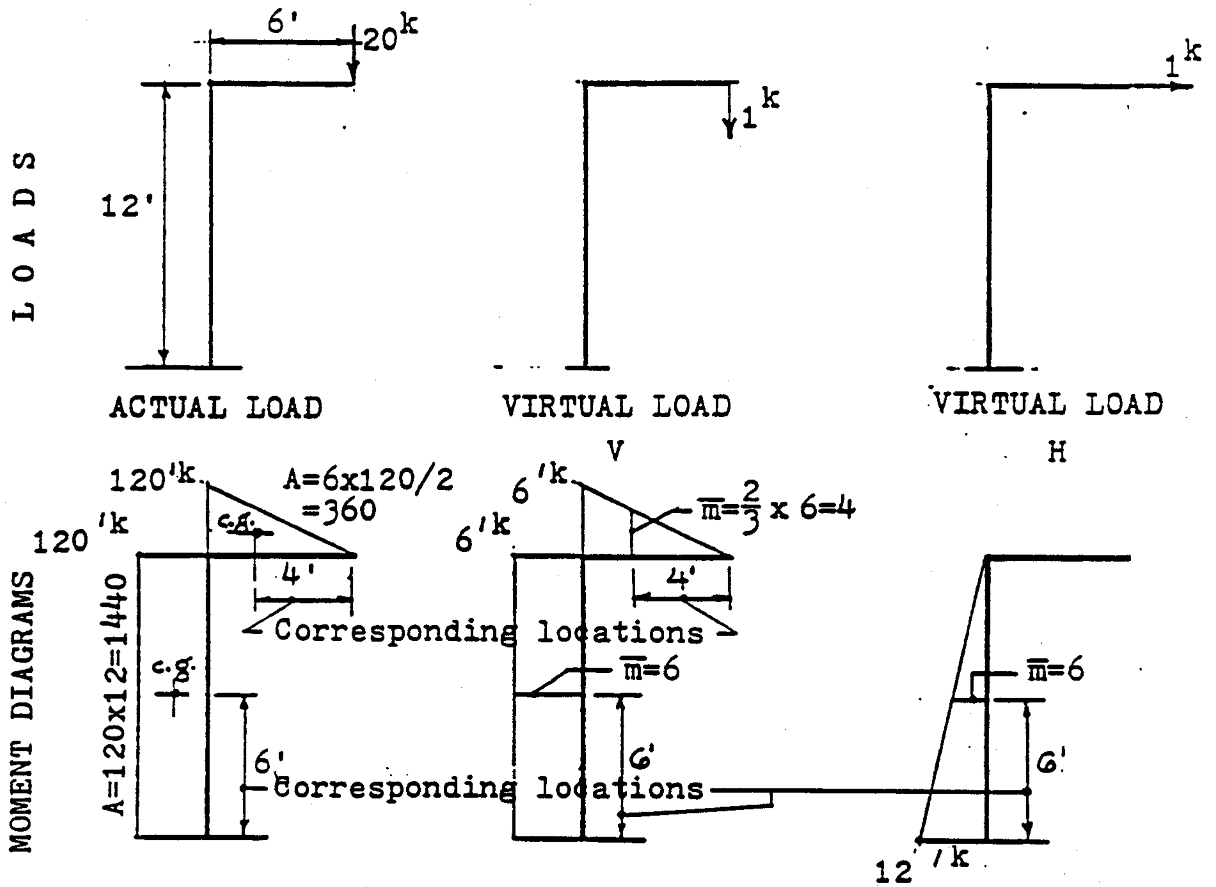
$$EI\,\Delta_{AH} = 1440y - 60y^2 \Big|_0^{12}$$

$$EI\,\Delta_{AH} = 17280 - 8640 = 8640$$

$$\Delta_{AH} = \frac{8640}{30000 \times 240} \times 1728 = 2.07\text{"} \text{ Ans.}$$

SEMI-GRAPHICAL METHOD OF INTEGRATION

A variation of the virtual work approach to this type of problem is a semi-graphical integration method. It works well for moment diagrams composed of easily defined geometrical shapes. Such is the case for this particular problem.



A_{RLM} = Areas of real load moment diagrams.

$\overline{m}$ = Ordinates taken on virtual load moment diagrams at location corresponding to centers of gravity of real load moment diagrams.

$$1 \times \Delta = \frac{A_{RLM} \times \overline{m}}{EI}$$

$\overline{m}$ for triangular moment diagram

$\overline{m}$ for rectangular mom. diag.

$$\Delta_V = \frac{360 \times 4 + 1440 \times 6}{1 \times 30 \times 10^3 \times 240} \times 1728 = 2.42''$$

$\overline{m}$ for top leg mom. diag.

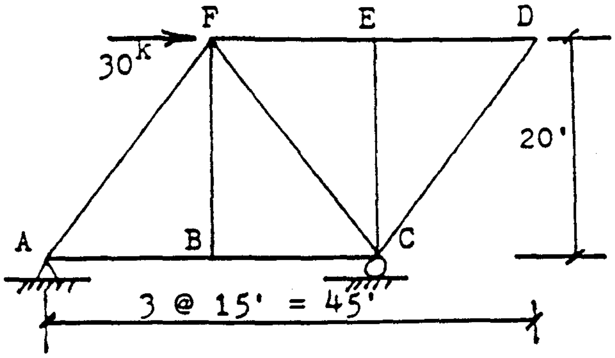
$\overline{m}$ for vert. leg mom. diag.

$$\Delta_H = \frac{360 \times 0 + 1440 \times 6}{1 \times 30 \times 10^3 \times 240} \times 1728 = 2.07''$$

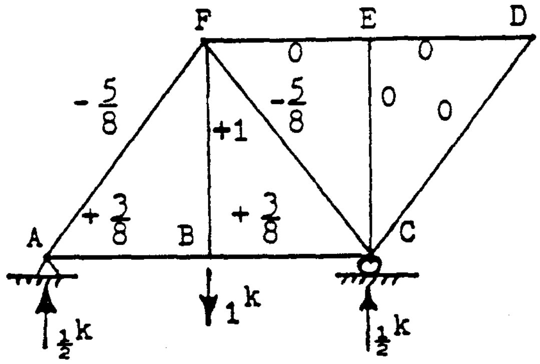
It can be seen that the graphical solution for this frame is much faster that the analytical one.

EXAMPLE

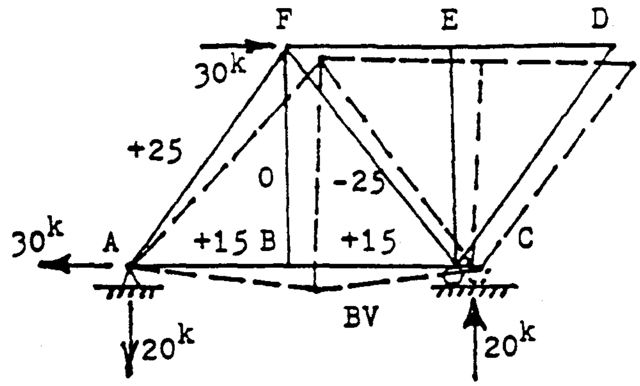
Compute the vertical deflection of joint B by virtual work.



Area of all bars = 5sq in.
$E = 30 \times 10^3$ksi
Support C settles $\frac{1}{4}$".
Bar FB fabricated 0.2" too long.



Q Forces



P Forces

$$W_e = W_i$$

$$Q\Delta_{BV} + W_R = \Sigma(F_Q \Delta L) = \frac{F_Q F_P L}{AE} + F_Q \Delta L$$

Note: The deltas on the right-hand side of the equation denote changes in length, not deflections.

	(ft)	(in²)	$(\frac{ft}{sq\ in})$	(k)	(k)	$(\frac{k^2 ft}{sq\ in})$	(in)	(in k)
	L	A	$\frac{L}{A}$	F_Q	F_P	$F_Q F_P (\frac{L}{A})$	L	$F_Q \Delta L$
AB	15	5	3	$+\frac{3}{8}$	+15	+16.875		
BC	15	5	3	$+\frac{3}{8}$	+15	+16.875		
AF	25	5	5	$-\frac{5}{8}$	+25	-78.125		
FC	25	5	5	$-\frac{5}{8}$	-25	+78.125		
FB	20	5	4	1	0	0	0.20	0.20
					$\Sigma =$	+33.750		0.20

Example (cont.)

$$(1)\,\Delta_{BV} + (0.25)(-\tfrac{1}{2}) = \frac{+33.75}{30000} \times 12 + 0.20$$

Note: The settlement at C is 0.25 inches and is positive since it is in the same direction as the assumed deflection.

The force at C due to the Q force is $\frac{1}{2}$ and it is negative because it is opposite in direction to the actual settlement.

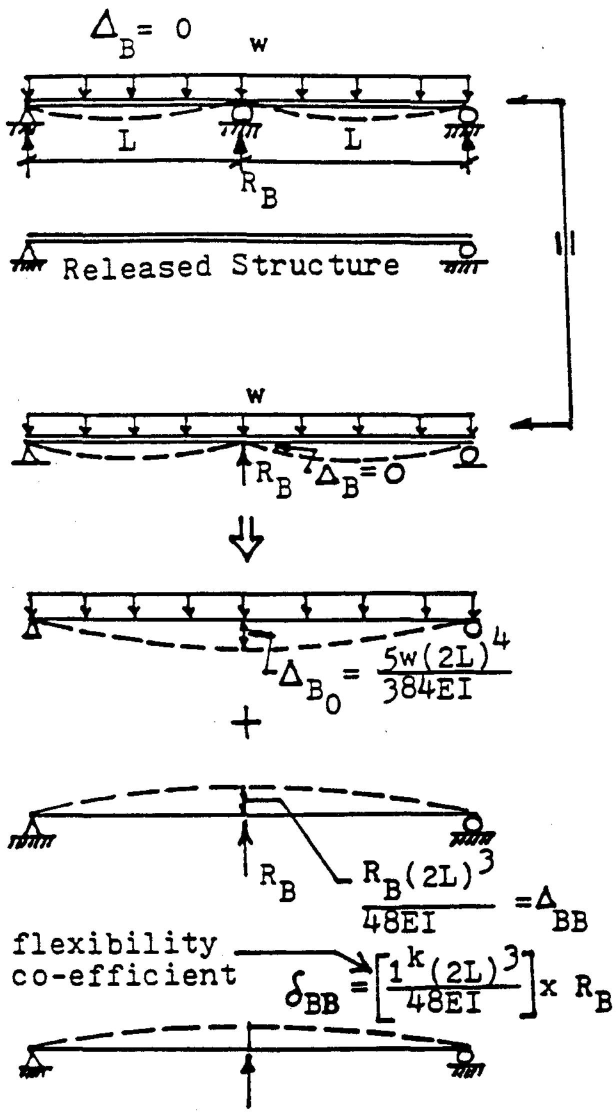
$$\Delta_{BV} - 0.125 = 0.0135 + 0.20 = 0.2135''$$

$$\therefore\ \Delta_{BV} = 0.2135 + 0.125 = 0.339''\ \text{Ans.}$$

INDETERMINATE ANALYSIS

FLEXIBILITY METHOD (Also called consistent deformation method)

Outline of analysis



1. Establish the degree of indeterminancy.
   4 react. - 3 eq. stat.=1°
2. Establish a RELEASED struct. (determinate and stable)
3. Equivalent to original structure in all respects.
4. Superimpose the displ's produced by the original load and that by the redundant R_B.

The deflection due to the redundant R_B may also be expressed by using flexibility coefficients.

SIGN CONVENTION: Forces and displacements in the direction of the redundant are positive.

Outline of analysis (cont.)

Compatibility or geometry requires ......

$\Delta_B = 0$

$\Delta_B = \Delta_{BO} + \Delta_{BB}$

$$0 = \frac{-5w(2L)^4}{384EI} + \frac{R_B(2L)^3}{48EI}$$

$R_B = 1.25$ wL Ans.

$\Delta_B = \Delta_{BO} + \delta_{BB} \cdot R_B$

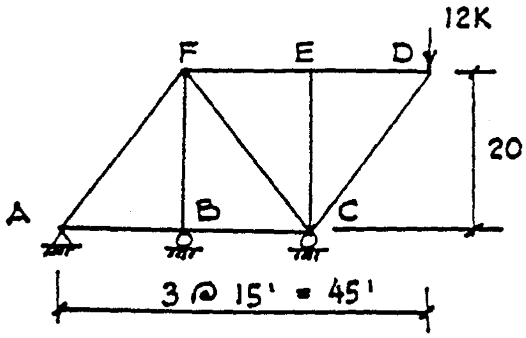
$$0 = \frac{-5w(2L)^4}{384\ EI} + \frac{R_B(2L)^3}{48\ EI}$$

$R_B = 1.25$ wL Ans.

Analysis of an indeterminate truss by Flexibility Method.

Determine the reactions for the given truss.

Area of all members = 5 sq in. E = 30,000 ksi

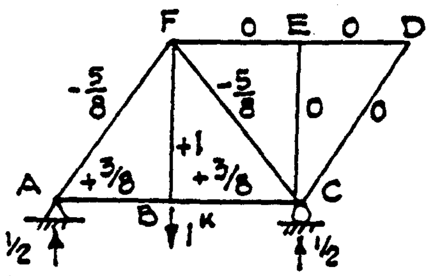


Use the reaction at B as the redundant.

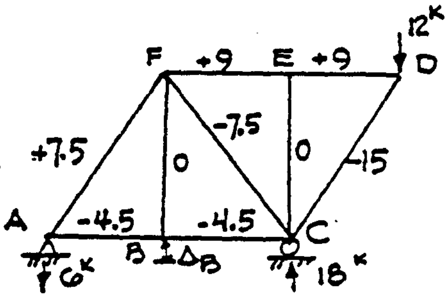
$$\Delta_B = 0 = \Delta_{BO} + \delta_{BB} R_B$$

Compute Δ_{BO} (The deflection of B in the released structure)

Q System

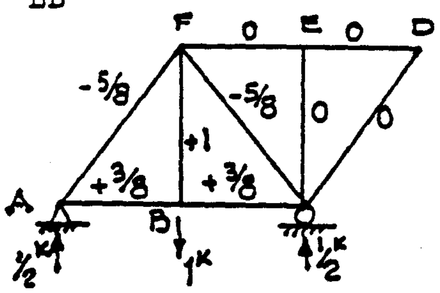


P System

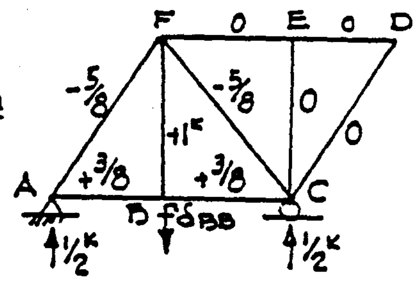


Compute δ_{BB} (The vertical defl. at B due to a unit load at B)

Q System



P System



ΔBO $\quad 1^k \Delta_{BO} = \sum F_Q \dfrac{F_P L}{AE}$

δBB $\quad 1^k \delta_{BB} = \sum F_Q \dfrac{F_Q L}{AE}$

Bar	L	F_Q	F_QF_QL	F_P	F_QF_PL
AB	15	$+\frac{3}{8}$	+ 2.11	- 4.5	- 25.31
BC	15	$+\frac{3}{8}$	+ 2.11	- 4.5	- 25.31
CD	25	0	0	-15	0
DE	15	0	0	+ 9	0
EF	15	0	0	+ 9	0
FA	25	$-\frac{5}{8}$	+ 9.77	+ 7.5	-117.19
FB	20	+ 1	+20	0	0
FC	25	$-\frac{5}{8}$	+ 9.77	- 7.5	+117.19
EC	20	0	0	0	0
		$\sum F_QF_QL=$	+43.76	$\sum F_QF_PL=$	- 50.63

$$\Delta_B = 0 = \Delta_{BO} + \delta_{BB}R_B$$

$$0 = -\frac{50.63}{AE} + \frac{43.76}{AE}R_B$$

$$AE \neq 0; \therefore -50.63 + 43.76R_B = 0$$

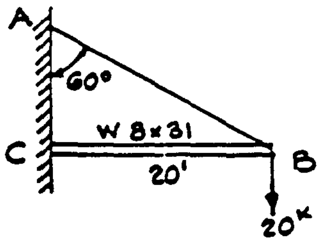
$$R_B = \frac{50.63}{43.76} = 1.16^k$$

R_B acts in the direction of the 1^k load, i.e. $\downarrow$

EXAMPLE

Determine the maximum stress in the beam and in the cable.

(Neglect the weight of the beam and the cable)



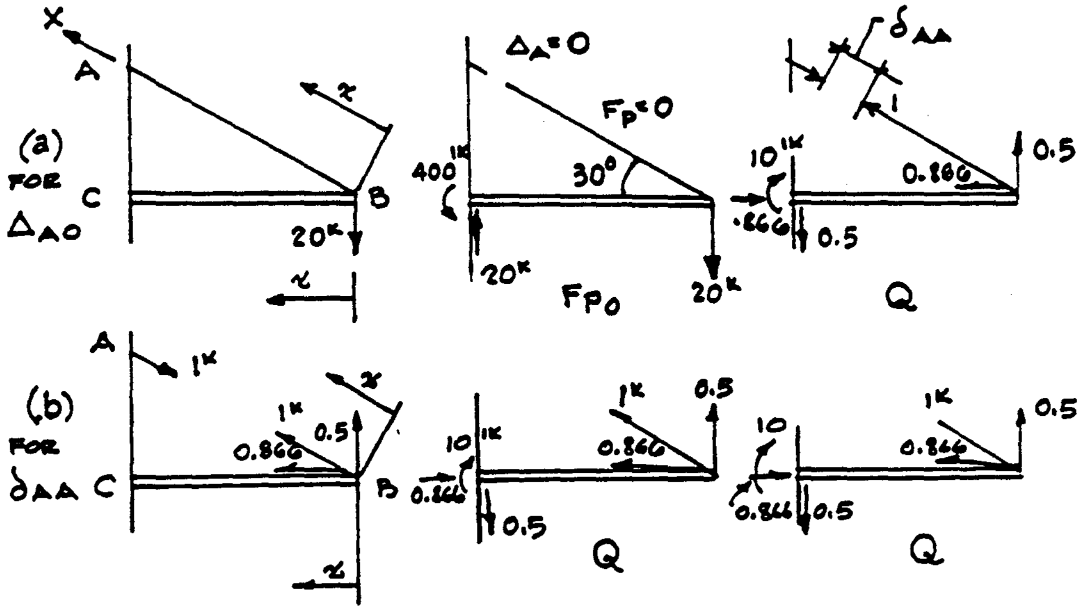
Area of the cable = 0.5 sq in

E of the cable = 22×10^3 ksi

W8x31 (AISC Manual 8th Ed.)

A = 9.12 sq in; I = 110 in^4;

S = 27.4 in^3; E = 30×10^3 ksi



Compatability Equation; $\Delta_A = 0 = \Delta_{AO} + X(\delta_{AA})$

By virtual work; for Δ_{AO} consider (a).

$$1^k \Delta_{AO} = \sum F_Q \frac{F_P L}{AE} + \sum \int_0^L M_Q M_P \frac{ds}{EI}$$

From B to C	L = 20'	$0 < x < 20$
	$F_P = 0$	$M_P = 20x$
	$F_Q = 0.866$	$M_Q = -(0.5)x$
From B to A	L = 23.09'	$0 < x < 23.09$
	$F_P = 0$	$M_P = 0$
	$F_Q = 1$	$M_Q = 0$

Example (cont.)

$$1^k\Delta_{AO} = 0 + \int_0^{20} -(0.5x)(20x)dx\ \frac{1728}{EI}$$

$$= -10\ \frac{x^3}{3}\Big]_0^{20} \left(\frac{1728}{30\ \text{x}1000\ \text{x}\ 110}\right)$$

$$\Delta_{AO} = -13.96"$$

For δ_{AA} consider (b).

$$1^k\delta_{AA} = \sum F_Q{}^2\ \frac{L}{AE} + \sum\int_0^L M_Q{}^2\ \frac{ds}{EI}$$

From B to C	$L = 20'$	$0 < x \leqslant 20$
	$F_P = -\ 0.866$	$M_P = -(0.5x)$
	$F_Q = -\ 0.866$	$M_Q = -(0.5x)$
From B to A	$L = 23.09'$	$0 < x \leqslant 23.09$
	$F_P = 1$	$M_P = 0$
	$F_Q = 1$	$M_Q = 0$

$$1^k\delta_{AA} = \frac{(-0.866)(-0.866)(20)(12)}{(9.12)(30\ \text{x}\ 10^3)} + \frac{(1)(1)(23.09)(12)}{(0.5)(22\ \text{x}\ 10^3)}$$

$$+ \int_0^{20} \frac{(0.5x)^2 dx}{(30\ \text{x}\ 10^3)(110)}\ \text{x}\ (1728)$$

$$= .0007 + .0252 + \left(\frac{.25\ x^3}{3}\right]_0^{20} \frac{1728}{(30\ \text{x}\ 1000\ \text{x}\ 110)}$$

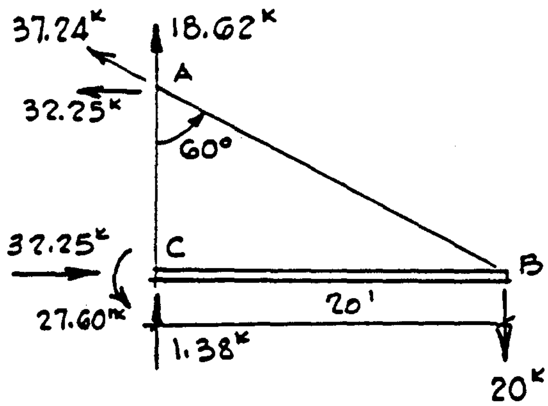
$$\delta_{AA} = .0007 + .0252 + .3491 = 0.375"$$

$$\therefore\ \Delta_A = \Delta_{AO} + X\,\delta_{AA} = 0$$

$$-\ 13.96 + 0.375\ X = 0$$

$$X = \frac{13.96}{0.375} = 37.24^k\ \text{Ans.}$$

Example (cont.)



Components of the cable tension:

$$A_V = 37.24 \sin 30^\circ = 37.24 \times 0.5 = 18.62^k$$

$$A_H = 37.24 \cos 30^\circ = 37.24 \times 0.866 = 32.25^k$$

Then for the whole structure;

$$+\uparrow F_y = 18.62 - 20 + C_V = 0$$

$$C_V = 1.38^k \uparrow$$

$$\overset{+}{\rightarrow} F_x = -32.25 + C_H = 0$$

$$C_H = 32.25^k \rightarrow$$

$$(+) M_C = (20 - 18.62)(20) = 27.60'^k$$

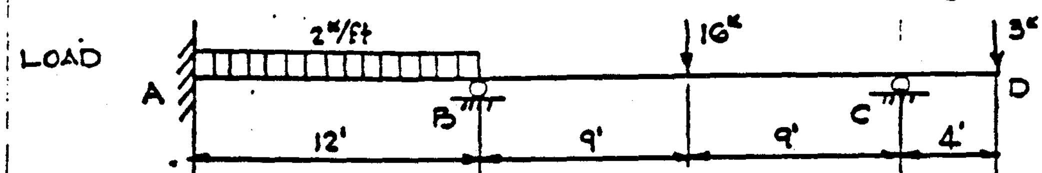
The stress in the beam due to bending $= \frac{M}{S} = \frac{(27.60)(12)}{27.4} = \pm 12.09$ ksi

The direct stress in the beam $= \frac{P}{A} = \frac{-32.25}{9.12} = -3.54$ ksi

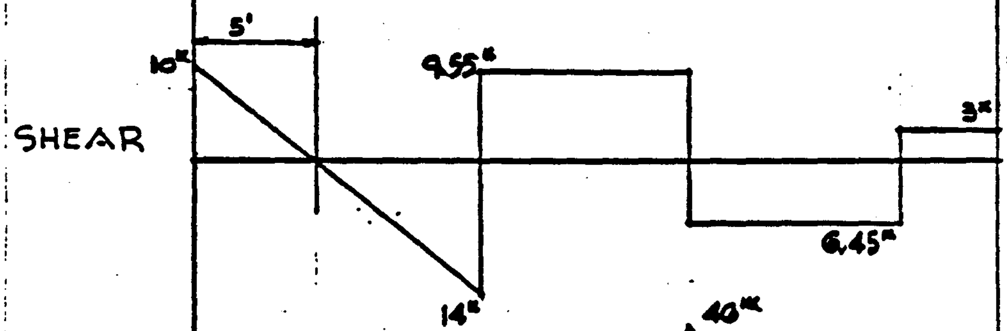
∴ The Max. stress in the beam is:

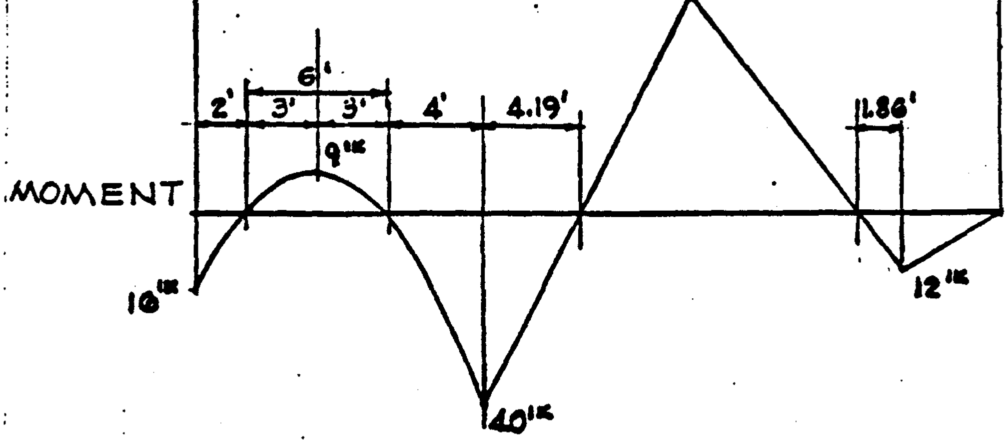
$$\frac{P}{A} \pm \frac{M}{S} = -3.54 \pm 12.09 = \begin{cases} 15.63 \text{ ksi compr.} & \text{Ans.} \\ 8.55 \text{ ksi tension} & \text{Ans.} \end{cases}$$

Draw the Shear and Moment curves for the beam below. Analyse by Moment Distribution.



	A	B		C	D
$K = \frac{I}{L}$	1/12		Modified $k = 3/4 \times 1/18 = 1/24$		0
d.f. $= \frac{K}{\Sigma K}$	0	.67	.33	1	0
FEM	-24	+24	-54		-12
dist.	—	+20	+10	+12	
c.o.	+10	–	+6		
dist.	—	-4	-2		
c.o.	-2				
dist.	—				
Σ-M	-16	+40	-40	+12	-12
V simple	12	12	8	8	3
V mom.	2	2	1.55	1.55	—
V adj.	10	14	9.55	6.45	3
React.	10	23.55		9.45	
V=0	5'→	←7'	9'→	←9'	





I = constant = 1

Sign convention

$+ \curvearrowleft \square \curvearrowright +$

$FEM_{AB} = -FEM_{BA}$

$-\frac{wL^2}{12} = -\frac{2 \times 12^2}{12} = -2$

AISC p 2-114 ff

$FEM_{BC} = -\frac{3PL}{16}$

$= -\frac{3}{16} \times 16 \times 18 = -54$

$FEM_{CB} = 0$

$FEM_{CD} = -PL$

$= -3 \times 4 = -12^{'k}$

+M (SPAN AB) = FROM END A

$\frac{1}{2}(10)(5) - 16 = 9^{'k}$

CHECK FROM END B

$\frac{1}{2}(14)(7) - 40 = 9^{'k}$ ✓

+M (SPAN BC) = FROM END B

$9.55 \times 9 - 40 = 46'$

CHECK FROM END C

$6.45 \times 9 - 12 = 46$

PTS. OF INFLECTI

SPAN AB

LENGTH OF +M SP

$L = \sqrt{\frac{8M}{w}}$

$L = \sqrt{\frac{8 \times 9}{2}} = 6'$

∴ PTS. OF INFL. A

$5 - 3 = 2'$; $7 - 3 = 4$

SPAN BC

$PI_{LEFT} = \frac{40}{86} \times 9 = 4.19'$

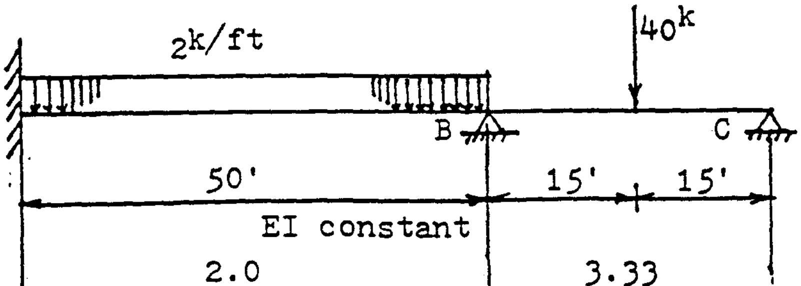
$PI_{RIGHT} = \frac{12}{58} \times 9 = 1.86$

MODIFIED STIFFNESS FACTORS FOR VARIOUS END CONDITIONS

1. One end simply supported: $K' = 3/4(K)$
2. One end symmetrical to other end: $K' = 1/2(K)$
3. One end antisymmetrical to other end: $K' = 3/2(K)$

e.g.

Case 1.



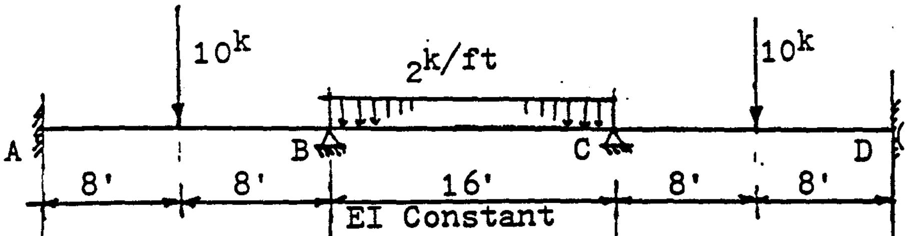
$$-FEM_{AB} = +FEM_{BA} = \frac{2 \times 50^2}{12} = 416.7'k$$

$$FEM_{BC} = -3/16(40 \times 30) = -225.0'k$$

(Diag. 13 p.2-118, AISC Manual 8th ed.)

	A		B (AB)	B (BC)	
K		2.0			3.33
K'		2.0		3/4(3.33)=2.50	
$D_f = \frac{K'}{\Sigma K'}$		2.0/4.5 =	0.44	0.56= 2.5/4.5	
FEM	-416.7		+416.7	-225.0	
d1			- 84.3	-107.4	
c.o.	- 42.2				
-M	-458.9		+332.4	-332.4	

Case 2.



$$FEM_{AB} = -\frac{10 \times 16}{8} = -20'k$$

$$FEM_{BA} = -FEM_{AB} = +20'k$$

(Diag. 16 p.2-119)

$$FEM_{BC} = -\frac{2 \times 16^2}{12} = -42.6'k$$

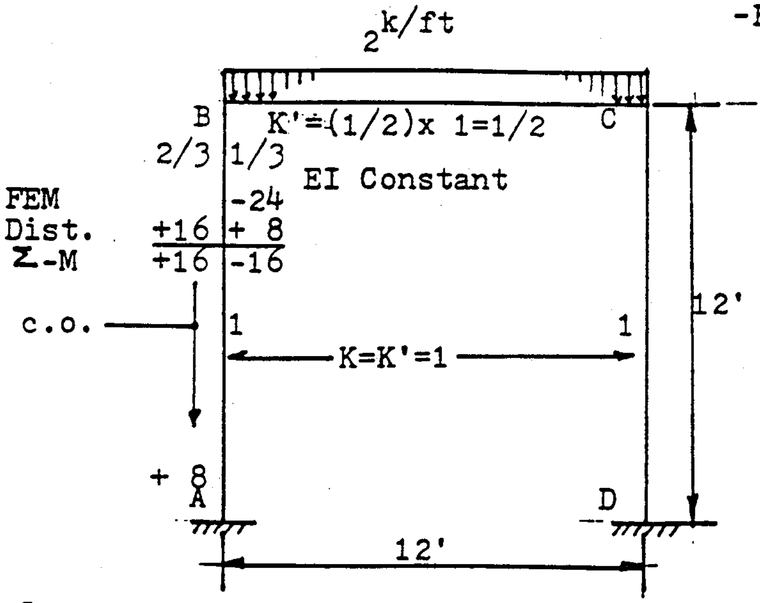
(Diag.15 p.2-119)

	A		B (BA)	B (BC)	C (CB)	C (CD)	D
K		1		1		1	
K'		1		1 x (1/2)=1/2		1	
$K' / \Sigma K'$	0		2/3	1/3			
FEM	-20.0		+20.0	-42.6			
Dist.			+15.1	+ 7.5			
c.o.	+ 7.5						
-M	-12.5		+35.1	-35.1	+35.1	-35.1	+12.5

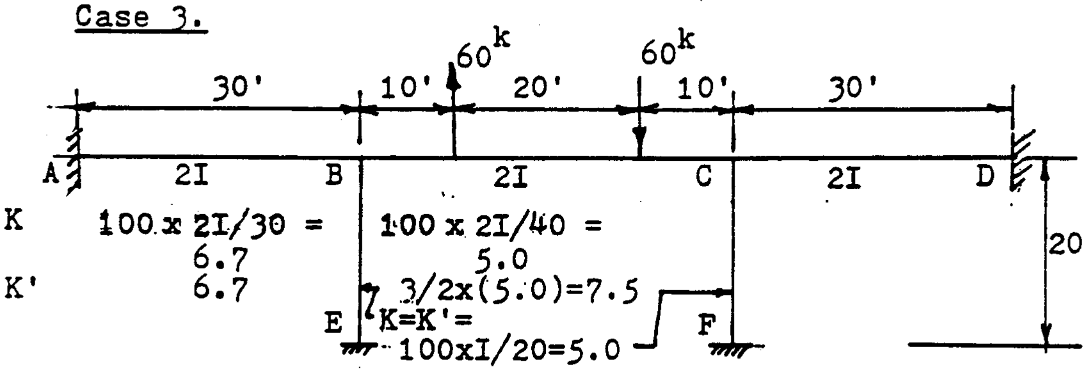
Case 2.

$$-FEM_{BC} = +FEM_{CB} = \frac{2 \times 12^2}{12} = 24'k$$

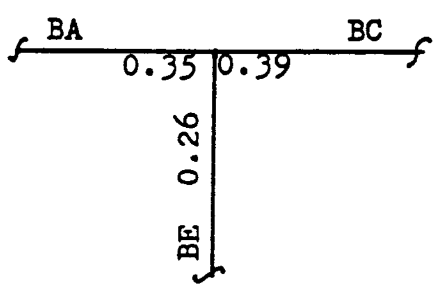
(Diag. 15 p.2-119)

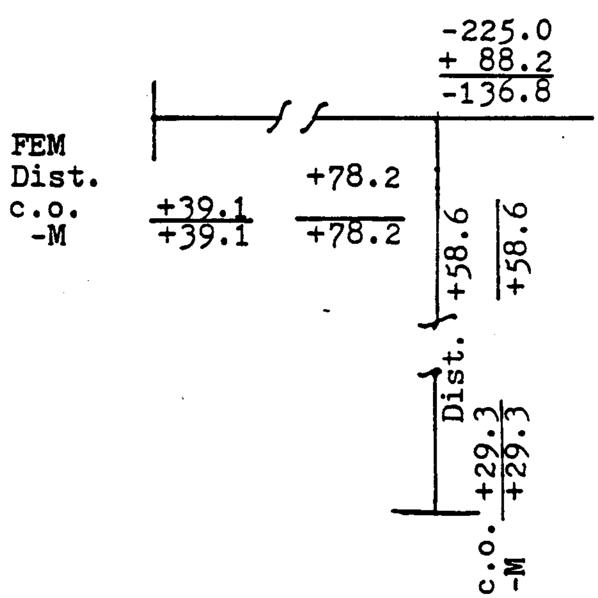


Case 3.

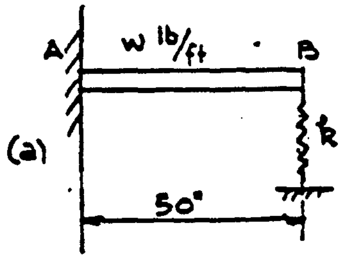


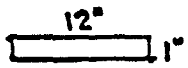
Dist. Factors





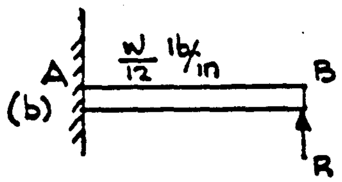
A spring placed under a cantilever beam as shown limits the deflection to one-half the value obtained if the beam could deflect freely. The beam is made of steel and is 12 inches wide by 1" deep. Find the spring constant.
Steel, E = 30,000,000 psi





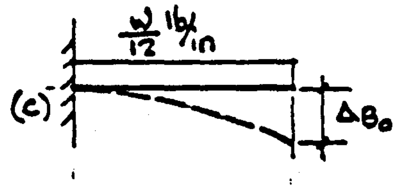
$$I = \frac{bd^3}{12} = \frac{12 \times 1^3}{12} = 1 \text{ in}^4$$

Consider the spring support as the redundant.
∴ The primary structure is as shown in (b).
The compatibility equation, based on the conditions shown in (c) and (d) and the initial deflection condition stated in the problem, is:-



$$\Delta_{Bo} - \delta_{bb} \times R = \tfrac{1}{2}\Delta_{Bo}$$

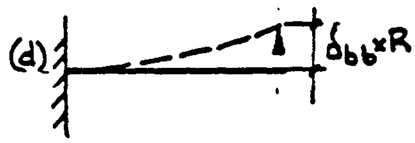
$$\therefore \delta_{bb} \times R = \tfrac{1}{2}\Delta_{Bo}$$



(1) $R = F$ (force in spring) $= \dfrac{\Delta_{Bo}}{2\delta_{bb}}$

$\Delta_{Bo} = \dfrac{wL^4}{8EI}$ AISC Tables p 2-120 No. 19

δ_{bb} = deflection at B due to a unit load acting upward at B.



$\therefore \delta_{bb} = -\dfrac{1}{3}\dfrac{L^3}{EI}$ AISC Tables p. 2-121 No. 22

Then, from eq. (1) above

$$F = \frac{\frac{1}{2}\frac{w}{12}\frac{L^4}{8EI}}{\frac{(1)\,L^3}{3EI}} = \frac{wL}{64}$$

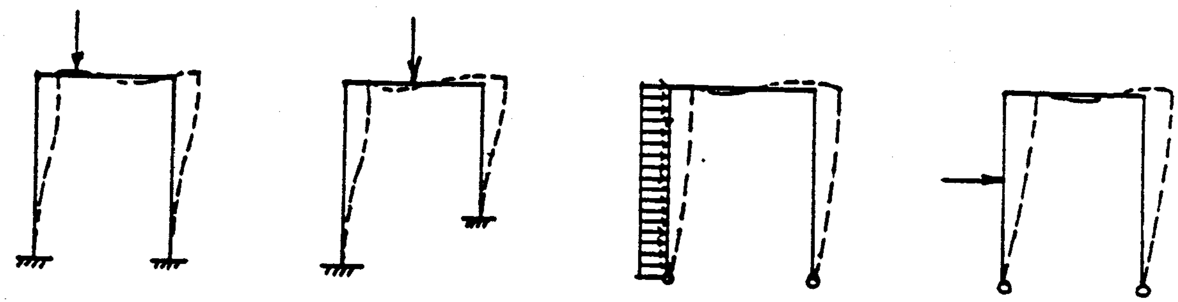
The force in the spring $= k\delta = k\tfrac{1}{2}\Delta_{Bo}$

$$\therefore \quad \frac{wL}{64} = k\,\frac{1}{2}\,\frac{w}{12}\,\frac{L^4}{8EI}$$

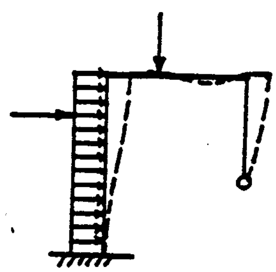
$$k = \frac{3EI}{L^3} = \frac{3 \times 30{,}000{,}000 \times 1}{50 \times 50 \times 50} = 720 \text{ lb/in}$$

FRAME SIDESWAY (By moment-distribution method)

Structural frames are subjected to sidesway because of unsymmetrical position of loads (Fig. a), unsymmetrical frame geometry (Fig. b), lateral loadings (Fig. c), or possibly a combination of these (Fig. d).



a b c



d

An analysis of these frames by an accepted method, say moment-distribution, will result in inconsistencies. When the values of the horizontal components at the supports are computed, the sum of these forces and any horizontal forces acting on the structure will not be equal to zero. This inequality, of course, means that the structure is not in equilibrium.

By using the principle of superposition it is possible to use an indirect approach to the solution of problems of this type. The method of moment-distribution lends itself readily to such a procedure.

FRAME SIDESWAY (cont.)

The general procedure for solution of problems of this type using the moment-distribution method is as follows:

1. Assume an imaginary support that prevents the structure from swaying.
2. Distribute the fixed-end moments due to the applied external loadings.
3. Compute and sum the shears at the bases of the columns. The unbalanced shear is the imaginary force required to prevent sidesway. If this force is removed the frame will undergo sidesway and the ends of the columns will rotate and induce moments at the joints.
4. Assume values for these moments. These values MUST BE such that the moments are proportional to the $\frac{I}{L^2}$ of the columns. This is especially important if the columns are of different lengths.
   Any moment may be assumed but the value should be consistent with the proportional requirements.
   It is generally easier if the magnitude is assumed to be of the same order as the moments due to the external loadings as figured in Step 2.
5. Analyse these moments and compute the shears at the bases of the columns, again you will have an unbalanced shear. Establish the ratio of

$$\frac{\text{Unbalanced shear in Step 3}}{\text{Unbalanced shear in Step 5}}$$

6. Multiply the moments in Step 5 by this ratio and add the result to the moments of Step 2. These are the correct moments
7. Figure the column base shears resulting from the final moments and isolate the frame members as free bodies. Apply all loadings, moments and shears. All members should be in equilibrium. This constitutes a final check.

EXAMPLE

Determine the end moments M_A and M_B for the given frame. (Fig. a)
Use the moment-distribution method.

Sign convention: Fixed-end Moments (+) .

$$FEM_{AB} = -FEM_{BA} = \frac{WL}{12} = \frac{(0.6)(10)(10)}{12} = -5'^k$$

$$FEM_{BC} = -FEM_{CB} = \frac{PL}{8} = \frac{20 \times 10}{8} = -25'^k$$

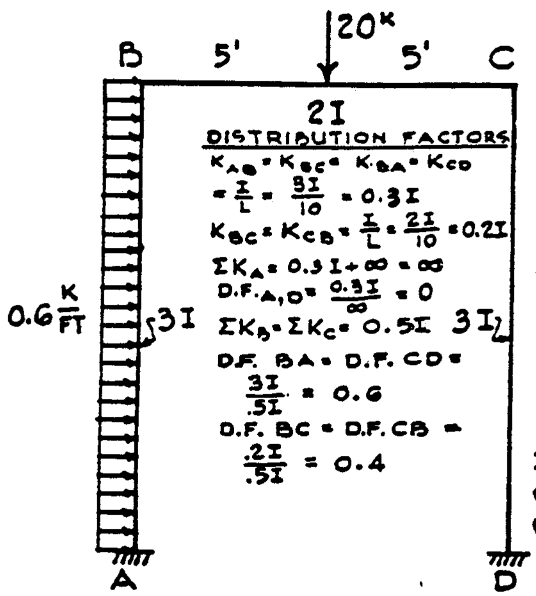


Fig. a

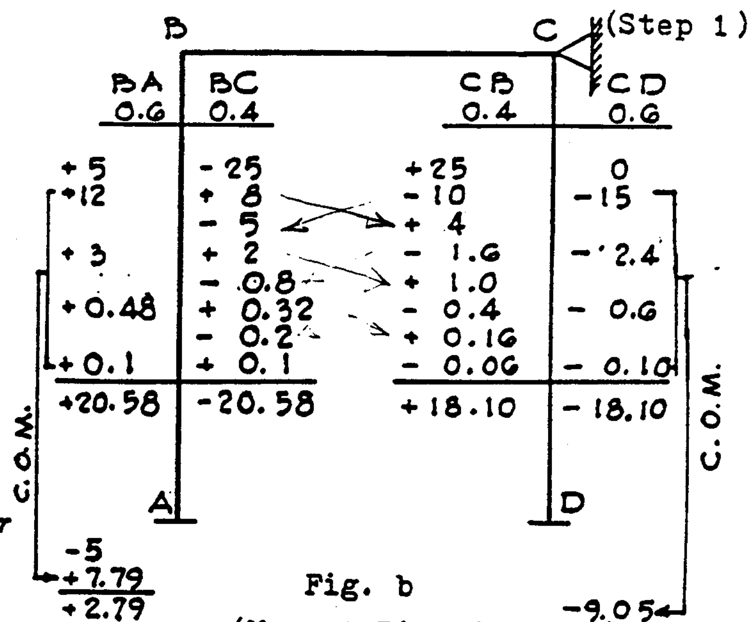


Fig. b
(Moment-Distribution)
(Step 2)

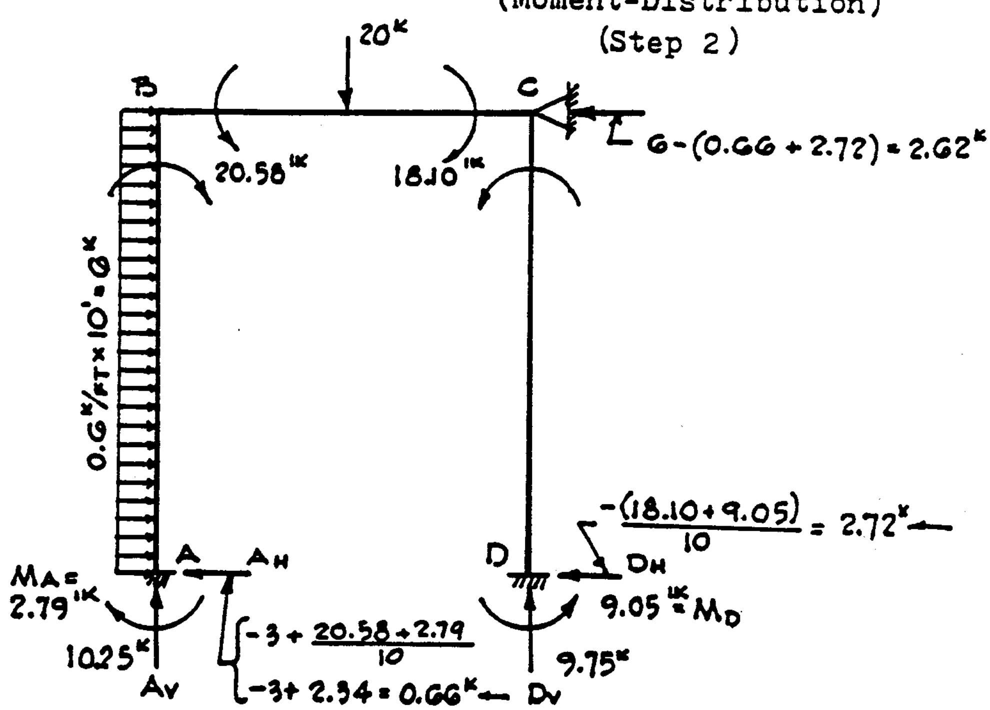


Fig. c
(Shears, moments,
and reactions)
(Step 3)

EXAMPLE (cont.)

Applying a force at A equal and opposite to the restraining force of 2.62 kips will cause the frame to deflect as shown. This will induce negative moments into the columns. Since the column inertias and lengths are the same the moments will be the same.

Assume a moment in each column equal to -100 ft-k.

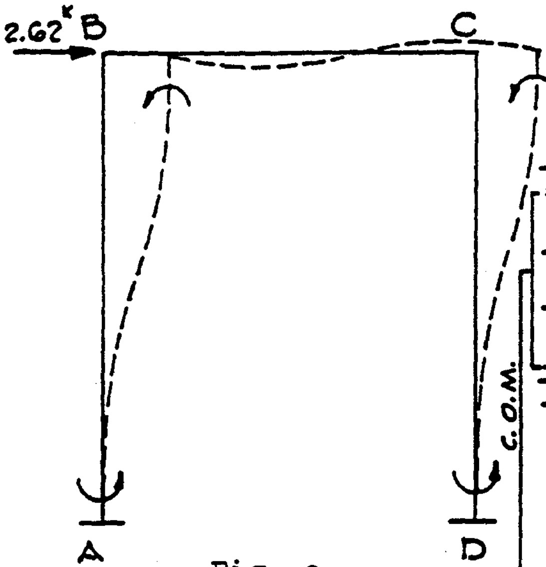


Fig. e
(Step 3)

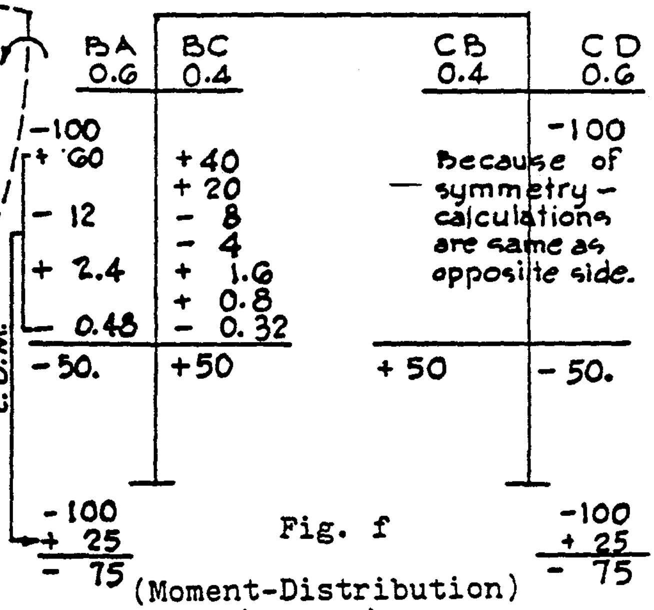
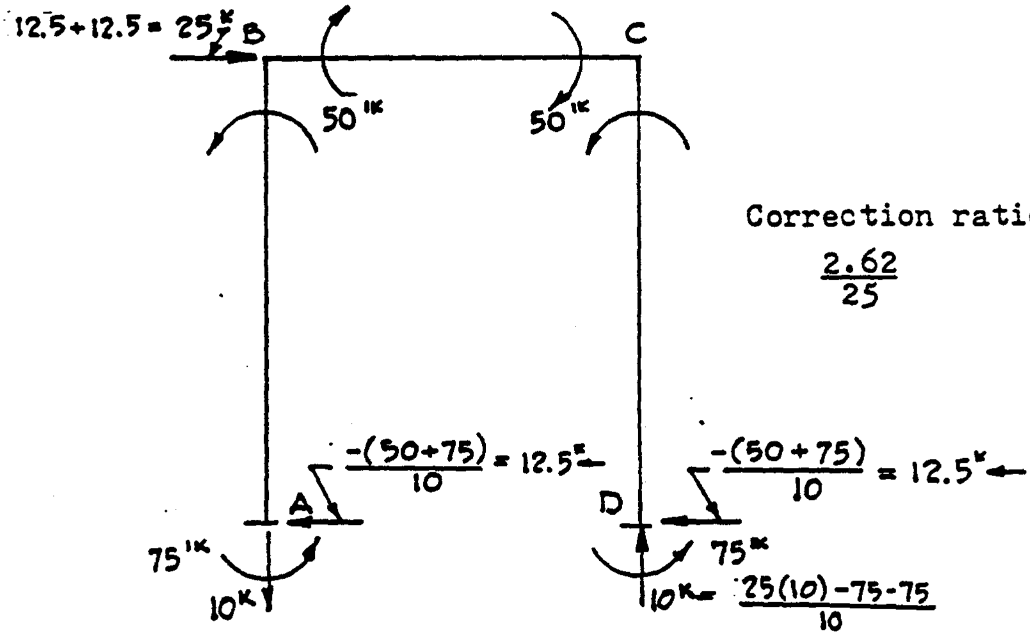


Fig. f
(Moment-Distribution)
(Step 4)



Correction ratio:

$\frac{2.62}{25}$

Fig. g
(Shears, moments and reactions)
(Step 5)

EXAMPLE (cont.)

Applying the correction factor to the assumed moments and shears will give the correct moments and shears for the restraining force.

$$25 \times \frac{2.62}{25} = 2.62^{k}$$

(a) $-50 \times \frac{2.62}{25} = -5.24'^{k}$ at the top of each column.

$-75 \times \frac{2.62}{25} = -7.86'^{k}$ at the bottom of each column.

Adding these to the moments and shears from the external loading (Step 2) gives:

(b)

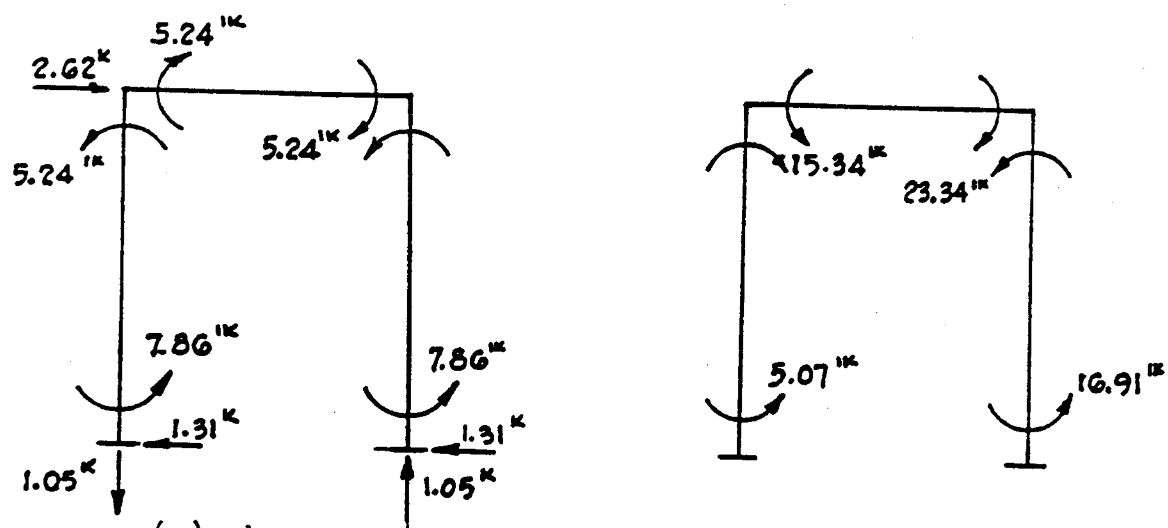
$+20.58 - 5.24 = +15.34'^{k}$ at the top of column AB.

$+ 2.79 - 7.86 = - 5.07'^{k}$ at the bottom of column AB.

$-18.10 - 5.24 = -23.34'^{k}$ at the top of column DC.

$- 9.05 - 7.86 = -16.91'^{k}$ at the bottom of column DC.

$+ 2.62 - 2.62 = 0$ thus eliminating the restraining force.

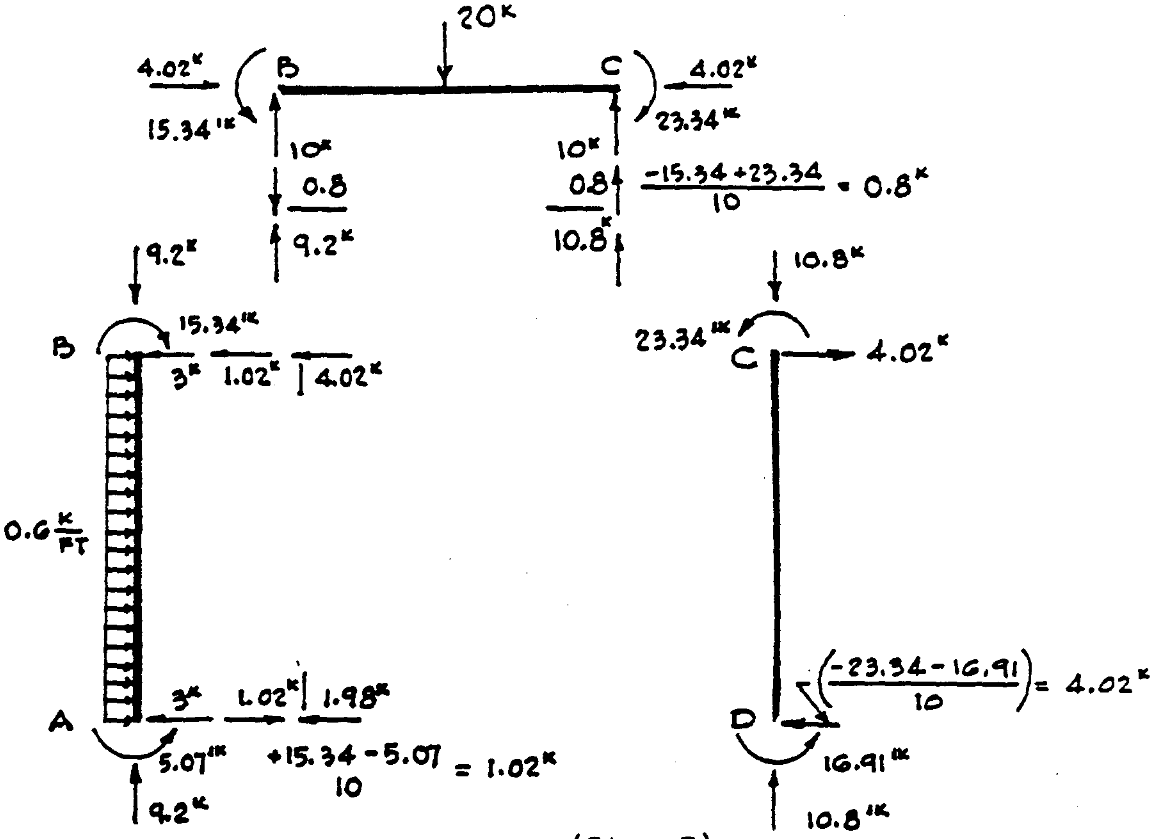


(a) above

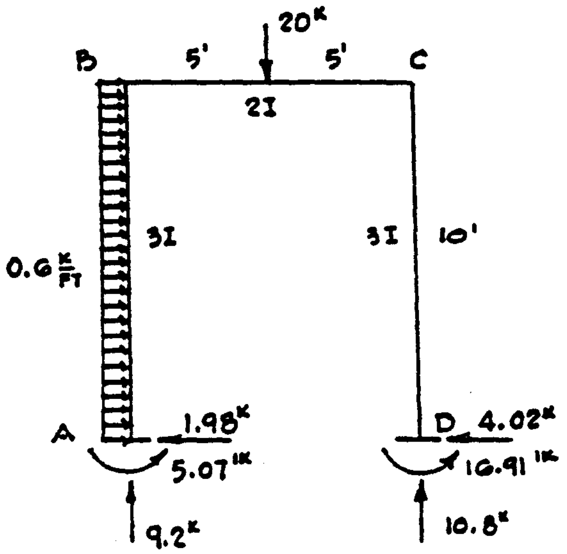
(b) above

(S t e p 6)

EXAMPLE (cont.)



Free-body Diagrams (Step 7)



Answer

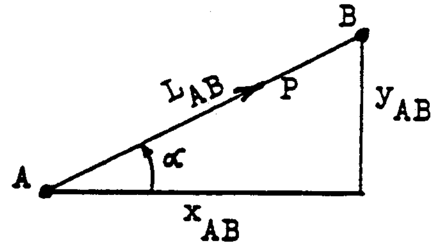
TENSION CO-EFFICIENTS

For over a half a century British engineers have used a variation of the method of joints which is easier to apply and which, for hand calculations, has no equal in solving space trusses. The procedure is called the method of tension co-efficients.

A tension co-efficient of a member is defined to be the force in the member divided by the length of the member. In using this approach the forces in the members are all considered to be tension and are therefore positive in the calculations. If the force is compression then the tension co-efficient will have a negative value.

Consider member AB:

$$t_{AB} = \frac{P_{AB}}{L_{AB}}$$



The horizontal and vertical components of the tension force P_{AB} are, $P_{AB}\cos\alpha$ and $P_{AB}\sin\alpha$ respectively. The horizontal and vertical projections of the length of the member are x_{AB} and y_{AB}.

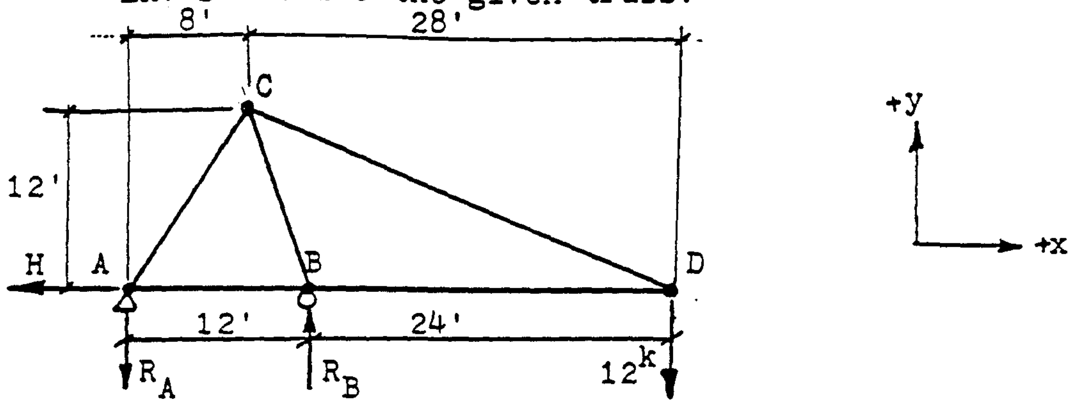
Writing the equations for horizontal and vertical equilibrium at pin A,

$$P_{AB}\cos\alpha = P\frac{x_{AB}}{L_{AB}} = t_{AB}x_{AB}$$

$$P_{AB}\sin\alpha = P\frac{y_{AB}}{L_{AB}} = t_{AB}y_{AB}$$

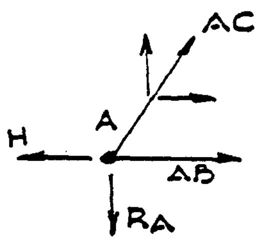
From these we can see that the equilibrium equations for ΣF_H and ΣF_V can be written in terms of tension co-efficients using the projected lengths of the members, which lengths we know, rather than sines and cosines of the angle α.

Ex. 1 Solve the given truss.



Solution:
Just as in the method of joints, each joint is isolated and equations for ΣF_x and ΣF_y are written.

Joint A $\Sigma F_x = 0$



Assuming all unknowns are tension, and summing forces in the x direction, for force AB_x we write its equivalent,

$$t_{AB}x_{AB} = t_{AB}12$$

for AC_x we write,

$$t_{AC}x_{AC} = t_{AC}8$$

and, since $\Sigma F_x = 0$;

$$12t_{AB} + 8t_{AC} - H = 0$$

The values for $12t_{AB}$ and $8t_{AC}$ are positive since they are directed in the positive x direction. The H is negative because its assumed direction is in the negative x direction. The remainder of the equations for the x and y directions at each joint are found in the same way and are recorded in Table I.

TABLE I

(1)	ΣA_x:	$12t_{AB} + 8t_{AC} - H = 0$
(2)	ΣA_y:	$12t_{AC} - R_A = 0$
(3)	ΣB_x:	$-12t_{AB} - 4t_{BC} + 24t_{BD} = 0$
(4)	ΣB_y:	$12t_{BC} + R_B = 0$
(5)	ΣC_x:	$-8t_{AC} + 4t_{BC} + 28t_{CD} = 0$
(6)	ΣC_y:	$-12t_{AC} - 12t_{BC} - 12t_{CD} = 0$
(7)	ΣD_x:	$-28t_{DC} - 24t_{BD} = 0$
(8)	ΣD_y:	$12t_{DC} - 12 = 0$

These equations are now solved for the respective values of the tension co-efficients of the various members and the values are recorded in Table II.

Example 1 (cOnt.)

The length L of each member is found from

$$L = (x^2 + y^2)^{\frac{1}{2}}$$

The length of each member is then multiplied by the respective tension co-efficient for that member and the product is the force in the member. The values of the reactions R_A, R_B, and H are found in the process of solving the equations in Table I.

TABLE II

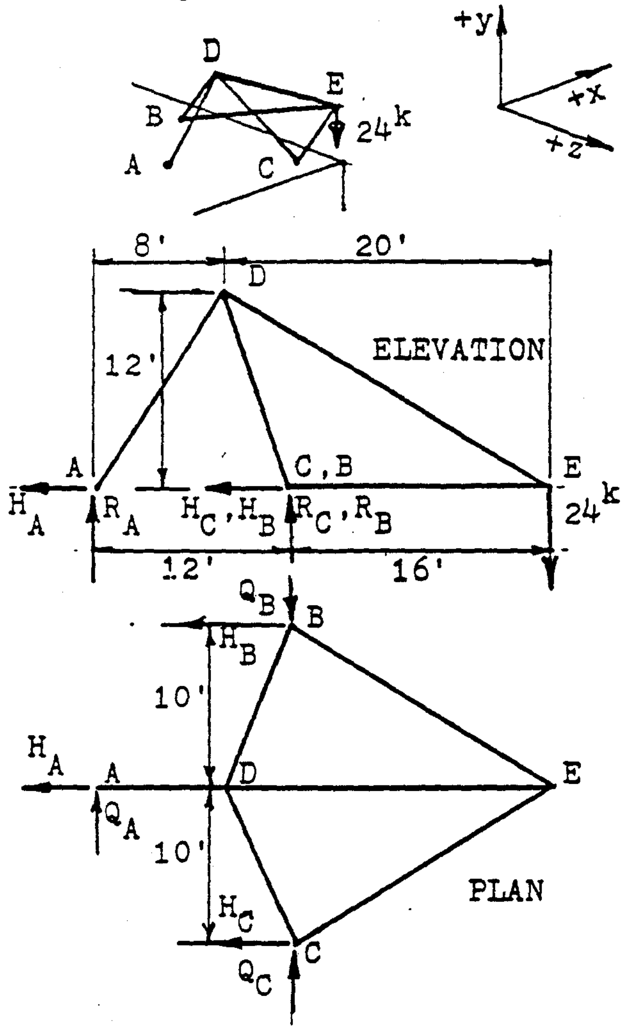
Member	Tens. cO-eff.	L	P
AB	-1.33	12.00	16.00 C
AC	2.00	14.42	28.84 T
BC	-3.00	12.65	37.95 C
BD	-1.17	24.00	28.00 C
CD	1.00	30.46	30.46 T
R_A			24 ↓
R_B			36 ↑
H			0

In solving the equations in Table I the following order was used:

(8), (7), (5) & (6) simultaneously, (4), (3), (2), and (1).

Tension co-efficients continued;

Example 2. The following tripod problem is solved in the same way.



Order of solving equations in Table I; $E_y; E_z; E_x; D_z; D_x \& D_y$ simultaneously $\therefore A_x;\ A_y$ etc. in any order.

TABLE I

A_x;	$8t_{AD} - H_A = 0$
A_y;	$12t_{AD} + R_A = 0$
A_z;	$-Q_A = 0$
B_x;	$-4t_{BD} + 16t_{BE} - H_B = 0$
B_y;	$12t_{BD} + R_B = 0$
B_z;	$10t_{BD} + 10t_{BE} + Q_B = 0$
C_x;	$-4t_{CD} + 16t_{CE} - H_C = 0$
C_y;	$12t_{CD} + R_C = 0$
C_z;	$-10t_{CD} - 10t_{CE} - Q_C = 0$
D_x;	$-8t_{DA} + 20t_{DE} + 4t_{DB} + 4t_{DC} = 0$
D_y;	$-12t_{DA} - 12t_{DC} - 12t_{DB} - 12t_{DE} = 0$
D_z;	$-10t_{DB} + 10t_{DC} = 0$
E_x;	$-16t_{EB} - 16t_{EC} - 20t_{ED} = 0$
E_y;	$12t_{ED} - 24 = 0$
E_z;	$-10t_{EB} + 10t_{EC} = 0$

The length of the frame members is obtained from;

$$L = (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

TABLE II

Member	Tens. co-eff.	Length	Force in member
AD	2.67	14.42	38.50 T
BD	-2.33	16.12	37.56 C
BE	-1.25	18.87	23.59 C
CD	-2.33	16.12	37.56 C
CE	-1.25	18.87	23.59 C
DE	2.00	23.32	46.64 T

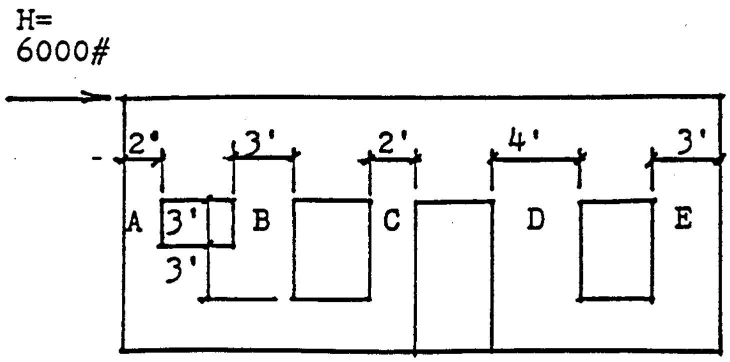
TABLE III

Reactions

H_A	21.33	←
R_A	32.00	↓
Q_A	0	
H_B	10.67	→
R_B	28.00	↑
Q_B	35.83	↓
H_C	10.67	→
R_C	28.00	↑
Q_C	35.83	↑

RIGIDITY OF MASONRY WALLS

Distribution of lateral forces to individual piers



t = wall thickness
(generally constant)

Fig. 1

H = horizontal load

P_n = force to each pier
(n = (A through E))
= design shear for each pier

M_n = design moment for each pier, where

$M_n = \frac{P_n h}{2}$ for top and bottom of fixed piers.

h = height of pier

d = width of pier

Rigidity of wall = $1/\Delta_F$ or $1/\Delta_C$

where $\Delta_F = 0.001\left[(h/d)^3 + 3(h/d)\right]$ for fixed ended piers.

$\Delta_C = 0.001\left[4(h/d)^3 + 3(h/d)\right]$ for fixed at one end only.

For example given in figure 1:

Pier	h (in.)	d (in.)	h/d	R	% of H to each pier	P (lbs)	M (in-lbs)
A	36	24	1.5	127	21.1	1266	22,788
B	36	36	1.0	250	41.4	2484	44,712
C	72	24	3.0	28	4.6	276	9,936
D	72	48	1.5	127	21.1	1266	45,576
E	72	36	2.0	71	11.8	708	25,488

$\Sigma R = 603$

DEFLECTION FORMULAS

The maximum deflection for a simply supported beam having a uniformly distributed load may be found from:

$$\Delta_{max} = \frac{180.00\ M\ L^2}{E\ I}$$

M=moment in ft-kips
L=span in feet
E=mod. of elast. in ksi
I=moment of inertia in in^4
Δ=deflection in inches

This formula is valid for beams of any material, except concrete, providing the beams have a constant cross section.

With the proper use of equivalent tabular load co-efficients, this formula provides a method of rapid calculation of maximum deflections for certain non-uniformly distributed load patterns.

To simplify the deflection formula, we may write:

$$\Delta_{max.} = M\ L^2 / k\ I$$

where k = E / 180.00.

Then k for steel and various E's for wood is as follows:

E	K
29000	161.111
1200	6.667
1300	7.222
1400	7.778
1500	8.333
1600	8.889
1700	9.444
1800	10.000
1900	10.556
2000	11.111

Note: Use one-half the values for k for dead load deflections for wood beams, since,for creep effect, the E_{creep} values are taken as E/2.

The maximum deflection for the wood design problem on page SD-30 may be found by using the formula above, with a modification to accommodate the concentrated load. The method is as follows:

The M in the formula above is the TOTAL MOMENT for a uniform load. The moment in the problem consists, by superposition, of two parts, the uniform load moment, and the concentrated load moment. By changing the concentrated load to an equivalent uniform load and adding its uniform load moment to the moment caused by the actual uniform load we would then have the total uniform load moment required for use in the deflection formula.

The concentrated load may be changed to an EUL (equivalent uniform load) by multplying it by an EUL co-efficient of 2.0 (See "f" co-eff. for n=2,on p.2-295 of AISC Manual of Steel Construction, 9th ed.); ASD; p.3-129 LRFD.

Using the total uniform load of W_{udl} = w x L, and a total EUL for the concentrated load of 2.0 x P, the moments are, for the UDL, $M=W_{udl} xL/8$, and for the concentrated load, $M_{EUL}=W_{EUL} xL/8$.

For the dead load:

UDL =0.060 k/ft	Conc. ld. = 0.600^k
$W_{udl}=0.060x18=1.080^k$	EUL = $2.0x0.600=1.2^k$
$M_{udl}=1.080x18/8=2.43^{ft-k}$	M_{EUL} = $1.2x18/8=2.70^{ft-k}$

Now, we must modify the EUL moment for the concentrated load by multiplying it by a co-efficent for deflection since the real load is a concentrated load.

The deflection co-efficient "g", from the reference above, is, 0.800.

$$\Delta_{DL(max)} = \frac{(2.43+(.800(2.70))(18)^2}{(4.444)(889.89)} = 0.38"$$

For the live load:

UDL =0.200 k/ft	Conc. ld. = 0.900^k
$W_{udl}=0.200x18=3.600^k$	EUL = $2.0x0.900=1.8^k$
$M_{udl}=3.600x18/8=8.10^{ft-k}$	M_{EUL} = $1.8x18/(8)=4.05^{ft-k}$

$$\Delta_{LL(max)} = \frac{(8.10+(.800)(4.05))(18)^2}{(8.889)(889.89)} = 0.46"$$

$$\Delta_{total} = 0.38 + 0.46 = 0.84"$$

To validate the procedure, the total uniform load moment of our modified beam loading is, by superposition:

$$M_{UDL}+M_{EUL} = (2.43+8.10)+((2.70+4.05)/1.15)$$
$$= 10.53 + 5.87 = 16.40^{ft-k}$$

which is exactly the maximum moment for the "with snow" condition on p. SD-30.

S T R U C T U R A L

D E S I G N

BEAM DESIGN by AISC Specs.

Reference: Manual of Steel Construction, American Institute of Steel Construction, Ninth Edition, ASD. Chapter F.

1. Basic Equation:

$$S = \frac{M}{F_b}$$

S = Section Modulus = I/c

M = Design Moment

F_b = Allowable bending stress

2. F_b (allowable bending stress)

The allowable value of F_b is a function of bending strength, which may be limited by plate buckling or lateral-torsional buckling (LTB).

2.1 Values of F_b are discussed below:

a. Compact Section

$$F_b = .66\ F_y,\ \text{but } F_y \le 65\ \text{ksi}$$

The cross-section can undergo plastification without local plate buckling or without LTB, (See Section B5.1., ASD Manual p.5-35). Generally, hot-rolled shapes of F_y = 36 ksi steel are compact except for W6x15.

b. Semi-compact Section

$$.66\ F_y > F_b > .6\ F_y$$

$$F_b = F_y\left(.79 - .002\frac{b_f}{2t_f}\sqrt{F_y}\right)$$

See Section F1.2 & Formula F1-3, p.5-46 ASD Manual.

All properties and bracing requirements are identical to those of a compact section except for the compression flange width to thickness ratio which is:

$$\frac{65}{\sqrt{F_y}} < \frac{b_f}{2t_f} < \frac{95.0}{\sqrt{F_y}}$$

See Sect. B5.1, p. 5-35 & Table B5-1, p.5-36, ASDM

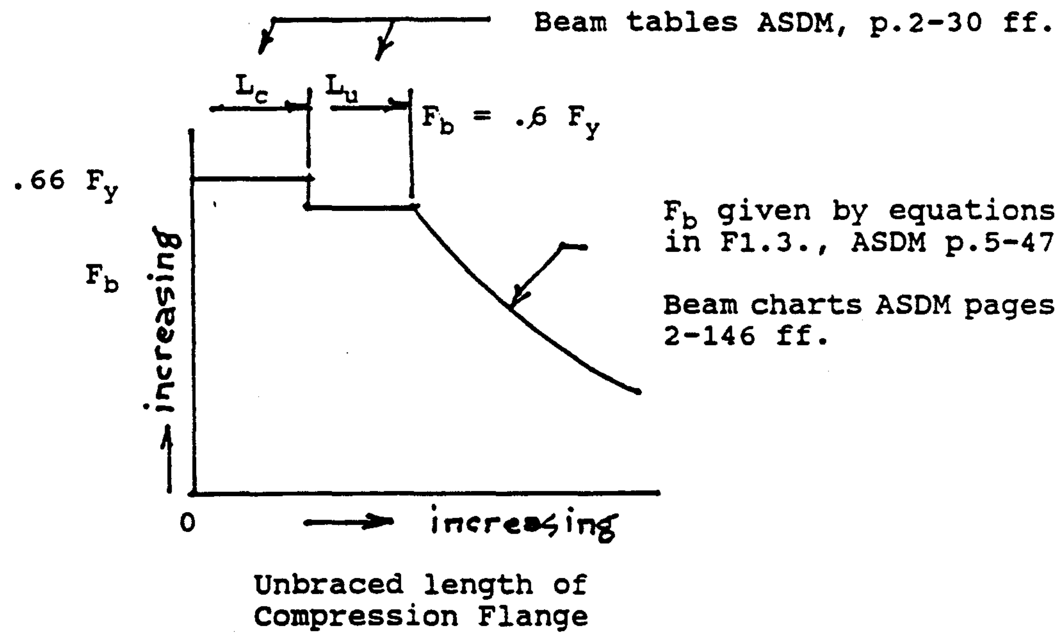
c. $F_b < .6F_y$ <u>LTB controls</u>

When the unbraced lenth of the compression flange is large, equations, F1-6,7,and 8, in Sect. F1.3., p.-47, ASDM, are used to determine the allowable bending stress, F_b. A simplified method is presented after this discussion.

d. If the compression flange width to thickness ratio is large, plate buckling may control F_b. In this case:

$F_b = .6F_yQ_s$ Equations for computing Q_s for slender compression members are given in Appendix B, page 5-98, ASDM. Normally, bending members in which Q_s must be calculated would be inefficient for use in structures.

The variation of allowable bending stress as a function of the unbraced length of the compression flange is shown below for a W16 x 36 of A-36 steel.



VARIATION OF F_b WITH UNBRACED LENGTH OF COMPRESSION FLANGE

<u>SIMPLIFIED APPROACH TO AISC BENDING FORMULAS</u>
Frank W. Stockwell Jr., AISC Eng. Journal, 3rd Quarter, 1974.

A new term, Q, is defined to be:

$$Q = \frac{F_y\left(\frac{\ell}{r_T}\right)^2}{510 \times 10^3 C_b}$$

If $Q \leq 0.2$	$F_b = 0.6\ F_y$
If $0.2 \leq Q \leq 1.0$	Use equation F1-6, ASDM p.5-47
If $Q > 1.0$	Use equation F1-7, ASDM p.5-47

Formula F1-6 is:

$$F_b = \left(\frac{2}{3} - \frac{F_y\left(\frac{\ell}{r_T}\right)^2}{1530\times10^3 C_b}\right)$$

The second term in the parentheses is Q/3, so F1-6 becomes:

$$F_b = \left(\frac{2 - Q}{3}\right) F_y$$

Formula F1-7 is:

$$F_b = \frac{170\times10^3 C_b}{\left(\frac{\ell}{r_T}\right)^2}$$

Solving for $(\ell/r_T)^2$ in the expression given above for Q, results in:

$$\left(\frac{\ell}{r_T}\right)^2 = \frac{510\times10^3 C_b Q}{F_y}$$

and, substituting this into F1-7:

$$F_b = \frac{170\times10^3\ {}_b}{\frac{510\times10^3 C_b Q}{F_y}} = \frac{F_y}{3Q}$$

Formula F1-8, ASDM p. 5-47, is:

$$F_b = \frac{12\times10^3 C_b}{\frac{\ell d}{A_f}}$$

This equation should be checked when the compression flange is solid, approximately rectangular in shape, and its area is not less than the area of the tension flange.

SIMPLIFIED APPROACH TO AISC BENDING FORMULAS (cont.)
Summary:

The four steps for finding F_b when the length of the compression flange is greater than L_u are:

1. Note: F_y is in ksi.

$$Q = \frac{F_y\left(\frac{\ell}{r_T}\right)^2}{510\times10^3 C_b}$$

2. (a) If $Q \leq 0.2$ $\quad F_b = .6\ F_y$

   (b) If $0.2 \leq Q \leq 1.0$ $\quad F_b = ((2-Q)/3)F_y$

   (c) If $Q \geq 1.0$ $\quad F_b = F_y/(3Q)$

3. If applicable, Check Formula F1-8.

$$F_b = \frac{12\times10^3 C_b}{\frac{\ell d}{A_f}}$$

4. F_b = the larger of the values from Steps 2 and 3 but not over 0.6 F_y.

If F_y = 36 ksi:

For Step 2(a) above;

$F_b = 0.6 \times 36 = 21.6$ ksi

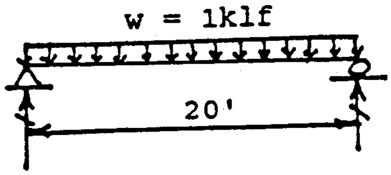
For Step 2(b) above:

$F_b = ((2 - Q)/3)(36) = 12(2 - Q)$

For Step 2(c) above:

$F_b = 36/(3Q) = 12/Q$

EXAMPLE Design of a laterally supported steel beam.



Select a W section to support a uniformly distributed load of 1^k per linear foot.
Steel is A36, full lateral support for top flange.

Solution:

Full lateral support: $\therefore F_b = 24$ ksi.

Assume beam weight = 50 plf.

Total w = 1.00 + 0.05 = 1.05 klf

$$M = wL^2/8 = ((1.05)(20)(20))/8 = 52.50'^k$$

Method 1

Compute S required.

$$S = M/F_b = (52.50 \times 12)/24 = 26.25 \text{ in}^3$$

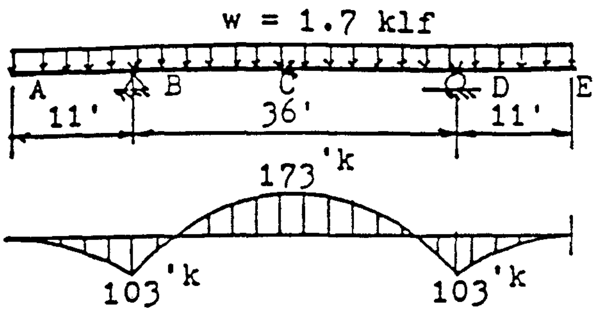
From table of S values, ASDM p. 2-12, try a W14x22, $S_x = 29.0$ in^3. We could also use the Moment of Resistance values, M_r, which are given in this table. For a W14x22, which is the lightest weight section of its group (bold type indication at top of group), the value of M_r is $58'^k$ which is greater than the applied moment of $52.50'^k$. Therefore, the W14x22 is satisfactory for bending.

Method 2

From the Beam Tables, Part 2, Beam and Girder Design, of the ASDM, on page 2-66 the allowable total uniform load for a W14x22 having a span of 22 feet is 23^k, since this is greater than the applied total load W = 20 x 1.05 = 21^k, the section is satisfactory for bending.

EXAMPLE

Select a W section to carry the uniform load. The beam is laterally braced at supports B and D, and at mid-span of BD.
$F_y = 36$ ksi



With lateral support at C, $L_u = \frac{36}{2} = 18'$

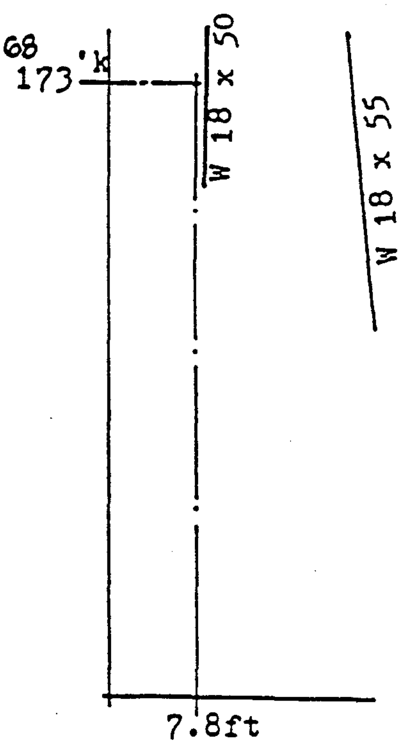
C_b for a cantilever is 1.0 ASDM p. 5-47

C_b for sections BC and CD =

$$C_b = 1.75 + 1.05\frac{M_1}{M_2} + 0.3\left(\frac{M_1}{M_2}\right)^2 \leqslant 2.3 \quad \text{ASDM p. 5-47}$$

$$C_b = 1.75 + 1.05\left(\frac{103}{173}\right) + 0.3\left(\frac{103}{173}\right)^2 = 2.43 \quad \text{Use } 2.3$$

We will use the beam diagram on page 2-168. Since the graph is drawn for C_b =1.0, divide the actual l_u by the actual C_b and enter the graph with this revised length and the moment. The nearest heavy line to the right of the intersection of length and moment will give a trial section.



$$L_{rev.} = \frac{18}{2.3} = 7.8'$$

The nearest trial size is a W18 x 50 which has an S_x = 88.9 in^3. p. 1-22

$$\therefore f_b = \frac{173 \times 12}{88.9} = 23.4 \text{ ksi}$$

The L_u for C_b = 1.0 for this section is 11 feet. The revised L_u for C_b = 2.3 is 11 x 2.3 = 25.4' > 18', so F_b = 21.6 ksi.

However, 21.6 ksi < 23.4 ksi, so the section is unsatisfactory.

EXAMPLE (cont.)

The next heavy line to the right is a W 18 x 55 the properties of which are:

$L_u = 12.1$ ft

$S = 98.3\ in^3$

$r_T = 1.95$ in

$\frac{d}{A_f} = 3.82$

$$f_b = \frac{173 \times 12}{98.3} = 21.12 \text{ ksi}$$

$$\frac{\ell}{r_T} = \frac{18 \times 12}{1.95} = 111$$

Using the simplified method for the allowable bending stress:

$$Q = \frac{F_y \left(\frac{\ell}{r_T}\right)^2}{510 \times 10^3\, C_b} = \frac{36\ (111)^2}{510{,}000 \times 2.3} = 0.38$$

$$F_b = 12\ (\ 2 - 0.38\ ) = 19.46 \text{ ksi}$$

Check Formula F1-8 p. 5-47

$$F_b = \frac{12000 \times 2.3}{(18 \times 12)(3.82)} = 33.45 \text{ ksi} \quad \text{Use } F_b = 21.6 \text{ ksi}$$

$$f_b = 21.12 < 21.6 \text{ ksi} = F_b$$

The W 18 x 55 is satisfactory.

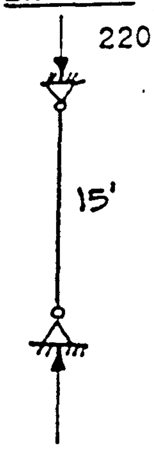
This could also have been determined by considering a revised unsupported length because of C_b.

$$\ell_u = 12.1 \times 2.3 = 27.8' > 18'$$

$$\therefore\ F_b = 21.6 \text{ ksi}$$

and, proceeding as above, the W 18 x 55 is found to be OK.

EXAMPLE



Determine the lightest weight W10 column, F_a, and P allowable for the column shown. F_y = 36 ksi.

The column has pinned ends, so the effective length factor, K, is equal to 1.0. Table C-C2.1 ASDM p. 5-135 From Column Tables ASDM p 3-30, with KL = 1 x 15 = 15'; a W10x49 has a P allowable of 235^k.
From ASDM p. 1-31, a W10x49 has the following properties:

r_y=2.54 in, r_x/r_y=1.71, A=14.4 sq in.

K_xL_x=1.0 x 15 = 15'

The equivalent minor axis effective length for the major axis is $K_xL_x/(r_x/r_y)=15/1.71=8.77' < K_yL_y=15'$

Then the minor axis length controls.

$$\frac{K_yL_y}{r_y} = 1.0 \times 15 \times 12/2.54 = 70.9$$

$$C_c = \left(\frac{2\pi^2 E}{F_y}\right)^{\frac{1}{2}} = \left(\frac{2\pi^2 29{,}000}{36}\right)^{\frac{1}{2}} = 126.1$$

or, ASDM p. 5-120, C_c for F_y=36, = 126.1

$$\frac{K_yL_y}{r_y} < C_c$$

By Formula E2-1 ASDM p. 5-42

$$F_a = \frac{\left[1 - \frac{(Kl/r)^2}{2C_c^2}\right] F_y}{\frac{5}{3} + \frac{3Kl/r}{8C_c} - \frac{(Kl/r)^3}{8C_c^3}}$$

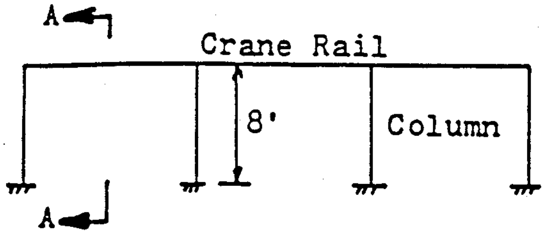
$$F_a = \frac{[1-(70.9)^2/2(126.1)^2]36}{1.67 + .375(70.9/126.1)-(70.9)^3/8(126.1)^3}$$

F_a = 16.34 ksi

Then P allowable = F_axA=16.34 x 14.4 = $235^k > 220^k$

Or, from Table C-36, ASDM p. 3-16
for Kl/r = 70.9, by interpolation,
F_a = 16.34 ksi, and P allow. is again, 235^k

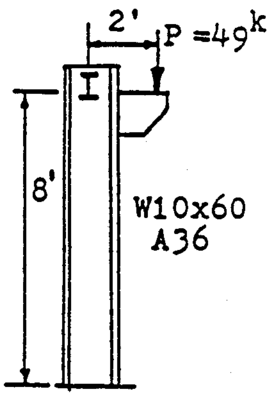
<u>EXAMPLE</u> Beam-column Design



Using AISC Specs., determine if the column has adequate strength to support the loads as shown.

Sidesway is not prevented in a direction perpendicular to the crane rail.

The column top is braced against sidesway in the direction along the crane rail.



SECTION A-A

Properties of W10x60: ASDM p. 1-30

$A=17.6$ sq in, $d=10.22$ in, $S_x=66.7$ in^3,

$r_x=4.39$ in, $r_y=2.57$ in, $r_T=2.77$ in,

$d/A_f=1.49$, $b_f/2t_f=7.4$

See Section H.1 ASDM p. 5-54

Determine which interaction equation controls.

Check f_a/F_a

$$f_a=P/A=49/17.6 = 2.78 \text{ ksi}$$

F_a depends on Kl/r, see p.5-135 for values of K, in this case we use condition (e) of Table C-C2.1, i.e. K=2.1.

$$\frac{K_x l_x}{r_x} = \frac{2.1 \times 8 \times 12}{4.39} = 45.92 \quad \underline{\text{Governs}}$$

$$\frac{K_y l_y}{r_y} = \frac{0.8 \times 8 \times 12}{2.57} = 29.9$$ Condition (b) of Table C-C2.1 gives K_y.

Interpolation in Table C-36, ASDM p.3-16, gives

$$F_a = 18.71 \text{ ksi}$$

$$f_a/F_a = 2.78/18.71 = 0.149 < 0.15$$

Then by Section H.1 we need only to check Formula H1-3

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} = \leq 1.0 \qquad \text{H1-3, ASDM p. 5-54}$$

EXAMPLE (cont)

Since we have no bending about the y-axis, the third term on the left of Formula H1-3 goes to zero.

$$f_b = \frac{M}{S} = \frac{49^k \times 24''}{66.7} = 17.63 \text{ ksi}$$

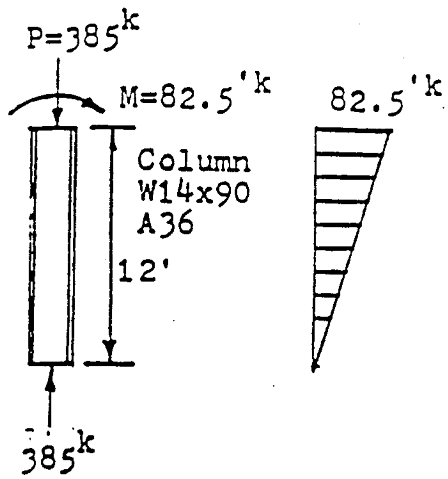
From p.3-30 ASDM, L_c = 10.6' which is greater than the actual length of 8'; therefore,

$$F_b = 24 \text{ ksi}$$

Substituting in Formula H1-3:

$$0.149 + \frac{17.63}{24} = 0.149 + 0.735 = 0.884 < 1.0 \quad \therefore \text{ OK}$$

EXAMPLE (Beam-column)



Can the W14x90 carry the load safely? Assume the column is hinged at both ends. See p.1-27 for properties of the W14x90.

$A=26.5\ in^2$

$d=14.02\ in$

$S_x=143\ in^3$

$r_x=6.14\ in$

$r_y=3.70\ in$

$r_T=3.99\ in$

$d/A_f=1.36$

From the Column Tables, p.3-23,

$L_c=15.3' > 12'$

$\therefore\ F_b = 24ksi$

Since column is hinged at both ends, $K_x=K_y=1.0$

$$\frac{K_x l_x}{r_x} = \frac{1 \times 12 \times 12}{6.14} = 23.4 \qquad F'_e = 273\ ksi \quad \text{(by interpolation from p.5-122)}$$

$$\frac{K_y l_y}{r_y} = \frac{1 \times 12 \times 12}{3.70} = 39.0 \qquad F_a = 19.27\ ksi \quad \text{(from p.3-16)}$$

$$f_a = \frac{P}{A} = \frac{385}{26.5} = 14.5\ ksi \qquad \frac{f_a}{F_a} = \frac{14.5}{19.27} = .75 > .15$$

$\therefore$ must check H1-1 and H1-2 p.5-54

H1-1

$$\frac{f_a}{F_a} + \frac{C_m f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right)F_{bx}} \leqslant 1.0$$

H1-2

$$\frac{f_a}{0.6F_y} + \frac{f_{bx}}{F_b} \leqslant 1.0$$

p.5-55; $C_m = 0.6 - 0.4\,\frac{\cancelto{0}{M_1}}{M_2} = 0.6$

$$f_{bx} = \frac{M}{S_x} = \frac{82.5 \times 12}{143} = 6.9\ ksi$$

Check H1-1 $\quad 0.75 + \frac{.6 \times 6.9}{\left(1 - \frac{14.5}{273}\right)24} = 0.75 + 0.18 = 0.93 < 1.0$ OK

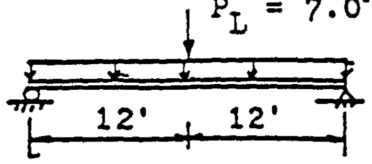
Check H1-2 $\quad (14.5/21.6) + (6.9/24) = 0.66 + 0.29 = 0.95 < 1.0$ OK

EXAMPLE

Proportion the beam below for moment requirements, the use condition is exterior exposure, and the loads shown are service (working) loads.

$f'_c = 3{,}000$ psi $\qquad f_y = 50{,}000$ psi

$P_D = 3.6^k$
$P_L = 7.0^k$



$w_D = 0.4$ klf
$w_L = .16$ klf

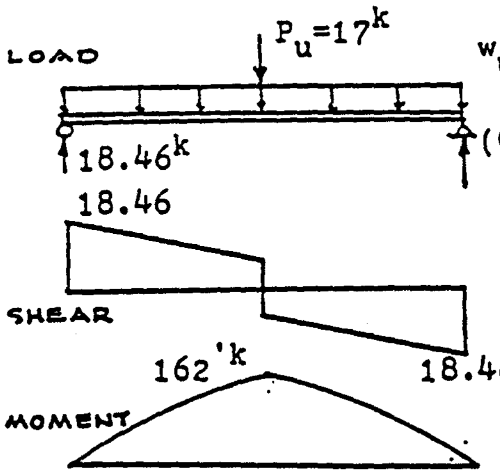
1. Establish the ultimate loads by multiplying service loads by the appropriate load factors found in Section 9.2 of the ACI Code, p. 92.

$$P_u = 3.6 \times 1.4 + 7.0 \times 1.7 = 16.94^k \text{ call } 17^k$$

$$w_u = 0.4 \times 1.4 + 0.16 \times 1.7 = 0.83 \text{ klf}$$

2. Shear and Moment Curves

$P_u = 17^k$ $\qquad w_u = 0.83$ klf



$$\begin{aligned}(0.83 \times 24)/2 &= 9.96\\ 17/2 &= \underline{8.50}\\ &\ 18.46\end{aligned}$$

$$M_{max} = [(18.46 + 8.50)/2](12) = 161.76'^k \text{ call } 162'^k$$

3. Establish the minimum depth.
   By Table 9.5(a) ACI Code p.96 see note at bottom of table "For f_y other than 60,000 etc."

$$h_{min} = \frac{\ell}{16}(0.4 + f_y/100{,}000)$$

$$h_{min} = \frac{24 \times 12}{16}\left(0.4 + \frac{50{,}000}{100{,}000}\right) = 16.2''$$

4. Establish b and d, at this point the designer selects a value for the percentage of steel.

$\rho \leq .75\ \rho_b$ Sections 10.3.2; 10.3.3 ACI p.109

$$\rho_b = \frac{0.85\beta_1 f_c'}{f_y}\left[\frac{87{,}000}{87{,}000+f_y}\right]$$

$$\rho_b = \frac{(.85)(.85)(3)}{50}\left[\frac{87}{87+50}\right] = 0.0275$$

$$0.75\rho_b = 0.75(0.0275) = 0.0204$$

Try $\rho = 0.018$, if this amount of steel is difficult to fit into the beam, a smaller ρ can be selected.

Using a capacity reduction factor $\phi = 0.90$, Sect. 9.3.2, p.94

$$M_u = \phi\rho f_y bd^2\left[1 - \frac{\rho f_y}{f_c'}\right] =$$

$$(162)(12) = (0.90)(0.018)(50\ bd^2)\left[1 - \frac{(0.018)(50)}{(1.7)(3)}\right]$$

$$2920 = bd^2$$

NOTE: f_c' and f_y are in KIPS PER SQ. IN.

If b = 12; d = 15.6 - Try
If b = 10; d = 17.0

Points to consider in selecting proportions:
a) deeper beam is more efficient, deflects less.
b) shallower beam provides more headroom

5. Area of steel required:
$A_s = \rho bd = (0.018)(12)(15.6) = 3.38$ sq in.

Use 2 - #10 and 1 - #9 rebars, A_s supplied = 3.53 sq in.

6. Check Z if $f_y > 40{,}000$ psi. This paramater limits the size of cracks. Section 10.6, ACI Code p.112

$$Z = f_s \sqrt[3]{d_c A}$$

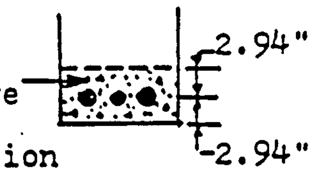
Where, $f_s = 0.6f_y = 0.6(50) = 30$ ksi
d_c is defined in Sect. 10.0, ACI Code, p.105
A is defined in the same section.

EXAMPLE (cont.)

6. (cont.)

$$
\begin{array}{lll}
d_c = & 2.0" & \text{Cover (ACI 7.7.1)} \\
 & .375" & \text{Stirrup dia.} \\
 & .5625" & = \frac{9}{8} \times \frac{1}{2} \\
\hline
 & 2.94" &
\end{array}
$$

A is the effective area of the tension concrete



$$A = \frac{A_t}{\text{No. bars*}}$$

*When the main reinforcement consists of several bar sizes, the number of bars shall be computed as the total steel area divided by the area of the largest bar.
(Def. of A, Sect.10.0
ACI Code p. 105

$A = (2.94 \times 2 \times 12)/(3.58/1.27)$

$A = 25.03$

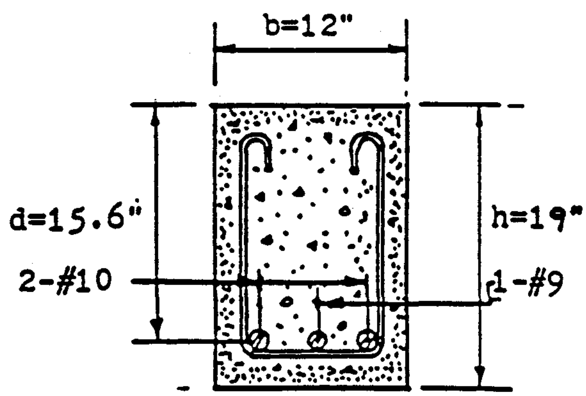
$$Z = 30 \sqrt[3]{2.94 \times 25.03}$$

$Z = 126 < 145 = Z_{max}$ for exterior exposure. (10.6.4 p. 113)

$\therefore$ The design is OK

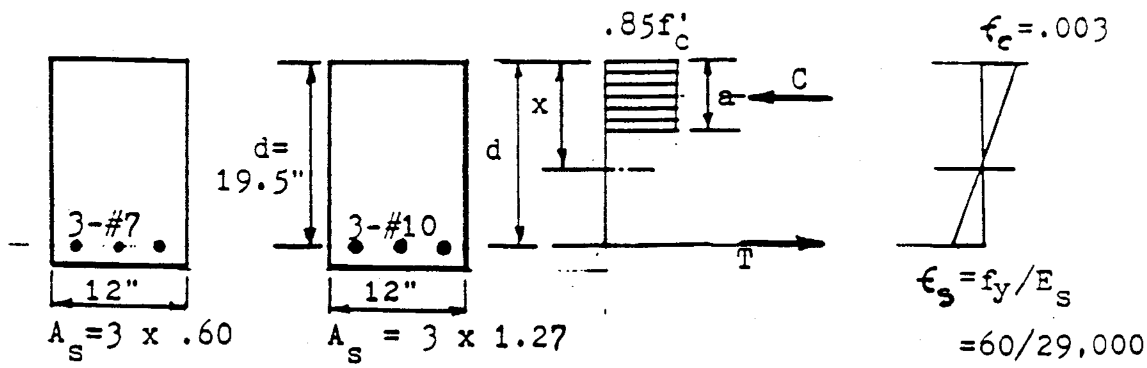
7. Sketch of the cross-section of the beam is:

$$
\begin{array}{ll}
h = & 15.6 \\
 & +2.94 \\
\hline
 & 18.54" \\
 & \text{SAY } 19"
\end{array}
$$



EXAMPLE

For each of the beams, using f'_c=4,000 psi, f_y=60,000 psi, determine (a) the theoretical capacity M_n, (b) if loading is 60% LL and the basic provision for dead load plus live load controls, what is the service moment capacity?



A_s=3 x .60 =1.80 sq in A_s = 3 x 1.27 = 3.81 sq in

Balanced condition:

$$x_b = \frac{.003}{\frac{60}{29,000} + .003}(19.5) = 11.52"$$

$$a_b = .85(11.52) = 9.80"$$

$$C_b = .85f'_c ba_b = (.85)(4)(12)(9.80) = 400^k$$

$$A_{sb} = \frac{C_b}{f_y} = \frac{400}{60} = 6.67 \text{ sq in}$$

$$A_{s_{max}} = 0.75\,A_{sb} = 0.75(6.67) = 5 \text{ sq in}$$

$$5 \text{ sq in} > 1.80 \text{ sq in or } 3.81 \text{ sq in} \quad \therefore \text{OK}$$

(a)

$$C = 0.85f'_c ab = 0.85(4)(12)(a) = 40.8a$$

$$T = A_s f_y = 1.80 \times 60 = 108^k$$

$$a = 108/40.8 = 2.65";\ \frac{a}{2} = 1.33"$$

$$x = \frac{a}{.85} = 2.65/.85 = 3.12"$$

$$M_n = T(d - \frac{a}{2}) = 108(19.5 - 1.33)/12 = 163.5^{'k}$$

call $164^{'k}$

EXAMPLE (cont.)

(b) $C = 40.8a$

$T = (3.81)(60) = 228.6^k$

$a = 228.6/40.8 = 5.60"$; $\frac{a}{2} = 2.80"$

$x = 5.60/.85 = 6.59"$

$M_n = 228.6(19.5 - 2.8)/12 = 318'^k$

Safe service load moment:

$$M_n = \frac{1.4(0.4M_w) + 1.7(0.6M_w)}{0.90} = \frac{0.56M_w + 1.02M_w}{0.90}$$

$$= 1.76M_w$$

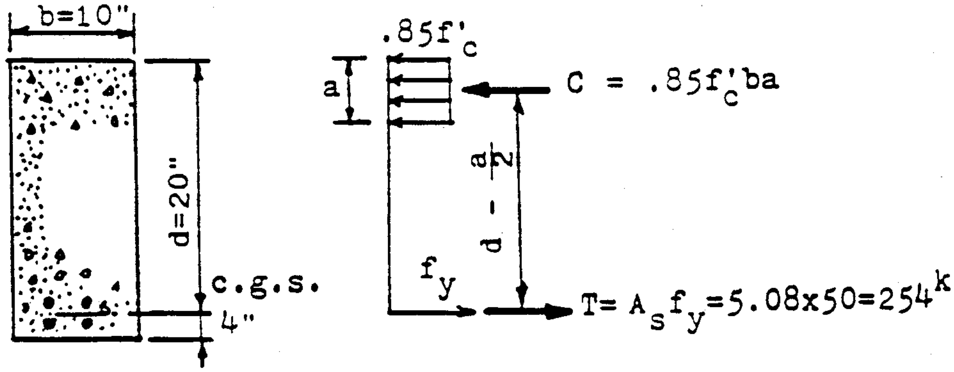
(a) $M_w = 164/1.76 = 93'^k$ for beam with 3 - #7

(b) $M_w = 318/1.76 = 181'^k$ for beam with 3 -#10

EXAMPLE

Using strength design, determine the maximum distance a simply supported beam can span if it has the cross-section shown below. Consider bending only. The beam must carry a uniform dead load of 0.8 klf and a live load of 0.9 klf.

f_y=50ksi; f'_c=3.75 ksi. Use ACI 318-89



4-#10 bars
A_s=1.27 x 4 = 5.08 sq in.

Equate C =T to get "a".

$$.85f'_c ba = A_s f_y \quad \text{which gives,} \quad a = \frac{A_s f_y}{.85f'_c b} = \frac{254}{.85x3.75x10} = 7.97"$$

$$M_u = \left[A_s f_y \left(d - \frac{a}{2}\right)\right]\phi \text{ where } \phi \text{, the reduction factor, } = 0.90.$$

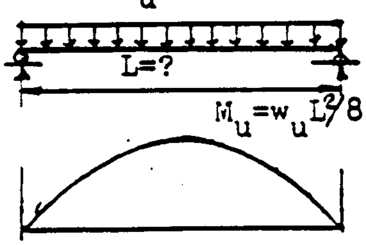
$$M_u = 0.90(254)\left(20 - \frac{7.97}{2}\right) = 3661"^k = 305'^k$$

The weight of the beam is (10x24)/144 (.150) = .25 klf.

$$w_u = 1.4w_D + 1.7w_L = 1.4(0.8+0.25) + 1.7(0.9)$$

$$w_u = 3.00 \text{ klf}$$

Equate the internal moment in terms of load and span length to M_u.



$$M_u = \frac{w_u L^2}{8}$$

$$306'^k = \frac{3.00L^2}{8}$$

$$816 = L^2$$

$$28.6' = L$$

EXAMPLE

Design a square tied concrete column to support an axial loading as follows:

$DL = 300$ kips; $LL = 120$ kips

$f'_c = 4{,}000$ psi, $f_y = 60{,}000$ psi

Section 10.3.5.2 ACI Code p. 110

$$P_u = \phi P_{n(max)} = 0.80\phi \left[0.85 f'_c (A_g - A_{st}) + f_y A_{st} \right]$$

Assume 2% of steel, i.e. $A_{st} = .02A_g$

$$P_u = 1.4(300) + 1.7(120) = 420 + 204 = 624^k$$

$$624 = (0.80)(0.70)\left[0.85(4)(A_g - 0.02A_g) + 60(0.02A_g)\right]$$

$$624 = 0.56(3.40A_g - 0.068A_g + 1.20A_g) = 0.56(4.532A_g)$$

$$624 = 2.538A_g$$

$$A_g = 245.9 \text{ sq in}$$

Use a 16" x 16" column, A = 256 sq in.

Area of steel, A_{st}, required:

$$624 = 0.56\ (3.40 \times 256) - 3.40A_{st} + 60A_{st}$$

$$624 = 487 + 31.7A_{st}$$

$A_{st} = 4.32$ sq in. Use 8 - #7, Area = 4.80 sq in.

Sections 7.10.5.1 and 7.10.5.2 ACI Code, p. 72

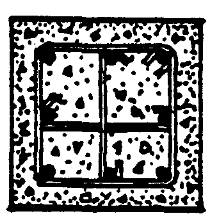
Using #3 ties, the spacing is:

48 x 3/8 = 18"

16 x 7/8 = 14" (Governs)

Least dimension of the column is 16"

Use #3 ties at 14 inches o.c.



16"x 16"column
8 - #7 vert.
#3 ties at 14"o.c.

EXAMPLE

Design a square column footing to support a 16 inch square column having a DL $=300^k$ and LL $=120^k$. The column re-bars are 8 -#7 vertical. The bottom of the footing is 5'-0" below ground. The concrete weight is 150pcf, the soil is 110, $f'_c = 4{,}000$ psi, $f_y = 60{,}000$ psi, and the soil capacity is three tons per sq ft.

Assume the footing depth = 24", then d = 24 -(3+1) = 20"

Section 15.2.2 ACI Code p.245

$$P_D=300^k \quad P_{LL}=120^k \quad P_{TOT}=300+120=420^k$$

The bearing pressure due to the footing and the soil is:

$$f_p=2.0\text{x}.150+3\text{x}.110=.300+.330=.630 \text{ ksf}$$

$$\text{Net } f_p=6^k - .630^k = 5.37 \text{ ksf}$$

$$\text{Area}_{req'd} = \frac{P_{TOT}}{\text{Net } f_p} = \frac{420}{5.37} = 78.2 \text{ sq ft}$$

Use footing 9' x 9'. Area = 81 sq ft.

Section 9.2.1 ACI Code p.91

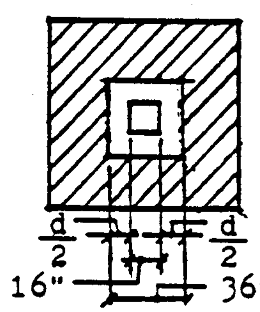
The design pressure on the base of the footing due to factored column loads is

$$f_{pu} = \frac{(1.4 \times 300) + (1.7 \times 120)}{81} = \frac{624}{81} = 7.70 \text{ ksf}$$

Depth for punching shear;

Sections 11.11.1.2; 11.12.2.1 ACI Code p.169

The shaded area is the loaded area for punching shear.



$b_0 = 4 \times 36=144"$

<u>EXAMPLE</u> (cont.)

$$V_u = (81 - 3^2)(7.70) = 554.4^k$$

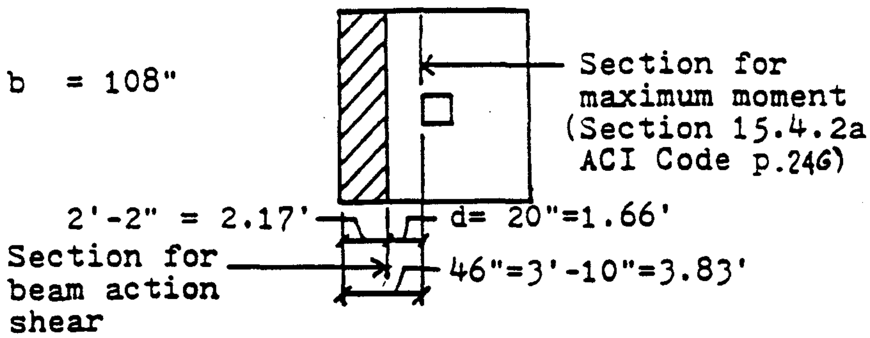
$$d = \frac{V_u}{\phi 4\sqrt{f'_c}\, b_0} = \frac{554{,}400}{(.85)(4)\sqrt{(4000)}(144)} = 17.90''$$

$17.90'' < 20''$ OK

Depth of footing for beam action:

Section 11.12.1.1 ACI Code p.168

b = 108"



$$V_u = 9.00 \times 2.17 \times 7.7 = 150.38^k$$

$$d = \frac{V_u}{\phi 2\sqrt{f'_c}\, b} = \frac{150{,}380}{(0.85)(2)\sqrt{(4000)}(108)} = 12.95'' < 20''\ \text{OK}$$

Steel Area:

$$M_u = (7.70)(9.0)(3.83)(3.83)/2 = 508.28\ 'k$$

$$R_u = \frac{M_u}{\phi b d^2} = \frac{508.28 \times 12000}{0.90 \times 108 \times 20 \times 20} = 156.88$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_u}{f_y}}\right) \quad \text{where } m = \frac{f_y}{.85\, f'_c} = \frac{60}{.85(4)} = 17.65$$

$$\rho = \frac{1}{17.65}\left(1 - \sqrt{1 - \frac{2 \times 17.65 \times 156.88}{60{,}000}}\right)$$

$$\rho = 0.0027; \quad \rho_{min} = \frac{200}{f_y} = \frac{200}{60{,}000} = 0.0033 \longleftarrow \text{Use}$$

$$A_{st} = A_{s(min)} = 0.0033 \times 108 \times 20 = 7.20\ \text{sq in}$$

Use 12 - #7 rebars $A_s = 12 \times 0.60 = 7.20$ sq in.

EXAMPLE (cont.)

Development length for footing rebars:
Section 12.2.2 ACI Code p. 182

$$\ell_d = \frac{.04\ A_b\ f_y}{f'_c} = \frac{.04 \times .60 \times 60{,}000}{4{,}000} = 23''$$

$$\ell_{min} = .0004\ d_b f_y = .0004 \times \frac{7}{8} \times 60{,}000 = 21''$$

$$\ell_{actual} = 46 - 3 = 43''\quad \text{OK}$$

Load transfer from base of column to footing:
Sections 10.15.1 ACI Code p. 133

Allowable load at the base of the column:

$P = \phi\, 0.85\ f'_c\ A_1$; where A_1 = area of column = 256 sq in.

$$P = (0.70)(0.85)(4)(256) = 609^k < 624^k$$

Allowable load on footing:

$P = 0.85\ f'_c\ A_1\sqrt{\frac{A_2}{A_1}}$; where A_2 is the supporting surface, and $\sqrt{\frac{A_2}{A_1}}$ need not exceed 2.

$$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{81}{1.77}} = 6.76 > 2 \quad \therefore \text{ Use } 2$$

$$P = (0.70)(0.85)(4)(256)(2) = 1218.6^k > 624^k \quad \text{OK}$$

Excess to be carried by dowels:
Section 15.8.1.2 ACI Code p. 249

$$624 - 609 = 15^k$$

$$A_s \text{ of dowels} = \frac{15}{60} = 0.25 \text{ sq in.}$$

$$A_{s(min)} = .005 \times 256 = 1.28 \text{ sq in.}$$

Use 4 - #6 rebars, area = 4 x .44 = 1.76 sq in.

EXAMPLE (cont.)

Development length of dowels:

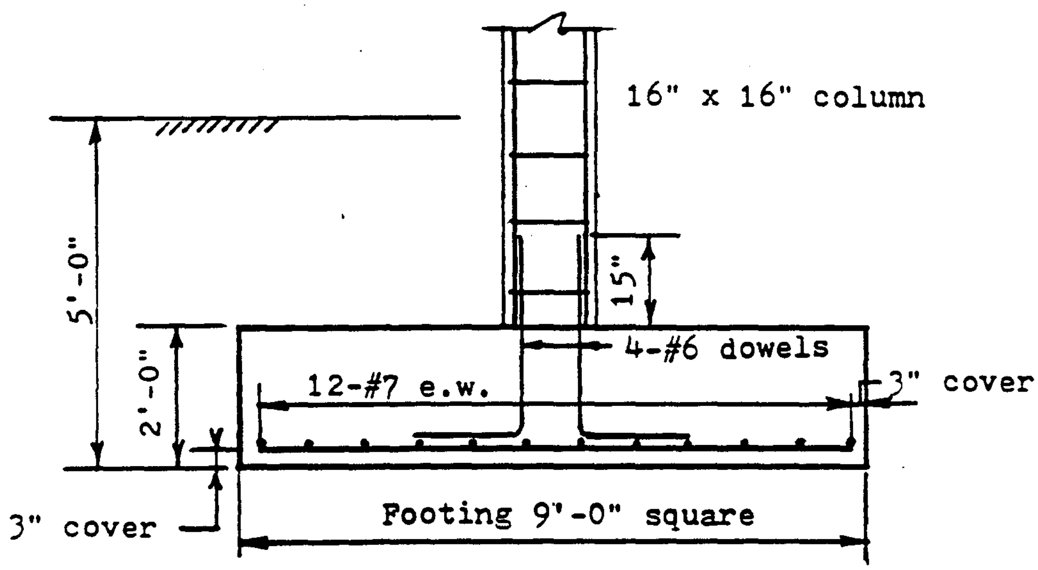
Section 12.3.2 ACI Code p. 188

$$\ell_d = \frac{0.02 f_y d_b}{\sqrt{f'_c}} = \frac{(.02)(60{,}000)(.75)}{\sqrt{4{,}000}} = 14.23" \longleftarrow \text{Governs}$$

$$\ell_d = 0.0003\, f_y d_b = (.0003)(60{,}000)(.75) = 13.5"$$

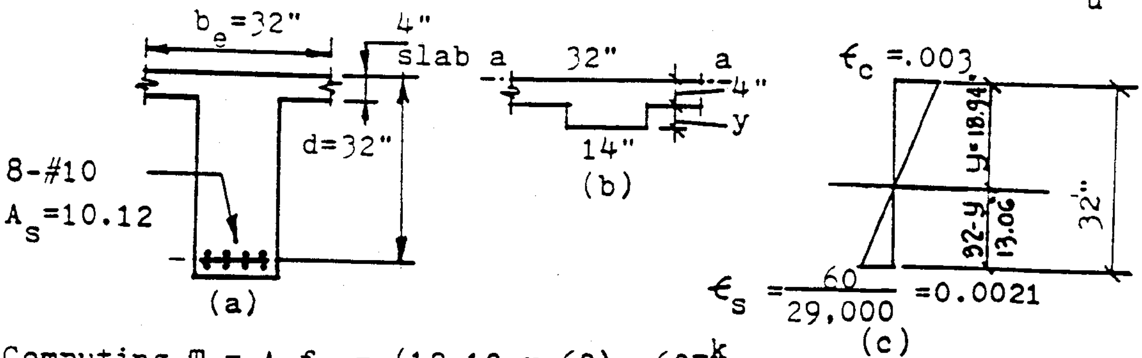
$$\ell_d = 8"$$

Use 4 - #6 dowels extending 15" up into the column and down into the footing so as to set on the rebar mat.



EXAMPLE

Given the concrete T beam shown, the span is 30 feet and f'_c = 4,000 psi, f_y = 60,000 psi. Considering bending only, find: (a) M_u = ? (b) what uniformly distributed load for M_u?



Computing $T = A_s f_y = (12.10 \times 60) = 607^k$

Since the total compression must be equal to T, and

$C = 0.85\, f'_c\, A_c$ (concrete in compression)

$$A_c = \frac{T}{0.85 f'_c} = \frac{607}{.85 \times 4} = 178.5 \text{ sq in}; \; 32 \times 4 = 128 \text{ sq in.}$$

∴ The neutral axis lies below the slab.

Using (b) above:

$$14 \times y = 178.5 - 128 = 50.5$$

$$y = 3.61"$$

Compute $\bar{y}$ with respect to axis a-a.

	Area	y_{a-a}	$A(y_{a-a})$
4 x 32	128	2	256
3.61 x 14	50.5	5.81	293.15
	178.5		549.15

$$\bar{y} = \frac{549.15}{178.5} = 3.08"$$

The lever arm from T to C = 32 - 3.08 = 28.92"

$$M_n = 607^k \times \left(\frac{28.92}{12}\right) = 1463'^k$$

$$M_u = \phi M_n = 0.90 \times 1463 = 1317'^k$$

EXAMPLE (Cont.)

Check $\rho_{max} = \frac{3}{4}\rho_b$. T is a function of ρ, then check T_{max}.

Using Fig.(c), for balanced conditions:

$$\frac{y}{32} = \frac{.003}{.003 + .0021}$$

$$y = 18.94" = y_b$$

$$32 - y = 13.06"$$

The depth of the compression stress block, $a_b = .85y_b$.

$$a_b = .85 \times 18.94 = 16.10"$$

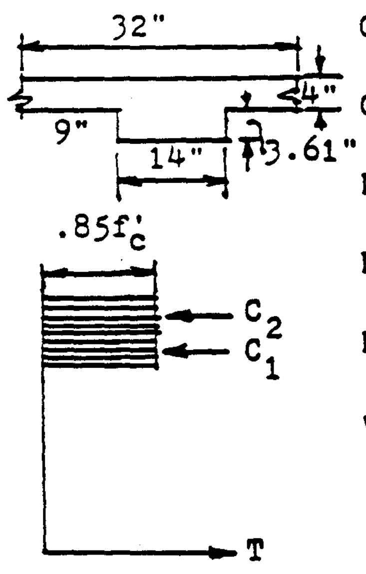
Then the area of the concrete compression block is;

$$A_c = 4(32) + (16.10 - 4)(14) = 297.4 \text{ sq in.}$$

$$C = (.85)(4)(297.4) = 1011^k$$

$$T_{max} = \frac{3}{4}C = 0.75 \times 1011 = 758^k > T_{actual} = 607^k \text{ OK}$$

Check on M_u by alternate method: see Fig.(d).



(d)

$$C_1 = (.85)(4)(14)(7.61) = 362.2^k$$

$$C_2 = (.85)(4)(2)(9)(4) = 244.8^k$$

$$M_n = [362.2(32 - \frac{7.62}{2}) + 244.8(32 - \frac{4}{2})]/12$$

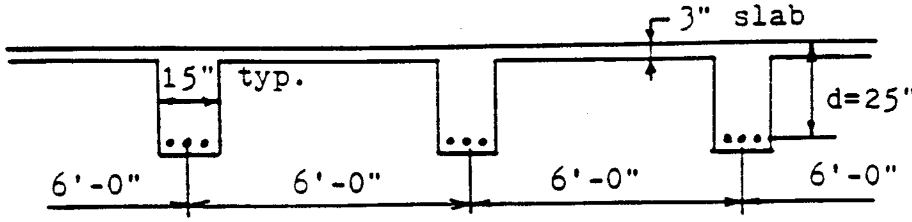
$$M_n = 1463'^k$$

$$M_u = \phi M_n = 0.90 \times 1463 = 1317'^k \text{ Ans.}$$

$$w_u = \frac{8M}{L^2} = \frac{8 \times 1317}{30 \times 30} = 11.70 \text{ klf Ans.}$$

EXAMPLE

Given the T beam floor system as shown, and $M_{DL} = 200'^{k}$, $M_{LL} = 250'^{k}$, f'_c=4,000 psi, f_y=60,000 psi, L = 20'0".
Find the area of steel required for a T beam in this system.



Section 8.10.2 ACI Code p. 85

Effective flange width b_e:

1) $b_e = \frac{1}{4} \times 20 \times 12 = 60"$ Governs

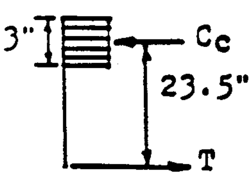
2) $b_e = (2)(8)(3) + 15 = 63"$

3) $b_e = (2)(\frac{57}{2}) + 15 = 72"$

$$M_u = 1.4(200) + 1.7(250) = 280 + 425 = 705'^{k}$$

$$M_n = \frac{M_u}{\phi} = \frac{705}{.90} = 783'^{k}$$

Assume a the same depth as the slab and compute moments about C_c.



$$T(23.5) = A_s f_y (23.5) = M_n$$

$$A_s(60)(23.5) = 783 \times 12$$

$$A_s = 6.66 \text{ sq in.}$$

Compute A_c:

$$A_c = \frac{C_c}{.85f'_c} = \frac{T}{.85f'_c} = \frac{6.66 \times 60}{(.85)(4)} = 117.6 \text{ sq in.}$$

$$a = \frac{A_c}{b_e} = \frac{117.6}{60} = 1.96" \quad a/2 = 0.98"$$

Since a<3" the neutral axis is in the flange.

EXAMPLE (Cont.)

$$M_n = T(d - \frac{a}{2}) = (A_s \times 60)(25 - 0.98)/12$$

$$783 = 120.10\ A_s$$

$$A_s = 6.52 \text{ sq in.}$$

Use 3 - #9 and 3 - #10 in two layers, A_s = 6.81 sq in.

Check a:

$$T = A_s f_y = 6.81 \times 60 = 408.6^k$$

$$A_c = \frac{408.6}{.85 \times 4} = 120.18$$

$$a = \frac{120.18}{60} = 2.00" \quad \frac{a}{2} = 1.00" \quad d - \frac{a}{2} = 24"$$

$$M_{n(allow.)} = T(d-\frac{a}{2}) = (408.6 \times 24)/12 = 817.2'^k > 783'^k \text{ OK}$$

$$A_s = \frac{M_{n(actual)}}{f_y(d-\frac{a}{2})} = \frac{783 \times 12}{60 \times 24} = 6.53 \text{ sq in} \sim 6.52 \text{ sq in OK}$$

$$M_u = \phi M_n = 0.90(817.2) = 735'^k > 705'^k \text{ OK}$$

Check $\rho_{max} = 0.75\rho_b$:

From ACI Tables, for f'_c = 4,000 psi and f_y = 60,000 psi,

$$0.75\rho_b = .0214$$

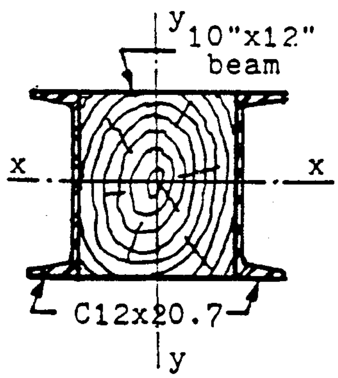
$$\rho_{actual} = \frac{A_s}{b_w d} = \frac{6.81}{15 \times 25} = .0182 < .0214 \quad \text{OK}$$

Check $\rho_{min} = 200/f_y$ = 200/60.000 = .0033 < .0182 OK

EXAMPLE

A pair of C12x20.7 channels are bolted to a beam 10"x 12" as shown. Determine the safe resisting moment if bending occurs about the x-axis.

Given: F_s=18,000 psi, F_w=1200 psi, and E_s/E_w=n=20.



From ASDM p.1-40 (9th Ed.)

C12x20.7 A=6.09 sq in., d=12", b_f=2.942"

Axis x - x	Axis y - y
$I = 129\ in^4$	$I = 3.88\ in^4$
$\bar{y} = 6$ in	$\bar{x} = 0.698$ in

10x12 A = 120 sq in.

Axis x - x	Axis y - y
$I=(10x12^3)/12$	$I=(12x10^3)/12$
$= 1440\ in^4$	$= 1000\ in^4$
$\bar{y}= 6$ in	$\bar{x}= 5$ in

Transform the section to an equivalent wood section by using the given value of n = 20.

For x-axis bending, since the section is symmetrical with re-pect to this axis, we may transform the moments of inertia directly.

$$I_{tr}= nI_s+I_w = (2)(20)(129)+1440=5160+1440=6600\ in^4$$

$$S_{tr}=I_{tr}/(\tfrac{d}{2})=6600/6 = 1100\ in^3$$

If F_w = 1200 psi, $F_s=nF_w=(20)(1200)=24000>F_{s(allow.)}=18,000$ NG

If F_s =18,000 psi, $F_w=F_s/n=18,000/20=900<F_{w(allow.)}=1200 \leftarrow$ Use

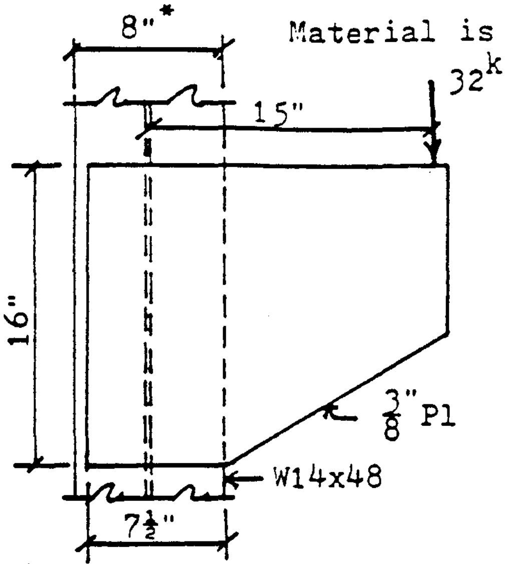
Then the moment capacity of the section is;

$$M_R=\frac{(F_w S_{tr})}{12,000} = 900\ \frac{lbs}{in^2}\ x 1100\ in^3 x\ \frac{1\ ft\ x\ 1^k}{12,000\ in\ lbs}$$

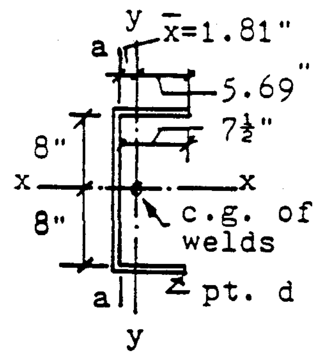
$$M_R= 82.50'^k$$

EXAMPLE

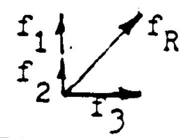
Determine the size of weld required for the bracket shown. Material is A36, and welding electrodes E70.



* Flange width for detailing, from ASDM p.1-26



Assume effective thickness of weld throat, t_e = 1".



Stress at Pt. d

Because of symmetry, the c.g. is half-way between the top and bottom welds. For $\bar{x}$ with respect to the y-axis: take moments about a-a.

$$\bar{x} = \frac{(2)(7.5)(1)(3.75)+(16)(1)(0)}{2(7.5)(1)+(16)(1)} = \frac{56.25}{31} = 1.81"$$

$$I_p = I_x + I_y = \underbrace{\frac{16^3}{12} + 2(7.5)(8)^2}_{I_x} + \underbrace{2\left(\frac{7.5^3}{12}\right) + 2(7.5)\left(\frac{7.5}{2} - 1.81\right) + 16(1.81)^2}_{I_y}$$

$$I_p = 1301.33 + 179.18 = 1480$$

$$e = 15 + 3\tfrac{1}{2} - 1.81 = 16.69"$$

$$f_1 = \frac{32}{(2x7.5)+16} = 1.03 \text{ ksi} \uparrow$$

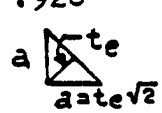
$$f_2 = \frac{Pex}{I_p} = \frac{32 \times 16.69 \times (7.50-1.81)}{1480} = 2.05 \text{ ksi} \uparrow$$

$$f_3 = \frac{Pey}{I_p} = \frac{32 \times 16.69 \times 8}{1480} = 2.89 \text{ ksi} \rightarrow$$

$$f_R = \sqrt{(f_1 + f_2)^2 + f_3^2} = \sqrt{(1.03+2.05)^2 + (2.89)^2} = 4.22 \text{ ksi}$$

$$D = \frac{4.22}{.928} = 4.55 \text{ sixteenths, use } \frac{5}{16}" \text{ fillet weld.}$$

Or:



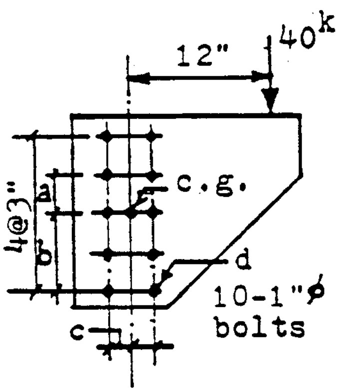
$$a = \frac{f_R\sqrt{2}}{F_{t(allow.)}} = \frac{5.97}{21} = 0.284", \text{ use } \frac{5}{16}".$$

$$\text{Min. plate thickness} = \frac{.707 \times .3125 \times 21}{14.5} = 0.320", \ \frac{3}{8}" \text{Pl} \quad \text{OK}$$

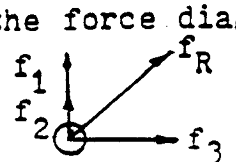
The numerator of the above fraction represents the shear force on the throat of a 5/16" fillet weld, and the denominator is the allowable shear stress for A36 steel which is in Section F4, ASDM p. 5-49 or, Table I p. 5-117.

EXAMPLE

Assume the plate is of sufficient size and strength to carry the applied loads. Using material having F_y=36 ksi, and 1"dia. A-325N high strength bolts, find the maximum load on the bolts.



By inspection, the center of gravity of the bolt group is as shown.
Since the resultant force in any bolt is perpendicular to the line which connects it to the center of gravity of the bolt group, the bolt having the largest load is farthest from the c.g. We will check bolt "d".
At "d", the force diagram is:



In this diagram, $f_1 = \frac{P}{n}$ Where n = total number of bolts.

$f_2 = \frac{Pex}{I_p}$; $f_3 = \frac{Pey}{I_p}$; and e =12".

$I_p = I_x + I_y$

I_x=No.bolts x a^2+No.bolts x $b^2 = 4(3^2 + 6^2) = 180$

I_y=No.bolts x $c^2 = 10 \times 3^2$ $= 90$

$I_p = 270$

$f_1 = \frac{40}{10} = 4^k$

$f_2 = \frac{40 \times 12 \times 3}{270} = 5.33^k$

$f_3 = \frac{40 \times 12 \times 6}{270} = 10.66^k$

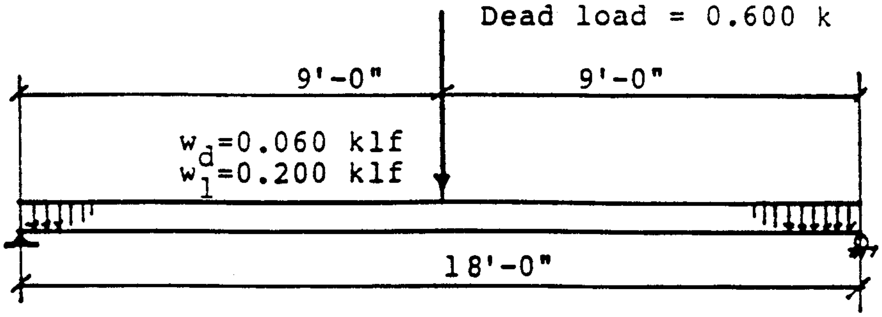
$f_R = \sqrt{(f_1 + f_2)^2 + f_3^2} = \sqrt{(4 + 5.33)^2 + (10.66)^2} = 14.17^k$

$R_{v(allow.)} = 16.5^k$, Table 1-D, ASDM p.4-5, and is the allowable load in single shear for a 1"diameter A-325N bolt. Therefore the maximum load on the given group of fasteners, 14.17^k, is satisfactory.

DESIGN OF WOOD BEAM

Design a wood beam for the given loads.

Lumber to be #2 kiln dried Southern Pine.
The beam is a floor beam, so the use condition is "dry".
The live load deflection is limited to L/360, and the dead load deflection is to be less than L/240.



Check with snow, and without snow.

With snow:

$$M_{max} = \frac{(.060+.200)(18)^2}{8} + \frac{(.900+.600)(18)}{4 \times 1.15 \text{ (snow duration factor)}}$$

$$= 10.53 + 5.87 = 16.4 \text{ ft-k}$$

W.O. snow:

$$M_{max} = 10.53 + \frac{.600 \times 18}{4} = 10.53 + 2.70 = 13.23 \text{ ft-k}$$

For single use, the bending stress F_b, for #2KD So. Pine, is 1.300 ksi.

$$S_{req.} = 16.4 \times 12/1.300 = 151.38 \text{ in}^3$$

Try multiple sticks:

Since we are using multiple sticks, the allowable bending stress F_b is increased to 1.500 ksi.

The new required section modulus is then:

$$S_{req.} = 151.38(1.3/1.5) = 131.20 \text{ in}^3$$

3 x 12's	S=52.734	No. req.= 131.20/52.734 = 2.49	Use 3
2 x 12's	S=31.641	No. req.= 131.20/31.648 = 4.15	Use 4

For the 2 x 12's; f_b=16.4 x 12/(4 x 31.648)=1.555 ksi.

This represent an overstress of ((1.555/1.5)-1.000)(100)= 3.667% which is acceptable.

The areas of the choices are:

$$4 - 2 \times 12's = 4 \times 16.875 = 67.5 \text{ in}^2$$

$$3 - 3 \times 12's = 3 \times 28.125 = 84.38 \text{ in}^2$$

Since the area of the 2 x 12 combination is less than that of the 3 x 12's, we will try the 4 -2x12's.

The maximum shear, at a distance from the support equal to the depth of the member, is:

With snow:

$$V_{max} = ((.900+.600)/2)+(.060+.200)((18/2)-(11.25/12))$$

$$= .750 + 2.096 = 2.846 \text{ k}$$

For snow duration:

$$V_{max} = 2.846/1.15 = 2.475 \text{ k}$$

W.O. snow:

$$V_{max} = (.600/2) + 2.096 = 2.396 \text{ k}$$

$$f_v = (3/2)(2.475/67.5) = 0.055 \text{ ksi} < F_v = .095 \text{ ksi} \quad \text{OK}$$

The dead load deflection is:

$$\Delta_{DL} = \frac{\dfrac{(5)(.060/12)(18\text{x}12)^4}{384} + \dfrac{.600(18\text{x}12)^3}{48}}{(1{,}600/2)(4)(177.979)}$$

(1/2)E for creep effect

$$\Delta_{DL} = 0.47"$$

The live load deflection is:

$$\Delta_{LL} = \frac{\dfrac{(5)(.200/12)(216)^4}{384} + \dfrac{.900(216)^3}{48}}{(1{,}600)(4)(177.979)}$$

$$\Delta_{LL} = 0.58"$$

$$0.58" = L/372 < L/360 \quad \text{OK}$$

The total load deflection is:

$$\Delta_{TOT} = 0.47 + 0.58 = 1.05" = L/206 > L/240 \quad \text{Not acceptable.}$$

Using the 3 - 3 x 12's:

The properties of this combination are:

Area = 3 x 28.125 = 84.375 in^2

Moment of inertia = 3 x 296.631 = 889.89 in^4

Section modulus = 3 x 52.734 = 158.202 in^3

The maximum shear is:

f_v = (3/2)(2.475)/84.375 = 0.044 ksi << F_v= .095 ksi OK

f_b = (16.4 x 12)/ 158.202 = 1.244 ksi < F_b= 1.500 ksi OK

Δ_{DL} = 0.47 x 711.96/889.89 = 0.38"

Δ_{LL} = 0.58 x 711.96/889.89 = 0.46"

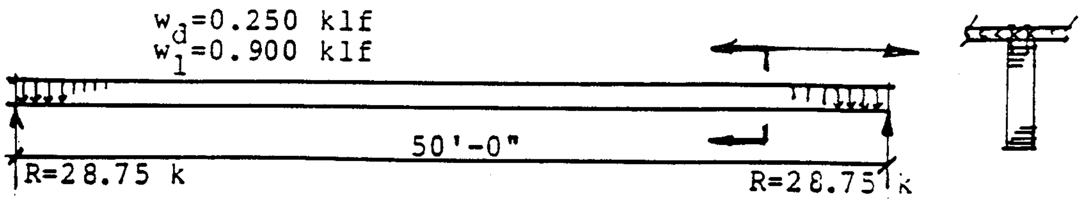
Δ_{TOT}= 0.84" = L/257 < L/240 OK

USE 3 - 3 x 12's

DESIGN OF GLULAM BEAM

Design a glulam roof beam for the given loads.

Lumber to be 22F Douglas Fir.
The use condition is "dry".
The live load deflection is limited to L/360, and the total load deflection is to be less than L/240.



By inspection, the total load condition will govern the design.

$$M_{max} = (0.250 + 0.900)(50)^2/8 = 359.4 \text{ ft-k}$$

The unsupported length of the beam is = 0, since the compression face is held by the roof planking.

Assume that the shape factor, $C_F = 0.90$,

$$F_b = 2.2 \times 0.90 \times 1.15 = 2.277 \text{ ksi}$$

where:

2.2 = F_b from allowable stress tables

0.90 = assumed shape factor

1.15 = snow load duration factor

Then, the required section modulus is:

$$S_{req.} = 359.4 \times 12 / 2.277 = 1{,}894 \text{ in}^3$$

Try a glulam beam 8 3/4 x 37.5 (25 - 1 1/2 lams)

Depth (in)	Area (in^2)	Moment of Inertia (in^4)	Section Modulus (in^3)
37.5	328.1	38,452.1	2050.8

$$C_F = (12/d)^{1/9} = (12/37.5)^{1/9} = 0.88$$

$$f_b = M/S = 354.4 \times 12 / 2050.8 = 2.103 \text{ ksi}$$

$$\text{Adj. } F_b = 2.2 \times .88 \times 1.15 = 2.226 > 2.103 \quad \text{OK}$$

Check shearing stress:

$$V' = 28.75 - (37.5/12)(1.150) = 25.16 \text{ k}$$

$$f_v = (3/2)(25.16)/328.1 = 0.115 \text{ ksi}$$

$$F_v = 0.165 \times 1.15 = 0.190 > 0.115 \qquad \text{OK}$$

Deflection calculations:

$$\Delta_{TL} = 180.0 \times M \times L^2 / E \times I$$

$$\Delta_{TL} = 180.0(359.4)(50)^2/(1800)(38{,}452.1)$$

$$= 2.34" = L/257 < L/240$$

$$\Delta_{LL} = 2.34\ (.900/1.150) = 1.83" = L/328 > L/360$$

Since this is about 10% over the allowable deflection value, a conservative approach would be to add an additional lamination, thus making the beam depth 39.0"

$$\text{The revised } I = 38{,}452.1 \times (39.0/37.5)^3$$

$$= 43{,}253.4 \text{ in}^4$$

$$\Delta_{LL} = 1.83 \times 38452.1/43253.4 = 1.63" = L/369 \text{ OK}$$

The dead load deflection, considering creep, and increasing the dead load deflection by 50%:

$$\Delta_{DL} = 1.5 \times 2.34 \times (.250/1.150) \times (38452.1/43253.4)$$

$$\Delta_{DL} = 0.68"$$

$$\Delta_{TL} = 0.68 + 1.63 = 2.31" = L/\ 260 < L/240 \text{ OK}$$

The bending and shear stresses are OK by inspection.

Provide a camber equal to 1.5 times the dead load deflection,

Camber = 1.5 x 0.68 = 1.02"

The required bearing length for the beam using F_c = 0.385 ksi, is:

Adj. F_c = 1.15 x 0.385 =

$\text{Area}_{req.}$ = 28.75/0.443 = 64.94 in^2

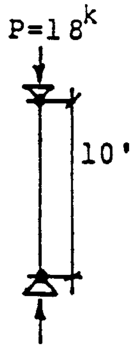
Bearing length, l_b= 64.94/8/75=7.42 say, 7 1/2"

USE 8 3/4"x39" GL beam (26 - 1 1/2" lams)

22F Douglas Fir, Camber 1"

WOOD COLUMN DESIGN

Design a solid section wood column for the given conditions. The column supports a roof, and the total live (snow) and dead load is 18 kips.
The bracing is the same for both the x and y axes.
The lumber is No. 1 Douglas fir-larch.



Try a 4 x 6 (3 1/2 x 5 1/2) (Joist Plank category)

Area = 19.25 in^2 $\quad$ $(l/d_y) = 10x12/3.5 = 34.3 < 50$

F_c = 1.25 ksi $\quad$ $K = 0.671(E/(F_c \times LDF))^{1/2}$

E = 1800 ksi $\quad$ $K = 0.671(1800/(1.25 \times 1.15))^{1/2}$

$K = 23.7 < 34.3$

Since the column l/d is greater than K and less than 50 it is a long column, and the allowable compressive strength parallel to the grain is :

$$F'_c = 0.3E/(l/d)^2 = 0.3(1800)/(34.3)^2 = 0.459 \text{ ksi}$$

Then: $P_{(allow.)} = F'_c x \text{ Area} = 0.459x19.25 = 8.84 \text{ k} < 18 \text{ k}$ N.G.

Try a 6 x 6 (5 1/2 x 5 1/2) (Post & Timber category)

Area = 30.25 in^2 $\quad$ $(l/d_y) = 10x12/5.5 = 21.8$

F_c = 1.00 ksi $\quad$ $K = 0.671(1600/(1.00 \times 1.15)^{1/2}$

E = 1600 ksi $\quad$ $K = 25.0 > 21.8$

The column l/d is between 11 (the limit of l/d for a short column) and K, therefore, it is an intermediate column, and the allowable compressive strength is equal to:

$$F'_c = F_c(LDF)(1-1/3((l/d)/K)^4)$$

$$F'_c = (1.00)(1.15)(1-1/3(21.8/25.0)^4) = 0.928 \text{ ksi}$$

The the allowable axial load on the column is:

$P_{(allow.)} = 0.928 \times 30.25 = 28.1 \text{ k} > 18 \text{ k}$ $\quad$ OK

USE 6 x 6, No.1 DF-L

In composite construction, the concrete slab is attached to the steel beam by means of steel shear studs which are welded to the beam flange. The resulting bonding of the concrete and steel permits the consideration of the system as a series of T-beams.

The AISC Code, Chapter I, ASDM p.5-56, sets the parameters for designing such members.

The methods of construction which may be considered in the design process are:

1. Shored construction, in which the steel beam and the formwork are set and shored before the concrete is placed.

2. Unshored construction, in this case, the steel beams and formwork are sized to support the dead load of the beams, concrete slab, formwork, and construction loads.

In this presentation only type 2 shall be considered.

Under system two, the steel beam is stressed and deflected to accommodate the applied loads.

To determine the properties of the composite section, the concrete is transformed to an equivalent steel section by reducing its effective width by the ratio of b_{eff}/n, where the portion of the effective width of the concrete slab on each side of the beam centerline shall not exceed the smallest of:

a. One eighth of the beam span, center to center of supports,
b. One-half the distance to the centerline of the adjacent beam,
c. The distance from the beam centerline to the edge of the slab. (ASDM Section I1, p.5-58)

The web and end connections of the steel beams are designed to carry the effects of the total load.

The allowable horizontal shear load for the stud connectors is given in ASDM Table I4.1, p.5-59.

In the following problem, the shear connectors are designed for the full composite action and they must carry the total horizontal shear to be resisted from the point of maximum moment to the points of zero mement.

Section I1, ASDM p.5-56 requires that the value of this shear be the smaller of:

$$V_h = \frac{0.85 f_c' A_c}{2}$$

$$V_h = \frac{A_s F_y}{2}$$

Where,

f_c' = the compressive strength of the concrete in ksi

A_c = the actual area, in square inches, of the effective concrete flange as defined in I1, ASDM p.5-56.

PROBLEM:

The floor framing for the lobby of an office building has beams spaced 9 feet on centers and spanning 28 feet.
Design a composite section using no shoring.
The steel is A36, the concrete f'_c=3,000 psi and is normal weight concrete, the allowable horizontal shear per 3/4" x 3" stud is 11.5^k per stud.
The LL = 100 psf, partitions = 20psf, and ceiling = 10psf.
The DL = 4" concrete slab @ 50psf, steel beams and metal formwork @ 30psf, (the metal formwork is considered as not conributing to the composite action of the system), and a construction load of 20 psf.
Limit the construction and dead load deflection to 1", and the LL deflection to L/360.
Because this is a composite beam, the metal form is welded to the top flange of the steel beam, F_b may then be taken as $0.66F_y$.

Loads

Before conc. hardens		Dead load		After conc. hardens	
Constr. load	.020 ksf			LL	.100 ksf
Slab	.050 ksf	Slab	.050 ksf	Part.	.020 ksf
St. bm & dk	.030 ksf	bm & dk	.030 ksf	Ceil.	.010 ksf
				Misc.	.005 ksf
	.100 ksf		.080 ksf		.135 ksf

	Unit ld (ksf)	Bm spcg (ft)	w/ft (klf)	W (k)	V (k)	M (k-ft)
w_{LL}	.135	9	1.215	34.02	17.01	119.07
w_{DL}	.080	"	0.720	20.16	10.08	70.56
w_{CL}	.100	"	0.900	25.20	12.60	88.20
w_{TL}	.215	"	1.935	54.18	27.09	189.63

$$S_{CL}\ \text{req.} = \frac{M_{CL} \times 12}{F_b} = \frac{88.20 \times 12}{.66(36)} = 44.10\ \text{in}^3$$

The minimum required transformed section modulus is:

$$S_{tr}\ \text{req.} = \frac{M_{TL} \times 12}{F_b} = \frac{189.63 \times 12}{24} = 94.82\ \text{in}^3$$

To limit the Δ_{CL} to 1", the required moment of inertia is:

$$I_{req} = \frac{M \times L^2}{161.11 \times \Delta} = \frac{88.20 \times 28^2}{161.11 \times 1} = 429\ \text{in}^4$$

$$E_c = 57\sqrt{f'_c} = 57\sqrt{3000} = 3{,}122 \text{ ksi}; \; E_s = 29{,}000 \text{ ksi}$$

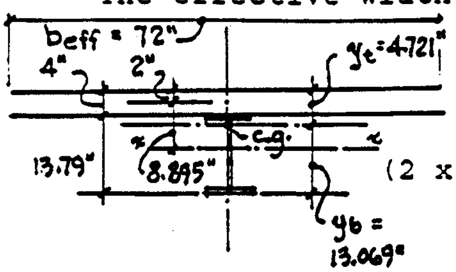
$$n = \frac{E_s}{E_c} = \frac{29{,}000}{3{,}122} = 9.2 \quad \therefore \quad n = 9$$

Select a trial section having an approximate section modulus equal to the average of the required S_{cL} and S_{tr}; in this case,

$$S_{trial} = \frac{44.10 + 94.82}{2} = 69.5 \text{ in}^3$$

Try a W14 x 48, $A=14.1$ in^2; $S=70.3$ in^3; $I=485$ in^4; $b_f=8.030$ in; $d=13.79$ in; $t_w=.340$ in; $t_f=0.595"$

The effective width b_e is the smallest of:



$$28 \times \frac{12}{4} = 84"$$

$$(2 \times 4 \times 8) + 8 = 72" \text{ (governs)}$$

$$9 \times 12 = 108"$$

$$b' = \frac{b_{eff}}{n} = \frac{72}{9} = 8"; \quad t = 4"$$

Item	A	y	Ay	Ay²	Io	Io +Ay²
I 8 x 4 =	32	8.895	284.64	2531.87	42.67	2574.54
II	14.1	0	0	0	485.00	485.00
	46.1		284.64		$I_{(x\ of\ bm)}$ =	3059.54
			$-Ay^2 = -(6.174 \times 284.64)$ =			-1757.37
					$I_{o\ (of\ comp\ bm)}$ =	1302.17

$$y_{bar} = \frac{284.64}{46.1} = 6.174"$$

$$y_t = (8.895+2)-6.174 = 4.721"$$

$$y_b = 6.895+6.174 = 13.069"$$

$$S_t = \frac{1302.17}{4.271} = 275.83 \text{ in}^3$$

$$S_{tr} = S_{bott} = \frac{1302.17}{13.069} = 99.64 \text{ in}^3 > \min S_{tr} = 94.82$$

$$\therefore\ f_{(CL)} = \frac{M_{CL}(12)}{S_s} = \frac{(88.20)(12)}{70.3} = 15.06 \text{ ksi} < F_b = 23.76 \text{ ksi} \quad \text{OK}$$

$$f_{(top)} = \frac{M_{LL}(12)}{nS_s} = \frac{(119.07)(12)}{(9)(275.83)} = 0.576 \text{ ksi} < 0.45\ f_c' \quad \text{OK}$$

$$f_{(bott)} = \frac{M_{TL}(12)}{S_{TR}} = \frac{(189.63)(12)}{99.64} = 22.84 \text{ ksi} < F_b = 23.76 \text{ ksi OK}$$

$$\Delta_{(const.\ ld)} = \frac{(M_{CL})(L^2)}{(161.11)(I_s)} = \frac{(88.20)(28^2)}{(161.11)(485)} = 0.89" < 1" \quad \text{OK}$$

$$\Delta_{(LL)} = \frac{(M_{LL})(L^2)}{(161.11)(I_{TR})} = \frac{(119.07)(28^2)}{(161.11)(1302.17)} = 0.45" < \frac{L}{360} = 0.93"$$

$$\Delta_{DL} = \frac{(M_{DL})(L^2)}{(161.11)(I_s)} = \frac{(70.56)(28^2)}{(161.11)(485)} = 0.71"$$

The required shear to be carried by the studs is the smaller of:

$$V_h = \frac{0.85\ f_c'\ bt}{2} = \frac{(0.85)(3)(72)(4)}{2} = 367.2^k$$

$$V_h = \frac{A_s F_y}{2} = \frac{(14.1)(36)}{2} = 253.8^k \quad \text{- Governs}$$

The stud diameter shall not be greater than 2.5 time the thickness of the steel beam flange, ASDM I4, p.5-60.

$(2.5)(0.595) = 1.49" > 0.75"$, $\therefore$ 3/4" dia studs are OK.

At 11.5^k per stud, the number of studs required to carry a shear of 253.8^k is:

$$N = \frac{253.8}{11.5} = 23.07, \text{ use 24 studs each side of beam center line.}$$

Use connectors in pairs, the spacing perpendicular to beam center line, per ASDM p.5-60, is four time the stud diameter.

$$\therefore \text{spacing} = (4)(.75) = 3"$$

As per ASDM p.5-60:

Min. longitudinal spacing of studs is 6 time the stud diameter, i.e.,

$$(6)(0.75) = 4.5"$$

Max. longitudinal spacing is 8 time the slab thickness, i.e.,

$$(8)(4) = 32"$$

The actual stud sapcing, longitudinally, is:

$$\frac{(28)(12)}{\left[\frac{24}{2}\right](2)} = 14" > 4.5", \text{ and } < 32"; \quad \therefore \text{ OK}$$

The total shear must be carried by the web of the steel beam, then f_v is:

$$f_v = \frac{V_{Tot}}{dt_w} = \frac{27.09}{(13.79)(0.340)} = 5.76 \text{ ksi} < F_v = 14 \text{ ksi} \quad \text{OK}$$

EXAMPLE

Using LRFD, select a beam to carry a simply supported uniformly distributed load of 1.0 ^{k}/FT LL and 0.5 ^{k}/FT DL. Use $F_y = 50$ ksi and consider beam as having full lateral support. The beam span = 40 FT.

Solution:

Estimated beam weight = 50 plf.

$\therefore\ w_u = 1.2(.50 + 0.05) + 1.6(1.0) = 0.66 + 1.6 = 2.26\ ^{k}/_{FT}$ LRFDM p. 6-25

$$M_u = \frac{w_u L^2}{8} = \frac{2.26(40)^2}{8} = 452^{1k}$$

$$Z_{x\,REQ} = \frac{M_u}{\phi_b F_y} = \frac{452 \times 12}{0.90(50)} = 121 \text{ in}^3$$

Try a W 24x55 $Z_x = 134 \text{ in}^3$; $I = 1350 \text{ in}^4$

Check for a LL deflection limited to $\frac{L}{360} = \frac{40 \times 12}{360} = 1.33''$

Using service load Moment for deflection check:-

$$M_{LL} = \frac{(1.01)(40)^2}{8} = 200^{1k}\text{; and } \Delta_{LL} = \frac{M_{LL} L^2}{161\, I} = \frac{(200)(40)^2}{161(1350)} = 1.47'' > 1.33 \text{ N.G.}$$

$$I \text{ req. for } \Delta_{LL} = 1.33'' = \frac{200 \times 40^2}{161 \times 1.33} = 1495 \text{ in}^4$$

Try W24x62 ; $Z_x = 153$; $I = 1550 \text{ in}^4$

w_u revised $= 1.2(0.50 + 0.062) + 1.6(1.0) = 2.27\ ^{k}/_{FT}$

$$M_u = \frac{2.27 \times 40^2}{8} = 454^{1k}$$

$$M_p = Z_x F_y = (153 \times 50)/12 = 638^{1k}$$

$$\phi M_p = 0.90(638) = 574^{1k} > 454^{1k} \quad \text{O.K.}$$

$$\text{actual } \Delta_{LL} = \frac{(200)(40)^2}{161(1550)} = 1.28'' < 1.33'' \quad \text{O.K.}$$

Check compact section limits

From p. 1-25 LRFDM

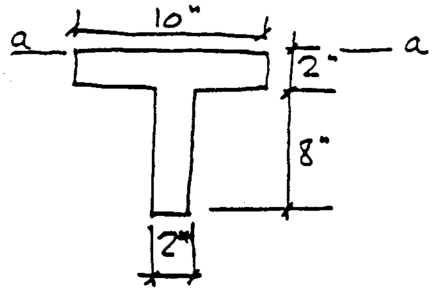
$\frac{b}{2t_f} = 6.0$; $\frac{h_c}{t_w} = 50.1$

Flange: $\lambda = \frac{b}{2t_f} \le \frac{65}{\sqrt{F_y}} \Rightarrow 6.0 < \frac{65}{\sqrt{50}} = 9.19$ O.K.

Web: $\lambda = \frac{h_c}{t_w} < \frac{640}{\sqrt{F_y}} \Rightarrow 50.1 < \frac{640}{\sqrt{50}} = 90.5$ O.K.

W24x62 is satisfactory

Determine M_y, Z and M_n for a steel T beam having the following section:- If the beam span is 20^{FT} find the nominal udl w_u — $F_y = 36$ ksi



M_y: —

$$A = (10\times 2) + (8\times 2) = 36^{\square''}$$

$$\bar{y}_{(a\text{-}a)} = \frac{(10\times 2\times 1) + (8\times 2\times 6)}{36} = \frac{20+96}{36} = 3.22'' = y_t \; ; \; y_b = 6.78''$$

$$I = \frac{1}{3}(2)(1.22^3 + 6.78^3) + \frac{1}{12}(10\times 2^3) + 20(2.22)^2$$

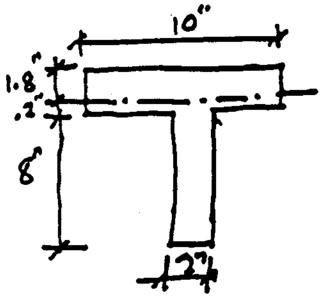
$$= 208.99 + 6.67 + 98.57 = 314.22 \text{ in}^4$$

$$\text{Smallest } S = \frac{I}{y_b} = \frac{314.22}{6.78} = 46.35 \text{ in}^3$$

$$M_y = SF_y = (46.35)(36)/12 = 139.04^{\,\text{K}}$$

Z:- Plastic areas above and below N.A. must be =.

$20 - 16 = 4$ ∴ Top area $= 20 - 2 = 18^{\square''}$
Bot area $= 16 + 2 = 18^{\square''}$



$$Z = (18\times 0.9) + (2.0\times 0.1) + (16\times 4.2) = 16.2 + 0.2 + 67.2 = 83.6 \text{ in}^3$$

$$M_n = F_y Z = (36)(83.6)/12 = 250.8 \text{ ft·K}$$

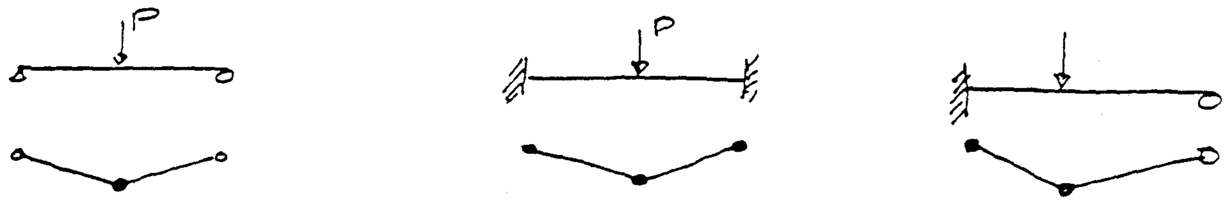
$$\text{Shape Factor} = \frac{M_n}{M_y} = \frac{250.8}{139.04} = 1.80$$

$$w_u = \frac{M_n 8}{L^2} = \frac{250.8\times 8}{20^2} = 5.02 \text{ K/FT}$$

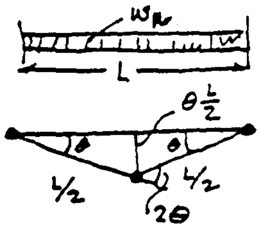
w_n

Plastic Hinges

○ = REAL HINGE ; ● = PLASTIC HINGE



VIRTUAL WORK METHOD:-



system is symmetrical, the end rotations, at plastic hinges, are equal. (θ)
By geometry:. the middle rotation is twice that of either end. (2θ)

work = Load or Force × distance = tot. load × av. deflect

The total load performs work which is equal to $(w_n L)\left[\left(\frac{\theta L}{2}\right)\left(\frac{1}{2}\right)\right]$

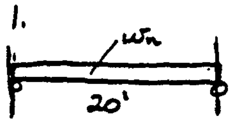
Virtual work says $\Rightarrow$ Σ ext. work = Σ int. work
$W_e = W_i$

W_i is the sum of the M_n at each plastic hinge times the angle through which it works.

$$\therefore \quad (w_n L)\left[\left(\frac{\theta L}{2}\right)\left(\frac{1}{2}\right)\right] = M_n(\theta + 2\theta + \theta)$$

$$\frac{w_n L^2}{16} = M_n$$

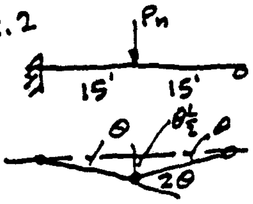
Ex. 1.



$$w_n(20)\left[\left(\frac{\theta \times 20}{2}\right)\left(\frac{1}{2}\right)\right] = M_n(4\theta)$$

$$w_n = \frac{M_n}{25}$$

Ex. 2



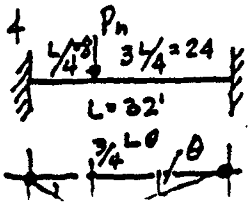
$$W_e = W_i$$

$$P_n\left(\theta\frac{L}{2}\right) = M_n(\theta + 2\theta)$$

$$\frac{P_n L}{6} = M_n$$

$$P_n = \frac{6 M_n}{30} = \frac{M_n}{5}$$

Ex. 4



$$W_e = W_i$$

$$P_n\left(3\theta \times \frac{L}{4}\right) = M_n(\theta + 4\theta + 3\theta)$$

$$P_n = \frac{M_n\, 8\theta \times 4}{L\,\theta\, 3} = \frac{10.67\, M_n}{L} = \frac{10.67\, M_n}{32} = .33\, M_n$$

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IV. Sanitary Engineering

Instructor: Ronald Sharpin

Tapes

6 7

PROFESSIONAL ENGINEER REFRESHER COURSE

SANITARY ENGINEERING REVIEW

OVERVIEW

A. WATER SUPPLY ENGINEERING

1. WATER CHEMISTRY

2. REGULATIONS

3. WATER TREATMENT

   - SIZE AND LOCATION
   - UNIT PROCESSES

B. WASTEWATER TREATMENT

C. SOLID WASTE - LANDFILL

WATER SUPPLY ENGINEERING

WATER CHEMISTRY

WATER CHEMISTRY

ELEMENT - SMALLEST PARTICLE THAT RETAINS CHARACTERISTICS OF THE ORIGINAL ELEMENT (ATOM)

- 100 KNOWN
- 92 NATURALLY OCCURRING

- IMPORTANT IN WATER CHEMISTRY

ATOM - THREE FUNDAMENTAL PARTICLES

- PROTON
- NEUTRON
- ELECTRON

- ELECTRONS = PROTONS = ELECTRICALLY STABLE
- ELECTRONS ≠ PROTONS = ELECTRICALLY UNSTABLE = ION

MATTER - ANYTHING THAT OCCUPIES SPACE AND HAS MASS (WEIGHT)

- SOLID
- LIQUID
- GAS

- PURE FORM (FEW EXIST - CARBON, OXYGEN)

- COMPOUND = TWO OR MORE ELEMENTS BONDED TOGETHER = MOLECULE (H_2O = WATER)
  = OVER 2 MILLION IDENTIFIED

Elements Important In Water Treatment

Element	Symbol	Element	Symbol	Element	Symbol
Aluminum	Al	Chromium	Cr	Oxygen	O
Arsenic	As	Fluorine†	F	Phosphorus	P
Barium	Ba	Hydrogen	H	Potassium	K
Boron	B	Iodine	I	Radium	Ra
Bromine	Br	Iron	Fe	Selenium	Se
Cadmium	Cd	Lead	Pb	Silicon	Si
Calcium	Ca	Magnesium	Mg	Silver	Ag
Carbon	C	Manganese	Mn	Sodium	Na
Chlorine	Cl	Mercury	Hg	Strontium	Sr
Copper	Cu	Nitrogen	N	Sulfur	S

- MIXTURE = TWO OR MORE ELEMENTS OR COMPOUNDS MIXED WITHOUT BONDING
  (CAN BE SEPARATED BY PHYSICAL MEANS SUCH AS SETTLING AND FILTERING)
  ($NaCl + H_2O$ = SALT WATER)

- VALENCE ELECTRONS = ELECTRONS IN THE OUTERMOST SHELL ON THE ATOM
  (NUMBER IS MOST IMPORTANT IN DETERMINING STABILITY AND THUS CHEMICAL REACTIONS)

- VALENCE = NUMBER RELATING TO AN ELEMENT'S VALENCE ELECTRONS THAT INDICATES THE ABILITY OF AN ELEMENT TO REACT

- COMMON ELEMENT VALENCES

- FORMULA = REPRESENTATION SHOWING:

  - WHAT ELEMENTS ARE PRESENT
  - NUMBER OF ATOMS OF EACH

ELEMENT

- $CaCO_3$

  - ONE ATOM OF CALCIUM (Ca)
  - ONE ATOM OF CARBON (C)
  - THREE ATOMS OF OXYGEN (O_3)

Oxidation Numbers of Various Elements

Element	Common Valences	Element	Common Valences
Aluminum (Al)	+3	Lead (Pb)	+2, +4
Arsenic (As)	+3, +5	Magnesium (Mg)	+2
Barium (Ba)	+2	Manganese (Mn)	+2, +4
Boron (B)	+3	Mercury (Hg)	+1, +2
Bromine (Br)	−1	Nitrogen (N)	+3, −3, +5
Cadmium (Cd)	+2	Oxygen (O)	−2
Calcium (Ca)	+2	Phosphorus (P)	−3
Carbon (C)	+4, −4	Potassium (K)	+1
Chlorine (Cl)	−1	Radium (Ra)	+2
Copper (Cu)	+1, +2	Selenium (Se)	−2, +4
Chromium (Cr)	+3	Silicon (Si)	+4
Fluorine (F)	−1	Silver (Ag)	+1
Hydrogen (H)	+1	Sodium (Na)	+1
Iodine (I)	−1	Strontium (Sr)	+2
Iron (Fe)	+2, +3	Sulfur (S)	−2, +4, +6

Oxidation Numbers of Common Radicals

Radical	*Common Valences*
Ammonium (NH_4)	+1
Bicarbonate (HCO_3)	−1
Hydroxide (OH)	−1
Nitrate (NO_3)	−1
Nitrite (NO_2)	−1
Carbonate (CO_3)	−2
Sulfate (SO_4)	−2
Sulfite (SO_3)	−2
Phosphate (PO_4)	−3

- $Ca(OH)_2$

  - ONE ATOM OF CALCIUM (Ca)
  - TWO HYDROXYL IONS $(OH)_2$ WITH:

    * TWO ATOMS OF OXYGEN (O_2)
    * TWO ATOMS OF HYDROGEN (H_2)

MOLE = MOLECULAR WEIGHT OF A SUBSTANCE EXPRESSED AS GRAMS

$$\text{Number of Moles} = \frac{\text{Total Weight}}{\text{Molecular Weight}}$$

EXAMPLE:

$Ca(HCO_3)_2$ +	$Ca(OH)_2 \rightarrow$	$2CaCO_3$ +	$2H_2O$
162.12	74.10	2(100.09)	2(18.02)
(1 Mole)	(1Mole)	(2 Moles)	(2 Moles)

PROBLEM: A LAB PROCEDURE CALLS FOR 6.0 MOLES OF SODIUM BICARBONATE ($NAHCO_3$) AND 0.20 MOLE OF POTASSIUM CHROMATE (K_2CrO_4). HOW MANY GRAMS OF EACH COMPOUND ARE REQUIRED?

	No. of Atoms		*Atomic Weight*		*Total Weight*
Sodium (Na)	1	X	22.99	=	22.99
Hydrogen (H)	1	X	1.01	=	1.01
Carbon (C)	1	X	12.01	=	12.01
Oxygen (O)	3	X	16.00	=	48.00
Molecular weight of NaHCO3				=	84.01

Therefore, 1 Mole of $NaHCO_3$ weighs 84.01 g.

Next, multiply the weight of 1 mole by the number of moles required:

6 Moles of $NaHCO_3$ are required.
6 Moles of $NaHCO_3$, weighs (6)(84.01 G) = 504.06 g.

	No. of Atoms		Atomic Weight		Total Weight
Potassium (K)	2	X	39.10	=	78.20
Hydrogen (H)	1	X	52.00	=	52.00
Carbon (C)	4	X	16.00	=	64.00

Molecular weight of K2CR04 = 194.20
Therefore, 1 Mole of K_2CrO_4 weighs 194.20 G.

Next, Multiply the Weight of 1 Mole by the Number of Moles Required:

0.20 Mole of K_2CrO_4 are Required.
0.20 Mole of K_2CrO_4 WEIGHS (0.20)(194.20 G) = 38.84 g.

MEASURE OF CONCENTRATION:

$$\text{Molarity} = \frac{\text{Moles of Solute}}{\text{Litres of Solution}}$$

PROBLEM: IF 0.4 MOLES OF NaOH IS DISSOLVED IN 2L OF SOLUTION, WHAT IS THE MOLARITY OF THE SOLUTION?

$$\text{Molarity} = \frac{\text{Moles of Solute}}{\text{Litres of Solution}}$$

$$= \frac{\text{0.4 Moles}}{\text{2 L Solution}}$$

$$= \text{0.2 Molarity}$$

$$= 0.2\ M \text{ Solution}$$

% element by weight = $\frac{\text{Weight of element in compound}}{\text{Molecular weight of compound}} \times 100$

PROBLEM: WHAT IS THE % (WEIGHT) OF Na AND Cl IN SODIUM CHLORIDE?

FORMULA = NaCl

- ONE ATOM OF SODIUM
- ONE ATOM OF CHLORIDE

ATOMIC WEIGHT: Na = 22.99
Cl = 35.45

MOLECULAR WEIGHT NaCl = 1 X 22.99
1 x 35.45
58.44

% (WEIGHT): Na = $\frac{22.99}{58.44} \times 100 = 39.3\%$

Cl = $\frac{35.45}{58.44} \times 100 = 60.7\%$

PROBLEM: DETERMINE THE POUNDS OF EACH ELEMENT IN 100 POUNDS OF CALCIUM BICARBONATE, $Ca(HCO_3)_2$

	No. of Atoms		*Atomic Weight*		*Total Weight*
Calcium (Ca)	1	X	40.08	=	40.08
Hydrogen (H)	2	X	1.01	=	2.02
Carbon (C)	2	X	12.01	=	24.02
Oxygen (O)	6	X	16.00	=	96.00

Molecular weight of $Ca(HCO_3)_2$ = 162.12

THE PERCENT OF WEIGHT:

$$\% Ca = \frac{\text{Weight of Ca in compound}}{\text{Molecular wgt. of compound}} \times 100$$

$$= \frac{40.08}{162.12} \times 100$$

$$= 0.247 \times 100$$

= 24.7% Ca by weight

$$\% H = \frac{\text{Weight of H in compound}}{\text{Molecular wgt. of compound}} \times 100$$

$$= \frac{2.02}{162.12} \times 100$$

$$= 0.012 \times 100$$

= 1.2% H by weight

$$\% C = \frac{\text{Weight of C in compound}}{\text{Molecular wgt. of compound}} \times 100$$

$$= \frac{24.02}{162.12} \times 100$$

$$= 0.148 \times 100$$

= 14.8% C by weight

$$\%O = \frac{\text{Weight of O in compound}}{\text{Molecular wgt. of compound}} \times 100$$

$$= \frac{96.00}{162.12} \times 100$$

$$= 0.592 \times 100$$

$= 59.2\%$ O by weight

POUNDS OF EACH ELEMENT PRESENT:

lbs. Ca	=	(0.247)(100)	=	24.7
lbs. H	=	(0.012)(100)	=	1.2
lbs. C	=	(0.148)(100)	=	14.8
lbs. O	=	(0.592)(100)	=	<u>59.2</u>
				99.9

CHEMICAL EQUATION = REPRESENTATION OF REACTION OF CHEMICALS EXPRESSED WITH CHEMICAL FORMULAS.

EXAMPLE:

calcium bicarbonate	*plus*	calcium hydroxide	react to form	calcium carbonate	*plus*	water
$Ca(HCO_3)_2$	+	$Ca(OH)_2$	$\rightarrow$	$2CaCO_3$	+	$2H_2O$

REACTANTS — *PRODUCTS*

PROBLEM: DETERMINE THE MOLECULAR WEIGHTS FOR REACTANTS AND PRODUCTS IN THE EQUATION.

THE MOLECULAR WEIGHT FOR $Ca(HCO_3)_2$:

	No. of Atoms		Atomic Weight		Total Weight
Calcium (Ca)	1	X	40.08	=	40.08
Hydrogen (H)	2	X	1.01	=	2.02
Carbon (C)	2	X	12.01	=	24.02
Oxygen (O)	6	X	12.00	=	96.00

Molecular weight of $Ca(HCO_3)_2$ = 162.12

The molecular weight for $Ca(OH)_2$:

	No. of Atoms		Atomic Weight		Total Weight
Calcium (Ca)	1	X	40.08	=	40.08
Oxygen (O)	2	X	16.00	=	32.00
Hydrogen (H)	2	X	1.01	=	2.02

Molecular weight of $Ca(OH)_2$ = 74.10

The molecular weight of $CaCO_3$:

	No. of Atoms		Atomic Weight		Total Weight
Calcium (Ca)	1	X	40.08	=	40.08
Carbon (C)	1	X	12.01	=	12.01
Oxygen (O)	3	X	16.00	=	48.00

Weight of one molecule $CaCO_3$	=	100.09
Weight of two molecules $CaCO_3$	=	(2)(100.09)
	=	200.18

The molecular weight for H_2O:

	No. of Atoms		*Atomic Weight*		*Total Weight*
Hydrogen (H)	2	X	1.01	=	2.02
Oxygen (O)	1	X	16.00	=	16.00

Weight of one molecule H_2O	=	18.02
Weight of two molecules H_2O	=	(2)(18.02)
	=	36.04

IN SUMMARY, THE WEIGHTS THAT CORRESPOND TO EACH TERM OF THE EQUATION ARE:

$Ca(HCO_3)_2$	+	$Ca(OH)_2$	$\longrightarrow$	$2CaCO_3$	+	$2H_2O$
162.12		74.10		200.18		36.04
236.22				236.22		

PROBLEM: IF 25 POUNDS OF CALCIUM HYDROXIDE WERE ADDED TO CALCIUM BICARBONATE, HOW MANY POUNDS OF CALCIUM BICARBONATE COULD REACT WITH THE CALCIUM HYDROXIDE? FROM THE PREVIOUS PROBLEM, THE EQUATION FOR THE REACTION IS:

$$Ca(HCO_3)_2 + Ca(OH) \dashrightarrow$$
$$162.12 \qquad\qquad 74.10$$

Known Ratio — *Desired Ratio*

$$\frac{74.10 \text{ lbs. } Ca(OH)_2}{162.12 \text{ lbs. } Ca(HCO_3)_2} = \frac{25 \text{ lbs. } Ca(OH)_2}{x \text{ lbs } Ca(HCO_3)_2}$$

$$\frac{74.10}{162.12} = \frac{25}{x}$$

$$\frac{(x)(74.10)}{162.12} = 25$$

$$x = \frac{(25)(162.12)}{74.10}$$

$$x = 54.7 \text{ lbs. } Ca(HCO_3)_2$$

MEASURE OF STRENGTH:

$$\text{Percent Strength (by weight)} = \frac{\text{Weight of Solute}}{\text{Weight of Solution}} \times 100$$

NOTE: Weight of solution = Weight of solute + Weight of solvent

PROBLEM: IF 50 LBS. OF CHEMICAL IS ADDED TO 1,000 LBS. OF WATER, WHAT IS THE PERCENT STRENGTH OF THE SOLUTION BY WEIGHT?

Weight of solution = Weight of solute + Weight of solvent

= 50 lb. 1000 lb.

= 1050 lb. solution

USING THIS INFORMATION, CALCULATE THE PRESENT CONCENTRATION:

$$\text{Percent strength (by weight)} = \frac{\text{Weight of Solute}}{\text{Weight of Solution}} \times 100$$

$$\frac{\text{50 lb. Chemical}}{\text{1050 lb. Solution}} \times 100$$

= 0.048 X 100

= 4.8% Strength Solution

PROBLEM: YOU WISH TO PREPARE 100 GAL. OF A 4 PERCENT STRENGTH SOLUTION. HOW MUCH WATER AND CHEMICAL SHOULD BE MIXED TOGETHER? (ASSUME THE SOLUTION WILL HAVE THE SAME DENSITY AS WATER: 8.34 LB/GAL.)

$$\text{Percent Strength (by weight)} = \frac{\text{Weight of Solute}}{\text{Weight of Solution}} \times 100$$

$$4\% = \frac{x \text{ lb. Chemical}}{(100)(8.34) \text{ lb. solution}} \times 100$$

$$4 = \frac{x}{(100)(8.34)} \times 100$$

$$\frac{4}{1} = \frac{(x)(100)}{(100)(8.34)}$$

$$\frac{(100)(8.34)(4)}{1} = (x)(100)$$

$$\frac{(100)(8.34)(4)}{100} = x$$

33.36 lb chemical $= x$

To Calculate the Pounds of Water, First Calculate the Total Pounds of Solution:

(100 Gal. Solution)(8.34 lb/gal) = 834 lb. solution

The 834 lb. of solution contains both water and chemical. Since the pounds of chemical are known, the pounds of water can be determined.

834 lb. solution	=	? lb. water	+	33.36 lb. chemical

lb. water = 800.64

EQUIVALENT WEIGHT = THE WEIGHT OF AN ELEMENT OR COMPOUND WHICH IN A CHEMICAL REACTION HAS THE SAME COMBINING CAPACITY AS 8 GRAMS OF OXYGEN OR 1 GRAM OF HYDROGEN (MOLECULAR WEIGHT DIVIDED BY THE OXIDATION NUMBER OR COMMON VALENCE)

$$\text{Number of Equivalent Weights} = \frac{\text{Total weight}}{\text{Equivalent wgt.}}$$

PROBLEM: IF 90 G. OF SODIUM HYDROXIDE (NaOH) WERE USED IN MAKING UP A SOLUTION, HOW MANY EQUIVALENT WEIGHTS WERE USED? USE 40.00 G. AS THE EQUIVALENT WEIGHTS FOR NaOH.

$$\text{Number of Equivalent Weights} = \frac{\text{Total weight}}{\text{Equivalent weight}}$$

$$= \frac{90\text{ g}}{40\text{ g}}$$

$$= 2.25 \text{ equivalent weights}$$

PROBLEM: A SURFACE WATER HAS THE FOLLOWING ANALYSIS: CALCIUM 72.0 MG/L, MAGNESIUM 48.8 MG/L, SODIUM 9.2 MG/L, BICARBONATE 305 MG/L, SULFATE 134.4 MG/L, CHLORIDE 7.1 MG/L

- CALCULATE THE NUMBER OF MILLIEQUIVALENTS PER LITER (MEQ/L) FOR EACH SUBSTANCE.

- USE THE EQUIVALENT WEIGHTS TO CALCULATE THE CONCENTRATION FOR EACH SUBSTANCE EXPRESSED AS MG/L OF $CaCO_3$

CONCENTRATION IN MEQ/L CAN BE CALCULATED BY THE EQUATION:

$$\text{meq/l} = \frac{\text{mg/l}}{\text{equivalent weight}}$$

CONCENTRATION EXPRESSED AS MG/L $CaCO_3$ CAN BE CALCULATED FROM THE EQUATION:

$$\text{mg/l } CaCO_3 = \text{mg/l} \times \frac{50}{\text{equivalent wgt.}}$$

The results are shown in the Table:

Component	mg/L	Equivalent Weight	meq/l	mg/L as $CaCO_3$
Calcium Ca^{++}	72.0	20.0	3.6	180
Magnesium Mg^{++}	48.8	12.2	4.0	200
Sodium Na^{+}	9.2	23.0	0.4	20
			8.0	400
Bicarbonate HCO_3^-	305	61.0	5.0	250
Sulfate SO_4^{--}	134.4	48.0	2.8	140
Chloride Cl^-	7.1	35.5	0.2	10
			8.0	400

REGULATIONS

REGULATIONS

BEFORE 1986

NATIONAL INTERIM PRIMARY DRINKING WATER REGULATIONS:

- INORGANICS (10)
- ORGANICS (7)
- RADIONUCLIDES (5)
- MICROBIAL (1)
- TURBIDITY

SECONDARY DRINKING WATER REGULATIONS:

- TASTE
- ODOR
- APPEARANCE

SAFE DRINKING WATER ACT (1974)

SDWA AMENDMENTS (1986):

- INORGANICS (13)
- ORGANICS (40)
- RADIONUCLIDES (3)

MAJOR ISSUES:

- SURFACE WATER TREATMENT RULE (SWTR)
- LEAD AND COPPER RULE (L/C)
- DISINFECTION BY-PRODUCTS RULE (DBP)
- WELLHEAD PROTECTION

SWTR	-	IMPROVE DISINFECTION (INCREASE CHLORINE AND DECREASE pH)
L/C -		REDUCE CORROSION (INCREASE pH)
DBP -		REDUCE TOTAL TRIHALOMETHANES (DECREASE CHLORINE)

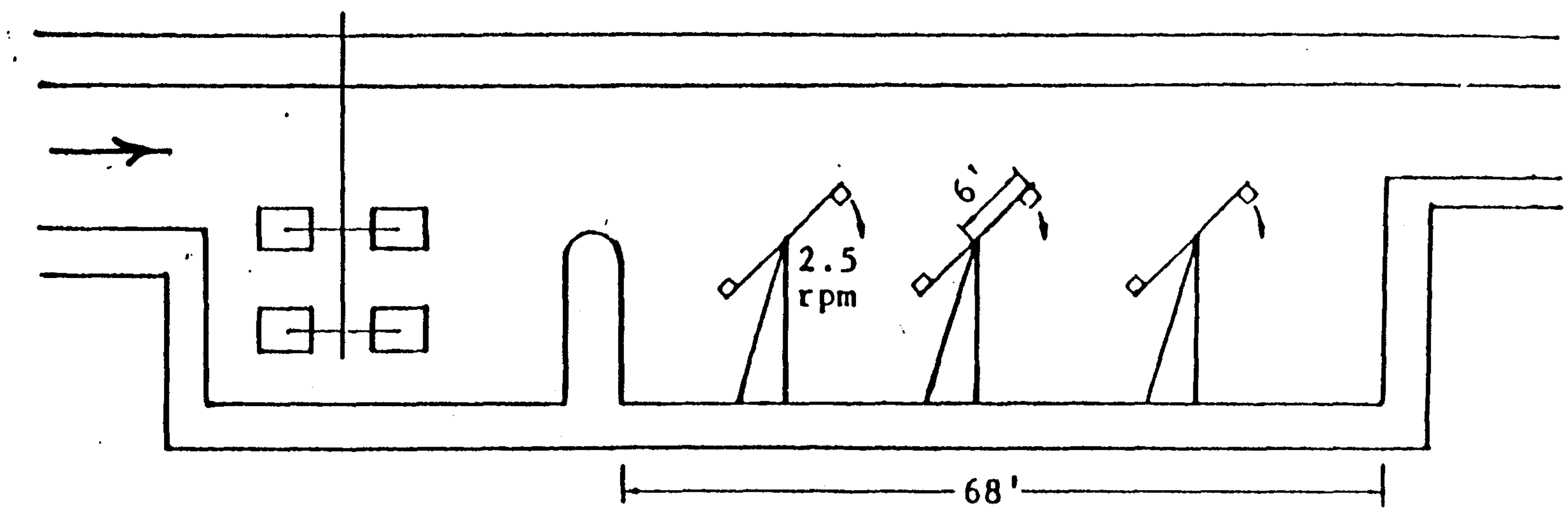
WATER TREATMENT

PROBLEM: FLOCCULATOR DESIGN

A FLOCCULATOR DESIGNED TO TREAT 15 MGD IS 68 FT. LONG, 31 FT. WIDE, AND 15 FT. DEEP. IT IS EQUIPPED WITH 12-INCH PADDLES SUPPORTED PARALLEL TO AND MOVED BY THREE HORIZONTAL SHAFTS WHICH ROTATE AT A SPEED OF 2.5 RPM. THE RADIUS OF THE PADDLES IS 6.0 FROM THE SHAFT, WHICH IS AT MID-DEPTH OF THE TANK. TWO PADDLES ARE MOUNTED ON EACH SHAFT, ONE OPPOSITE THE OTHERS AS SHOWN IN THE FIGURE. THE MEAN VELOCITY OF THE WATER IS APPROXIMATELY 1/4 OF THE VELOCITY OF THE PADDLES AND THEIR DRAG COEFFICIENT IS 1.8. ASSUME A WATER TEMPERATURE OF 50°F, AND DYNAMIC VISCOSITY = 2.74×10^{-5} (1 LB. FORCE) (SEC)/SQ. FT.

DETERMINE:

1. THE VELOCITY DIFFERENTIAL BETWEEN THE PADDLES AND THE WATER.



Section of Flocculator tank

2. THE USEFUL POWER INPUT AND THE ENERGY CONSUMPTION.

3. THE DETENTION TIME, T.

4. THE VALUE OF VELOCITY GRADIENT, G, AND THE PRODUCT, GT.

5. THE FLOCCULATOR LOADING.

SOLUTION:

1. ROTATION SPEED:

$$V_p = 2\pi rn = \frac{2\pi \times 6 \times 2.5}{60} = 1.57 \text{ fps}$$

THE VELOCITY DIFFERENTIAL BETWEEN PADDLES AND FLUID IS:

$$(1 - 0.25)(1.57) = 1.17 \text{ fps}$$

2. POWER INPUT:

$$P = C_D \times A \times \frac{D}{} \times \frac{V^3}{2g}$$

$C_D = 1.8$; $D = 62.4$ lbs/cu. ft.

Paddle area, $A = 3 \times 2 \times (31 - 1) \times \frac{12}{12} = 180$ sq. ft.

$V = 1.17$ fps

$$P = 1.8 \times 180 \times \frac{62.4}{62.4} \times (1.17)^3 = 502 \text{ ft-lbs/sec.}$$

$$\text{POWER CONSUMPTION} = \frac{502}{550} = 0.91 \text{ HP}$$

3. DETENTION TIME:

$$t = \frac{68 \text{ ft} \times 31 \text{ ft} \times 15 \text{ ft} \times 7.48 \text{ gal/cu ft} \times 24 \text{ hrs/day} \times 60 \text{ min/hr.}}{15 \times 10^6 \text{ gal.}}$$

$= 22.7$ min.

4. VELOCITY GRADIENT G AND PRODUCT GT:

$$G = \sqrt{\frac{P}{V\text{-}u}}$$

WHERE:

P = POWER
V = BASIN VOLUME
U = DYNAMIC VIS.

$$G = \left[\frac{502 \times 10^5}{68 \times 31 \times 15 \times 2.74}\right]^{1/2} = 24.1 \text{ fps/ft.}$$

$Gt = 24.1$ fps/ft $\times$ 22.7 min. $\times$ 60 sec/min.

$= 3.2 \times 10^4$ (unitless)

(This is satisfactory since the acceptable value is between 10^4 - 10^5.)

5. FLOCCULATOR LOADING:

$$F.L. = \frac{Q}{V} = \frac{15 \times 10^6 \text{ gals/day}}{68 \text{ ft.} \times 31 \text{ ft.}}$$

$$= 7{,}116 \text{ gpd/sq. ft.}$$

SIZE AND LOCATION - SITING

- SOURCE OF WATER
- LAND AVAILABILITY/CONTROL
- COST
- TAXES
- POWER SUPPLY
- SEWERAGE AVAILABILITY
- ENVIRONMENTAL IMPACT
- TRAFFIC IMPACT

- ELEVATION
  - 15 FEET ENERGY HEAD FOR GRAVITY FLOW THROUGH TRADITIONAL TREATMENT PLANT
  - 20 FEET ENERGY HEAD FOR GRAVITY FLOW THROUGH WTP WITH BAC AND OZONE CONTACT
- ABOVE FLOODPLAIN

SIZING - INFRASTRUCTURE (50 YEAR LIFE)
EQUIPMENT (15-20 YEAR LIFE)

AVERAGE WATER USE PER PERSON PER DAY -

$$\text{gpcd} = \frac{\text{Water used (gpd)}}{\text{Total number of people}}$$

PROBLEM: THE TOTAL WATER USE ON A PARTICULAR DAY WAS 4,503,000 GAL. IF THE WATER SYSTEM SERVED A POPULATION OF 19,800, WHAT WAS THIS WATER USE EXPRESSED IN GPCD?

$$\text{gpcd} = \frac{\text{Water used (gpd)}}{\text{Total number of people}}$$

$$= \frac{\text{4,503,000 gpd flow}}{\text{19,800 people}}$$

$$= \text{227.42 gpcd water used}$$

PRINCIPAL USES OF MUNICIPAL WATER:

- DOMESTIC
- COMMERCIAL
- INDUSTRIAL
- PUBLIC
- "UNACCOUNTED FOR"

CALCULATING THE DAILY USE FOR THE FAMILY

Laundry	11.43 gpd
Shower	40 gpd
Tub Bath	30 gpd
Dishwashing	15 gpd
Toilet	35 gpd
Drinking Water	0.75 gpd
Garbage Disposal	5 gpd
Car Washing	3.57 gpd
Lawn Watering	+ 42 gpd
TOTAL GPD	182.75

Rounded to the nearest gallon, the water used daily by this family is 183 gals.

AVERAGE WATER USE OVER ALL SECTORS:

(Excluding Fire Fighting)

Residential	75-130
Commercial & Industrial	70-100
Public	10-20
Loss & Waste	10-20
TOTAL GPCD	165-270

DOMESTIC WATER USE:

Household Fixture Use Rate

Use	Approximate Rate
Laundry	20 to 45 gals. per load
Shower	20 to 30 gals. per shower
Tub Bath	30 to 40 gals. per bath
Dishwashing	15 to 30 gals. per load
Toilet	3.5 to 7 gals. per flush
Drinking water	1-2 qt./day/person
Garbage disposal	5 gpm*
Car washing	5 gpm*
Lawn watering	7 to 43 gals. per 100 ft^2 per week*

*Assuming that a faucet draws 5 gpm fully open.

EXAMPLE: FOR A FAMILY OF THREE:

Laundry .... 4 loads per week
Showers .... 2 per day
Tub baths .... 1 per day
Dishwashing .... 1 load per day
Toilet .... 10 flushes per day
Drinking water .... 3 people
Garbage disposal .... 1 minute per day
Car washing .... 5 minutes per week
Lawn watering .... 4200 ft^2

$$\text{AVERAGE DAILY FLOW (ADF)} = \frac{\text{Sum of all daily flows}}{\text{Total number of daily flows used}}$$

PROBLEM: A WATER TREATMENT PLANT REPORTED THAT THE TOTAL VOLUME OF WATER TREATED FOR THE CALENDAR YEAR 1977 WAS 152,655,000 GALS. WHAT WAS THE ANNUAL AVERAGE DAILY FLOW FOR 1977?

$$\text{ADF} = \frac{\text{Sum of all daily flows}}{\text{Total number of daily flows used}}$$

$$= \frac{152{,}655{,}000 \text{ gals.}}{365 \text{ days}}$$

$$= 418{,}233 \text{ gpd}$$

$$\text{ANNUAL AVERAGE DAILY FLOW (AADF)} = \frac{\text{Sum of all monthly ADF's}}{\text{Total number of monthly ADF's used}}$$

PROBLEM: THE AVERAGE DAILY FLOW (IN MILLION GALLONS PER DAY) AT A TREATMENT PLANT FOR EACH MONTH IN THE YEAR IS GIVEN BELOW. USING THIS INFORMATION, CALCULATE THE ANNUAL AVERAGE DAILY FLOW.

January	10.71	July	11.96
February	9.89	August	12.24
March	10.32	September	11.88
April	10.87	October	11.53
May	11.24	November	11.36
June	11.58	December	10.98

IF YOU KNEW ALL 365 FLOWS FOR THE YEAR, THE ANNUAL AVERAGE DAILY FLOW (AADF) WOULD BE CALCULATED USING THE FORMULA:

$$\text{AADF} = \frac{\text{Sum of all daily flows}}{\text{Total number of daily flows used}}$$

IN THIS PROBLEM, HOWEVER, AVERAGE DAILY FLOWS FOR EACH MONTH OF THE YEAR ARE GIVEN. THEREFORE, THE AVERAGE DAILY FLOW IS CALCULATED USING THE FORMULA:

$$\text{AADF} = \frac{\text{Sum of all monthly ADF's}}{\text{Total number of monthly ADF's used}}$$

$$AADF = \frac{134.56 \text{ mgd}}{12}$$

$$= 11.21 \text{ mgd}$$

VARIATIONS IN THE AVERAGE DAILY FLOW ARE:

DEMAND MULTIPLIERS

Consumption time/period	multiplier
winter	0.80
summer	1.30
maximum daily	1.50 - 1.80
maximum hourly	2.00 - 3.00
early morning	0.25 - 0.40
noon	1.50 - 2.0

PROBLEM: A WATER TREATMENT PLANT IS TO TREAT 25 GALLONS PER DAY OF WATER FROM A LARGE RIVER. TREATMENT CONSISTS OF RAPID MIXING, FLOCCULATION, SEDIMENTATION, STANDARD RATE SAND FILTRATION, AND CHLORINATION. ALUM IS ADDED AT THE RATE OF 1.5 GRAINS PER GALLON.

Calculate the following:

(a) Design capacity and size of each treatment unit.

(b) The amount of alum and chlorine required on a daily basis.

FLASH MIXING

Design for minimum detention period of 30 seconds.

$$\text{Tank Capacity} = \frac{26 \times 106 \text{ gal.}}{24 \text{ hrs.} \times 60} \times 0.5 \text{ min.} \times \frac{ft^3}{7.48 \text{ gal.}}$$

$$= \quad 1160 \text{ ft}^3$$

For square tank 10 feet deep, Area = 116 ft2

Tank Dimensions: 10.5 ft. x 10.5 ft. x 10 ft. deep

FLOCCULATION

Use horizontal paddle mixer in flocculation basins. Design for 30 minutes detention time (minimum required).

$$\text{Total Capacity} = \frac{25 \times 106 \text{ gal.}}{24 \text{ hrs.}} \times 0.5 \text{ hr.} \times \frac{\text{ft}^3}{7.48 \text{ gal.}}$$

$$= 557{,}000 \text{ ft}^3$$

For 12 foot tank depth:

Total Area = 46,420 ft2

Surface loading = 538 gpd/ft2

Use eight basins, each 5803 ft2

Assume length: width ratio of 4:1 for rectangular basin

Tank Dimensions: 12 ft. deep X 38 ft. X 152 ft.

FILTRATION

Design for filtration rate of 2 gpm/ft2.

Assuming each filter unit will treat 2.5 mgd, 10 beds will berequired.

Assume 5 filters are backwashed for 20 minutes each day at a rate of 18 gpm/sq. ft. Also provide for surface wash. Total backwash time is 100 minutes.

Calculate required filter surface area.

$$\frac{25 \times 10^6 \text{ gal/day}}{2 \text{ gpm/ft}^2 \times 60 \text{ min/hr} \times (24 \text{ hr.} - 10/60 \text{ hr.})} = 8724 \text{ ft}^2$$

= 872 ft2/filter

Assume length: width ratio of 2:1.

Filter Dimension: 21 ft. x 42 ft.

(b) Calculate alum required at dosage rate of 1.5 grains/gallon

1 grain/gallon =142.5 lbs./million gals.

1.5 x 142.5 x 25 = 5344 lbs/day alum

Calculate chlorine required. Assume maximum dosage of 10 mg/l:

10 mg/L x 8.34 x 25 mgd = 2085 lbs/day

Design for chlorination capacity equal to 150% of maximum demand or 3128 lbs/day

TYPICAL WATER TREATMENT PROCESSES

Raw Water Sources → Pretreatment → Screening → Presedimentation

↓

Coagulation ← Aeration ← Flow Measuring ← Microstraining

↓

Flocculation → Sedimentation → Softening → Filtration → Adsorption

↓

Clear Well ← Disinfection ← Stabilization ← Fluoridation

↓

Distribution/Storage → Residuals Management

UNIT PROCESSES

PRELIMINARY TREATMENT - PHYSICAL, MECHANICAL AND/OR CHEMICAL TREATMENTS THAT PRECEDE THE MAJOR WATER TREATMENT PROCESSES.

- SOURCE PRETREATMENT
- SCREENING
- PRESEDIMENTATION
- MICROSTRAINING
- "AERATION"
- "CHLORINATION"

SOURCE PRETREATMENT

- ALGAE PROBLEMS
  - TASTE AND ODORS
  - FILTER CLOGGING
  - SLIME BUILD-UP
  - COLOR
  - CORROSION
  - INTERFERENCE WITH MAJOR WATER TREATMENT PROCESSES
  - TOXICITY

- ALGAE CONTROL
  - COPPER SULFATE - ALKALINITY ($CaCO_3$) LESS THAN 50 MG/L, DOSE AT 0.9 LB/ACRE-FOOT OF COMMERCIAL PRODUCT - ALKALINITY ($CaCO_3$) MORE THAN 50 MG/L, DOSE AT 5.4 LB/ACRE OF COMMERCIAL PRODUCT.
    - POWDERED ACTIVATED CARBON
    - COVERS
    - HARVESTING
    - DEWATERING
    - DREDGING
    - LINING

SCREENING	-	BAR SCREENS (60 - 80 DEGREE ANGLE) WIRE-MESH SCREENS
	-	MANUALLY CLEANED AUTOMATICALLY CLEANED
PRESEDI-MENTATION	-	IMPOUNDMENTS (24-HR. DETENTION)
	-	SAND TRAPS (WET WELL DEPRESSION)
	-	MECHANICAL SAND-AND-BRIT REMOVAL DEVICES (CYCLONE DEGRITTERS)
MICRO-SCREENING	-	VERY FINE SCREEN (20 µM) TO REMOVE ALGAE AND SMALL ORGANISMS AND DEBRIS
	-	COMMONLY A ROTATING (4-7 RPM) DRUM LINED WITH FINELY WOVEN MATERIAL (FLOW FROM INSIDE TO OUTSIDE)
	-	ALGAE REMOVED BY BACKWASH
	-	ALGAE REMOVAL EFFICIENCY (50-90%)

FLOW MEASUREMENT - PRESSURE DIFFERENTIAL METER

- VENTURI TUBE
- FLOW TUBE
- ORIFICE PLATE

- VELOCITY METER

- PROPELLER METER
- TURBINE METER

- MAGNETIC FLOW METER

- ULTRASONIC FLOW METER

AERATION - TREATMENT BY:

- SCRUBBING ACTION (AIR/WATER MIXING)
- OXIDATION

- CONTROL OF OPERATION BY TESTING:

- DISSOLVED OXYGEN
- pH
- TEMPERATURE

- CONSTITUENTS COMMONLY AFFECTED:

RAPID MIXING

$$\bar{G} = \left[\frac{P}{\mu V}\right]^{1/2}$$

$\bar{G}$ = mean velocity gradient
P = power dissipated
V = reactor volume
μ = viscosity of water @ 10° C - 1.31 cp
@ 20° C - 1.01 cp

Backmix Reactor Design @ 10-30 seconds contact time (others @ 1-2 minutes)

$\bar{G}$ = 700 - 1000 per second
HPinput = 0.9 - 1.2 hp/mgd

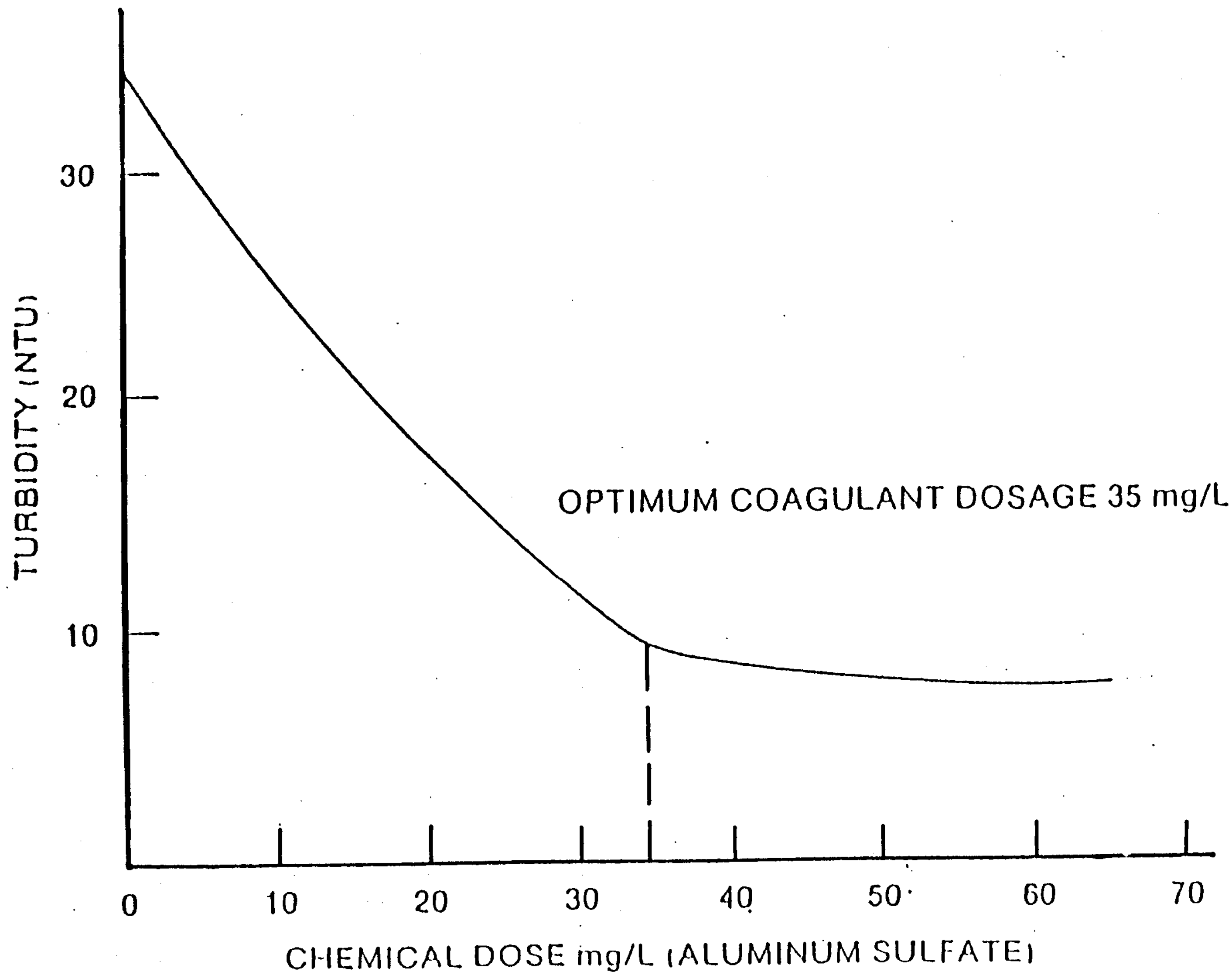
In-Line Blender Design @ 0.5 - 1.0 seconds

$\bar{G}$ = 3000 - 5000 per second

HPinput = 0.5 hp/mgd
Headloss = 1.3 feet
(additional HP = 0.16 - 0.51 hp/mgd)

SELECTION OF CHEMICALS - JAR TEST

- TEMPERATURE
- pH



Typical Jar Test Results

- ALKALINITY
- TURBIDITY
- COLOR

PROBLEM: A FLASH MIXING BASIN HAS A CAPACITY OF 1800 GALS. IF THE FLOW TO THE MIXING BASIN IS 37 GPS, WHAT IS THE DETENTION TIME?

SINCE THE FLOW RATE IS EXPRESSED IN SECONDS, THE DETENTION TIME IS EXPRESSED IN SECONDS:

$$\text{Detention time} = \frac{\text{Volume of Tank}}{\text{Flow Rate}}$$

$$= \frac{1800 \text{ gals.}}{37 \text{ gps}}$$

$$= 48.65 \text{ sec.}$$

- CARBON DIOXIDE (4.5 MG/L)
- HYDROGEN SULFIDE
- METHANE
- IRON (14 MG/L DISSOLVED OXYGEN/1 MG/L IRON)
- MANGANESE (0.27 MG/L DISSOLVED OXYGEN/1 MG MANGANESE)
- TASTE AND ODOR CAUSING CHEMICALS
- DISSOLVED OXYGEN

\- TYPES OF AERATORS

- WATER - INTO - AIR
- AIR - INTO - WATER
- COMBINATION

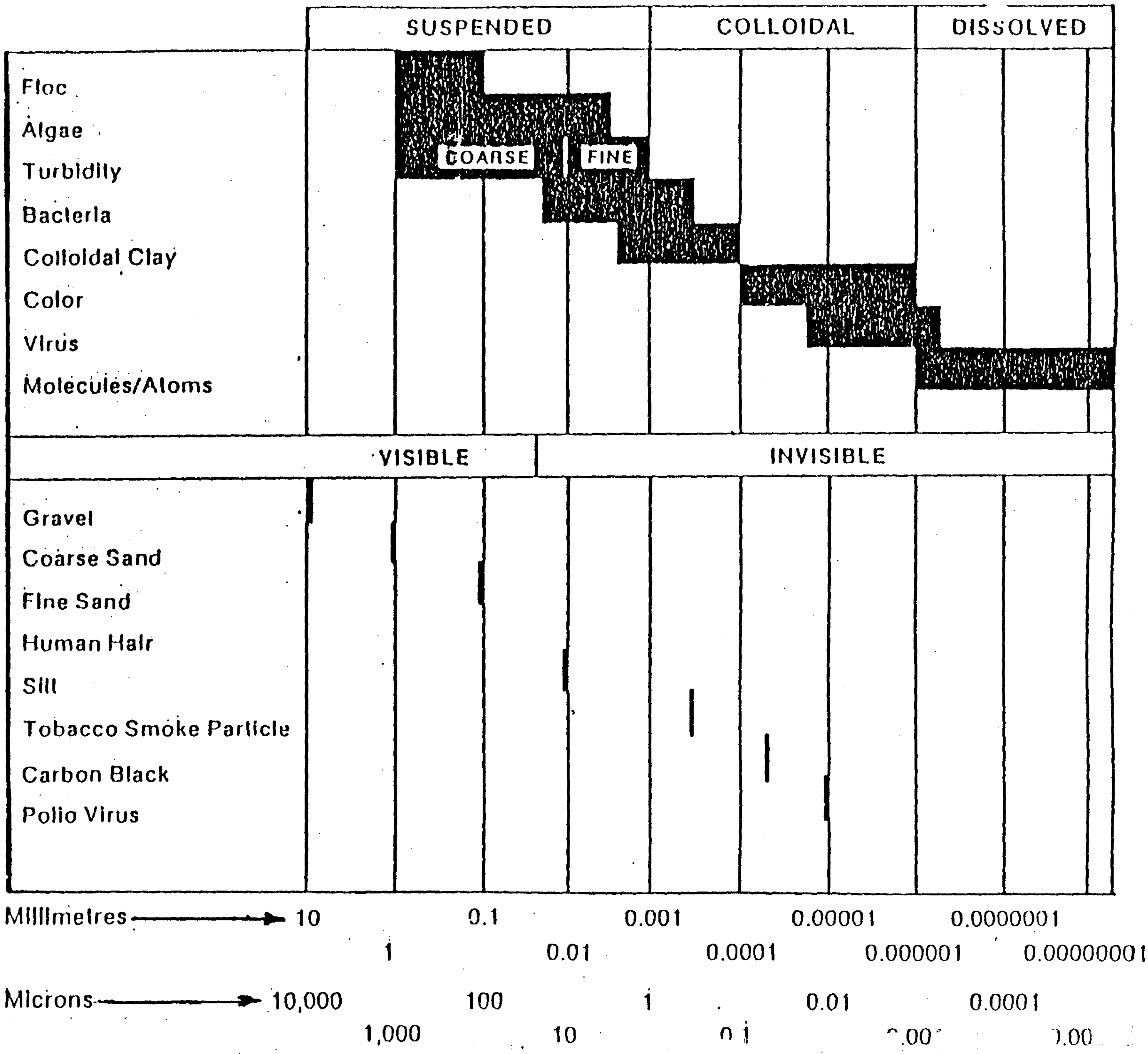
COAGULATION/ FLOCCULATION

\- COAGULATION - FEEDING AND RAPID MIXING OF ONE OR MORE CHEMICAL COAGULANTS INTO WATER CAUSING FLOC PARTICLES

\- FLOCCULATION - GENTLE MIXING OF WATER AND COAGULANTS CAUSES FORMATION OF LARGER, HEAVIER, MORE SETTLEABLE FLOC PARTICLES

\- PARTICLE SIZE - 1-10,000 MICRONS

Size Range of Solids



Natural Settling Rates for Small Particles

Particle Diameter mm	Representative Particle	Time Required to Settle in 1 ft (0.3 m) Depth
		Settleable
10	Gravel	0.3 sec
1	Coarse sand	3 sec
0.1	Fine sand	38 sec
0.01	Silt	33 min
		Considered nonsettleable
0.001	Bacteria	55 hour
0.0001	Color	230 day
0.00001	Colloidal particles	6.3 year
0.000001	Colloidal particles	63-year minimum

Source: *Water Quality and Treatment*. AWWA, Denver, Colo. (3rd ed., 1971)

- SUSPENDED
- COLLOIDAL (0.001 - 1.0 MICRONS)
- DISSOLVED (0.00001 - 0.001 MICRONS)

- RAPID MIXING

  - 30-60 SECONDS DETENTION TIME

- FLOCCULATION

  - 20-60 MINUTES DETENTION TIME

PROBLEM: COAGULATED/FLOCCULATED WATER ENTERS A SEDIMENTATION BASIN 25 FT. WIDE AND 100 FT. LONG AT A RATE OF 3.3 MGD. THERE ARE 200 FT. OF OVERFLOW WEIR AND THE BASIN HAS A 10 FT. DEPTH.

$$\text{DETENTION TIME} = \frac{(25)(100)(10)(7.48)(24)}{(3.3)(10^6)}$$

$$= 1.36 \text{ HOURS}$$

$$\text{SURFACE OVERFLOW RATE} = \frac{(3.3)(10^6)}{(25)(100)}$$

$$\text{WEIR OVERFLOW RATE} = \frac{(3.3)(10^6)}{200}$$

$$= 16{,}500 \text{ GPD/FT}$$

Dosage	Flow Rate	Conversion Factor		Feed Rate
(ppm)	(mgd)	(8.34 lb/gal)	=	(lb/day)

PROBLEM: THE DRY ALUM DOSAGE RATE IS 12 MG/L AT A WATER TREATMENT PLANT. THE FLOW RATE AT THE PLANT IS 3 MGD. HOW MANY POUNDS PER DAY OF ALUM ARE REQUIRED?

(mg/L)(mgd)(8.34 lb/gal) = lb/day

$$(12\ mg/L)(3\ mgd)(8.34\ lb/gal) = x\ lb/day$$
$$300.34\ lb/day = x\ lb/day$$

PROBLEM: THE CHLORINE DOSAGE RATE AT A WATER TREATMENT PLANT IS 4 MG/L. THE FLOW RATE AT THE PLANT IS 700,000 GPD. HOW MANY POUNDS PER DAY OF CHLORINE ARE REQUIRED?

(mg/L)(mgd)(8.34 lb/gal) = lb/day

700,000 gpd = 0.7

$$(4\ mg/L)(0.7\ mgd)(8.34\ lb/gal) = x\ lb/day$$
$$23.36\ lb/day = x$$

PROBLEM: HOW MANY POUNDS OF HYPOCHLORITE (65 PERCENT AVAILABLE CHLORINE) ARE REQUIRED TO DISINFECT 8000 FT. OF 24-IN. WATER LINE IF AN ADDITIONAL DOSE OF 20 MG/L IS REQUIRED?

(0.785)(2 ft)(8000 ft)(7.48 gal/cu ft) = 187,898 gal.

THE VOLUME MUST BE EXPRESSED IN TERMS OF MILLION GALLONS:

187,898 gal = 0.188 mil gal.

SEDIMENTATION - RECTANGULAR BASINS (RECTILINEAR FLOW)
CIRCULAR BASINS (RADIAL FLOW)

ZONES IN A SEDIMENTATION BASIN -

- INLET ZONE - DECREASES THE VELOCITY OF THE INCOMING WATER AND DISTRIBUTES THE FLOW EVENLY ACROSS THE BASIN.
- SETTLING ZONE - PROVIDES THE CALM AREA NEEDED FOR THE SUSPENDED MATERIAL TO SETTLE.
- OUTLET ZONE - PROVIDES A SMOOTH TRANSITION FROM THE SETTLING ZONE TO THE EFFLUENT AREA.
- SLUDGE ZONE - RECEIVES THE SETTLED SOLIDS AND KEEPS THEM SEPARATE FROM OTHER PARTICLES IN THE SETTLING ZONE.

DESIGN CRITERIA - DETENTION TIME = 2-6 HOURS
OVERFLOW RATE = 500 GPD/SQ. FT. (COMMON)
- WEIR OVERFLOW RATE = 15,000 - 20,000 GPD/FT.

SOFTENING - REMOVAL OF CALCIUM AND MAGNESIUM

- TYPES OF HARDNESS

  - CALCIUM AND MAGNESIUM
  - CARBONATE AND NONCARBONATE

$\left.\begin{array}{l} Ca^{++} \\ Mg^{++} \end{array}\right\}$ TOTAL HARDNESS

$\left.\begin{array}{l} CO_3^{--} \\ HCO_3^{-} \\ OH^{-} \end{array}\right\}$ CARBONATE HARDNESS

$\left.\begin{array}{l} SO_4^{--} \\ Cl^{-} \\ NO_3^{--} \end{array}\right\}$ NONCARBONATE HARDNESS

METHODS OF WATER SOFTENING:

1. LIME TREATMENT
2. LIME - SODA ASH TREATMENT
3. EXCESS LIME - SODA ASH TREATMENT
4. ION EXCHANGE

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Comparative Classification of Water for Softness/Hardness

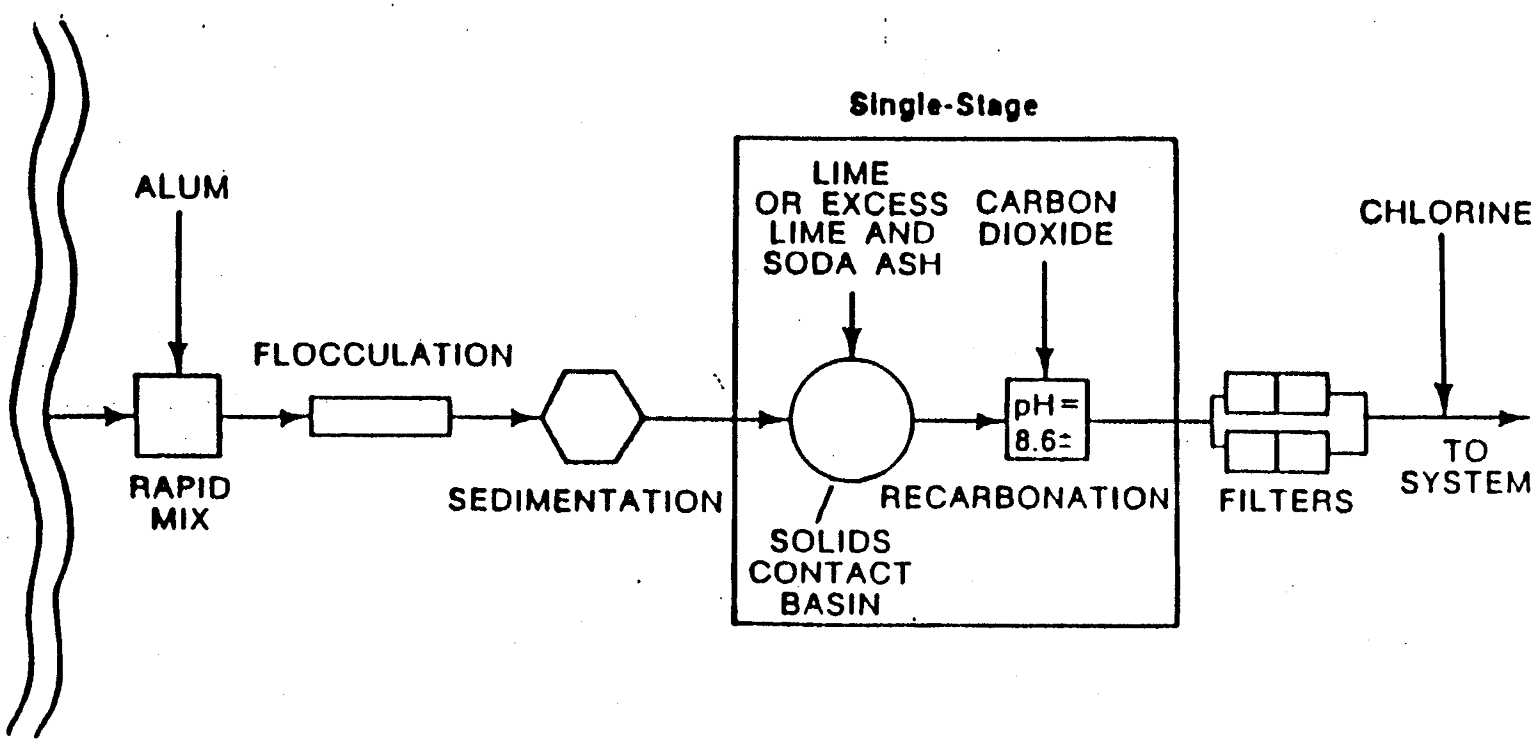
Classification	mg/L as $CaCO_3$*	mg/L as $CaCO_3$†
Soft	0-75	0-60
Moderately Hard	75-150	61-120
Hard	150-300	121-180
Very Hard	over 300	over 180

BASIC EQUATIONS

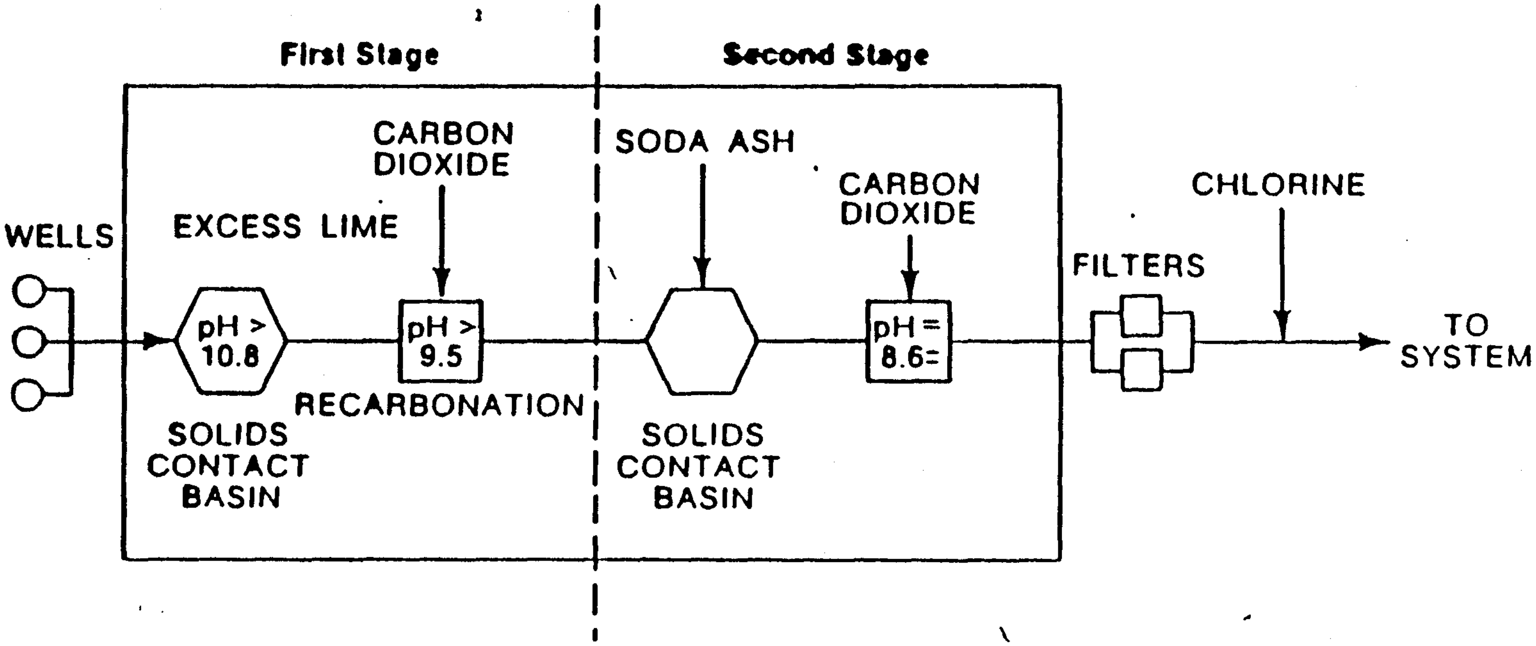
1. $CO_2 + Ca(OH)_2 \rightarrow CaCO_3 \downarrow + H_2O$
2. $Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2\,CaCO_3 \downarrow + 2\,H_2O$
3. $Mg(HCO_3)_2 + Ca(OH)_2 \rightarrow MgCO_3 + CaCO_3 \downarrow + 2\,H_2O$

   $MgCO_3 + Ca(OH)_2 \rightarrow Mg(OH)_2 \downarrow + CaCO_3 \downarrow$
4. $CaSO_4 + Na_2CO_3 \rightarrow Na_2SO_4 + CaCO_3 \downarrow$
5. $MgSO_4 + Ca(OH)_2 \rightarrow CaSO_4 + Mg(OH)_2 \downarrow$

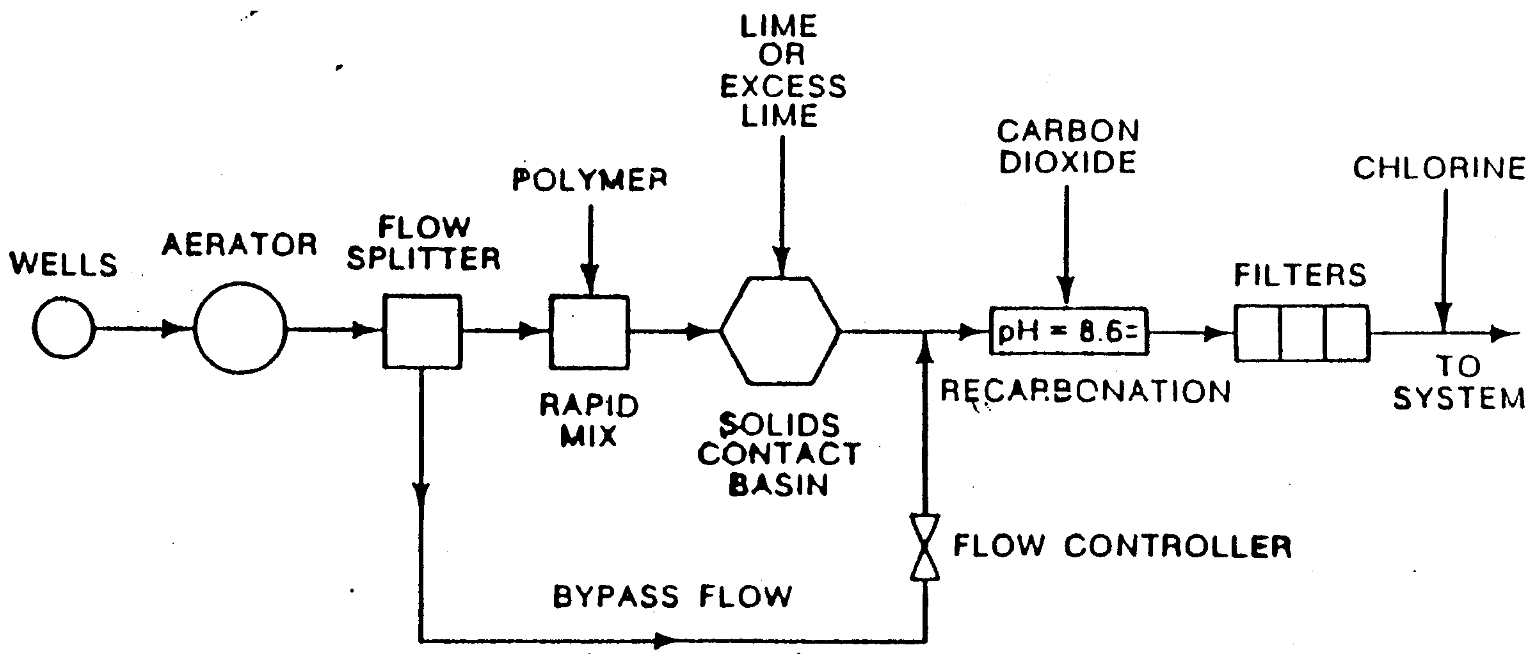
   $CaSO_4 + Na_2CO_3 \rightarrow Na_2SO_4 + CaCO_3 \downarrow$



Single-Stage
ALUM
FLOCCULATION
RAPID MIX
SEDIMENTATION
LIME OR EXCESS LIME AND SODA ASH
CARBON DIOXIDE
pH = 8.6±
SOLIDS CONTACT BASIN
RECARBONATION
FILTERS
CHLORINE
TO SYSTEM



Double-Stage Process



Split Treatment

PROBLEM: A RAW WATER WITH AN ANALYSIS AS SHOWN IS TO BE TREATED USING A LIME SODA-ASH SOFTENING PROCESS. EXCESS LIME AND SODA ASH IS ADDED TO ACHIEVE A RESIDUAL HARDNESS TO THE PRACTICAL LIMIT OF 30 MG/L AS $CaCO_3$ AND 10 mg/L $Mg(OH)_2$ as $CaCO_3$.

(a) Discuss the chemical reactions that occur in the lime soda-ash softening process.

(b) List several advantages and disadvantages of the process.

(c) Calculate the quantity of chemicals required for softening and recarbonation.

RAW WATER ANALYSIS FOR PROBLEM

	mg/L	Equivalent Weight	meq/L	mg/L as $CaCO_3$
Calcium	80	20.0	4.0	200
Magnesium	30	12.2	2.5	125
Sodium	19	23.0	0.8	40
Chloride	18	35.5	0.5	25
Sulfate	64	48.0	1.3	65
Bicarbonate	336	61.0	5.5	275
Carbon dioxide *(free)*	15	22.0	0.7	35

(a) Basic reactions for the lime-soda ash water softening process using hydrated lime, $Ca(OH)_2$ are written as:

Lime

$CO_2 + Ca(OH)_2 = CaCO_3 + H_2O$

$Ca(HCO_3)_2 + Ca(OH)_2 = 2CaCO_3 + 2H_2O$

$Mg(HCO_3)_2 + 2Ca(OH)_2 = 2CaCO_3 + Mg(OH)_2 + 2H_2O$

In these reactions, carbonate hardness in the form ofcalcium and magnesium bicarbonate is removed by reaction with lime to form insoluble $CaCO_3$ and Mg(OH). Lime also reacts with free CO_2 in the water to form a $CaCO_3$ precipitate.

Soda Ash - Na_2CO_3

$CaSO_4 + Na_2CO_3 = CaCO_3 + Na_2SO_4$

$MgSO_4 + Na_2CO_3 + Ca(OH)2 = Mg(OH)_2 + CaCO_3 + Na_2SO_4$

(b)

Advantages	Disadvantages
Large quantities of water can be softened economically.	Large quantities of chemical sludge are produced creating disposal problems.

(b)

Advantages	Disadvantages
Alkalinity and total solids are reduced. Water is low in color turbidity.	Complete softening to low hardness levels cannot be achieved.
Disinfection is achieved by the effect of lime and high pH.	Chemical feed rates must be controlled closely.
Reduction in silica can be achieved.	Operating problems may be experienced in filtration of softened water.

(c) A bar graph of the raw water is shown. The required amount of lime is calculated as:

$$\text{Lime} = \text{meq/L}\ [CO_2 + Ca(HCO_3)_2 + Mg(HCO_3)_2 + MgSO_4]$$

$$= 0.7 + 4.0 + 1.5 + 1.0$$

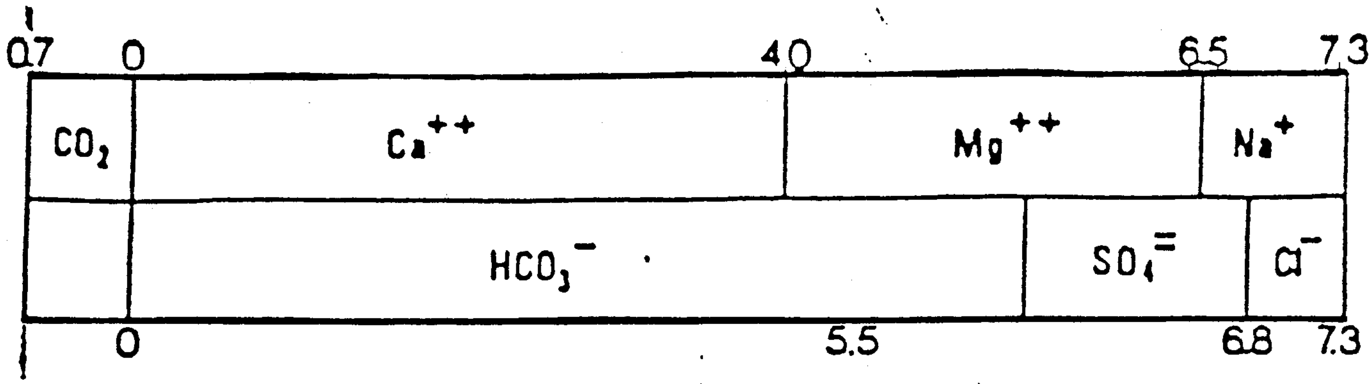
$$= 7.2\ \text{meq/L}$$

Equivalent weight of lime (CaO) = 28.
Use 35 mg/L excess lime.
Lime dosage = (7.2 x 28.0) + 35 = 237 mg/L
= 1976 lbs/million gallons.

The required amount of soda ash is calculated as:

$$\text{meq/L}\ Na_2CO_3 = \text{meq/L}\ MgSO_4 = 1.0$$

Equivalent weight soda ash = 53.



Raw water bar diagram.

Soda ash dosage = 1.0 x 53 = 53 mg/L
= 442 lbs/million gallons.

Calculate carbon dioxide required for recarbonation. Carbon dioxide neutralizes excess lime and OH according to the equation.

$$Ca(OH_2) + CO_2 = CaCO_3 + H_2O$$

Equivalent weight CO_2 = 22.0

Excess lime = 1.25 meq/L

OH^- = 0.20 meq/L

CO_2 = (1.25 + 0.20) x 22 = 31.9 mg/L

Additional carbon dioxide converts remaining alkalinity as carbonate ion to bicarbonate ion according to the equation:

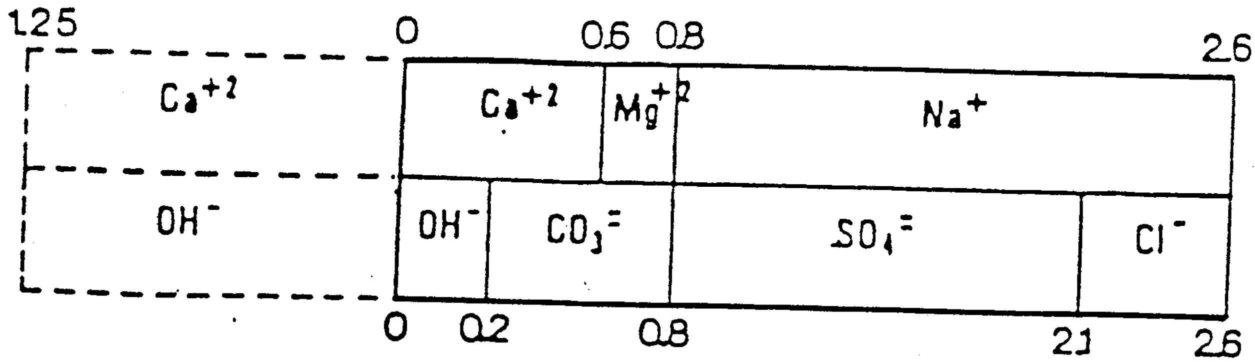
$$CaCO_3 + CO_2 + H_2O = Ca(HCO_3)_2$$

Assuming all of remaining alkalinity is converted the bicarbonate form.

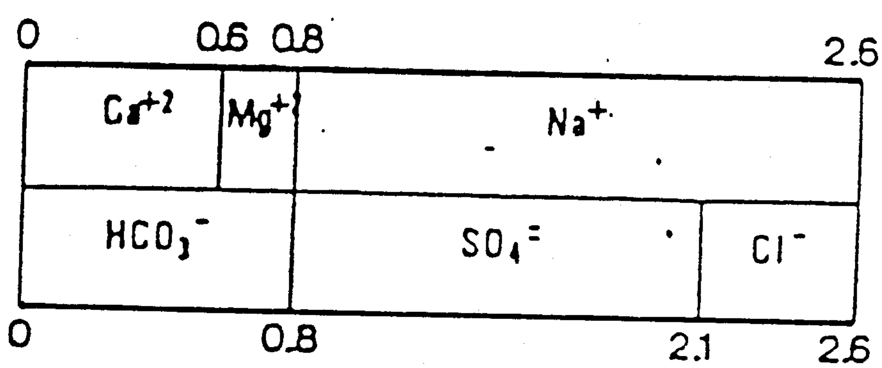
CO_2 = 0.6 meq/L x 22 = 13.2 mg/L

Total CO_2 = 45.1 mg/L

Bar diagrams of the softened water before recarbonation and the finished water are shown:



(a)



(b)

ION EXCHANGE SOFTENING - REMOVAL OF HARDNESS IONS BY A CATION EXCHANGE

ION EXCHANGE MATERIALS -

- ZEOLITES (2,800 - 11,000 GRAINS/CUBIC FOOT)
- RESINS (11,000 - 35,000 GRAINS/CUBIC FOOT)

NOTE: 1 GRAIN = 64.8 Milligrams
1 GRAIN/GAL = 17.12 mg/L
= 142.86 lb/mg

PROBLEM: A TREATMENT UNIT CONTAINS 20 CU. FT. OF CATION EXCHANGE MATERIAL WHICH HAS A RATED REMOVAL CAPACITY OF 20,000 GRAINS PER CUBIC FOOT. WHAT IS THE TOTAL HARDNESS REMOVAL CAPACITY? GIVE ANSWER IN GRAINS.

Total removal capacity = (20,000 grain/cu ft)(20 cu ft)

= 400,000 grains removed per cycle

THE UNIT TREATS WATER WITH A HARDNESS OF 253 MG/L. HOW MANY GALLONS OF WATER CAN BE SOFTENED BEFORE THE EXCHANGE MUST BE REGENERATED?

$$\text{gpg} = \frac{\text{mg/L}}{17.12}$$

$$\text{Hardness} = \frac{253 \text{ mg/L}}{17.12}$$

$$= 14.78 \text{ grains per gallon}$$

$$\text{Volume of Water Softened} = \frac{\text{Removal capacity per cycle}}{\text{Hardness}}$$

$$\text{Volume of Water Softened} = \frac{400{,}000 \text{ grains}}{14.78 \text{ gpg}}$$

$$= 27{,}063 \text{ gal. per cycle}$$

IF 0.35 LB. OF SALT WILL RESTORE 100 GRAINS OF REMOVAL CAPACITY, HOW MUCH SALT IS REQUIRED FOR REGENERATION?

$$\text{Salt Requirement} = (\text{Removal Capacity})(\text{Regeneration Requirement})$$

$$= (\text{Removal Capacity}) \left(\frac{\text{Salt requirement in lb}}{1000 \text{ grains of hardness}}\right)$$

$$\text{Salt Requirement} = (400{,}000 \text{ grain}) \left(\frac{0.35 \text{ lb}}{1000 \text{ grain}}\right)$$

$$= (400)(0.35)$$

$$= 140 \text{ lb. salt}$$

FILTRATION - REMOVAL OF SUSPENDED MATERIAL (TURBIDITY) FROM WATER, INCLUDING:

- FLOC (FROM COAGULATION/ FLOCCULATION)
- MICROORGANISMS
- PRECIPITATES (FROM SOFTENING)
- IRON AND MANGANESE (FROM GROUNDWATER)

- REMOVAL BY:

  - PHYSICAL TRAPPING
  - ADSORPTION

- TYPES OF FILTERS:

  - GRAVITY
    - SLOW SAND
    - RAPID SAND
    - HIGH RATE
      * DUAL-MEDIA
      * MULTI-MEDIA

- PRESSURE

  - SAND OR MULTI-MEDIA
  - DIATOMACEOUS EARTH

DESIGN CRITERIA:

FILTER LOADING RATE = $\frac{\text{GPM FLOW}}{\text{SQ. FT. FILTER AREA}}$

FILTER LOADING RATE = $\frac{\text{INCHES OF WATER FALL}}{\text{MINUTES}}$

Type of Filter	Common Loading Rate
Slow sand filters	0.016 to 0.16 gpm/sq. ft.
Rapid sand filters	2 gpm/sq. ft.
Dual media (coal/sand)	2 to 4 gpm/sq. ft.
Multi-media (coal/sand/ garnet or coal/sand/ilmenite	5 to 10 gpm/sq. ft.

PROBLEM: A RAPID SAND FILTER IS 10 FT. WIDE AND 15 FT. LONG. IF THE FLOW THROUGH THE FILTER IS 450,000 GPD, WHAT IS THE FILTER LOADING RATE IN GALLONS PER MINUTE PER SQUARE FOOT?

FIRST, CONVERT THE GALLONS PER DAY TO GALLONS PER MINUTE:

$$\frac{450{,}000 \text{ gpd}}{1440 \text{ min/day}} = 312.5 \text{ gpm}$$

THEN EXPRESS THE FILTER LOADING RATE MATHEMATICALLY AS:

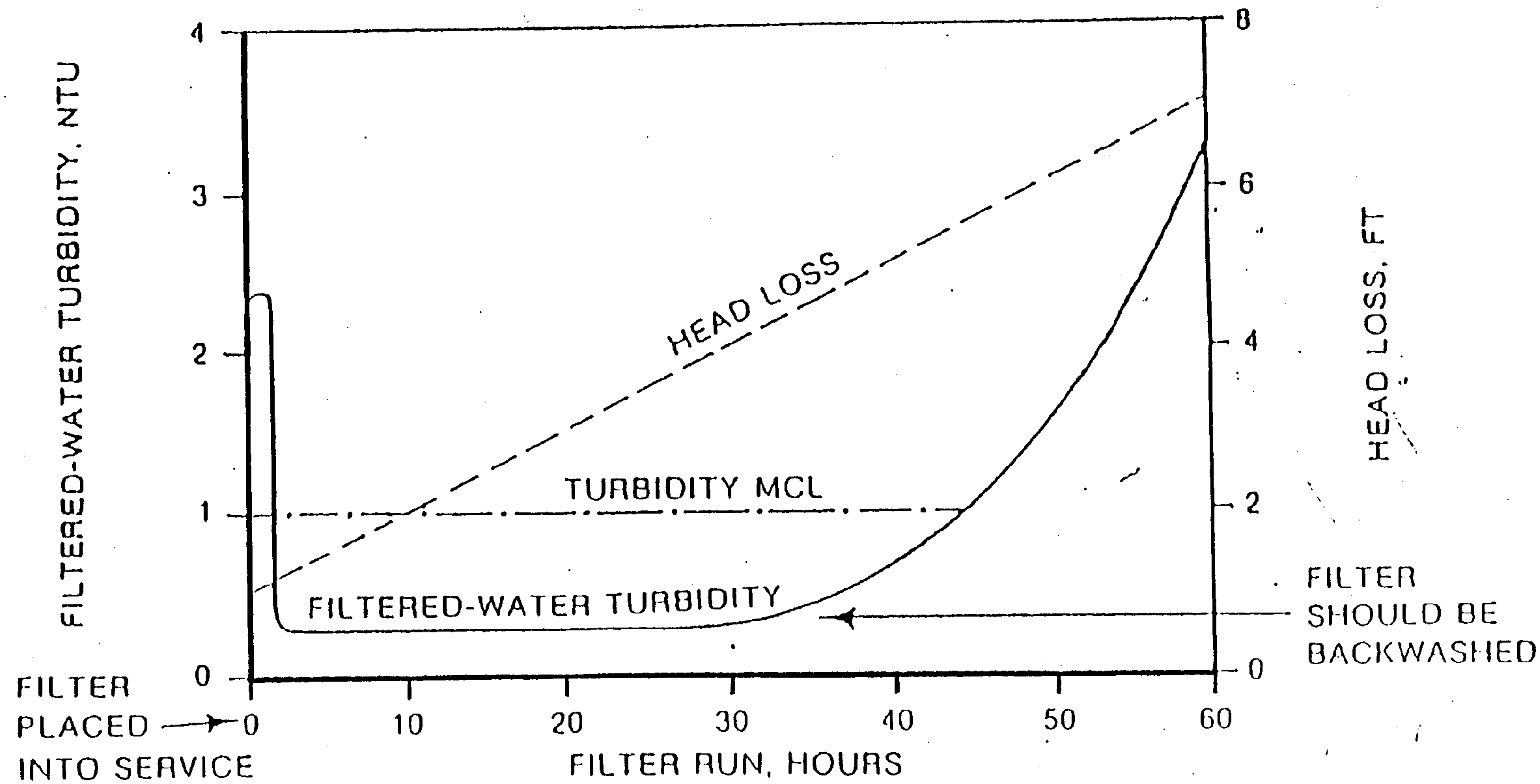
$$\text{Filter loading rate} = \frac{\text{gpm flow}}{\text{sq. ft. filter area}}$$

$$= \frac{312.5 \text{ gpm}}{150 \text{ sq. ft.}}$$

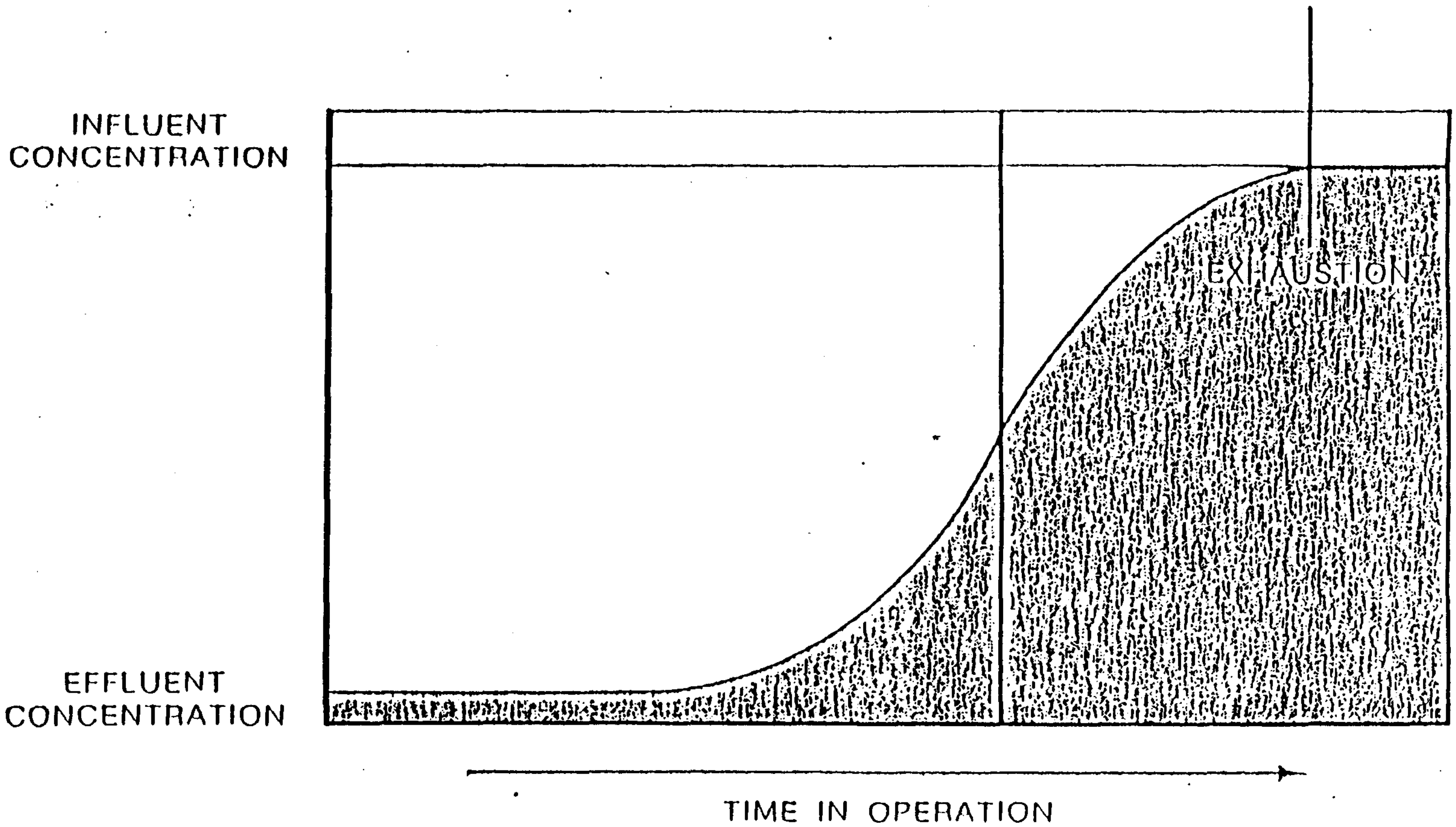
$$= 2.08 \text{ gpm/sq ft.}$$

ADSORPTION

- REMOVAL OF DISSOLVED ORGANIC MATERIAL AND INORGANIC IONS OF FLUORIDE AND ARSENIC PRIMARILY BY ADHESION.
  - ACTIVATED CARBON AND SYNTHETIC RESINS REMOVE ORGANICS
  - ACTIVATED ALUMINA REMOVES INORGANIC IONS OF FLUORIDE AND ARSENIC.
- REMOVES:
  - TASTE AND ODOR
  - COLOR
  - TOXICS



Typical Filter Run



Breakthrough Pattern for 30-In. GAC Bed

DISINFECTION - TREATMENT PROCESS USED TO DESTROY DISEASE-CAUSING (PATHOGENIC) ORGANISMS

WATERBORNE DISEASES -

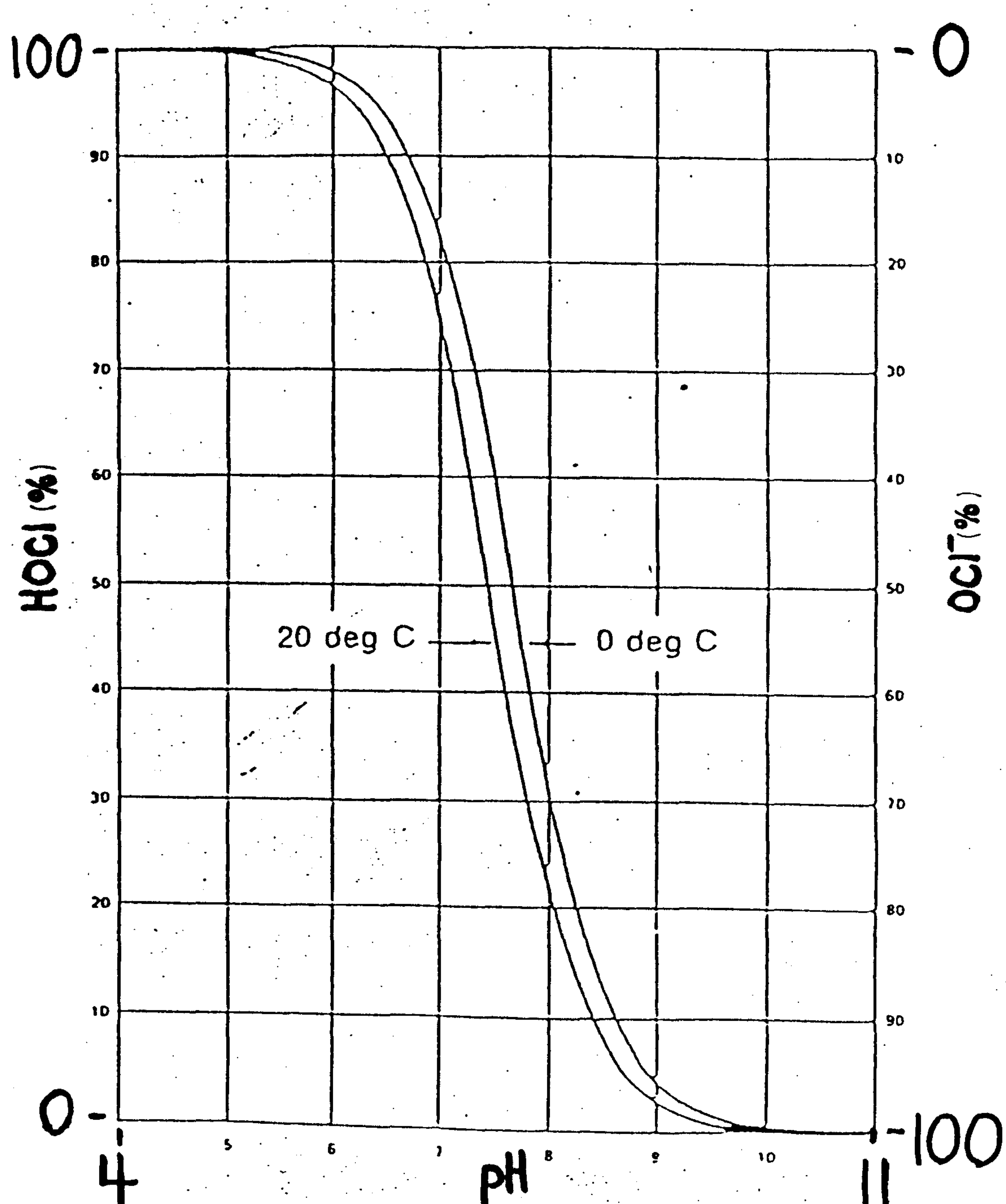
- GASTROENTERITIS
- TYPHOID
- DYSENTERY
- CHOLERA
- INFECTIOUS HEPATITIS
- AMEBIC DYSENTERY
- GIARDIASIS

TYPES OF DISINFECTION

- HEAT TREATMENT
- RADIATION TREATMENT
- CHEMICAL TREATMENT
  - BROMINE
  - IODINE
  - OZONE
  - CHLORINE
    - CHLORINE (100% Cl)
    - CALCIUM HYPOCHLORITE (65% Cl)
    - SODIUM HYPOCHLORITE (1-15% Cl)

EFFECTIVENESS OF CHLORINATION -

- pH
- TEMPERATURE
- CONTACT TIME
- CONCENTRATION
- SUBSTANCES IN THE WATER



Relationship Between HOCl, and OCl⁻, and pH

REACTION OF CHLORINE AND WATER -

Cl_2	+	H_2O	$\rightarrow$	$HOCl$	+	HCl
Chlorine		Water		Hypochlorous acid		Hydrochloric acid

$HOCl$	$\rightarrow$	H^+	+	OCl
Hypochlorous acid		Hydrogen		Hypochlorite ion

HCl	$\rightarrow$	H^+	+	Cl^-
Hypochloric acid		Hydrogen		Chloride ion

REACTION WITH AMMONIA -

NH_3	+	$HOCl$	$\rightarrow$	NH_2Cl	+	H_2O
Ammonia		Hypochlorous Acid		Monochloramine		

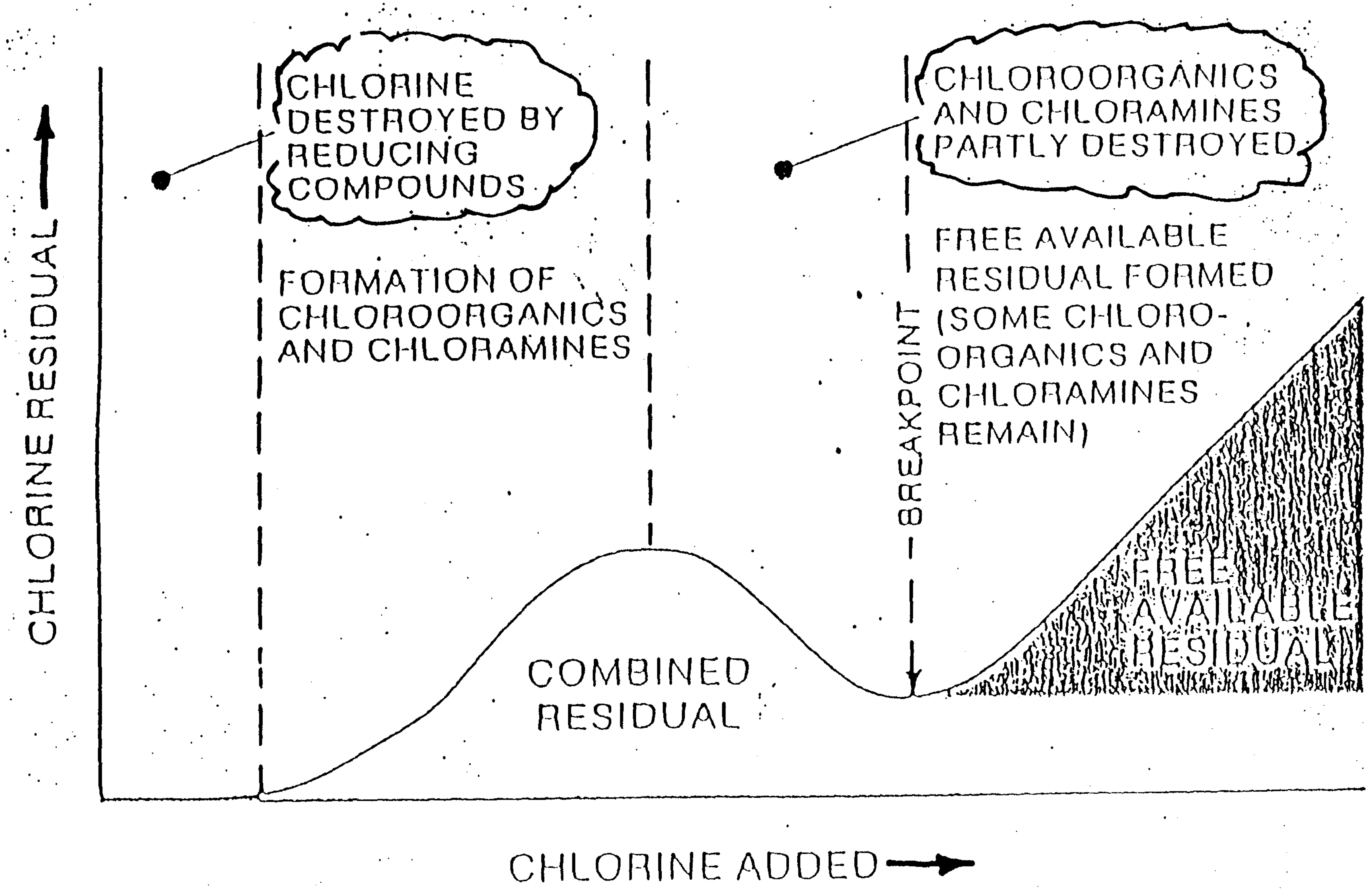
NH_2Cl	+	$HOCl$	$\rightarrow$	$NHCl_2$	+	H_2O
Monochloramine		Hypochlorous Acid		Dichloramine		

$NHCl_2$	+	$HOCl$	$\rightarrow$	NCl_3	+	H_2O
Dichloramine		Hypochlorous		Trichloramine		

PROBLEM: THE CHLORINE DEMAND OF A WATER IS 5.5 MG/L. A CHLORINE RESIDUAL OF 0.3 MG/L IS DESIRED. HOW MANY POUNDS OF CHLORINE WILL BE REQUIRED DAILY FOR A FLOW OF 28 MGD?

$$\text{Dosage} = 5.5 \text{ mg/L} + 0.3 \text{ mg/L}$$
$$= 5.8 \text{ mg/L}$$

BREAKPOINT CHLORINATION CURVE



CONVERT TO POUNDS PER DAY:

(5.8 mg/L)(28 mgd)(8.34 lb/gal) = 1354 lb/day chlorine feed rate

NOTE: PRECHLORINATION -

CHLORINE DOSAGE = CHLORINE DEMAND

CHICK-WATSON THEORY:

$$\ln \frac{N}{No} = kC^{n}t$$

N = pathogens at time "t"

No = pathogens at time "o"

k = coefficient of specific lethality

t = time

n = dilution coefficient

C = concentration of disinfectant

FLUORIDATION - DOSING THE DRINKING WATER SUPPLY WITH THE FLUORIDE ION (REDUCTION OF TOOTH DECAY IN CHILDREN)

- SODIUM FLUORIDE (90-98% PURITY)
- HYDROFLUOSILCIC ACID (98-99% PURITY)
- SODIUM SILICOFLUORIDE (22-30% PURITY)

FEED MIXTURE -

- NaF - 18.8 lbs/mg = 1.0 ppm F
- H_2SiF_6 - 35.2 lbs/mg = 1.0 ppm F
- Na_2SiF_6 - 14.0 lbs/mg = 1.0 ppm F

PROBLEM: THE NATURAL FLUORIDE ION CONCENTRATION OF A WATER SUPPLY IS 0.2 MG/L. HOW MANY POUNDS OF SODIUM SILICOFLUORIDE MUST BE USED EACH DAY TO TREAT 0.5 MGD OF WATER TO A LEVEL OF 1 MG/L? (THE SODIUM SILICO FLUORIDE IS TYPICALLY 98.5 PERCENT PURE.)

TO RAISE THE FLUORIDE LEVEL FROM 0.2 MG/L TO 1 MG/L:

1 mg/L - 0.2 mg/L = 0.8 mg/L

Required dosage pure fluoride ion = (0.8 mg/L)(0.5 mgd)(8.34 lb/day)

= 3.34 lb/day

SINCE 98.5 PERCENT Na_2SiF_6 CONTAINS 60% FLUORIDE ION:

(0.60)(lb/day Na_2SiF_6, 98.5%) = 3.34 lb F/day

lb/day of Na_2SiF_6 = $\frac{3.34}{0.60}$

= 5.57 lb/day

Optimum Fluoride Concentrations and Fluoride MCLs

Annual Average of Maximum Daily Air Temperature F	C	Recommended Control Limits of Fluoride Concentration mg/L Lower	Optimum	Upper	Maximum Contaminant Level mg/L
53.7 and below	12.0 and below	0.9	1.2	1.7	2.4
53.8-58.3	12.1-14.6	0.8	1.1	1.5	2.2
58.4-63.8	14.7-17.6	0.8	1.0	1.3	2.0
63.9-70.6	17.7-21.4	0.7	0.9	1.2	1.8
70.7-79.2	21.5-26.2	0.7	0.8	1.0	1.6
79.3-90.5	26.3-32.5	0.6	0.7	0.8	1.4

(20 mg/L)(0.188 mil gal)(8.34 lb/gal) =
x lb/day

Chlorine Required = 31.4 lb/day

SINCE 65 PERCENT HYPOCHLORITE IS TO BE USED, MORE THAN 31.36 LB/DAY OF HYPOCHLORITE WILL BE REQUIRED.

(0.65)(x lb) = 31.36 lb.

$$x = \frac{31.36}{0.65}$$

HYPOCHLORITE REQUIRED: 48.25 lb.

PROBLEM: A PUMP DISCHARGES 800 GPM. WHAT CHLORINE FEED RATE (POUNDS-PER-DAY) IS REQUIRED TO PROVIDE A DOSAGE OF 2.5 MG/L?

(800 gpm)(1440 min/day) = 1,152,000 gpd

= 1.152 mgd

(2.5 mg/L)(1.152 mgd)(8.34 lb/gal) = 24.0 lbs/day

PROBLEM: HOW MANY POUNDS PER DAY OF HYPOCHLORITE (70 PERCENT AVAILABLE CHLORINE) ARE REQUIRED FOR DISINFECTION IN A PLANT WHERE THE FLOW RATE IS 0.7 MGD AND THE CHLORINE DOSAGE IS 5.0 MG/L?

(mg/L)(mgd)(8.34 lb/gal) = lb/day

(5.0 mg/L)(0.7 mgd)(8.34 lb/gal) = x lb/day
available chlorine required = 29.19

(70%) (total lb/day) = 29.19 lb/day

THIS EQUATION CAN BE RESTATED AS:

(0.7)(x lb/day) = 29.19 lb/day

$$x = \frac{29.19}{0.7}$$

TOTAL HYPOCHLORATE = 41.7 lb/day

STABILIZATION - TREATMENT PROCESS USED TO REDUCE OR ELIMINATE THE PROBLEMS OF CORROSION AND SCALING.

STABILITY OF WATER INFLUENCED BY:

- DISSOLVED OXYGEN
- TOTAL DISSOLVED SOLIDS
- pH
- ALKALINITY
- TEMPERATURE
- TYPE OF MATERIAL

STABILIZATION METHODS:

- pH AND ALKALINITY ADJUSTMENT
- PROTECTIVE COATINGS
- INHIBITORS AND SEQUESTERING AGENTS

Galvanic Series For Metals Used In Water Systems

Corroded End (Anode)	MOST ACTIVE
Magnesium	+
Magnesium alloys	
Zinc	
Aluminum	
Cadmium	
Mild steel	Corrosion Potential
Wrought (black) iron	
Cast iron	
Lead-tin solders	
Lead	
Tin	
Brass	
Copper	
Stainless steel	−
Protected End (Cathode)	LEAST ACTIVE

Comparison of Common Stability Indices*

Stability Characteristics	*Stability Index*		
	Langelier Index (LI)	*Aggressive Index (A.I.)*	*Ryznar Index (R.I.)*
Highly aggressive	<−2.0	<10.0	>10.0
Moderately aggressive	−2.0 to <0.0	10.0 to <12.0	6.0 to <10.0
Nonaggressive	>0.0	>12.0	<6.0

$$LI = pH - pH_s$$

$$AI = pH + \log_{10}(A) + \log_{10}(Ca)$$

$$RI = 2\,pH_s - pH$$

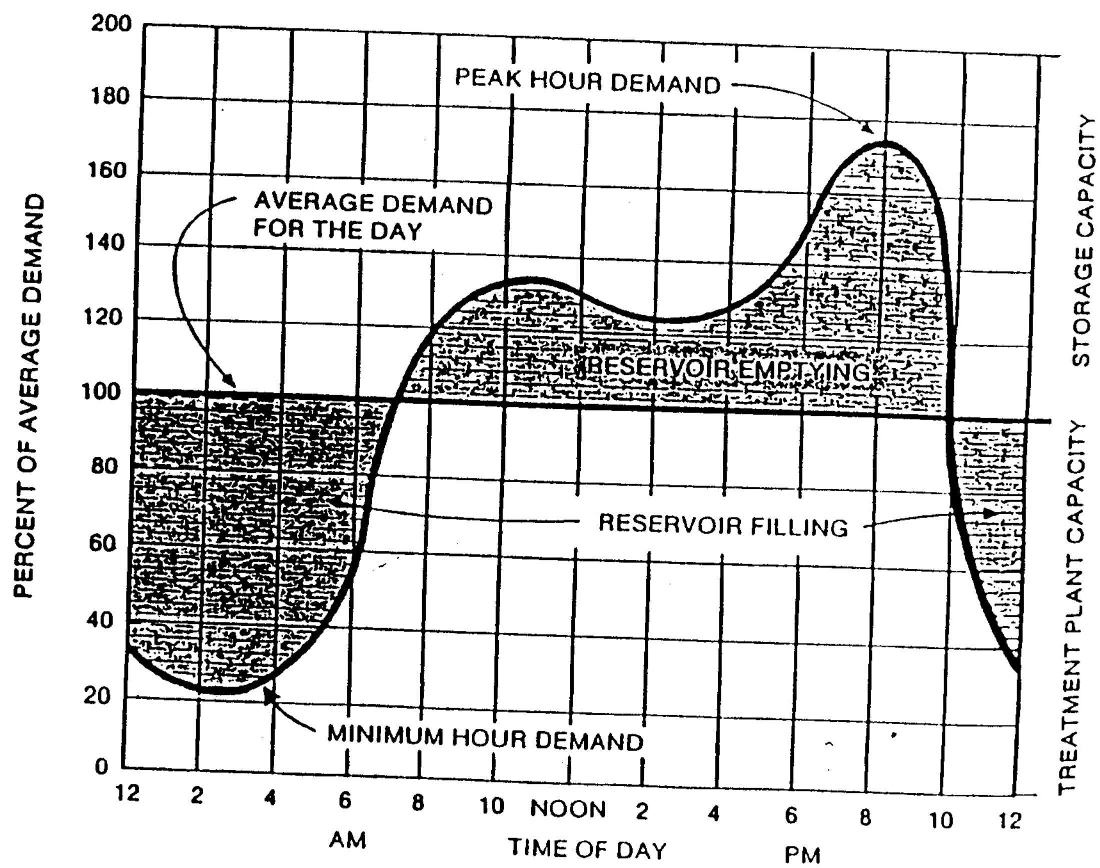
pH_s = saturation pH

A = total alkalinity (mg/L $CaCO_3$)

Ca = calcium (mg/L $CaCO_3$)

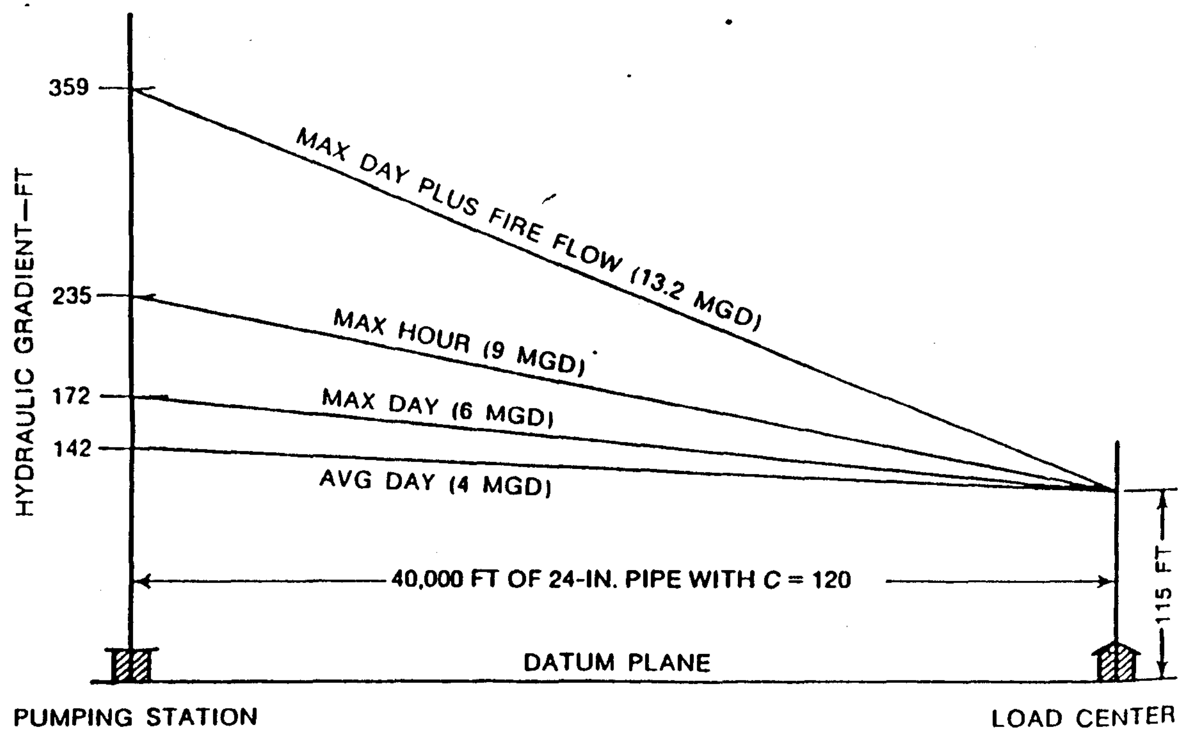
Purpose of Water Storage

- Equalizing supply and demand
- Increasing operating convenience
- Leveling out pumping requirements
- Providing water during source or pump failure
- Providing water to meet fire demands
- Providing surge relief
- Increasing detention times
- Blending water sources.

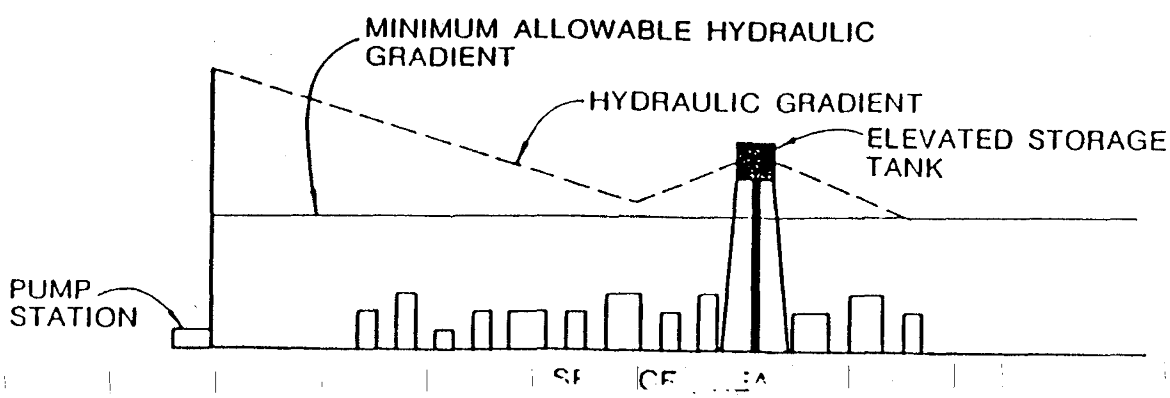


Daily Variation of System Demand

System Demand



Location of Elevated Storage



FIRE DEMAND - RESIDENTIAL AREA (1-2 FAMILY, 2 STORY)

Distance Between Houses (ft)	Required Flow (gpm)	Duration (hrs)
Greater 100	500	2
31 - 100	750	2
11 - 30	1000	2
Less than 11	1500	2
	less than 3000	2
	3000 - 3500	3
	more than 3500 (maximum 12,000)	4

GROUNDWATER

$$\text{Well yield} = \frac{\text{Gallons}}{\text{Minutes}}$$

PROBLEM: IF IT TAKES A WELL PUMP 82 SECONDS TO FILL A 55-GAL. BARREL, WHAT IS THE WELL YIELD IN GALLONS PER MINUTE?

THE EQUATION USED IN CALCULATING WELL YIELD:

$$\text{Well yield} = \frac{\text{Gallons}}{\text{Minutes}}$$

BEFORE FILLING IN THE EQUATION WITH THE GIVEN INFORMATION, THE SECONDS MUST BE EXPRESSED AS MINUTES:

$$\frac{82\text{ sec.}}{60\text{ sec/min.}} = 1.37\text{ minutes}$$

THE WELL YIELD PROBLEM CAN NOW BE SOLVED BY FILLING IN THE GIVEN INFORMATION AND COMPLETING THE DIVISION INDICATED.

$$\text{Well yield} = \frac{55\text{ gals}}{1.37\text{ min.}}$$

$$= 40.15\text{ gpm}$$

Drawdown = Pumping water level - Static water level

PROBLEM: WHEN A PUMP IS NOT IN OPERATION, THE WATER LEVEL IN THE WELL IS 39 FT. BELOW GROUND SURFACE. THE WATER LEVEL DROPS TO 67 FEET WHEN THE PUMP IS IN OPERATION. WHAT IS THE DRAWDOWN IN FT.?

Drawdown = Pumping water level - Static water level

= 67 ft - 39 ft.

= 28 ft.

$$\text{Specific capacity} = \frac{\text{Well yield in gpm}}{\text{Drawdown in ft.}}$$

IT TAKES A WELL PUMP 0.8 MIN. TO FILL A 55-GAL. BARREL. IF THE DRAWDOWN WHILE THE PUMP IS IN OPERATION IS 11 FT., WHAT IS THE SPECIFIC CAPACITY OF THE WELL?

TO CALCULATE THE SPECIFIC CAPACITY OF THE WELL, YOU MUST KNOW THE GALLONS-PER-MINUTE WELL YIELD AND THE FEET OF DRAWDOWN.

$$\text{Well Yield} = \frac{55\text{ gal.}}{0.8\text{ min.}}$$

$$= 68.75\text{ gpm}$$

USING THE GALLONS-PER-MINUTE YIELD AND DRAWDOWN INFORMATION, CALCULATE THE SPECIFIC CAPACITY OF THE WELL:

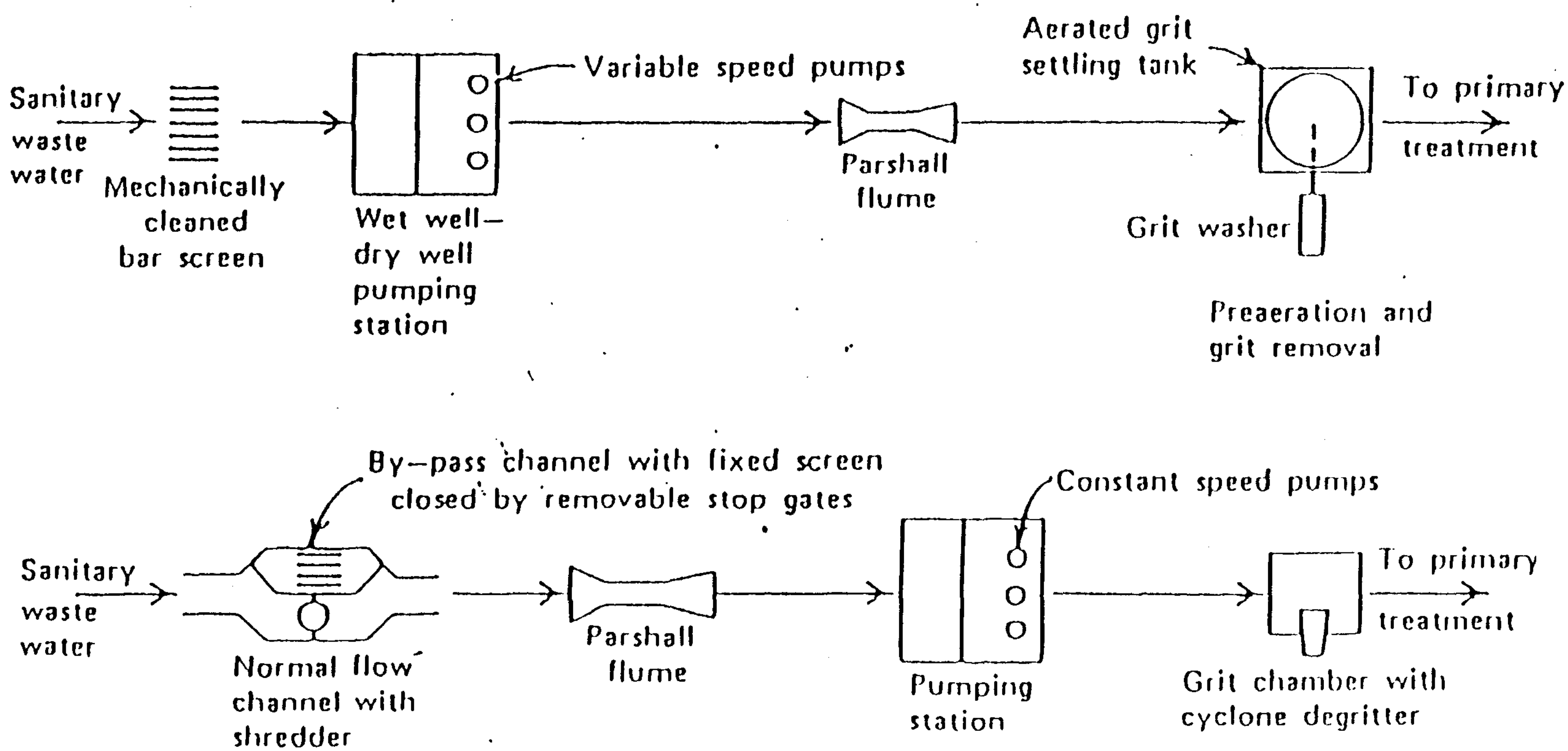
$$\text{Specific capacity} = \frac{\text{Well yield in gpm}}{\text{Drawdown in ft.}}$$

$$= \frac{68.75 \text{ gpm}}{11 \text{ ft.}}$$

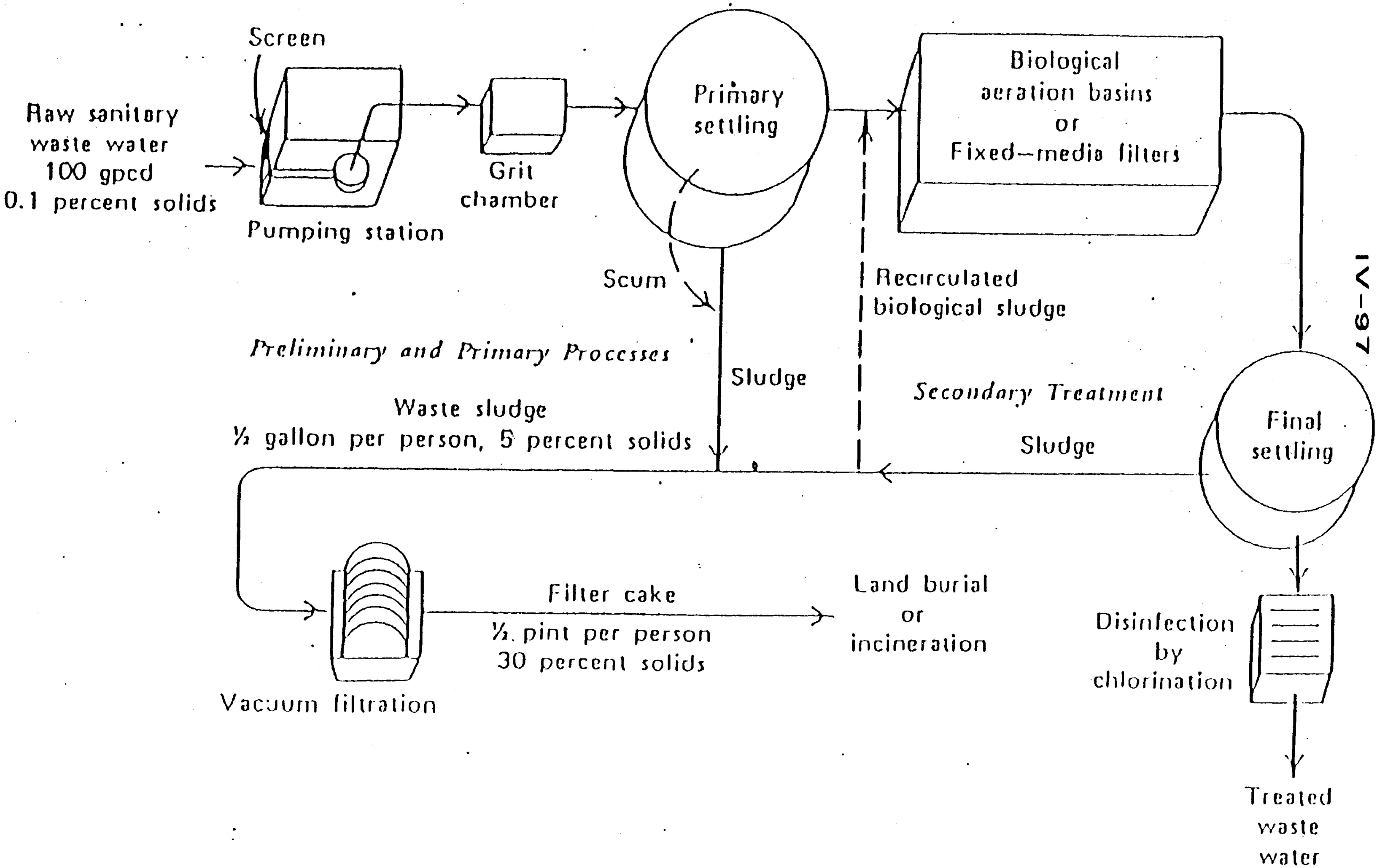
$$= 6.25 \text{ gpm/ft.}$$

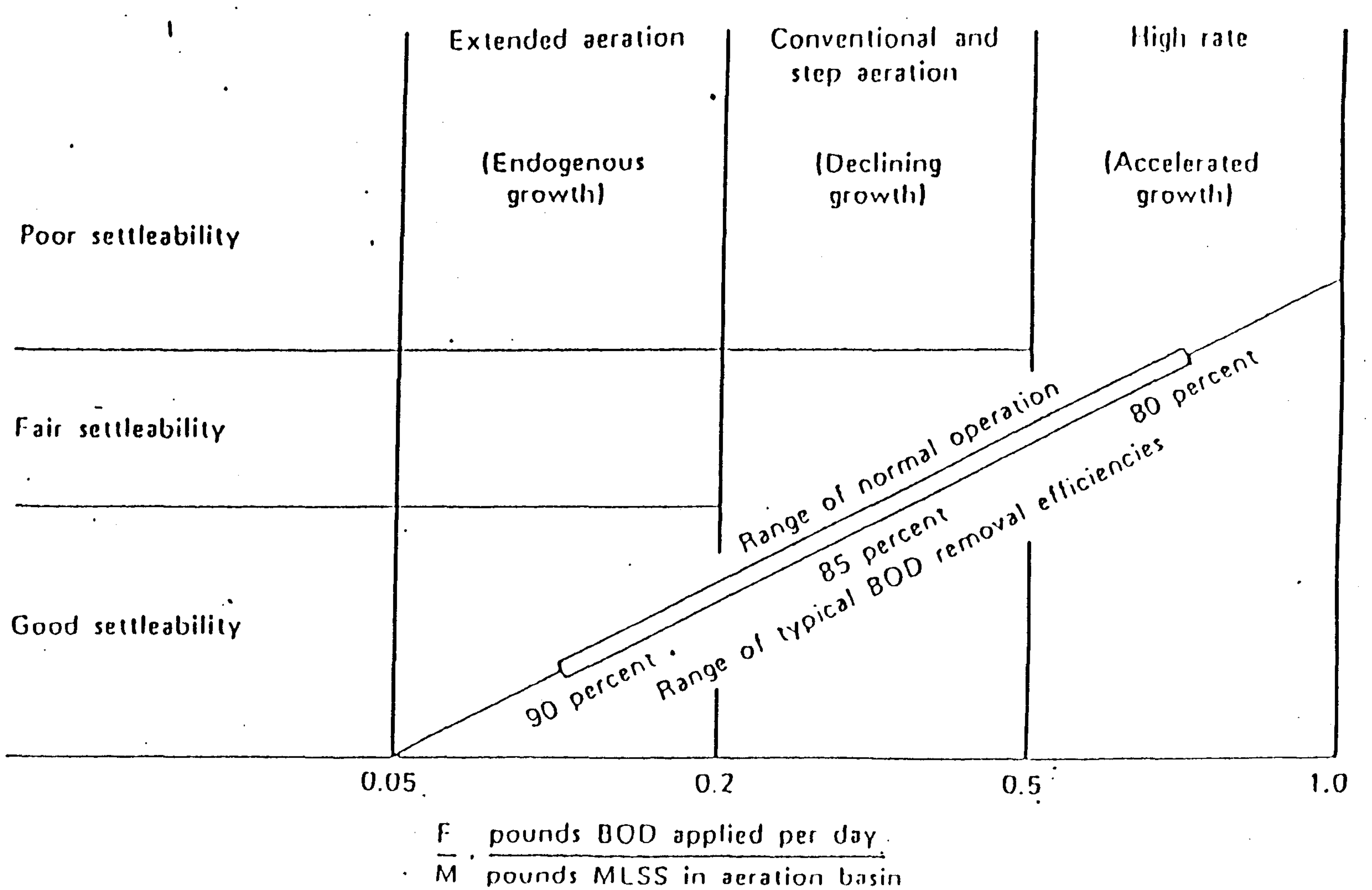
WASTEWATER TREATMENT

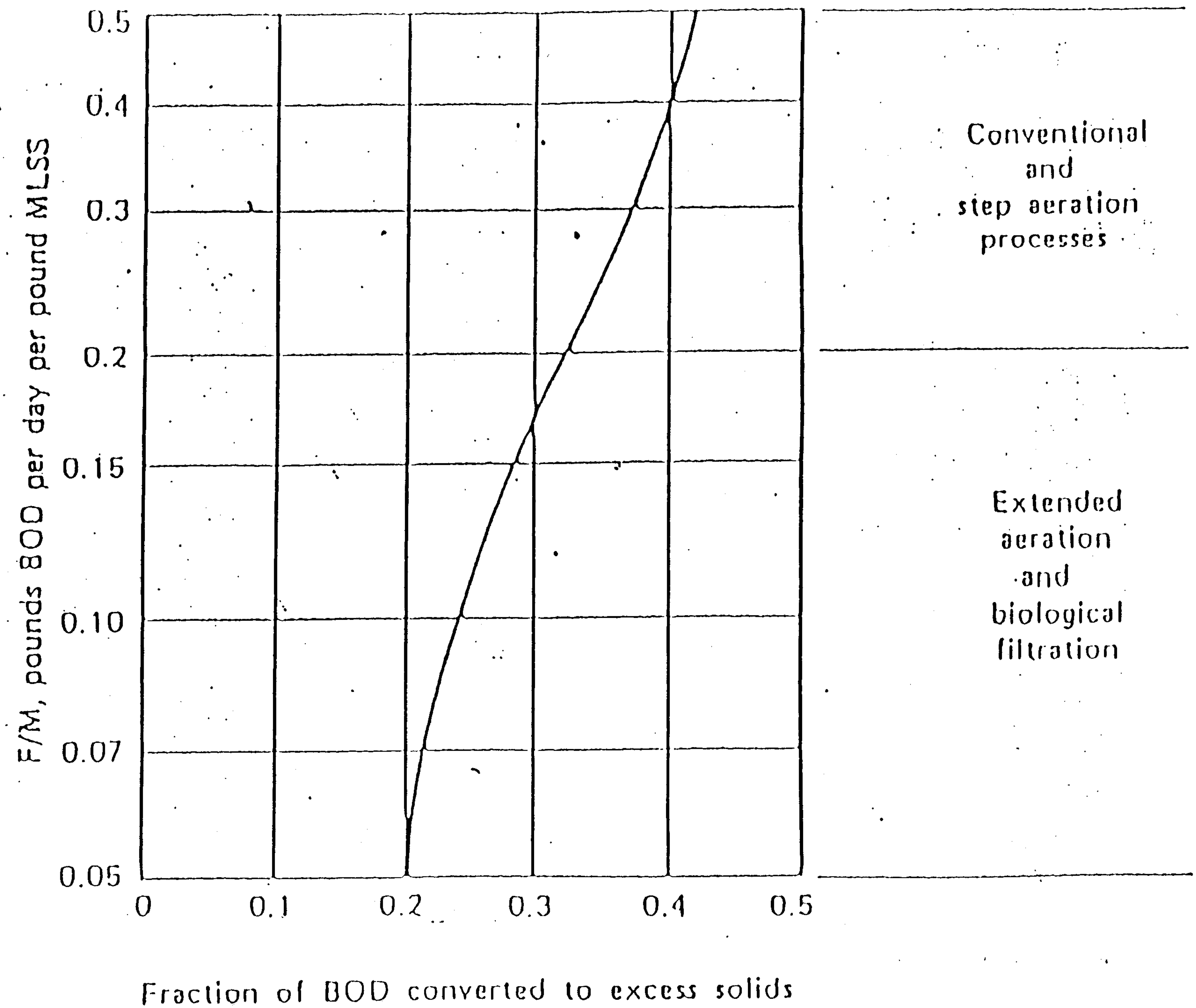
Preliminary Treatment —



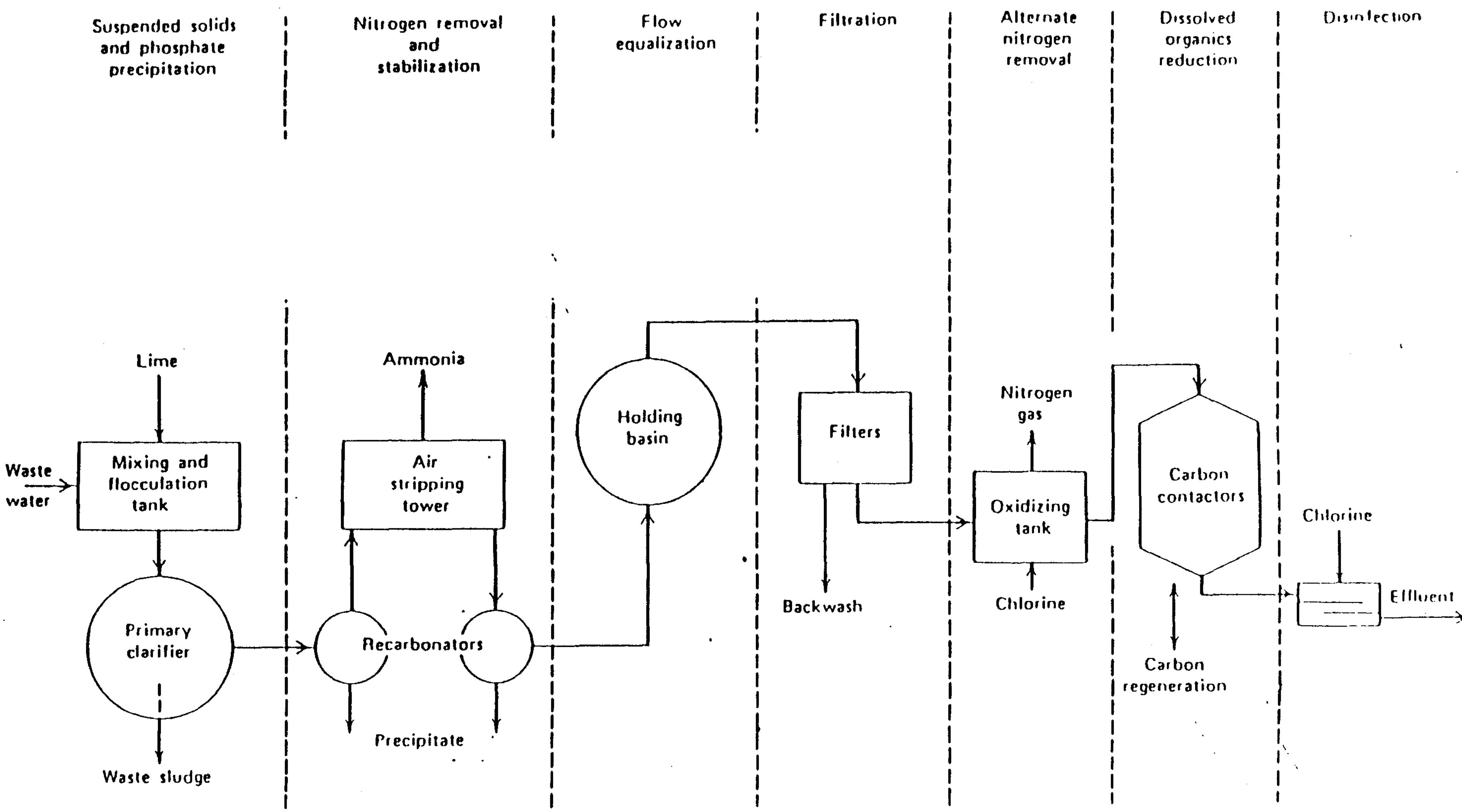
Typical arrangements of preliminary treatment units in municipal waste-water processing. The lower sequence is common to smaller plants.

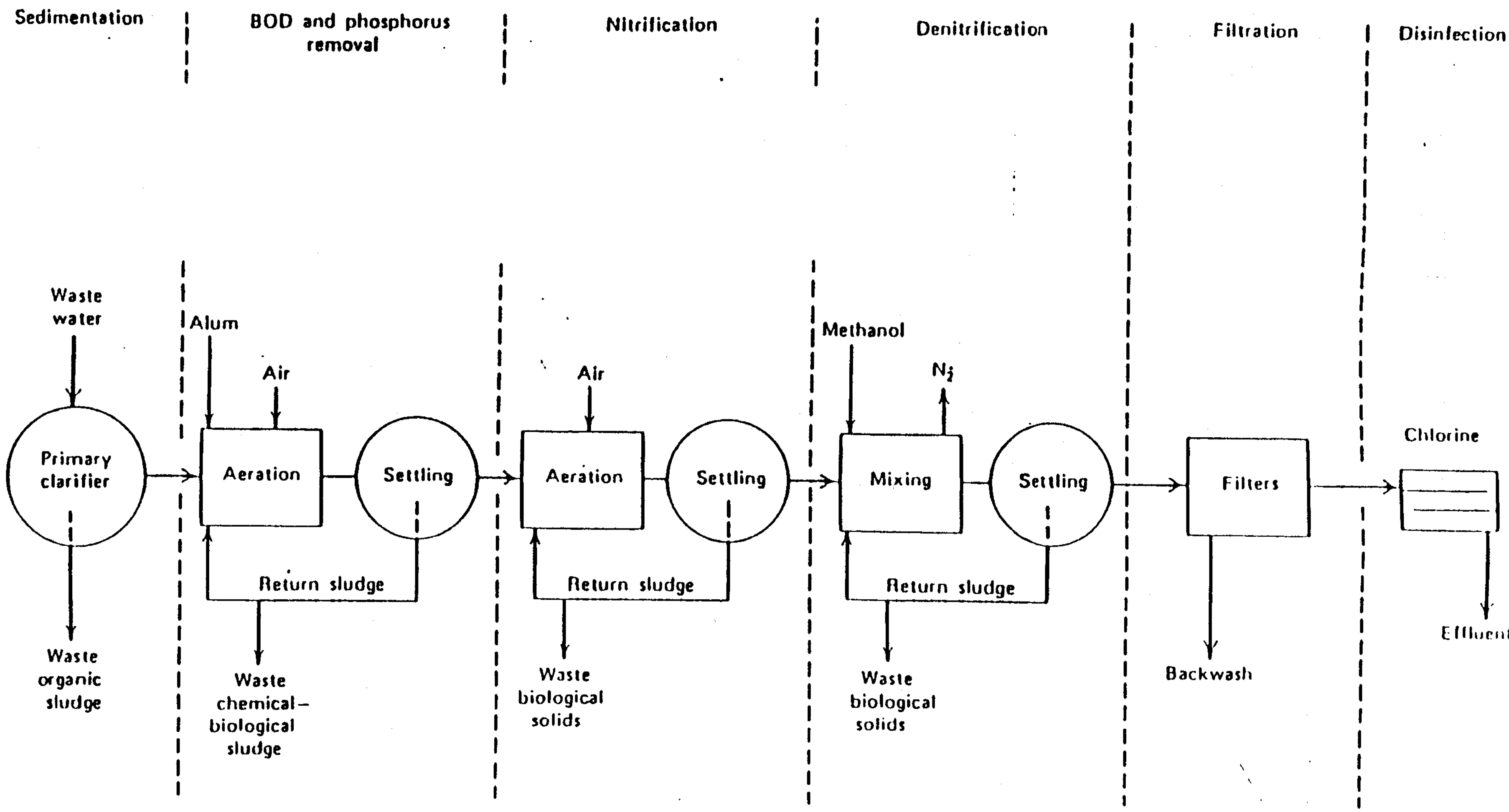

Raw sanitary waste water
100 gpcd
0.1 percent solids
Screen
Pumping station
Grit chamber
Primary settling
Scum
Sludge
Biological aeration basins or Fixed–media filters
Recirculated biological sludge
Preliminary and Primary Processes
Secondary Treatment
Waste sludge
½ gallon per person, 5 percent solids
Sludge
Final settling
Vacuum filtration
Filter cake
½ pint per person
30 percent solids
Land burial or incineration
Disinfection by chlorination
Treated waste water


Extended aeration
Conventional and step aeration
High rate
(Endogenous growth)
(Declining growth)
(Accelerated growth)
Poor settleability
Fair settleability
Good settleability
Range of normal operation
90 percent
85 percent
80 percent
Range of typical BOD removal efficiencies
0.05
0.2
0.5
1.0
F/M, pounds BOD applied per day / pounds MLSS in aeration basin



F/M, pounds BOD per day per pound MLSS
0.5
0.4
0.3
0.2
0.15
0.10
0.07
0.05
0
0.1
0.2
0.3
0.4
0.5
Fraction of BOD converted to excess solids
Conventional and step aeration processes
Extended aeration and biological filtration


Suspended solids and phosphate precipitation
Nitrogen removal and stabilization
Flow equalization
Filtration
Alternate nitrogen removal
Dissolved organics reduction
Disinfection
Lime
Waste water
Mixing and flocculation tank
Primary clarifier
Waste sludge
Ammonia
Air stripping tower
Recarbonators
Precipitate
Holding basin
Filters
Backwash
Nitrogen gas
Oxidizing tank
Chlorine
Carbon contactors
Carbon regeneration
Chlorine
Effluent


Sedimentation
BOD and phosphorus removal
Nitrification
Denitrification
Filtration
Disinfection
Waste water
Primary clarifier
Waste organic sludge
Alum
Air
Aeration
Settling
Return sludge
Waste chemical–biological sludge
Air
Aeration
Settling
Return sludge
Waste biological solids
Methanol
N_2
Mixing
Settling
Return sludge
Waste biological solids
Filters
Backwash
Chlorine
Effluent

DESIGN CRITERIA (ACTIVATED SLUDGE)

PRIMARY CLARIFIER -

REMOVAL

- BOD = 30%
- SS = 60%

OVERFLOW RATES -

	Flow Rate (gpd/ft^2) Average	Peak
Primary	600 - 1000	1800
Primary & Secondary	800 - 1500	3000
Primary & A.S. Return	600 - 1000	1800

SECONDARY TREATMENT -

	% BOD_5 Removal	(lb. BOD_5/ 1000 ft^3)	(lb. BOD_5/ lb SS)
Conventional	88 - 92	20 - 40	0.25 - 0.35
Step Aeration	88 - 92	40 - 60	0.25 - 0.35
Contact Stabil.	85 - 90	40 - 60	0.25 - 0.35
Extended Aeration	92 - 95	10 - 15	0.04 - 0.06
Modified Aeration	60 - 70	90 - 110	2.0 - 5.0

SECONDARY TREATMENT -

	MLSS (mg/L)	Hydraulic Detention Time (Hours) Avg Flow	Peak Flow
Conventional	1200 - 2200	5-8	2
Step Aeration	2000 - 3000	3-5	2
Contact Stabilization	2000 - 3500	3-5	
Extended Aeration	4000 - 5000	18-30	
Modified Aeration	400 - 600	2-4	

SECONDARY TREATMENT -

	O_2 Required/ lb. BOD_5 Applied	Review Sludge % Avg. Flow
Conventional	1.0	15 - 75
Step Aeration	1.0	20 - 75
Contact Stabilization	1.0	50 - 150
Extended Aeration	1.5	50 - 200
Modified Aeration	1.0	5 - 25

SECONDARY CLARIFIER -

OVERFLOW RATES:

	Flow Rate (gpd/ft_2) Average	Peak
Conventional	400 - 800	1200
Step Aeration	400 - 800	1200
Extended Aeration	200 - 400	800
Trickling Filter	800 - 1000	1500
Modified Aeration	600 - 800	1400

Solids Loading - ≤ 20 lb/square foot-day
(including recycle)

TYPICAL COMPOSITION OF DOMESTIC SEWAGE
(All values except settleable solids are expressed in mg/liter)

Components	Concentration		
	Strong	Medium	Weak
Solids, total	1,200	700	350
Dissolved, total	850	500	250
Fixed	525	300	145
Volatile	325	200	105
Suspended, total	350	200	100
Fixed	75	50	30
Volatile	275	150	70
Settleable solids, (ml/L)	20	10	5
Biochemical oxygen demand 5-day, 20°C	300	200	100
Total organic carbon (TOC)	300	200	100
Chemical oxygen demand (COD)	1,000	500	250
Nitrogen, (total as N)	85	40	20
Organic	35	15	8
Free ammonia	50	25	12
Nitrites	0	0	0
Nitrates	0	0	0

Components	Concentration		
	Strong	Medium	Weak
Phosphorus (total as P)	20	10	6
Organic	5	3	2
Inorganic	15	7	4
Chlorides* (Infiltration)	100	50	30
Alkalinity (as $CaCO_3$)	200	100	50
Oil & Grease	150	100	50

PROBLEM: A CONVENTIONAL ACTIVATED SLUDGE TREATMENT PLANT TREATS 1.0 MGD. THE INFLUENT BOD_5 IS 300 MG/L AND THE SS ARE 300 MG/L. DESIGN THE PLANT WITH A FOOD-TO-MICROORGANISM RATIO (F/M) OF 0.3. ASSUME REMOVAL THROUGH THE PRIMARY CLARIFIER OF 30% FOR BOTH BOD_5 AND SUSPENDED SOLIDS (SS).

Loading to Aeration = 300 (0.7) 1 (8.34) = 1750 lb. BOD_5/day

$$MLSS = \frac{1750}{0.3} = 5840 \text{ lb. SS}$$

Assume Loading = 35 lb. BOD5/1000 cubic feet

Aeration Volume = $\frac{1750}{35}$ x 1000 = 50,000 cf

Assume Depth = 15 feet

Surface Area = $\frac{50,000}{15}$ = 3333 square feet

Assume Length/Width Ratio = 5:1

$$(5W)(W) = 3333$$
$$W_2 = 667$$
$$W = 26' \qquad L = 130'$$

ASSUME UNDERFLOW FROM SECONDARY CLARIFIER = 7500 MG/L

ASSUME NO GENERATION OF SOLIDS DURING AERATION

MASS BALANCE –

(300 X 0.7) Q + 7500 R = 1850 (Q + R)

210 Q + 7500 R = 1850 Q + 1850 R

Q = 1.0

5650 R =1640

R = 0.34 mgd = 34%

ASSUME GENERATION OF SOLIDS DURING AERATION:

1 lb. BOD_5 = 0.5 lb. SS

REMOVAL = 90% BOD5 (CONVENTIONAL ACTIVATED SLUDGE)

$0.90 = 0.30 + 0.70\,(x)$

TOTAL = Primary + Secondary

x = 0.857 = 85.7% BOD5 removal

Secondary Treatment = 0.857 x (300 x 0.7)
= 180 mg/L BOD5

Generation of SS = 0.5 x (180) = 90 mg/L

MASS BALANCE -

$(300 \times 0.7)\,Q + 7500\,R + 90\,Q = 1850\,(Q + R)$

$210\,Q + 7500\,R + 90\,Q = 1850\,Q + 1850\,R$

Q = 1.0 $\qquad 5650\,R = 1550$

$R = 0.275$ mgd = 27.5%

OXYGEN REQUIREMENTS -

Assume: 1.0 lb. O_2 for each 1.0 lb. BOD_5 applied.

Required: $O_2 = (1) \times (1750 \text{ lb/day}) = 1750$ lb. O_2/day

Air = 20% O_2

Required Air = $\frac{1750}{0.20}$ = 8750 lb. per day

Assume Transfer Efficiency = 10%

Required Air = $\frac{8750}{0.10}$ = 87,500 lb. per day

PRIMARY CLARIFIER -

Average Overflow = 1000 gpd/square feet

Q = 1.0

Required Surface Area = $\frac{1.0 \times 10^6}{1000} = 1000$ s.f.

Area = $\frac{3.1416}{4} (D)^2 = 1000$

D = 36 feet

Solids Loading = (300 x .3) x (8.34) x 1.0) = 750 lbs.

= $\frac{750}{1000}$ = 0.75 lb. per square foot

SECONDARY CLARIFIER -

Average Overflow = 600 gpd/square feet

Q = 1.0

Required Surface Area = $\frac{1.0 \times 10^6}{600}$ = 1670 s.f.

Area = 3.1416 $(D)^2$ = 1670

D = 46 feet

Solids Loading = (210 x 1.0) + (7500 x 0.275) + (90 x 1.0) =

$\frac{19703}{1670}$ = 11.8 lb. per s.f.

DESIGN CRITERIA TRICKLING FILTER

CLASSIFICATION -

HYDRAULIC

- LOW RATE
- HIGH RATE

ORGANIC

- LOW RATE
- HIGH RATE

PARAMETER	LOW RATE	HIGH RATE
Hydraulic Loading (MGAD)	1-4	10-40
Organic Loading (#BOD5/A-Ft-Day)	300-1000	1000-5000
Depth (ft)	6-10	3-8
Recirculation Basin	None	1:1 - 4:1
Energy Requirements	None	10-50 HP/MG
Filter Flies	Many	Few (larvae are washed out)
Sloughing	Intermittent	Continuous
Operation	Simple	Some operator skill
Effluent	Highly Nitrified	Partially Nitrified at Low Loading

VELZ EQUATION -

$$\frac{L_D}{L} = 10^{-KD}$$

$$L_a = \frac{L_o + R\,L_e}{1 + R}$$

where:

L	=	applied BOD_L (removable) £ 0.90 L_o
L_D	=	removable portion of BOD_L remaining at depth D
K	=	rate of removable coefficient - 0.175 for low-rate filter - 0.150 for high-rate filter
D	=	depth (feet)
L_a	=	applied BOD_L after dilution by recirculation
L_o	=	BOD_L of applied untreated wastewater
L_e	=	effluent BOD_L
R	=	recirculation ratio (Qr/Q)

NRC EQUATIONS -

$$E_1 = \frac{1}{1 + .0085\sqrt{W/VF}} \quad \text{(Single-stage)}$$

$$E_2 = \frac{1}{1 + \frac{.0085}{1-E_1}\sqrt{\frac{W_1}{VF}}} \quad \text{(Second-stage)}$$

where:

E_1 = Fractional efficiency of BOD_5 removal for process including recirculation and sedimentation.

W = BOD_5 filter loading (lbs/day)

V = Volume of filter media (Acre-Feet)

F = Recirculation Factor:

$$\left(\frac{1 + R}{[1 + R/10]^2}\right) \text{ with } R = \frac{Qr}{Q}$$

E_2 = Fractional efficiency of BOD_5 removal for second-stage process including recirculation and sedimentation.

W_1 = BOD_5 loading to second stage (lbs/day).

PROBLEM: USE THE VELZ EQUATION

DETERMINE THE FILTER DEPTH FOR A LOW-RATE FILTER IF THE SETTLED SEWAGE ULTIMATE BOD DEMAND, L_0, IS 250 MG/L AND THE EFFLUENT BOD_5 IS 30 MG/L. ASSUME THAT THE EFFLUENT BOD_5 IS EQUAL TO 0.7 BOD_L AND THAT L = 0.9 L_0.

BOD_L of the Effluent: $BOD_L = \frac{30}{0.7} = 43$ mg/L

BOD that is not removable:

$BOD_{HR} = 250\ (0.1) = 25$ mg/L

Therefore: $L_D = 43 - 25 = 18$ mg/L (removable BOD remaining)

$L = 250\ (0.9) = 225$ mg/L (initial removable BOD)

$\frac{L_D}{L} = \frac{18}{225} = 0.08$

$0.08 = 10^{-.175\ (D)}$

$12.5 = 10^{.175D}$

$D = \frac{1.097}{.175} = 6.3$ feet

PROBLEM: USE THE NRC EQUATIONS

AN INDUSTRY WITH A WASTE BOD_5 OF 600 MG/L IS TO BE TREATED BY A TWO-STAGE TRICKLING FILTER. THE DESIRED EFFLUENT QUALITY IS 50 MG/L OF BOD_5. IF THE FILTER DEPTHS ARE EACH 6 FEET AND THE

RECIRCULATION RATIO IS 4:1, DETERMINE THE REQUIRED FILTER DIAMETERS. Q = 2 MGD AND ASSURE EQUAL EFFICIENCIES. USE THE NRC METHOD.

$$\text{Overall Efficiency} = \frac{600-50}{600} \times 100 = 91.7\%$$

Therefore: $E_1 + E_2 \ (1 - E_1) = 0.917$
$E_1 = E_2 = 0.713$

Recirculation Factor:

$$F = \frac{1 + R}{(1 + R/10)^2} = \frac{1 + 4}{(1.4)^2} = 2.55$$

Loading (1st filter) -

$W_1 = 600\ (8.34)(2) = 10{,}000$ lb. BOD_5/day

Volume (1st stage):

$$E_1 = \frac{1}{1 + .0085\sqrt{W/V_1F}}$$

$V_1 = 1.75$ acre-feet

Diameter (1st stage):

$$A_1 = \frac{V_1}{d} = \frac{1.75}{6} = 0.292 \text{ acre} = 12{,}700 \text{ s.f.}$$

$\emptyset_1 = 127$ feet

Loading (2nd filter):

$W_2 = (1-E_1)\ W_1 = .287\ (10{,}000) = 2870$ lbs. BOD_5/day

Volume (2nd stage):

$$E_2 = \frac{1}{1 + \frac{.0085}{(1-E_1)}\sqrt{\frac{W_2}{V_2F}}}$$

$V_2 = 6.08$ acre-foot
$A_2 = 1.013$ acre $= 44{,}200$ square feet
$Ø_2 = 237$ feet (or two 2nd stage filters)
at $Ø'_2 = 168'$

PROBLEM: A DOMESTIC SEWAGE HAS A SUSPENDED SOLIDS CONCENTRATION OF 300 MG/L. THE FIRST PHASE OF A TREATMENT PROCESS FOR THIS WASTE IS PRIMARY SEDIMENTATION. THE PRIMARY SEDIMENTATION TANK HAS A DETENTION TIME OF 2 HOURS, AND THE AVERAGE SEWAGE FLOW RATE IS 4 MGD.

(a) Determine the volume of the primary sedimentation tank.
(b) Determine the expected suspended solids removal.
(c) Determine the quantity of sludge (in gallons) which would be produced per day, assuming the sludge contains 95% water and has a specific gravity of 1.03.

(a) Volume = flowrate (cfs) x detention time (sec.)

$$\text{Volume} = (4\ \text{MGD})\left(1.547\ \frac{\text{cfs}}{\text{MGD}}\right)(2\ \text{hrs})\left(3600\ \frac{\text{sec.}}{\text{hr.}}\right)$$

Volume = 44,550 cu. ft.

(b) A well designed clarifier may be expected to remove 50 to 60 percent of the suspended solids from a domestic wastewater.

(c) Assuming a removal of 55%.

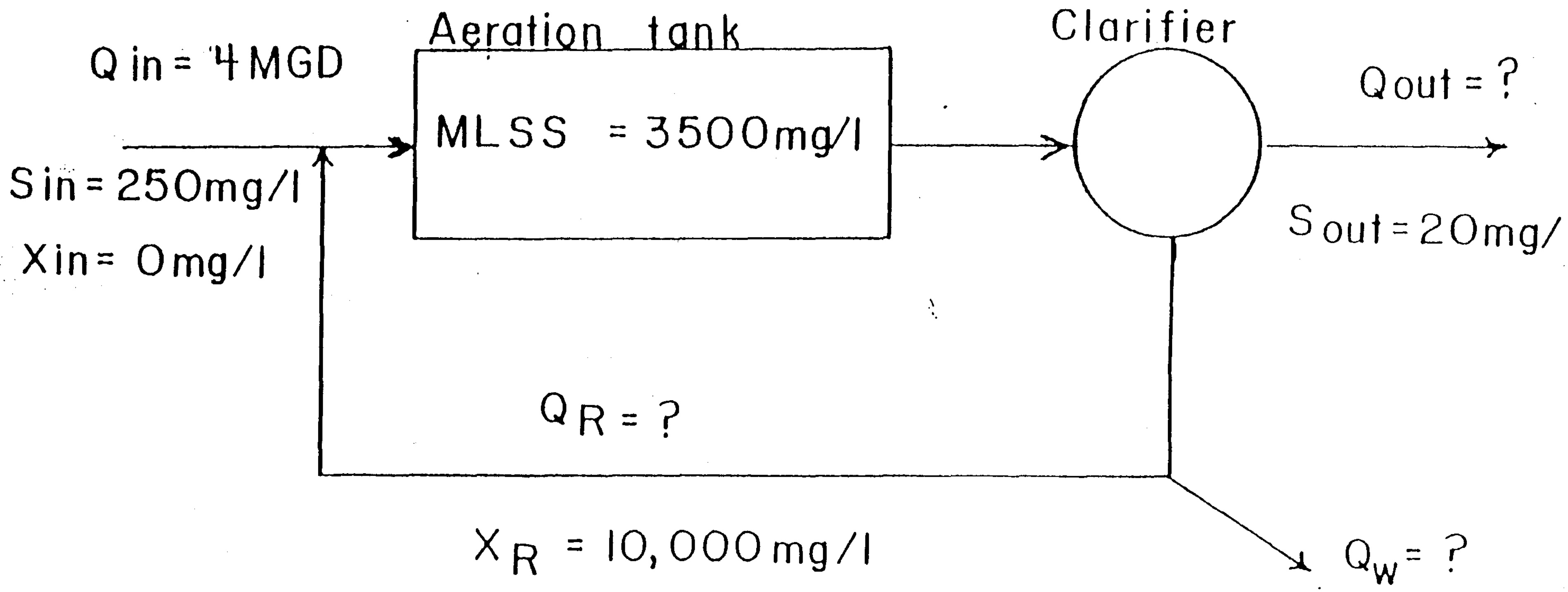
$$\text{Volume of Sludge} = \frac{(0.55)(300\ \text{mg/L})\left(8.34\ \frac{\text{lb/MG}}{\text{mg/L}}\right)(4\ \text{MGD})(7.48\ \text{gal/cu. ft.})}{(1.03)(0.05)(62.4\ \text{lb./cu. ft.})}$$

= 12,800 gals/day

PROBLEM: THE FLOW SHEET REPRESENTS A COMPLETE MIXACTIVATED SLUDGE SYSTEM. THE INFLUENT FLOW RATE, Q_{IN}, IS 4 MGD, WITH AN INFLUENT WASTE CONCENTRATION, S_{IN}, OF 250 MG/L. A MLSS OF 3500 MG/L IS TO BE MAINTAINED IN THE AERATION TANK WITH A RETURN SLUDGE CONCENTRATION, X_R, OF 10,000 MG/L. ASSUMING THAT THE INFLUENT SUSPENDED SOLIDS CONCENTRATION, X_{IN}, IS NEGLIGIBLE, THE GROWTH-YIELD COEFFICIENT IS:

$$0.5\left(\frac{\text{lb cells}}{\text{lb BOD utilized}}\right)$$

AND THE EFFLUENT WASTE CONCENTRATION, S_{OUT}, MUST BE NO GREATER THAN 20 MG/L, DETERMINE:


Aeration tank
Clarifier
Q in = 4 MGD
MLSS = 3500mg/l
Qout = ?
Sin = 250mg/l
Xin = 0mg/l
Sout = 20mg/l
QR = ?
XR = 10,000mg/l
QW = ?

(a) the effluent flowrate, Q_{OUT}
(b) the sludge wastage rate, Q_W
(c) the return sludge flowrate, Q_R

FIND Q_W FIRST -

(a) Determine the quantity of excess sludge produced:

excess sludge = (yield)(lb. BOD utilized)

$$= 0.5 \ \frac{\text{lb cells}}{\text{lb BOD utilized}} \times (250 \text{ mg/L} - 20 \text{ mg/L}) \times$$

$$(8.34 \ \frac{\text{lb/mg}}{\text{mg/L}}) \times (4 \text{ mgd})$$

$$= 3836 \text{ lb/day}$$

$$\text{Volume of sludge, } Q_W = \frac{(3836 \text{ lb/day})(7.48 \text{ gal/cu. ft.})}{(0.01)(62.4 \text{ lb/cu. ft.})(1.00)}$$

$$= 46{,}000 \text{ gal/day}$$

FIND Q_{OUT}

$$Q_{OUT} = Q_{IN} - Q_W$$

$$= (4 \text{ mgd}) - (46{,}000 \text{ gal/day})$$
$$= 3.95 \text{ mgd}$$

MASS BALANCE:

$$Q_{IN}(0) + Q_R(X_R) = (Q_{IN} + Q_R) \text{ MLSS}$$

$$Q_R(X_R1 - Q_R(\text{MLSS}) = Q_{IN}(\text{MLSS})$$

$$Q_R \frac{(X_R}{MLSS} - 1) = Q_{IN}$$

FIND Q_R

$$Q_R \approx \left[\frac{1}{\left(\frac{X_R}{MLSS}\right) - 1}\right] Q_{IN}$$

$$Q_R \approx \left[\frac{1}{\left(\frac{10,000}{3500}\right) - 1}\right] (4)$$

$Q_R \approx 2.15$ mgd

PROBLEM: A MUNICIPAL WASTEWATER TREATMENT PLANT HAS AN AVERAGE FLOW OF 50 MGD AND APPLIES CHLORINE TO THE SEWAGE EFFLUENT AT 12.5 PPM. CHLORINE IS PURCHASED IN 16-TON CARS AT $2.70 PER 100 POUNDS. CHLORINATION IS ONLY REQUIRED FROM MAY 15 THROUGH SEPTEMBER 15. A SWITCHING AND DELIVERY CHARGE OF $92 PER CAR IS INCURRED. CHLORINE IS APPLIED AS A 0.2% SOLUTION AND WATER COST $0.16 PER 1000 GALLONS. WHAT IS THE ANNUAL COST FOR CHLORINATION?

May 15 - September 15 = 124 days

Pounds of Chlorine = 50 x 12.5 x 8.34 x 124
= 646,000 lbs.

Number of Tank Cars = $\frac{646,000}{200,000 \times 16} = 20.2$
or 21 cars

Cost per Tank Car = $\frac{32,000}{100} \times 2.70 + 92 = \$956$

Cost of the Water:

$$\frac{\text{Gallons Water}}{10^6} \times 2000 \times 8.34 = 646,000$$

Gallons = 38.7×10^6

Cost = $\frac{38.7 \times 10^6}{1000} \times 0.16 = \$6,190$

Total Chlorination Cost =

$956 \times 21 + 6190 = \$26,266$

Note: 0.2% = 2000 ppm

PROBLEM: A WASTEWATER TREATMENT PLANT PROVIDES PRIMARY TREATMENT FOR 30 MGD OF SANITARY SEWAGE HAVING AN AVERAGE SS CONTENT OF 200 PPM, 60% OF WHICH IS REMOVED IN THE CLARIFIERS. SLUDGE PUMPED DAILY TO THE DIGESTERS CONTAINS 4% DRY SOLIDS, 65% OF WHICH ARE VOLATILE. DIGESTION AT 90°F REDUCES VOLATILE SOLIDS 60%, PRODUCING GAS HAVING A HEAT VALUE OF 600 BTU PER FT^3

OF THE RATE OF 15 FT^3 PER POUND OF VOLATILE MATTER DESTROYED.

ALL OF THE GAS IS USED AS FUEL IN ENGINES. HEAT RECOVERED FROM ENGINE JACKETS AND EXHAUST GAS IS 50% OF INPUT AND IS USED TO HEAT THE DIGESTERS IN A SYSTEM HAVING AN EFFICIENCY OF 75%.

HOW MUCH ADDITIONAL HEAT FROM OTHER SOURCES IS REQUIRED WHEN DIGESTION TANK LOSSES ARE 1,000,000 BTU PER HOUR AND PRIMARY SLUDGE HAS A TEMPERATURE OF 45°F?

SS removed by clarifiers = W_{SS} = 30 x 200 x 8.34 x .60 = 30,000 lbs/day

Volatile material = W_{VS} = .65 x 30,000 = 19,500 lbs/day

Volatile material digested = W_{VSD} = .60 x 19,500 = 11,700 lbs/day

Heat from the Sludge Gas = Qg =

$$\frac{11{,}700 \times 600 \times 15}{24} = 4.39 \times 10^6 \text{ BTU/hr.}$$

Amount of heat produced by engines = Q1 = $0.50 \times 4.39 \times 10^6 = 2.196 \times 10^6$ BTU/hour

Amount of Primary Sludge =

$$W_{PS} = \frac{30,000}{.04 \times 24} = 31,200 \text{ lb/hr.}$$

Assume specific heat of sludge is the same as water or 1 BTU/lb/°F

Heat required to raise temperature of sludge from 45°F to 90°F =

$Q_2 = 31,200 \times 1\ (90-45) = 1.404 \times 10^6$ BTU/hour

Heat loss from digester = $Q_L = 10^6$ BTU/hour

Heat required by digester = $Q_L + Q_2 = (1 + 1.404) \times 10^6 = 2.404 \times 10^6$ BTU/hr.

Let: Q_0 = needed heat from outside sources

Efficiency of Heating System =

$$\frac{\text{Output}}{\text{Input}} = \frac{Q_L + Q_2}{Q_0 + Q_1} = 7.5$$

$$.75 = \frac{2.404 \times 10^6}{Q_0 + 2.196 \times 106}$$

$$Q_0 = 1.01 \times 106 \text{ BTU/hr.}$$

BOD EQUATION —

$$\frac{dL}{dt} \propto L$$

$$\frac{dL}{dt} = -K_1 L$$

$$\int_{L_u}^{L_t} \frac{dL}{L} = -K_1 \int_0^t dt$$

$$\ln L_t - \ln L_u = K_1 t$$

$$\frac{L_t}{L_u} = 2.7^{-K_1 t} = 10^{-k_1 t}$$

= fraction BOD remaining

$$\% \text{ remaining} = 100 \times \frac{L_t}{L_U} = 100 \times 10^{-k_1 t}$$

$$\log\left(100 \times \frac{L_t}{L_U}\right) = 2 - k_1 t$$

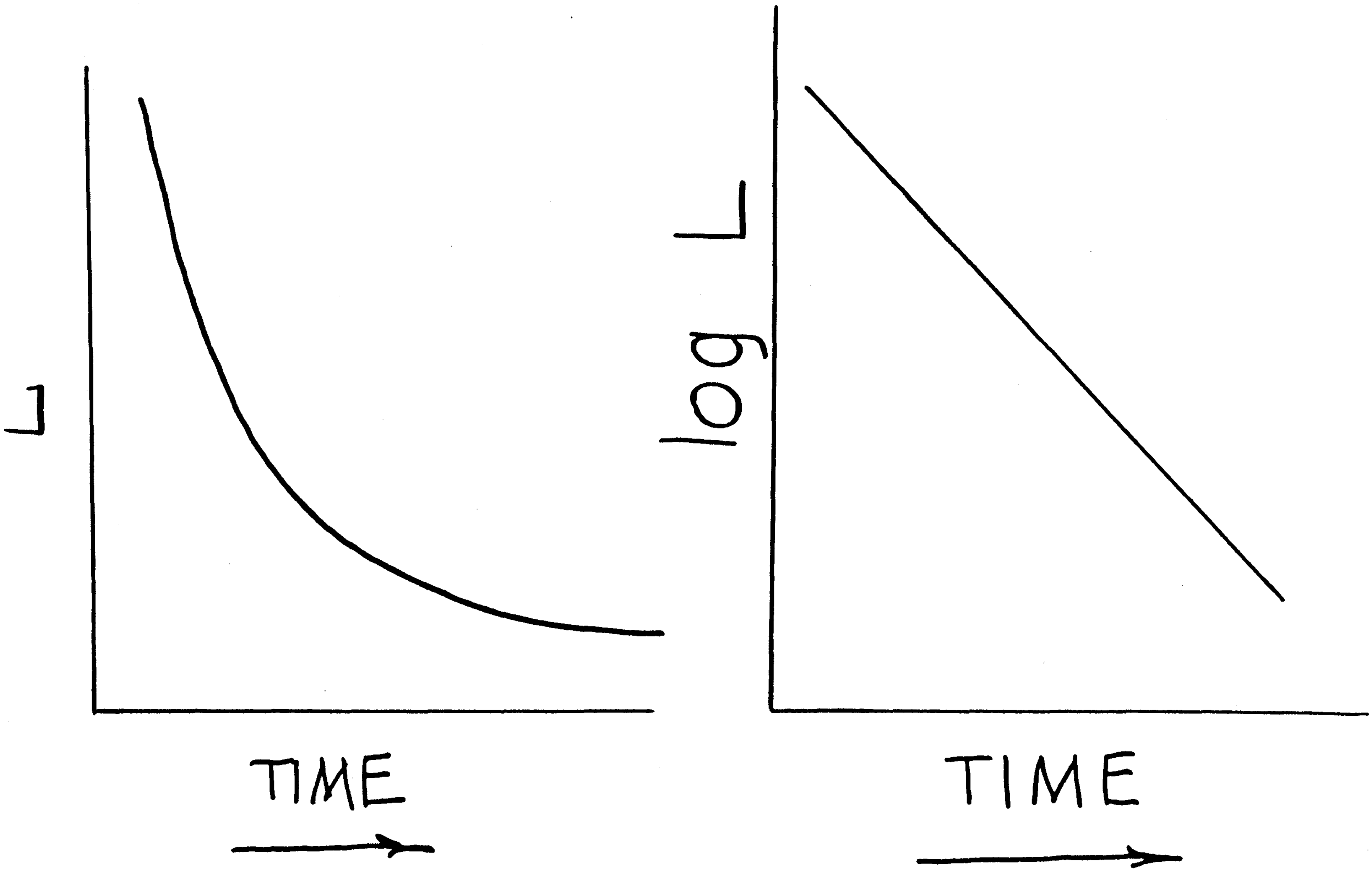
$t = 0$ % remaining $= 100$

$t = ?$ % remaining $= 32$

$$\log(32) = 2 - k_1 t$$

$$1.5 - 2 = -k_1 t$$

IV–125


L
TIME
log L
TIME

$$t = \frac{-0.5}{-0.1} = 5 \text{ day}$$

Effect of Temperature =

$$k_T = k_{20} \times 1.047^{(T-20)}$$

$T = 0°C \qquad k_0 = 0.04$

$T = 30°C \qquad k_{30} = 0.158$

PROBLEM: TOWN A LOCATED ON A RIVER AND DISCHARGING A WASTE OF 100,000 POPULATION EQUIVALENTS PER DAY. INDUSTRY B IS LOCATED 4 DAYS TIME OF TRAVEL DOWNSTREAM OF A. ASSUME THE RIVER LOAD UPSTREAM OF A IS ZERO; RIVER K, AT 20°C IS 0.1, AND THE RIVER WATER TEMPERATURE IS 25°C; THE INDUSTRY HAS A WASTE FLOW OF 2 MGD AT 489 MG/L BOD_5 WITH AN EFFLUENT k_1 AT 20°C OF 0.1. DETERMINE THE RESIDUAL POPULATION EQUIVALENTS IN THE RIVER AT 10 DAYS TIME-OF-TRAVEL BELOW A AND FRACTION ORIGINATED AT TOWN A.

River k_1 value:

$$K_{25^\circ} = 0.1 \times 1.047^{(25-20)} = 0.13$$

Industrial Load:

$$\text{P.E.} = \frac{489}{.68} \times 8.34 \times 2 \times \frac{1}{.24} = 49{,}979$$

Where: 1 population equivalent = 0.24 lb. BODu/person-day

Assume: $k_R = k_1$ (i.e., no settling, sludge deposits, etc.

$$\log(Y) = 2 - .13 \times 4$$

$$Y = 30\% \text{ remaining after 4 days}$$

Population Equivalent (PE) =

100,000 x 0.3 = 30,000 remaining at
Town A

Industry B outfall:

River P.O. = 30,000 + 49,979 = 79,979

log(Y) = 2 - .13 x 6

Y = 17% remaining after 6 more days

P.E. = 79,979 x .17 = 13,600 remaining

Town A Residual:

log(Y) = 2 - .13 x 10

Y = 5% remaining after 10 days

P.E. = 100,000 x .05 = 5000 remaining

Therefore:

Town A fraction = $\frac{5000}{13,600}$ = .37

or 37% of residual at 10 days originated at Town A

OXYGEN SAG EQUATION —

DEOXYGENATION $$\frac{dL}{dt} = -K_1 L$$

REOXYGENATION $$\frac{dC}{dt} = K_2(C_s - C)$$

$$\frac{dD}{dt} = -K_2 D$$

$$\frac{dD}{dt} = K_1 L - K_2 D$$

$$D = \frac{k_1 L_u}{k_2 - k_1}\left(10^{-k_1 t} - 10^{-k_2 t}\right) + D_a \, 10^{-k_2 t}$$

SAG POINT —

$$\frac{dD}{dt} = 0$$

SAG EQUATION –

$$t_c = \frac{1}{k_2 - k_1} \log\left[\frac{k_2}{k_1}\left(1 - \frac{D_a(k_2 - k_1)}{L_u \times k_1}\right)\right]$$

$$D_c = \frac{k_1}{k_2} L_u \times 10^{-k_1 t_c}$$

PROBLEM: A WASTEWATER TREATMENT PLANT IS BEING CONSIDERED FOR A CITY OF 25,000. THE RECEIVING STREAM HAS A MINIMUM 7 CONSECUTIVE DAY, 10 YEAR LOW FLOW OF 15 CUBIC FEET PER SECOND. THE STREAM ABOVE THE OUTFALL HAS A RESIDUAL LOAD OF 2 PPM BOD5 AND 9.0 PPM D.O. AT A TEMPERATURE OF 20°C; THE RIVER k1 IS 0.13 AND k2 OF 0.18. IT IS DESIRED TO MAINTAIN THE MINIMUM D.O. ≥ 3.0 PPM AT ALL TIMES. WHAT DEGREE OF TREATMENT IS NEEDED? (EFFLUENT D.O. IS ASSUMED AT 6.0 PPM).

Assume: 100 gallons/capita-day

Therefore:

(Plant Effluent) $Q_1 = 25{,}000 \times 100 = 2.5$ mgd

(Stream Flow) $Q_2 = 15 \times 7.5 \times 86{,}400 = 9.72$ mgd

$G_s = 9.2$ ppm (20°C)

Therefore: $D_c = 9.2 - 3.0 = 6.2$ ppm

Initial Conditions:

$$\frac{9.72 \times 9.0 + 2.5 \times 6.0}{12.22} = 8.4 \text{ ppm}$$

Therefore: $D_a = 9.2 - 8.4 = 0.8$ ppm

$$t_c = \frac{1}{.18 - .13} \log \left[\frac{.18}{.13} \left(1 - .8 \frac{(.18 - .13)}{.13 \, L_u}\right)\right]$$

$$D_c = \frac{.13}{.18} L_u \times 10^{-13t_c}$$

Solve for proper t_c and L_u to give $D_c = 6.2$

Trial	Assumed L_u	Calc. t_c	Calc. D_c	Conclusions
1	200	2.78	52	Too large
2	20	2.66	6.5	Too large
3	19	2.65	6.2	O.K.

Therefore:

L_u = 19 ppm (ultimate stream BOD at t = 0)

$\log(Y) = Z - .13 \times 2.65$
= 49% remaining at t_c

$\log(Y) = 2 - .13 \times 5$ (Assuming effluent k_1 = 13)
= 22% remaining (78% exerted)

Therefore:

$BOD_5 = 19 \times .78 = 14.8$ ppm

Assume raw domestic sewage: BOD_5 = 250 ppm

Continuity:

Raw Wastewater
$Q_0 = 2.5$ MGD
$BOD_5 = 250$ ppm

→ Wastewater Treatment Plant

Treated Wastewater
$Q_1 = 2.5$ MGD
$BOD_5 = ?$

STREAM

$Q_2 = 9.72$ MGD
$BOD_5 = 2.0$ ppm

$Q_3 = 12.22$ MGD
$BOD_5 = 14.8$ ppm

$$9.72 \cdot 2 + 2.5 \cdot X = 12.22 \cdot 14.8$$

$$X = 64.6 \text{ ppm}$$

Required Treatment $= \dfrac{250 - 64.6}{250} = 74\%$

SOLID WASTE

SOLID WASTE

LANDFILL PROBLEM:

GIVEN:

1. A SITE IS PROJECTED TO SERVE AS A SANITARY LANDFILL FOR A CITY OF 10,000 PEOPLE. THE CITY OF FORECAST TO GROW TO A POPULATION OF 24,000 WITHIN 15 YEARS.

2. PER CAPITA SOLID WASTE GENERATION IS ESTIMATED TO BE 1,200 LBS/YR.

3. THE ESTIMATED FINAL DENSITY OF THE TRASH IS 620 LBS/CU YD.

4. THE SITE CAN BE EXCAVATED DOWN TO A 7-FT. DEPTH.

FIND:

1. GENERAL REQUIREMENTS FOR A SANITARY LANDFILL (REGULATIONS).

2. WHAT ARE THE GENERAL CONSIDERATIONS TO CHOOSE A SITE FOR A SANITARY LANDFILL?

3. GIVE THE INCREMENTAL AREA PER YEAR NEEDED FOR LANDFILL'S LIFE FOR A SELECTED DEPTH OF FILL.

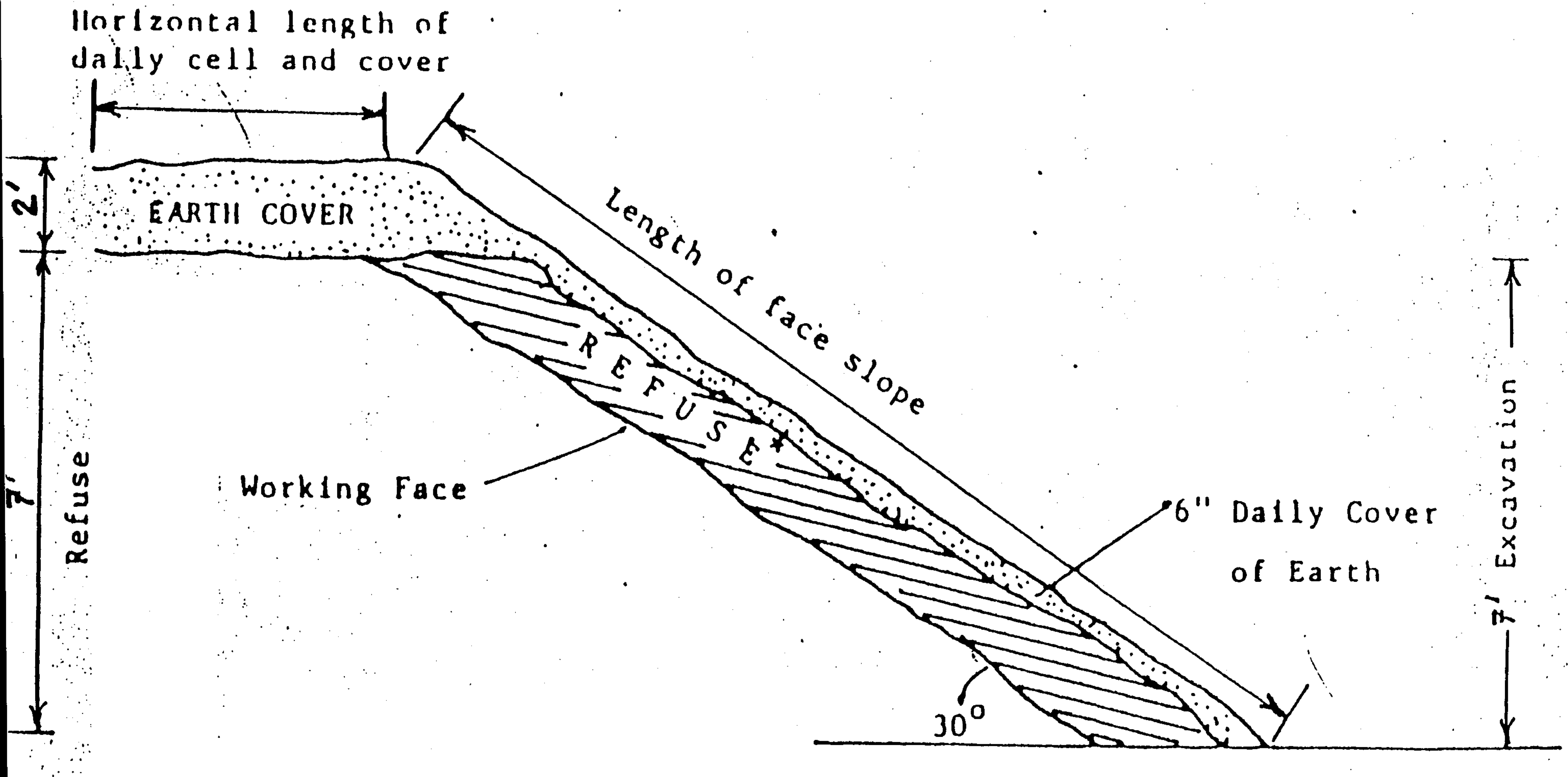
4. GIVE THE AREA NEEDED FOR LANDFILL PER YEAR IN ACRES FOR A 7-FT. LIFT.

SITE SELECTION:

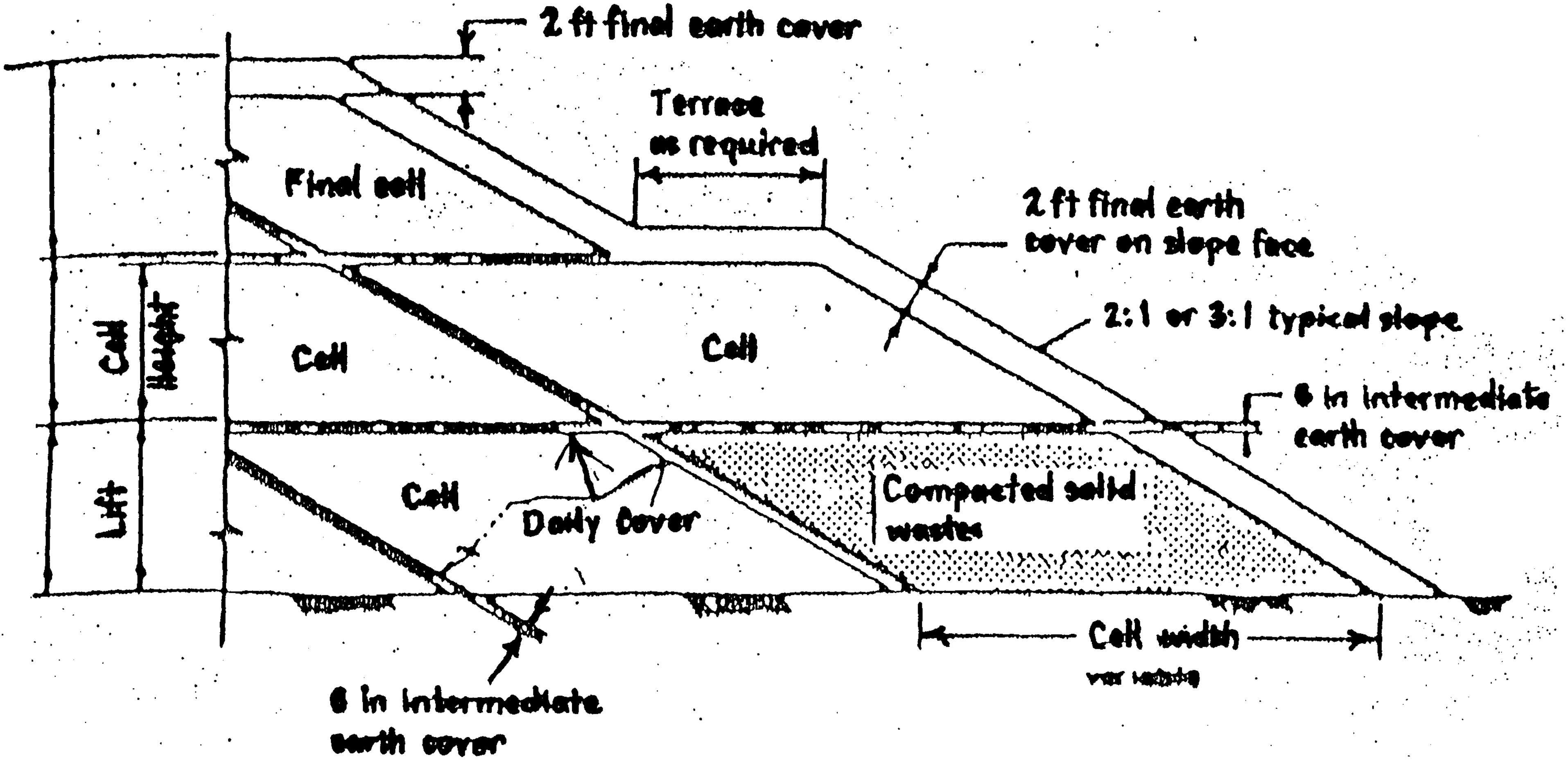
SITE SELECTION CONSIDERS SOIL CONDITIONS, GROUNDWATER LEVELS, LOCATION RELATIVE TO POPULATED AREAS AND FUTURE LAND-USE PLANNING. CONDITIONS MUST BE SUCH THAT GAS LEACHATE, WATER SEEPAGE AND RUN-OFF DO NOT CAUSE POLLUTION NUISANCE OR HEALTH HAZARDS. A MONITORING PROGRAM SHOULD BE ESTABLISHED TO ENSURE THAT AN ADEQUATE ENVIRONMENT IS MAINTAINED AT THE SITE. PROJECTED LAND USE MAY BE A PARK WITH RECREATIONAL FACILITIES THAT ARE NOT AFFECTED BY GRADUAL SUBSISTENCE AT THE GROUND SURFACE.

GENERAL REQUIREMENTS:

1. EACH DAY'S WORK SHOULD FORM ONE CELL OR LIFT BUT THE MAXIMUM HEIGHT OF A CELL OR LIFT SHOULD BE 10 FT. (7 FT. < 10 FT. THEREFORE OK).



* Refuse Depth - 2 ft. (compacted)


2 ft final earth cover
Terrace as required
Final cell
2 ft final earth cover on slope face
2:1 or 3:1 typical slope
Cell
Cell
Cell
Height
Lift
Cell
6 in intermediate earth cover
Compacted solid wastes
Daily Cover
Cell width
6 in intermediate earth cover

2. SPREAD AND COMPACT WASTE IN LAYERS 2 FT. OR LESS COMPACTED THICKNESS.

3. COVER WORK AT THE END OF EACH DAY WITH 6 TO 12 IN. OF EARTH.

4. FINAL TOP AND SIDE SURFACES SHOULD BE COVERED WITH AT LEAST 2 FT. OF EARTH.

5. BOTTOM OF LANDFILL SHOULD BE AT LEAST 5 FT. ABOVE THE MAXIMUM ELEVATION OF GROUNDWATER TABLE.

6. BOTTOM OF LANDFILL SHOULD BE AT LEAST 5 FT. ABOVE BEDROCK.

(a) Year	(b) Population	(c) Solid Waste (Millions, lbs.)	(d) Per Year (1000 cu. yds)	(e) Acres
1	10,000	12	19.4	2.15
2	11,000	13.2	21.4	2.37
3	12,000	14.4	23.4	2.49
4	13,000	15.6	25.4	2.61
5	14,000	16.8	27.4	2.73
6	15,000	18.0	29.4	2.85
7	16,000	19.2	31.4	2.97
8	17,000	20.4	33.4	3.09
9	18,000	21.6	35.4	3.21
10	19,000	22.8	37.4	3.33
11	20,000	24.0	39.4	3.45
12	21,000	25.2	41.4	3.57

(a) Year	(b) Population	(c) Solid Waste (Millions, lbs.)	(d) Per Year (1000 cu. yds)	(e) Acres
13	22,000	26.4	43.4	3.69
14	23,000	27.6	45.4	3.81
15	24,000	28.8	47.4	3.93

Total Acreage Required in 15 Years 46.25 Acres

THE REFUSE IS LAID ON THE SLOPE IN LAYERS 12 IN. TO 18 IN. THICK. EACH LAYER IS COMPACTED BY FOUR OR FIVE PASSES OF THE TRACTOR UP AND DOWN THE SLOPE. ASSUME THAT FOR EACH DAY'S WORK THE FINAL COMPACTED THICKNESS OF THE REFUSE IS:

Compacted Thickness = 2 ft.

TO CALCULATE AREAS REQUIRED, CALCULATE VOLUME AND WEIGHT OF REFUSE PER FT^2 OF AREA. FOR EVERY 2 FT. OF COMPACTED REFUSE ON THE SLOPE FACE, THERE IS 6 IN. OF EARTH COVER. THEREFORE, IN A VERTICAL COLUMN OF LANDFILL 7 FT. DEEP, THE DEPTH OCCUPIED BY REFUSE IS:

$$\text{Refuse Depth} = \frac{2.0}{2.5}(7) = 5.6 \text{ ft.}$$

THE VOLUME OF COMPACTED REFUSE PER ACRE IS:

$$\text{Volume} = \frac{5.6}{27}(43{,}560) = 9{,}035 \text{ yd}^3/\text{acre}$$

AND THE WEIGHT PER ACRE IS:

$$\text{Weight} = 9035(620) = 5{,}602(10)^6 \text{ lb/acre}$$

V.
Transportation

Instructor: Rick Bryant

Tape

8

Traffic Volumes

- Definitions:

  -- ADT: Average Daily Traffic
  -- DHV: Design Hour Volume (30th Highest Hour)
  -- K-Factor: DHV/ADT
  -- D-Factor: Directional Distribution (%)
  -- T-Factor: Truck Percentage
  -- ATR: Automatic Traffic Recorder
  -- TMC: Turning Movement Count

SAMPLE PROBLEM:

- Given: ATR Volume, Percent Trucks

  -- ATR = 300 vehicles
  -- Trucks = 5 percent (five axles per truck)

- Determine: Actual Volume

  Hint: Total of vehicles is less than 300.

Solution:

- 301 ATR Reading = 602 axles
- Let T = # of Trucks; and P = # of cars
- (T x 5) + (P x 2) = 602; also,
- T/(T + P) = 5/100 (5 percent)
- Solve for P:

$$\begin{aligned} 100\,(T) &= 5\,(T + P) = 5T + 5P \\ 95\,T &= 5\,P \\ 19\,T &= P \end{aligned}$$

- Now: (T x 5) + (P x 2) = 602
- Substitute for P: (T x 5) + (19 T x 2) = 602
- 43 T = 602
- T = 602/43 = 14
- P = T x 19 = 266
- T + P = 280, not 301

Traffic Volumes (Peak Hour Factor) (Cont'd.)

SAMPLE PROBLEM (Cont'd.)

- Problem: Determine Peak Hour Factor
- Given: Traffic Volumes
- Reference: PHF = Hourly Volume/(4 x Highest 15-Minute Volume)
- Solution:

Time	Volumes	Volume	Hour
4:15 - 4:30	600		
4:30 - 4:45	625		
4:45 - 5:00	530		
5:00 - 5:15	880	2635	4:15 - 5:15
5:15 - 5:30	720	2755*	4:30 - 5:30
5:30 - 5:45	600	2730	4:45 - 5:45
5:45 - 6:00	510	2710	5:00 - 6:00
6:00 - 6:15	500	2330	5:15 - 6:15

$$\text{PHF} = \frac{\text{hourly volume}}{4 \text{ x peak 15-minute volume}} = \frac{2755}{4 \text{ x } 880} = 0.78$$

TRANSPORTATION

- References
- Traffic Volumes
- Traffic Signal Warrants
- Freeway Interchanges
- Level of Service
- Freeway Analysis
- Signalized Intersections
- Vehicle Speeds
- Accident Data
- Trip Generation
- Queuing

Transportation References

1. Manual of Uniform Traffic Control Devices for Streets and Highways by U.S. Department of Transportation Federal Highway Administration, 1978.

2. Transportation and Traffic Engineering Handbook, by the Institute of Transportation Engineers, 1976.

3. Highway Capacity Manual, Highway Research Board Special Report 209, 1985.

4. Highway Engineering, Wright and Paquette, 5th Edition, 1985.

5. A Policy on the Geometric Design of Highways and Streets, American Association of State Highway and Transportation Officials, Washing, DC, 1990.

Traffic Signal Warrants

- Definition: Minimum Criteria for Provision of a Signal.
- Source: MUTCD.

Warrant	Warrant
1 - Minimum Volume	6 - Accidents
2 - Interruption of Traffic	7 - Systems
3 - Pedestrian	8 - Combination
4 - School Crossing	9 - Delay*
5 - Progressive Movement	10 - Four Hour*
	11 - Peak Hour*

- Warrant 1 -- Minimum Vehicular Volume:

  -- Major Street more than 500 vph for 8 hours; and

  -- Minor Street more than 150 vph for 8 hours
  (Reduce by 30 percent for rural location)

- Warrant 2 -- Interruption of Continuous Traffic:

  -- Major Street more than 750 vph for 8 hours; and

  -- Minor Street more than 75 vph for 8 hours
  (Reduce by 30 percent for rural locations)

Traffic Signal Warrants (Cont'd.)

SAMPLE PROBLEM:

- Given: Hourly Traffic Volumes, Urban Location.
- Determine: If Signal Warrants are Met.

Hour Begins	Major Street EB	Major Street WB	Minor Street NB	Minor Street SB
6:00	400	100	35	60
7:00	750	300	100	170
8:00	600	400	100	190
9:00	500	350	80	90
10:00	400	400	90	110
11:00	450	500	80	120
12:00	500	500	175	150
1:00	350	400	125	160
2:00	3:00	350	80	140
3:00	350	600	135	105
4:00	400	700	200	175
5:00	300	600	180	195

Solution:

- Step 1: Determine Total Main Street Volumes.
- Step 2: Select Appropriate Volume Criteria.
- Step 3: Compare Actual Volumes to Selected Criteria.
- Step 4: Count Number of Hours for Which Criteria are Satisfied.

• **Solution:** **Warrant Worksheet**

Time	Eastbound Approach	Westbound Approach	Total	Minimum Vehicle Volume %	Interruption of Continuous Traffic %	Northbound Approach	Southbound Approach	Minimum Vehicle Volume %	Interruption of Continuous Traffic %
6:00 - 7:00	400	100	500	100	66	35	60	40	80
7:00 - 8:00	750	300	1,050	210	140	100	170	113	226
8:00 - 9:00	600	400	1,000	200	133	100	190	127	253
9:00 - 10:00	500	350	850	170	113	80	70	53	106
10:00 - 11:00	400	400	800	160	106	90	110	73	146
11:00 - 12:00	450	500	950	190	126	80	120	80	160
12:00 - 1:00	500	500	1,000	200	133	175	150	116	233
2:00 - 3:00	300	350	650	130	86	80	140	93	186
3:00 - 4:00	400	700	1,100	220	146	200	175	133	266
5:00 - 6:00	300	600	900	180	120	180	195	300	260

Freeway Interchanges

SAMPLE PROBLEM:

- Problem: Design Appropriate Interchange
- Given: Traffic Volumes
- Reference: AASHTO -- Chapter X

Solution

- Step 1: Determine Roadway Classifications

Local Road	-- ADT = 2,000 vpd
Collector Road	-- ADT = 2,000 - 12,000 vpd
Arterial Road	-- ADT = 12,000 - 40,000 vpd
Freeway	-- ADT = 30,000 vpd and up

- Step 2: Consider:

  * Cost
  * Left Turns
  * Right-of-Way
  * Weaves

<u>Left Turns At-Grade</u>

Restricted: Cloverleaf

Unrestricted: Diamond

<u>Weave Volumes</u>

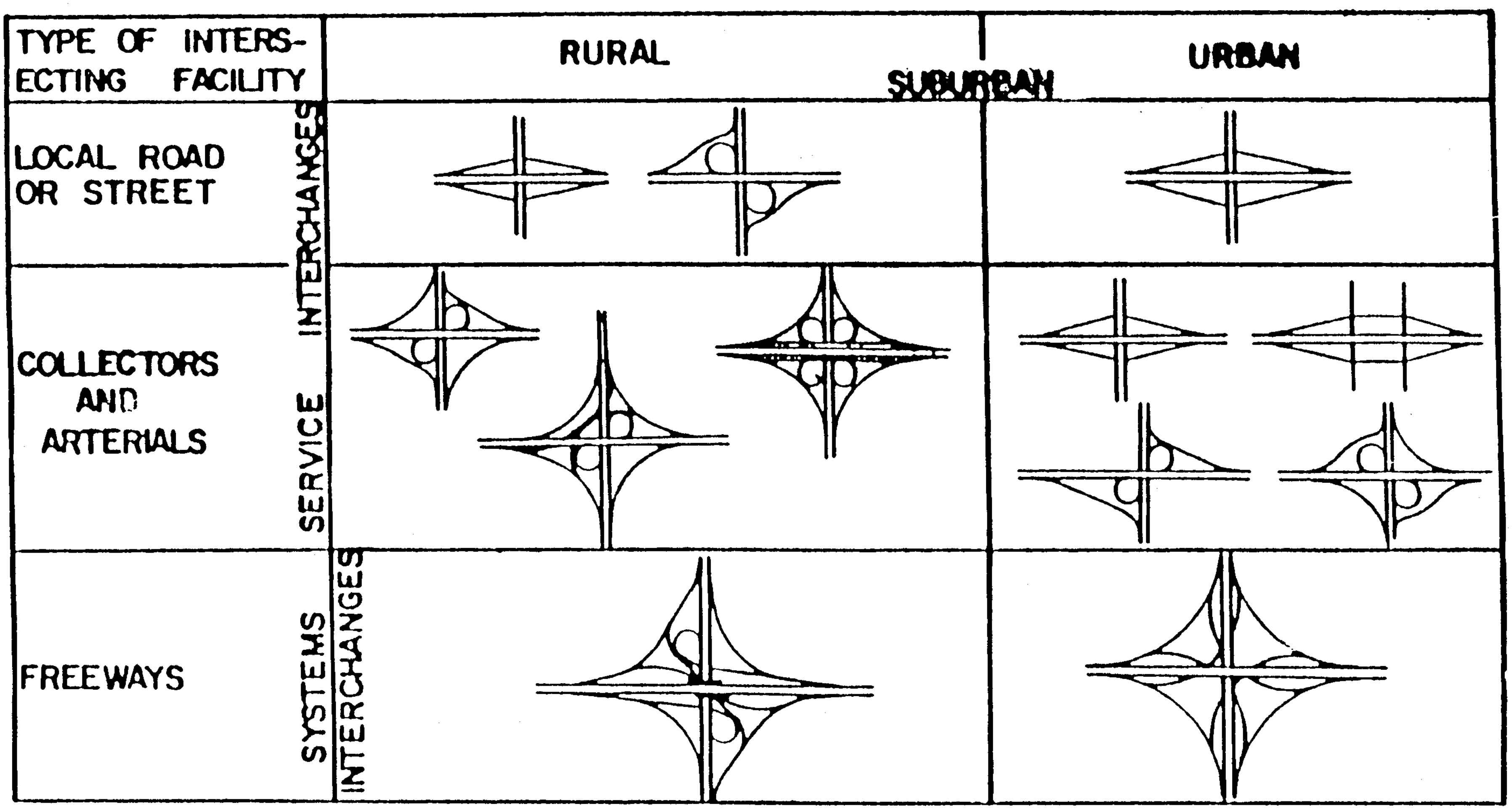
High: All-Directional Ramps

Low: Cloverleaf

- Step 3: Select Alternatives

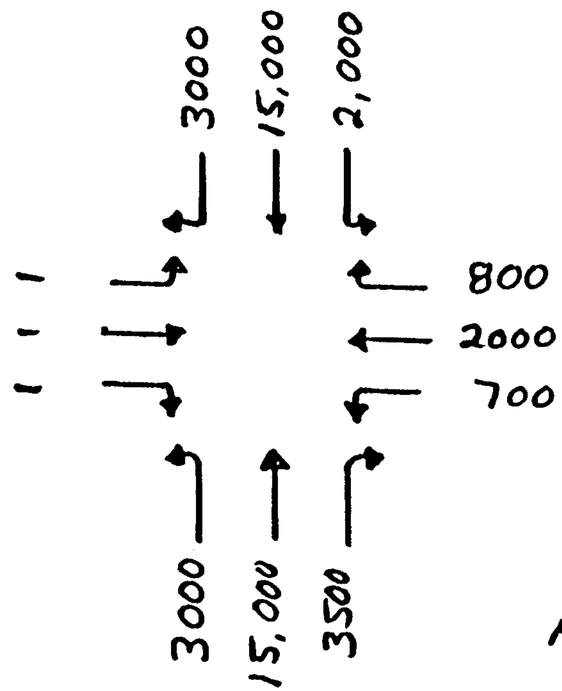
Scenario	Class	Lefts	Weave	Design
A	Collector/Local	Low	Low	At-Grade
B	Freeway/Arterial	High	High	All-Directional
C	Freeway/Local	Medium	Low	Diamond
D	Freeway/Local	High	Low	Partial Cloverleaf

Alternative Designs



SAMPLE PROBLEMS

C.



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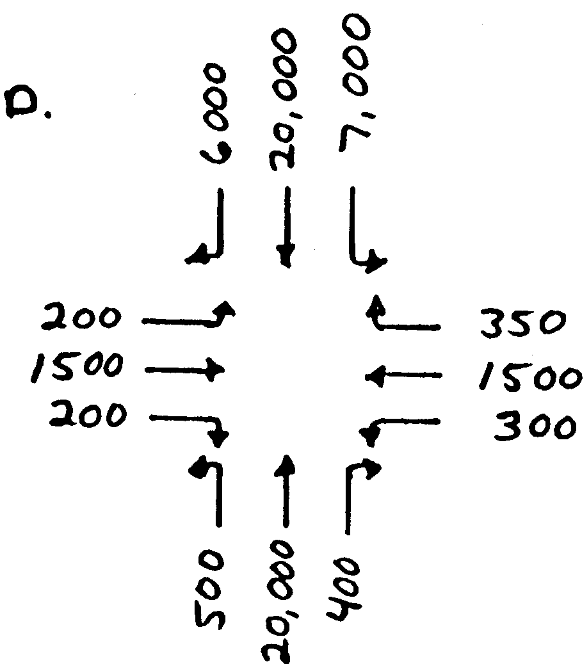
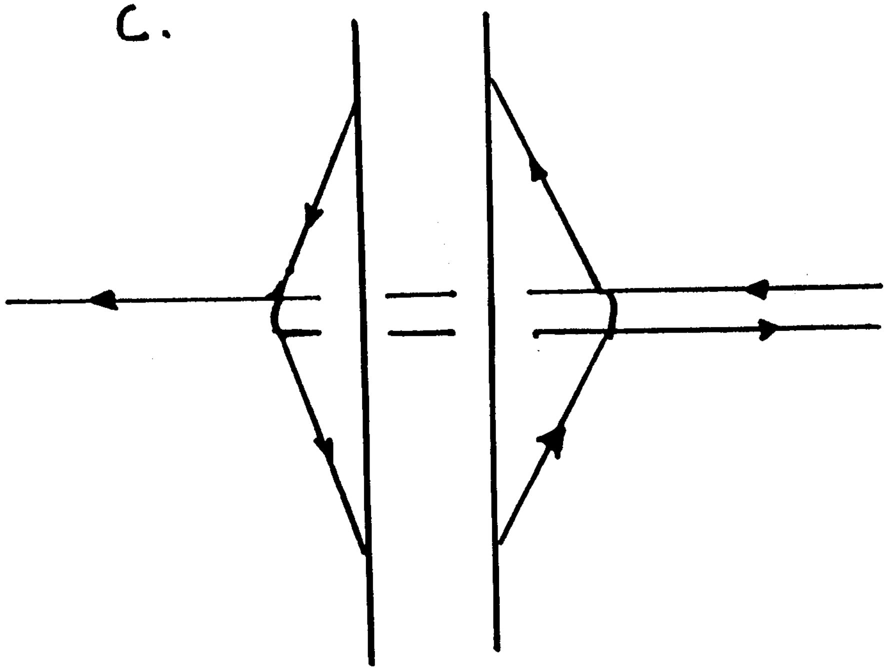


ADT's

Solutions

C.



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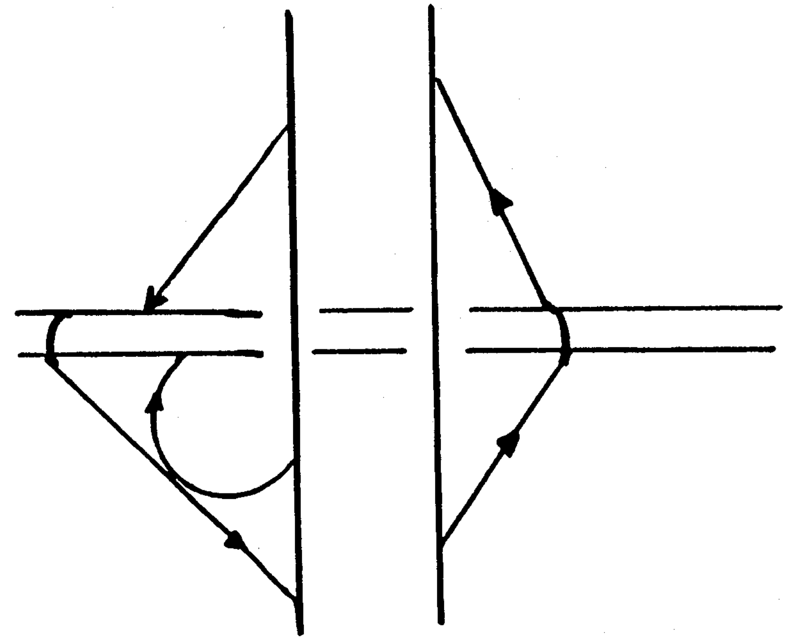


Level of Service

- Quality of Flow
- Based on Volume, Delay, Density or Volume-to-Capacity
- Scale A - F

FREEWAY ANALYSIS

- Source: HCM, Chapter 3
- Service Flow = $SF_i = C_j \times (V/C)_i \times N \times f_w \times f_{HV} \times f_p$
- C_j = Ideal Capacity Based on Design Speed j
- V/C_i = Maximum Volume-to-Capacity Ratio at LOS i
- N = Number of Lanes
- f_w = Factor for Width
- f_{HV} = Factor for Heavy Vehicles
- f_p = Factor for Population

- Given: ADT = 60,000 vpd

  -- K-Factor = 9.8%
  -- D-Factor = 75%
  -- Trucks = 5% (Heavy)
  -- Grade = 4% for 1 mile
  -- Lateral Clearance = 10' right; 6' left; 12' lanes
  -- Design Speed = 60 mph
  -- Population = Commuters
  -- LOS for Design = LOS D

- Determine: Number of Lanes Needed

Sample Problem: Freeway Analysis (Cont'd.)

- Solution:

  -- $SF =$ ADT x D-Factor x K-Factor
  $= 60{,}000 \times .75 \times .098 = 4{,}410$

  -- $C_{60} =$ 2,000 vph (Table 3.1)

  -- $V/C_D =$ 0.84 (Table 3.1)

  -- $N =$ Assume equals 1

  -- $f_w =$ 1.0 (Table 3.2)

  -- $f_{HV} =$ 0.69 (Table 3.6, Eq. 3.4) $= 1/(1 + .05\ (10-1))$
  also, $E_T = 10$

  -- $f_p =$ 1.0 (Table 3.10)

- $SF_D = C_j \times (V/C_D) \times N \times f_w \times f_{HV} \times f_p$

- Maximum Flow/Lane at LOS D = 2,000 (.84) (1.0) (.69) (1.0)

- SF_D/Lane = 1,159

- N = 4,410/1,159 = 3.8 (say 4 lanes/direction)

TABLE 3-1. LEVELS OF SERVICE FOR BASIC FREEWAY SECTIONS

LOS	DENSITY (PC/MI/LN)	70 MPH DESIGN SPEED			60 MPH DESIGN SPEED		
		SPEED[b] (MPH)	v/c	MSF[a] (PCPHPL)	SPEED[b] (MPH)	v/c	MSF[a] (PCPHPL)
A	≤ 12	≥ 60	0.35	700	—	—	—
B	≤ 20	≥ 57	0.54	1,100	≥ 50	0.49	1,000
C	≤ 30	≥ 54	0.77	1,550	≥ 47	0.69	1,400
D	≤ 42	≥ 46	0.93	1,850	≥ 42	0.84	1,700
E	≤ 67	≥ 30	1.00	2,000	≥ 30	1.00	2,000
F	> 67	< 30	[c]	[c]	< 30	[c]	[c]

TABLE 3-2. ADJUSTMENT FACTOR FOR RESTRICTED LANE WIDTH AND LATERAL CLEARANCE

DISTANCE FROM TRAVELED PAVEMENT[a] (FT)	ADJUSTMENT FACTOR, f_w — OBSTRUCTIONS ON ONE SIDE OF THE ROADWAY — LANE WIDTH (FT) 12	11	10	9	OBSTRUCTIONS ON BOTH SIDES OF THE ROADWAY — LANE WIDTH (FT) 12	11	10	9
	4-LANE FREEWAY (2 LANES EACH DIRECTION)							
≥ 6	1.00	0.97	0.91	0.81	1.00	0.97	0.91	0.81
5	0.99	0.96	0.90	0.80	0.99	0.96	0.90	0.80
4	0.99	0.96	0.90	0.80	0.98	0.95	0.89	0.79
3	0.98	0.95	0.89	0.79	0.96	0.93	0.87	0.77
2	0.97	0.94	0.88	0.79	0.94	0.91	0.86	0.76
1	0.93	0.90	0.85	0.76	0.87	0.85	0.80	0.71
0	0.90	0.87	0.82	0.73	0.81	0.79	0.74	0.66
	6- or 8- LANE FREEWAY (3 or 4 LANES EACH DIRECTION)							
≥ 6	1.00	0.96	0.89	0.78	1.00	0.96	0.89	0.78
5	0.99	0.95	0.88	0.77	0.99	0.95	0.88	0.77
4	0.99	0.95	0.88	0.77	0.98	0.94	0.87	0.77
3	0.98	0.94	0.87	0.76	0.97	0.93	0.86	0.76
2	0.97	0.93	0.87	0.76	0.96	0.92	0.85	0.75
1	0.95	0.92	0.86	0.75	0.93	0.89	0.83	0.72
0	0.94	0.91	0.85	0.74	0.91	0.87	0.81	0.70

TABLE 3-6. PASSENGER-CAR EQUIVALENTS FOR HEAVY TRUCKS (300 LB/HP)

GRADE (%)	LENGTH (MI)	PASSENGER-CAR EQUIVALENT, E_T															
		4-LANE FREEWAYS								6–8 LANE FREEWAYS							
PERCENT TRUCKS		2	4	5	6	8	10	15	20	2	4	5	6	8	10	15	20
< 1	All	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	0–1/4	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	1/4–1/2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	1/2–3/4	4	4	4	4	3	3	3	3	4	4	4	3	3	3	3	3
	3/4–1	5	4	4	4	3	3	3	3	5	4	4	4	3	3	3	3
	1–1½	6	5	5	5	4	4	4	3	6	5	5	4	4	4	3	3
	> 1½	7	5	5	5	4	4	4	3	7	5	5	5	4	4	3	3
2	0–1/4	4	4	4	3	3	3	3	3	4	4	4	3	3	3	3	3
	1/4–1/2	7	6	6	5	4	4	4	4	7	5	5	5	4	4	4	4
	1/2–3/4	8	6	6	5	5	4	4	4	8	6	6	6	5	5	4	4
	3/4–1	8	6	6	6	5	5	5	5	8	6	6	6	5	5	5	5
	1–1½	9	7	7	7	6	6	5	5	9	7	7	6	5	5	5	5
	> 1½	10	7	7	7	6	6	5	5	10	7	7	6	5	5	5	5
3	0–1/4	6	5	5	5	4	4	4	3	6	5	5	5	4	4	4	3
	1/4–1/2	9	7	7	6	5	5	5	5	8	7	7	6	5	5	5	5
	1/2–3/4	12	8	8	7	6	6	6	6	10	8	7	6	5	5	5	5
	3/4–1	13	9	9	8	7	7	7	7	11	8	8	7	6	6	6	6
	> 1	14	10	10	9	8	8	7	7	12	9	9	8	7	7	7	7
4	0–1/4	7	5	5	5	4	4	4	4	7	5	5	5	4	4	3	3
	1/4–1/2	12	8	8	7	6	6	6	6	10	8	7	6	5	5	5	5
	1/2–3/4	13	9	9	8	7	7	7	7	11	9	9	8	7	6	6	6
	3/4–1	15	10	10	9	8	8	8	8	12	10	10	9	8	7	7	7
	> 1	17	12	12	10	9	9	9	9	13	10	10	9	8	8	8	8
5	0–1/4	8	6	6	6	5	5	5	5	8	6	6	6	5	5	5	5
	1/4–1/2	13	9	9	8	7	7	7	7	11	8	8	7	6	6	6	6
	1/2–3/4	20	15	15	14	11	11	11	11	14	11	11	10	9	9	9	9
	> 3/4	22	17	17	16	13	13	13	13	17	14	14	13	12	11	11	11
6	0–1/4	9	7	7	7	6	6	6	6	9	7	7	6	5	5	5	5
	1/4–1/2	17	12	12	11	9	9	9	9	13	10	10	9	8	8	8	8
	> 1/2	28	22	22	21	18	18	18	18	20	17	17	16	15	14	14	14

NOTE: If a length of grade falls on a boundary condition, the equivalent from the longer grade category is used. For any grade steeper than the percent shown, use the next higher grade category.

TABLE 3-10. ADJUSTMENT FACTOR FOR THE CHARACTER OF THE TRAFFIC STREAM

TRAFFIC STREAM TYPE	FACTORS, f_p
Weekday or Commuter	1.0
Other	0.75–0.90[a]

[a] Engineering judgment and/or local data must be used in selecting an exact value.

Signalized Intersections

- LOS Based on Delay

- Delay Based on:
  - -- cycle length
  - -- green time
  - -- volume
  - -- capacity
  - -- platooning characteristics

- Ideal Capacity (saturation flow = 1,800 vph)

- Source: HCM

$$S = S_o \, N f_w \, f_{HV} \, f_g \, f_p \, f_{bb} \, f_a \, f_{RT} \, f_{LT}$$

S_o = Ideal Saturation Flow = 1,800 vph

N = Number of Lanes

f_w = Width Factor (Table 5.8)

f_{HV} = Truck Factor (Table 5.9)

f_g = Grade Factor (Table 5.10)

f_p = Parking Factor (Table 5.11)

f_{bb} = Bus Factor (Table 5.12)

f_a = Area Factor (Table 5.13)

f_{RT} and f_{LT} = Right- and Left-Turn Factors

SAMPLE PROBLEM:

- Given: Signal Timing, Geometry, Location, Volumes

- Determine: Capacity
  - -- Two 11' Lanes
  - -- 10% Trucks
  - -- 2% Downgrade
  - -- CBD Location
  - -- No Buses
  - -- No Parking
  - -- Random Arrivals
  - -- No Turns
  - -- 60-Second Cycle
  - -- Green Ratio = .45

- Solution:

$$S = S_o \; N \; f_w \; f_{HV} \; f_g \; f_p \; f_{bb} \; f_a \; f_{RT} \; f_{LT}$$

S = 1,800 x 2 x 0.97 x 0.95 x 1.01 x 0.90 (all others = 1.0)

S = 3,015 vehicles/hour of green time

Capacity = 3,015 x .45 = 1,357 vph

TABLE 5-12
ADJUSTMENT FACTOR FOR BUS BLOCKAGE

Number of Lanes in Lane Group	*Number of Buses Stopping per Hour, N_B*				
	0	*10*	*20*	*30*	*40*
1	1.00	0.96	0.92	0.88	0.83
2	1.00	0.98	0.96	0.94	0.92
3	1.00	0.99	0.97	0.96	0.94

SOURCE: *Highway Capacity Manual.* Transportation Research Board Special Report No. 209 (1985).

TABLE 5-13
ADJUSTMENT FACTOR FOR AREA TYPE

Type of Area	*Factor f_a*
Central business district	0.90
All other areas	1.00

SOURCE: *Highway Capacity Manual.* Transportation Research Board Special Report No. 209 (1985).

TABLE 5-14
LEVEL OF SERVICE CRITERIA FOR SIGNALIZED INTERSECTIONS

Level of Service	*Stopped Delay per Vehicle (sec)*
A	≤5.0
B	5.1 to 15.0
C	15.1 to 25.0
D	25.1 to 40.0
E	40.1 to 60.0
F	>60.0

TABLE 5-8
ADJUSTMENT FACTOR FOR LANE WIDTH

Lane Width, ft	8	9	10	11	12	13	14	15	≥16
Lane Width Factor, f_w	0.87	0.90	0.93	0.97	1.00	1.03	1.07	1.10	Use 2 lanes

SOURCE: *Highway Capacity Manual.* Transportation Research Board Special Report No. 209 (1985).

TABLE 5-9
ADJUSTMENT FACTOR FOR HEAVY VEHICLES

Percent Heavy Vehicles, %HV	0	2	4	6	8	10	15	20	25	30
Heavy Vehicle Factor f_{HV}	1.00	0.99	0.98	0.97	0.96	0.95	0.93	0.91	0.89	0.87

SOURCE: *Highway Capacity Manual.* Transportation Research Board Special Report No. 209 (1985).

TABLE 5-10
ADJUSTMENT FACTOR FOR GRADE

	Downhill			*Level*	*Uphill*		
Grade, %	−6	−4	−2	0	+2	+4	+6
Grade, Factor, f_g	1.03	1.02	1.01	1.00	0.99	0.98	0.97

SOURCE: *Highway Capacity Manual.* Transportation Research Board Special Report No. 209 (1985).

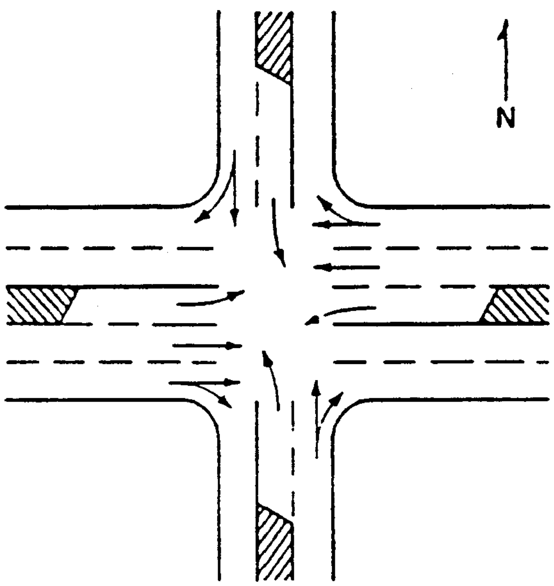
TABLE 5-11
ADJUSTMENT FACTOR FOR PARKING

Number of Lanes in Lane Group	*No Parking*	*Number of Parking Maneuvers per Hour, N_m*				
		0	*10*	*20*	*30*	*40*
1	1.00	0.90	0.85	0.80	0.75	0.70
2	1.00	0.95	0.92	0.89	0.87	0.85
3	1.00	0.97	0.95	0.93	0.91	0.89

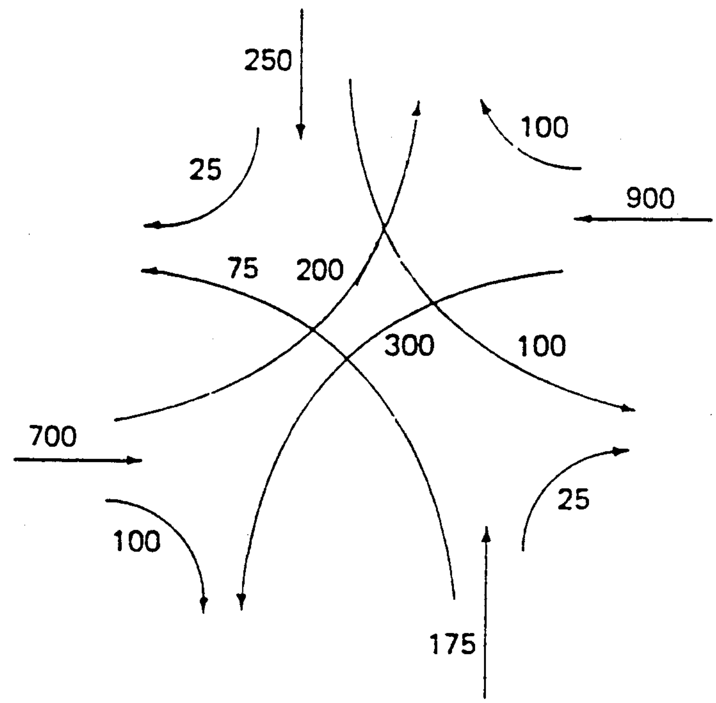
SOURCE: *Highway Capacity Manual.* Transportation Research Board Special Report No. 209 (1985).

Sample Problem: Signalized Intersection (Planning Method)

- Problem: Determine Level of Service
- Given: Geometry, Volumes
- Reference: HCM, Chapter 9



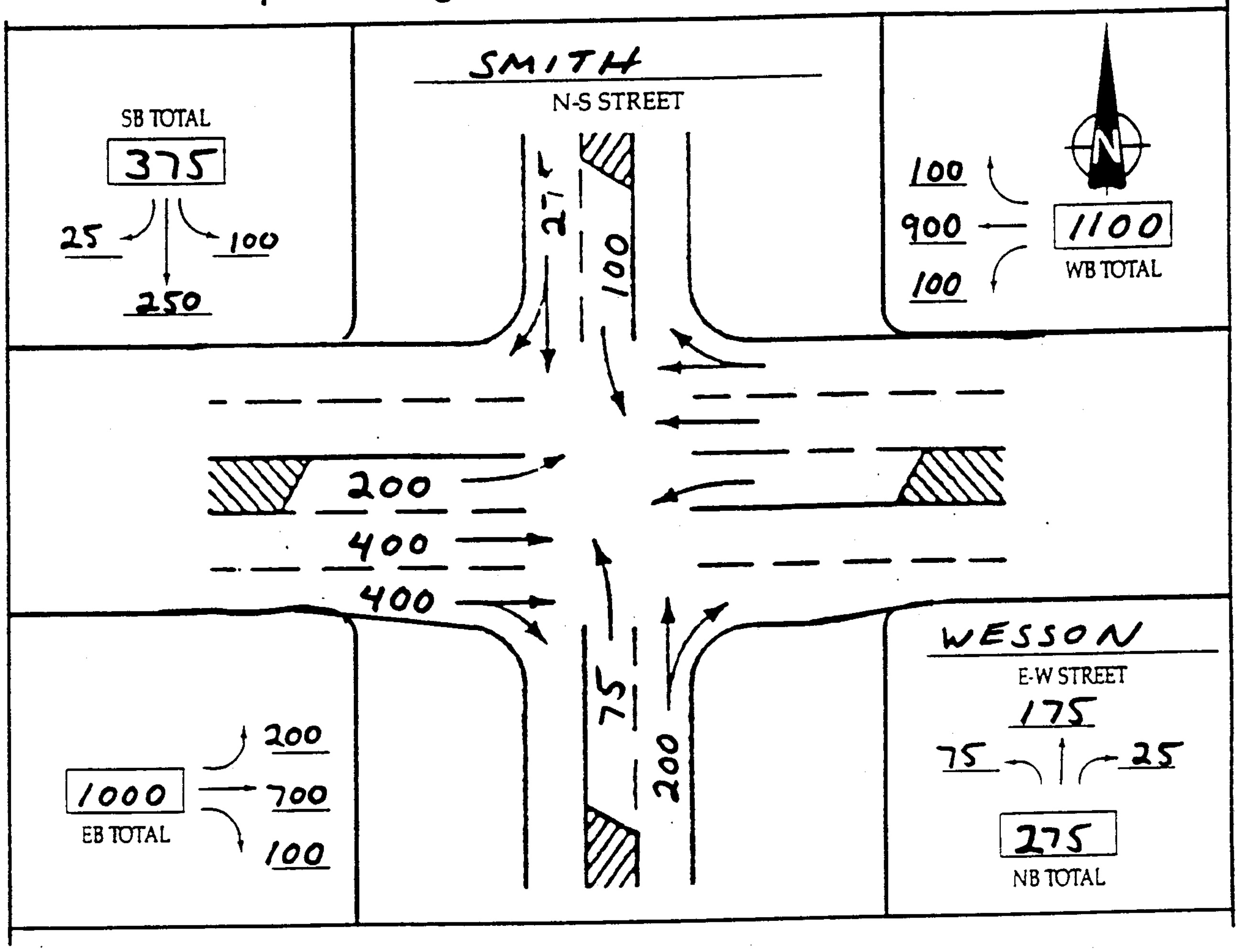
Intersection Geometrics



- Solution:

  -- Step 1: Assign Volumes to Lanes

-- **Step 1: Assign Volumes to Lanes**



Sample Problem: Signalized Intersection (Planning Method)

- Solution (Cont'd.):

  -- Step 2: Sum Conflicting Volumes (through plus opposing left)

EB LT	= 200	NB LT	= 75
WB TH	= 500	SB TH	= 275
	$\boxed{700}$		$\boxed{350}$
	OR		OR
WB LT	= 100	SB LT	= 100
EB TH	= 400	NB TH	= 200
	$\boxed{500}$		$\boxed{300}$

  -- Step 3: Select Critical Conflicting [redacted] Flows + Combine

700 (E-W CRITICAL) + 350 (N-S CRITICAL) = 1050

  -- Step 4: Determine Status Per Table

MAXIMUM SUM OF CRITICAL VOLUMES	CAPACITY LEVEL
0 TO 1,200	UNDER
1,201 to 1,400	NEAR
> 1,400	OVER

Signalized Intersections--Timing

- Minimum Cycle Length = C = L x X/[X - $(V/S)_1$ - $(V/S)_2$ - $(V/S)_3$ - . . .]

  v/s = volume/saturation

  X = desired overall V/C ratio

  L = Lost Time (typically 3 to 5 seconds per phase)

- Optimum Green Time = (C-L) x (V/S)/$(V/S)_1$ + $(V/S)_2$ + $(V/S)_3$ + ...

SAMPLE PROBLEM:

- Given: Phasing, Volumes, Lost Time
- Determine: Timing
- Source: HCM

-- Two Phase
-- N-S Volume = 500 vph
-- N-S Capacity = 1,500 vph
-- E-W Volume = 600 vph
-- E-W Capacity = 1,200 vph
-- Lost Time = 4 seconds/phase
-- Desired V/C Ratio = .90
-- $V/S_{N\text{-}S}$ = 500/1,500 = .33
-- $V/S_{E\text{-}W}$ = 600/1,200 = .50

Total V/S = .33 + .50 = .83
C = (4 + 4) .90/(.90 - .83)
= 103 seconds

$$G/C_{N\text{-}S} = \frac{.33}{.50 + .33} \times (103 - 8) = 38 \text{ seconds}$$

$$G/C_{E\text{-}W} = \frac{.50}{.50 + .33} \times (103 - 8) = 57 \text{ seconds}$$

Vehicle Speeds

- Use of Speed Data:
  - -- Law Enforcement
  - -- Design Criteria
  - -- Setting Speed Limits
  - -- Sight Distance Evaluation
- Definitions:

  Mean Speed = Average

  Median Speed = 50th Percentile

  85th Percentile = Design Standard

  Mode = Most Frequently Observed Speed

  Pace = 10 mph Speed Range Containing Most Observations

SAMPLE PROBLEM:

- Given: Speed Frequencies
- Determine: Mean, Median, 85th Percentile, Mode; Pace
- Source: T+TE Handbook

Speed Group (mph)	Frequency
11-15	1
16-20	3
21-25	21
26-30	41
31-35	38
36-40	10
41-45	10
46-50	2

Vehicle Speeds

- Solution:

-- Step 1: Calculate Mean Speed

Speed Group (mph)	Frequency	Assumed Speed	Total (mph)
11-15	1	13	13
16-20	3	18	48
21-25	21	23	483
26-30	41	28	1,148
31-35	38	33	1,254
36-40	10	38	380
41-45	4	43	172
46-50	2	48	96
Total	120		3,594

Mean = 3,594/120 = 29.95 (say 30 mph)

-- Step 2: Calculate Median Speed.
Multiply 120 x .50 = 60. By inspection, median speed = 28 mph.

-- Step 3: Calculate 85th Percentile Speed.
Multiply 120 x .85 = 102. By inspection, 85th percentile speed = 38 mph

-- Step 4: By inspection, mode = 28 mph.

-- Step 5: By inspection, pace = 26 mph to 35 mph.

Accident Data

- Use of Accident Data:
  - -- Identify Hazardous Locations
  - -- Establish Safety Improvement Priorities
  - -- Cost-Benefit Analysis
- Classifications:
  - -- Property Damage Only
  - -- Personal Injury
  - -- Fatality

SAMPLE PROBLEM:

- Given: Number of Accidents, Volume
- Determine: Improvement Priorities. Rate vs. Number of Methods, Why.
- Source: T+TE Handbook

Intersection	ADT	Accidents/ Year	Number	Rate
A	820	4	4	4/820 = .0049
B	1,200	5	5	5/1,200 = .0042
C	1,070	7	7	7/1,070 = .0065
D	1,400	6	6	6/1400 = .0043

Link	ADT	Accidents/ Year	Length (miles)	Rate
A	19,000	7	1.50	7/(19 x 1.5) = .25
B	20,000	14	1.35	14/(20 x 1.35) = .52
C	55,000	18	4.5	18/(55 x 4.5) = .07
D	40,000	30	2.48	30/(40 x 2.48) = .30

Traffic Accident (Skidding)

SAMPLE PROBLEM:

- Stopping Distance Traveled = Distance Traveled During Reaction + Distance Traveled During Deceleration

S = Distance (feet)
V = Velocity (mph)
f = Coefficient of friction (no units; varies with speed)
g = Grade (percent)
T = Time (seconds)

$$S = 1.47\,V\,T + V^2/30\,(f + g)$$
$$S = 0 + V^2/30\,(f + g)$$
$$S = V^2/30\,(f - .03)$$

- Given: Length of Skid, Grade, Final Speed, Pavement Condition.
- Determine: Initial Speed
- Source: T+TE Handbook, AASHTO (Table III-1)

Given	Initial Speed (mph)	Wet Pavement Coefficient of Friction (f)
3% downgrade	20	.40
185-foot skid	25	.38
Wet pavement	30	.35
	35	.34
	40	.32
	45	.31

Final Speed = 20 mph

Sample Problem: Traffic Accident (Skidding)

- Distance Traveled During Reaction (does not apply; considering breaking distance only)
- Skid Length = Braking Distance for Initial Velocity--Braking Distance for Final Velocity
- 185 Feet = $SD_i - SD_f$
- $185 = SD_i - SD_f$
- $SD_f = \dfrac{V^2}{30\,(f - .03)} = \dfrac{20^2}{30\,(.40 - .03)} = 36$ FT
- $185 = SD_i - SD_f = SD_i - 36$
  $SD_i = 185 + 36 = 221$
- $221 = \dfrac{V_i^2}{30\,(f - .03)}$
- By trial and error
- $221 = 45^2/30\,(.31 - .03) \neq 241$
- $221 = 43^2/30\,(.31 - .03) = 220$

Initial Speed = 43 mph

- Check: Table III-1

  43 mph = 191
  20 mph = 33
  = 191 - 33 = 158 on level pavement

Problem statement = 185 on 3% downgrade

Design Speed (mph)	Assumed Speed for Condition (mph)	Brake Reaction		Coefficient of Friction f	Braking Distance on Level (ft)	Stopping Sight Distance	
		Time (sec)	Distance (ft)			Computed (ft)	Rounded for Design (ft)
20	20-20	2.5	73.3-73.3	0.40	33.3-33.3	106.7-106.7	125-125
25	24-25	2.5	88.0-91.7	0.38	50.5-54.8	138.5-146.5	150-150
30	28-30	2.5	102-7-110.0	0.35	74.7-85.7	177.3-195.7	200-200
35	32-35	2.5	117.3-128.3	0.34	100.4-120.1	217.7-248.4	225-250
40	36-40	2.5	132.0-146.7	0.32	135.0-166.7	267.0-313.3	275-325
45	40-45	2.5	146.7-165.0	0.31	172.0-217.7	318.7-382.7	325-400
50	44-50	2.5	161.3-183.3	0.30	215.1-277.8	376.4-461.1	400-475
55	48-55	2.5	176.0-201.7	0.30	256.0-336.1	432.0-537.8	450-550
60	52-60	2.5	190.7-220.0	0.29	310.8-413.8	501.5-633.8	525-650
65	55-65	2.5	201.7-238.3	0.29	347.7-485.6	549.4-724.0	550-725
70	58-70	2.5	212.7-256.7	0.28	400.5-583.3	613.1-840.0	625-850

Table III-1. Stopping sight distance (wet pavements).

Sample Problem: Trip Generation

- Problem: Estimate ADT

- Given: Roadway is Sole Access to:

  A. 350 Homes
  B. 250 SF Retail

- Solution:

  T = Trips
  X = Size (# of Homes; # of KSF)
  Ln ( ) = Natural Log

A.

$$\begin{aligned} Ln\,(T) &= 0.921\ Ln\,(X) + 2.6 \\ Ln\,(T) &= 0.921\ Ln\,(350) + 2.6 \\ Ln\,(T) &= 0.921\ (5.85) + 2.6 \\ LN\,(T) &= 5.4 + 2.6 = 8.0 \\ T &= 2{,}966 \end{aligned}$$

B.

$$\begin{aligned} Ln\,(T) &= 0.625\ Ln\,(X) + 5.985 \\ Ln\,(T) &= 0.625\ Ln\,(250) + 5.985 \\ Ln\,(T) &= (0.625)\ 5.52 + 5.985 \\ Ln\,(T) &= 3.45 + 5.985 = 9.4359 \\ T &= 12{,}530 \end{aligned}$$

Queuing

- λ = Arrival Rate ➡ Queue ➡ μ = Service Rate
- Average Arrival Rate/Distribution
- Average Service Rate/Distribution
- Random = negative exponential distribution
- Formulas (Source: T+TE Handbook):
  1. Utilization Factor: $p = \lambda/\mu$
  2. Number in System (n): $p(n) = p^n P(0)$; $p(0) = 1 - p$
  3. Average Queue Length: $E(m) = \dfrac{p^2}{1 - p}$
  4. Average Number in System: $E(n) = \dfrac{p}{1 - p}$

SAMPLE PROBLEM:

- Given: Arrival Rate, Service Rate, Distribution for Parking Lot Entrance
- Determine: Average and 95th Percentile Number in System

VI.
Soils

Instructor

Tapes

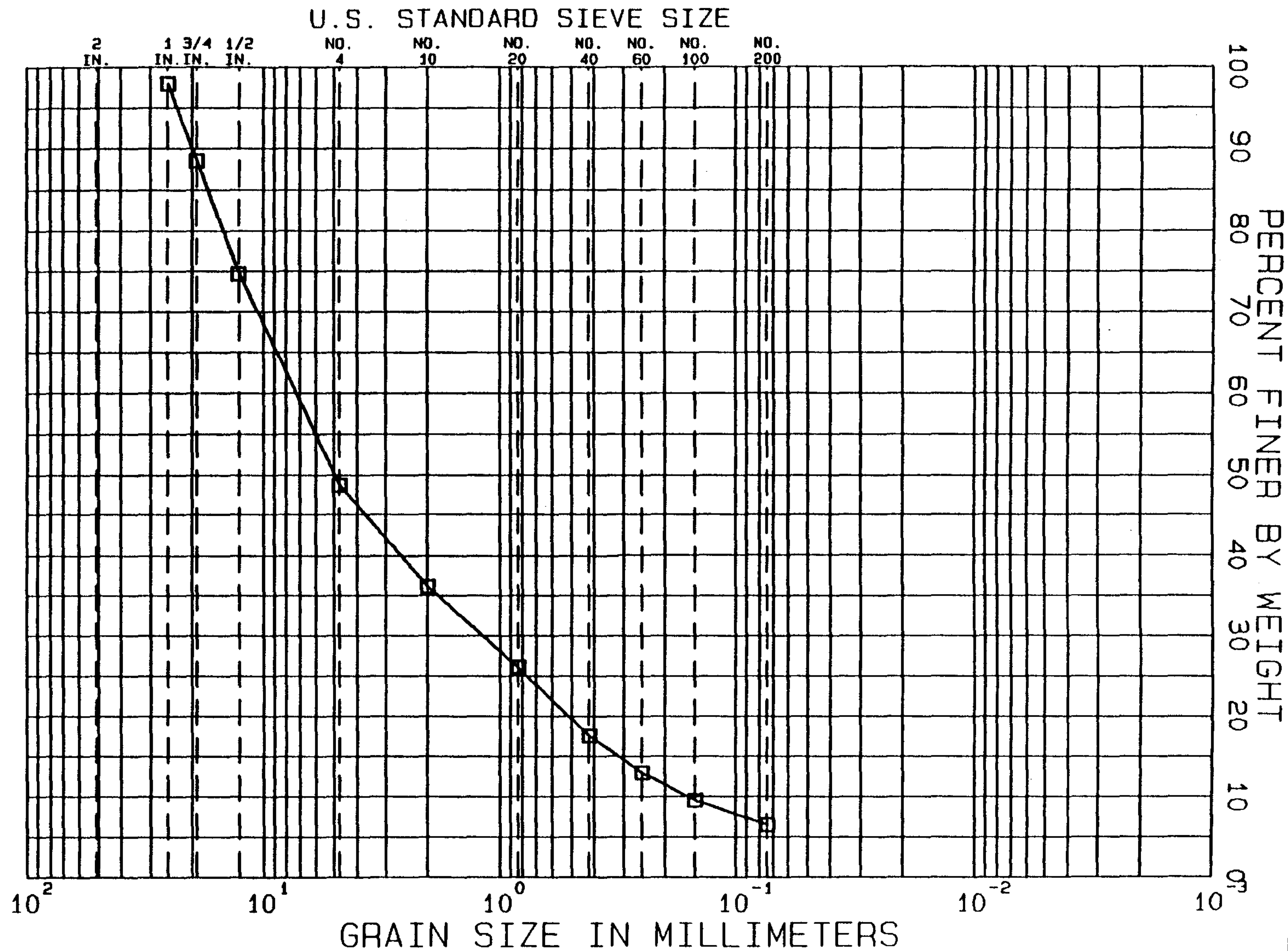
9 **10**

SOIL MECHANICS

LECTURE 1

- SOIL CLASSIFICATION
- PHASE RELATIONSHIPS
- EFFECTIVE STRESS
- CONSOLIDATION

GRADATION ANALYSIS



GRAVEL		SAND			SILT	CLAY
COARSE	FINE	COARSE	MEDIUM	FINE		

TEST NO.	MATERIAL SOURCE	REMARKS
S12.1	Sample No. S-4 Andover, Mass.	Brown f-c GRAVEL and f-c SAND, trace Silt

GZA GEOENVIRONMENTAL, INC.
ENGINEERS AND SCIENTISTS

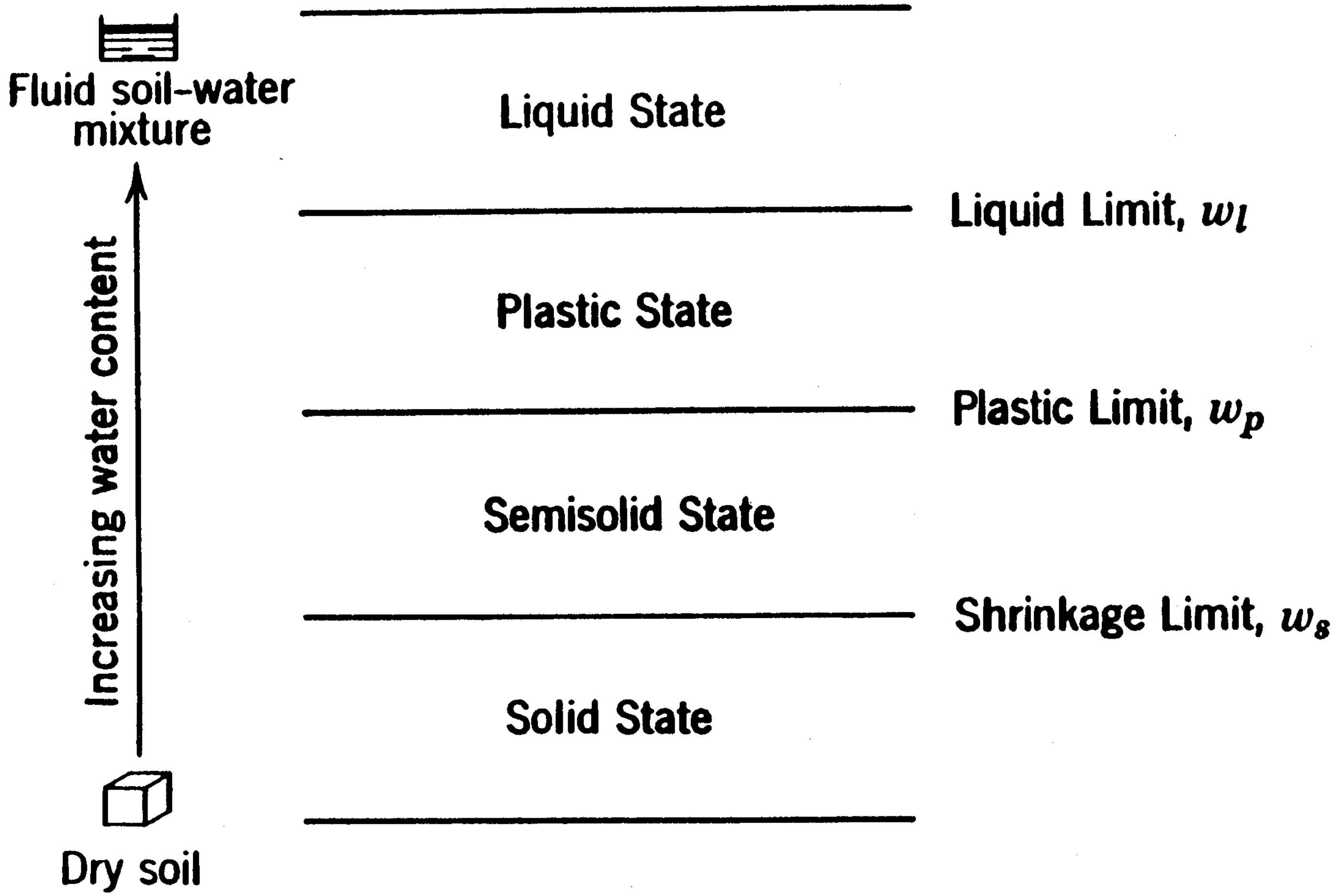
REDSTONE SHOPPING CENTER
STONEHAM, MASS.
GRADATION TESTS

BORING NO.
SAMPLE S-4
DEPTH
TECH. BF
REVIEWER DAS

TEST SERIES
NO. 12
DATE Aug. 81
FILE 12457.1

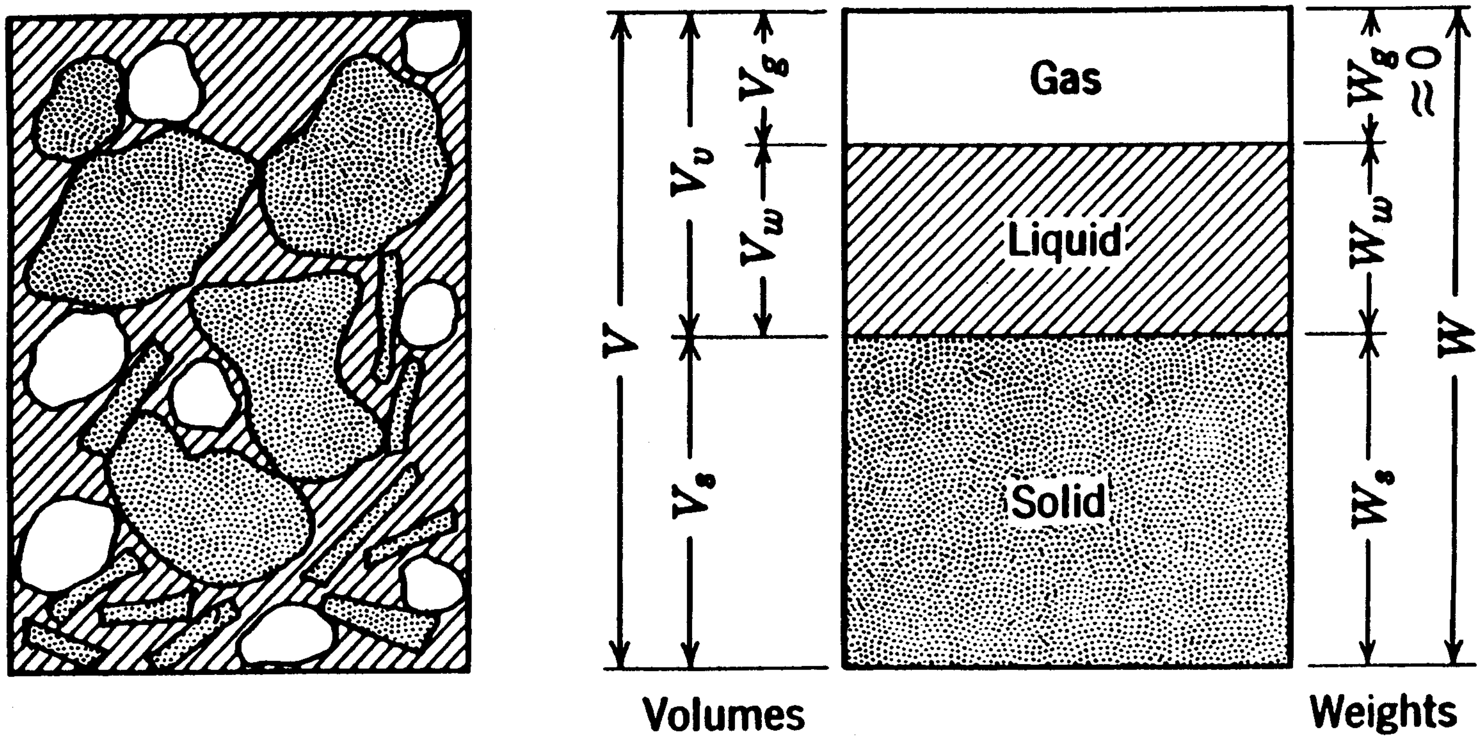
APPENDIX E-9

VI-2



Atterberg limits and related indices.

PHASE RELATIONSHIPS

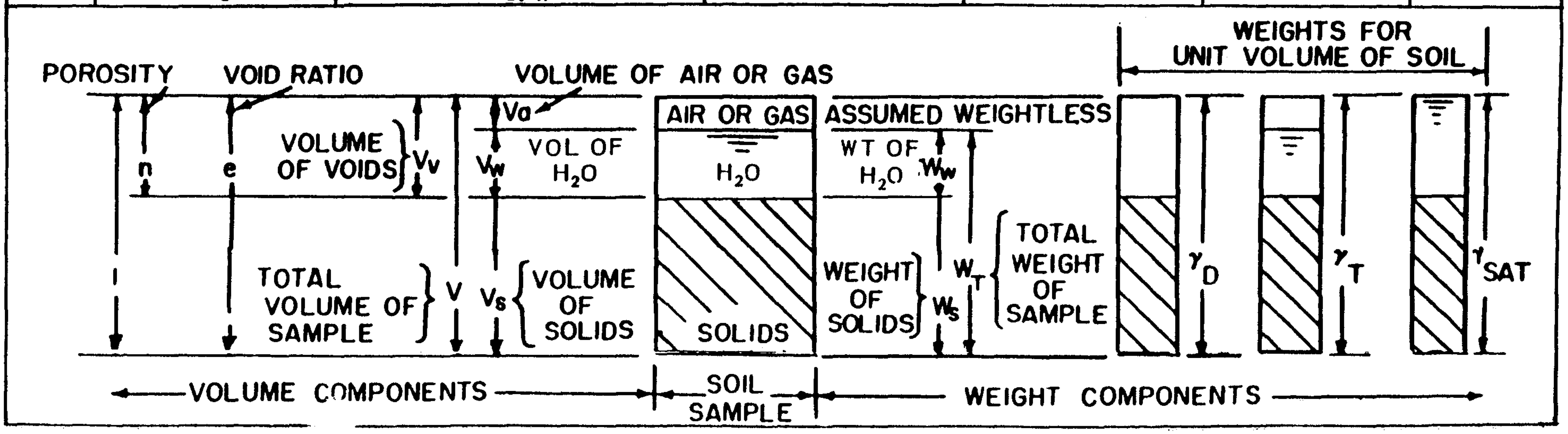


VOLUME AND WEIGHT RELATIONSHIPS

	Property		Saturated sample (W_s, W_w, G, are known)	Unsaturated sample (W_s, W_w, G, V are known)	Supplementary formulas relating measured and computed factors			
Volume components	V_s	volume of solids	$a\ \frac{W_s}{G\gamma_w}$		$V - (V_a + V_w)$	$V(1 - n)$	$\frac{V}{(1 + e)}$	$\frac{V_v}{e}$
	V_w	volume of water	$\frac{W_w}{\gamma_w{}^*}$		$V_v - V_a$	SV_v	$\frac{SV_e}{(1 + e)}$	$SV_s\ e$
	V_a	volume of air or gas	zero	$V - (V_s + V_w)$	$V_v - V_w$	$(1 - S)V_v$	$\frac{(1 - S)V_e}{(1 + e)}$	$(1 - S)V_s\ e$
	V_v	volume of voids	$\frac{W_w}{\gamma_w{}^*}$	$V - \frac{W_s}{G\gamma_w}$	$V - V_s$	$\frac{V_s\ n}{1 - n}$	$\frac{V_e}{(1 + e)}$	$V_s\ e$
	V	total volume of sample	$V_s + V_w$	measured	$V_s + V_a + V_w$	$\frac{V_s}{1 - n}$	$V_s(1 + e)$	$\frac{V_v(1 + e)}{e}$
	n	porosity	$\frac{V_v}{V}$		$\frac{1 - V_s}{V}$	$1 - \frac{W_s}{GV\gamma_w}$	$\frac{e}{1 + e}$	
	e	void ratio	$\frac{V_v}{V_s}$		$\frac{V}{V_s - 1}$	$\frac{GV\gamma_w}{W_s} - 1$	$\frac{W_w G}{W_s S}$	$\frac{n}{1 - n}$; $\frac{wG}{S}$
Weights for specific sample	W_s	weight of solids	measured		$\frac{W_T}{(1 + w)}$	$GV\gamma_w(1 - n)$	$\frac{W_w G}{eS}$	
	W_w	weight of water	measured		wW_s	$S\gamma_w V_v$	$\frac{eW_s S}{G}$	
	W_t	total weight of sample	$W_s + W_w$		$W_s(1 + w)$			

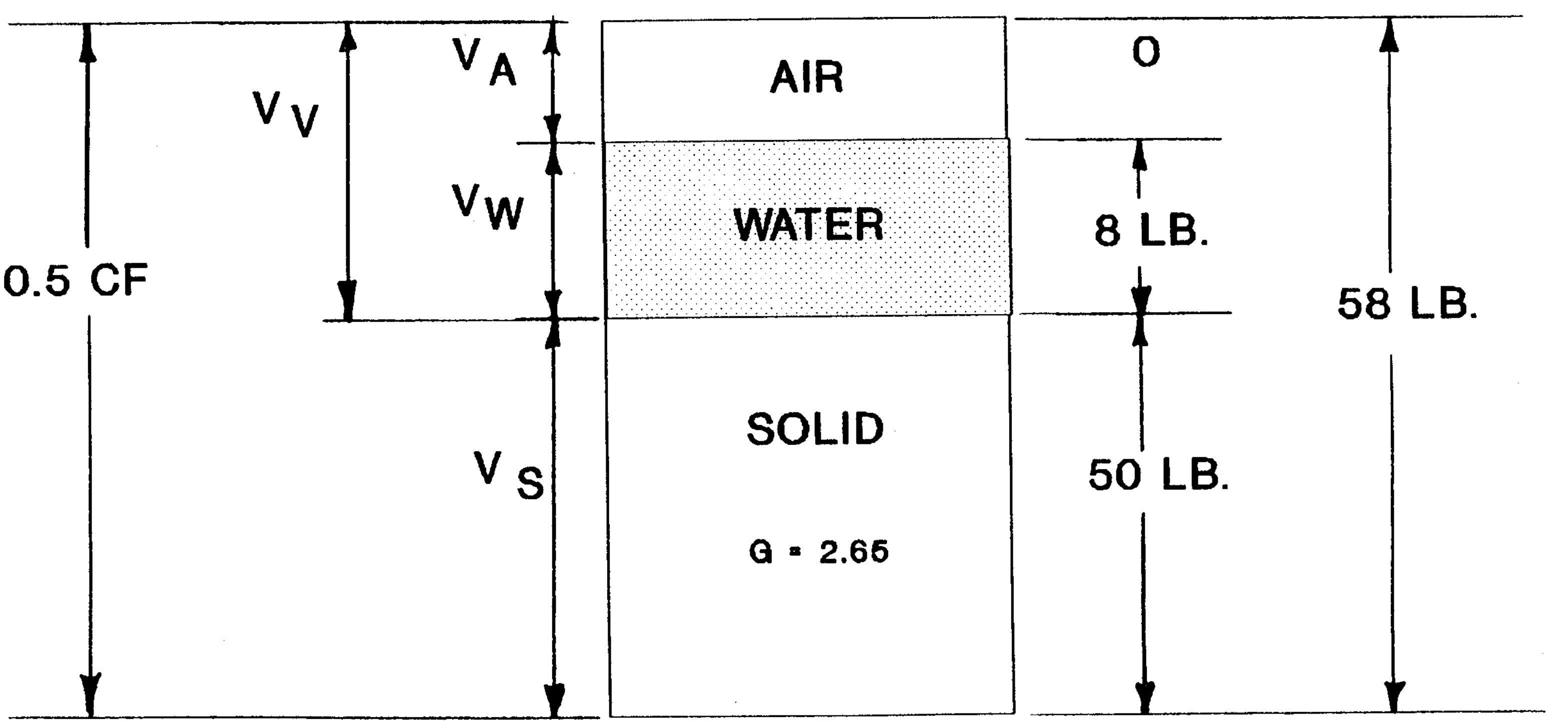
VOLUME AND WEIGHT RELATIONSHIPS (continued)

Weights for sample of unit volume	γ_D	dry unit weight	$\frac{W_s}{V_s + V_w}$	$\frac{W_s}{V}$	$\frac{W_t}{V(1+w)}$	$\frac{G\gamma_w}{(1+e)}$	$\frac{G\gamma_w}{1+wG/S}$	
	γ_T	wet unit weight	$\frac{W_s + W_w}{V_s + V_w}$	$\frac{W_s + W_w}{V}$	$\frac{W_T}{V}$	$\frac{(G+Se)\gamma_w}{(1+e)}$	$\frac{(1+w)\gamma_w}{w/S + 1/G}$	
	γ_{SAT}	saturated unit weight	$\frac{W_s + W_w}{V_s + V_w}$	$\frac{W_s + V_v\gamma_w}{V}$	$\frac{W_s}{V} + \left(\frac{e}{1+e}\right)\gamma_w$	$\frac{(G+e)\gamma_w}{(1+e)}$	$\frac{(1+w)\gamma_w}{w+1/G}$	
	γ_{SUB}	submerged (buoyant) unit weight	$\gamma_{SAT} - \gamma_w{}^*$		$\frac{W_s}{V} - \left(\frac{1}{1+e}\right)\gamma_w{}^*$	$\left(\frac{G+e}{1+e} - 1\right)\gamma_w{}^*$	$\left(\frac{1-1/G}{w+1/G}\right)\gamma_w{}^*$	
Combined relations	w	moisture content	$\frac{W_w}{W_s}$		$\frac{W_t}{W_s} - 1$	$\frac{Se}{G}$	$S\left[\frac{\gamma_w{}^*}{\gamma_D} - \frac{1}{G}\right]$	
	S	degree of saturation	1.00	$\frac{V_w}{V_v}$	$\frac{W_w}{V_v\gamma_w{}^*}$	$\frac{wG}{e}$	$\frac{w}{\left[\frac{\gamma_w{}^*}{\gamma_D} - \frac{1}{G}\right]}$	
	G	specific gravity	$\frac{W_s}{V_s\gamma_w}$		$\frac{Se}{w}$			



PROBLEM II-1

A SOIL SAMPLE WEIGHING 58 POUNDS WAS TAKEN FROM A HIGHWAY EMBANKMENT UNDER CONSTRUCTION. THE VOLUME OF THE SAMPLE AS DETERMINED BY FILLING THE HOLE WITH DRY SAND WAS 0.5 CUBIC FEET. AFTER OVEN DRYING, THE SAMPLE WEIGHED 50 POUNDS. ASSUMING A SPECIFIC GRAVITY OF THE SOIL PARTICLES OF 2.65:



PROBLEM II-1 (continued)

PART A: WHAT WAS THE ORIGINAL MOISTURE CONTENT AND VOIDS RATIO OF THIS SOIL?

$$\omega = \textit{Moisture Content, \%} = \frac{(\textit{Wgt. Water})\ (100)}{(\textit{Wgt. Solids})}$$

$$\omega = \frac{8\ (100)}{50} = 16\%$$

$$e = \textit{Void Ratio} = \frac{\textit{Vol. Voids}}{\textit{Vol. Solids}} = \frac{V_v}{V_s}$$

$$G_s = \frac{\frac{W_s}{V_s}}{62.4\ pcf} \quad \therefore \quad (2.65 \times 62.4) = \frac{50\ lbs}{V_s}$$

$$or \quad V_s = \frac{50}{165.4} = 0.302\ \textit{cubic feet}$$

$$V_{total} = V_s + V_v \quad \therefore \quad 0.5 = 0.302 + V_v \qquad V_v = 0.198\ c.f.$$

$$e = \frac{V_v}{V_s} = \frac{0.198}{0.302} = 0.656$$

PROBLEM II-1 (continued)

PART B: WHAT WOULD BE THE MAXIMUM MOISTURE CONTENT THIS SOIL COULD ATTAIN WITHOUT SWELLING?

$$G_{water} = 1.00 = \frac{\dfrac{W_w}{V_w}}{62.4}$$

for $V_v = V_w$=0.198 *cubic feet*, max *moist. without swelling.*

$$1.00 = \frac{\dfrac{W_w}{0.190}}{62.4}$$

$$W_w = (1.00)\ (62.4)\ (0.198) = 12.35 \textit{ pounds}$$

$$\omega = \frac{(W_w)\ (100)}{W_s}$$

$$\omega = \frac{(12.35)\ (100)}{50.0} = 24.7\ \% \quad \text{max } \textit{without swelling}$$

ALT: $V_v = V_{water}$

$$W_w = V_w\ (62.4) = 12.35 \textit{ pounds}$$

PROBLEM II-3

SOIL FROM A BORROW PIT IS TO BE USED IN THE CONSTRUCTION OF A HIGHWAY FILL. THE FOLLOWING DATA FOR THIS SOIL WERE OBTAINED FROM THE STANDARD PROCTOR COMPACTION TEST IN WHICH A ONE-THIRTIETH (1/30) CUBIC FOOT SOIL SAMPLE CONTAINED IN A STEEL MOLD IS COMPACTED BY A DROP HAMMER:

Moisture Content	Sample Wet Weight
5%	3.10 pounds
9%	3.42 pounds
14%	3.78 pounds
22%	3.85 pounds

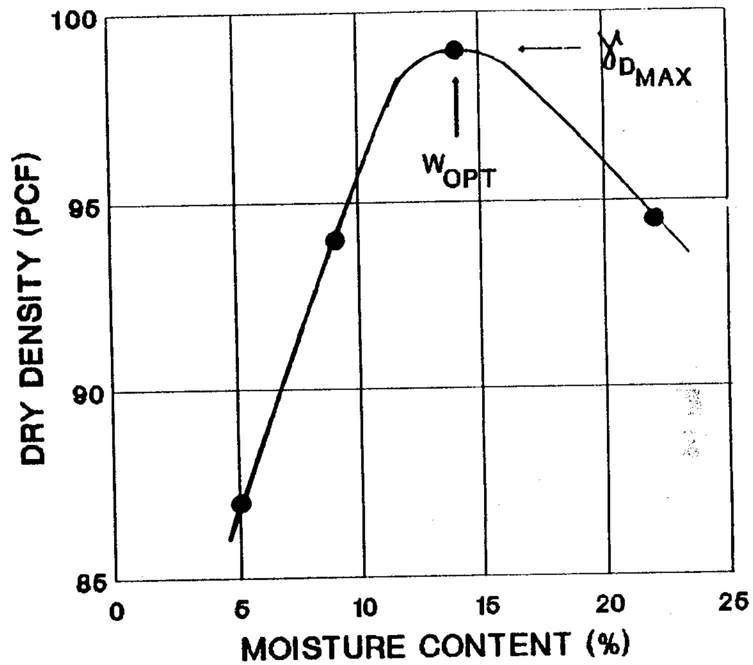
SPECIFICATIONS STATE THAT THE SOIL IS TO BE COMPACTED AT THE OPTIMUM MOISTURE CONTENT AND TO AT LEAST 95% OF MAXIMUM DENSITY AS DETERMINED FROM THE STANDARD PROCTOR COMPACTION TEST.

(A) IN TERMS OF DRY UNIT WEIGHT, WHAT IS THE MINIMUM DENSITY WHICH MUST BE OBTAINED IN THE EMBANKMENT TO MEET SPECIFICATIONS?

(B) IF THE INITIAL WATER CONTENT OF THE SOIL IS 9%, HOW MANY GALLONS OF WATER PER CUBIC YARD SHOULD BE ADDED TO THE FILL MATERIAL TO REACH 100% MAXIMUM DENSITY AND OPTIMUM MOISTURE CONTENT (DRY WEIGHT BASIS)?

PROBLEM II-3 (continued)

ω (%)	γ_{wet}	γ_{dry}
5	93.0 pcf	88.6
9	102.6	94.1
14	113.4	99.5
22	115.5	94.7



(A) 0.95 (99.5) = 94.5 pcf

(B) ADD 14-9 = 5% WATER

$$\Delta\ W_w = 0.05\ (99.5)\ = 5.0\ pcf$$

$$5.0\ pcf\ x\ 27\ \frac{ft^3}{c.y.} = 135\ pounds/c.y.$$

$$= \frac{135\ pounds}{62.4}\ x\ 7.48\ \frac{gallons}{ft^3} = 16.2\ gallons/c.y.$$

PROBLEM V-3 (continued)

- <u>BEARING CAPACITY</u>:

$\phi = 30°$ $\quad \gamma = 120\ pcf$

$$NET\ q_{ULT} = \gamma D\ (N_q - 1) + \frac{1}{2} B \gamma N_\gamma = (0.120)\ (4)\ (20 - 1) + \frac{1}{2}\ (10)\ (0.120)\ (17) = 19.31^{ksf}$$

$$F.S. = \frac{19.31}{3.14} = 6.16 > 3.0\ (O.K.)$$

- <u>SLIDING: (F.S. ≥ 1.5)</u>

SHEAR RESISTANCE AVAILABLE ALONG BASE

$$S = \sum V \tan\phi = 18.12\ \tan 30° = 10.45^{K}$$

PASSIVE FORCE @ TOE : $\quad P_P = \frac{1}{2}\ (0.120)\ (3.0)\ (4)^2 = 2.88^{K}$

$$MIN.\ F.S. = 10.45 \div 6.48 = 1.61 > 1.5\ (O.K.)$$

$$MAX.\ F.S. = (10.45 + 2.88) \div 6.48 = 2.06\ (O.K.)$$

PROBLEM V-3 (continued)

- <u>FACTOR OF SAFETY AGAINST OVERTURNING: (F.S. ≥ 2)</u>

$$F.S. = \frac{RESISTING\ MOMENT}{OVERTURNING\ MOMENT} = \frac{107.3}{38.8} = 2.77 > 2.0\ (O.K.)$$

- <u>LOCATION OF RESULTANT: (MIDDLE THIRD)</u>

$$FROM\ POINT\ \text{"}A\text{"} \quad \frac{\sum M_A}{\sum V} = \frac{68.5}{18.12} = 3.78\ ft$$

$$THEN\ e = \frac{10}{2} - 3.78 = 1.22' < \frac{10'}{6}\ (O.K.)$$

- <u>SOIL PRESSURE AT BASE:</u>

$$q = \frac{\sum V}{B}\left(\frac{1 \pm 6e}{B}\right)$$

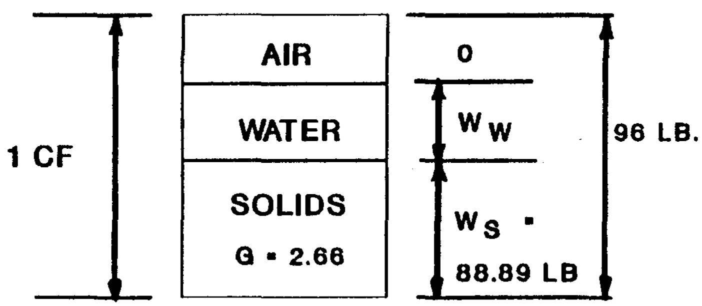
$$AT\ TOE \quad q_{max} = \frac{18.12}{10}\left(1 + \frac{6 \times 1.22}{10}\right) = 1.812\ (1 + 0.732) = 3.14^{ksf}$$

$$AT\ HEEL \quad q_{min} = 1.812\ (1 - 0.732) = 0.49^{ksf}$$

PROBLEM II-4

BORROW MATERIAL TO BE USED TO CONSTRUCT A HIGHWAY EMBANKMENT HAS A MOIST UNIT WEIGHT OF 96.0 pcf, A WATER CONTENT OF 8.0%, AND THE SPECIFIC GRAVITY OF THE SOIL SOLIDS IS 2.66. THE SOIL IS TO BE PLACED AS FILL WITH A DRY UNIT WEIGHT OF 112.0 pcf AND WATER CONTENT OF 13.0%.

BORROW



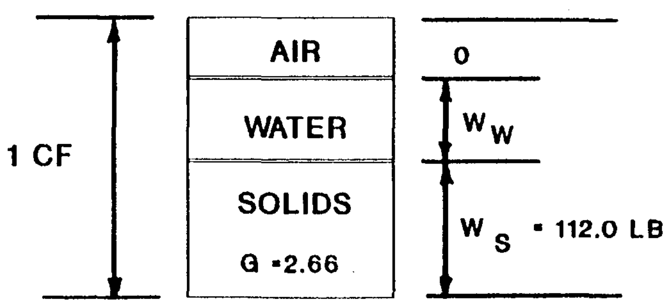
$$\omega = 8\% = 0.08 = \frac{W_w}{W_s} \quad or \quad W_w = 0.08\ W_s$$

ALSO $\quad W_s + W_w = W_T = 96$ *pounds*

SUBSTITUTING: $W_s + 0.08\ W_s = 96$ *pounds* *or* $W_s = 88.89$ *lbs*

$\therefore\ W_w = 96$ *pounds* $- 88.89$ *pounds* $= 7.11$ *lbs*

FILL



$$\omega = 13\% = 0.13 = \frac{W_w}{W_s} = \frac{W_w}{112.0}$$

$W_w = 14.56$ *pounds*

VI-14

PROBLEM II-4 (continued)

(A) HOW MANY CUBIC YARDS OF BORROW ARE REQUIRED TO CONSTRUCT A 250,000-CUBIC-YARD EMBANKMENT?

FOR 1 C.F. OF EMBANKMENT FILL, YOU NEED 112.0 POUNDS SOLID. THEREFORE, YOU NEED MORE THAN 1 C.F. OF BORROW TO MAKE 1 C.F. OF FILL:

$$\frac{112 \textit{ pounds required}}{X \textit{ c.f.}} = \frac{88.89 \textit{ pounds}}{1 \textit{ c.f.}} \quad \textit{or} \quad x = 1.26 \textit{ c.f. borrow required for } 1 \textit{ c.f. fill}$$

FOR 250,000 C.Y. BORROW:

$$\frac{1 \textit{ c.f. fill}}{1.26 \textit{ c.f. borrow}} = \frac{250{,}000 \textit{ c.y. fill}}{X \textit{ c.y. borrow}}$$

X = 315,000 C.Y. BORROW REQUIRED

PROBLEM II-4 (continued)

(B) HOW MANY GALLONS OF WATER MUST BE ADDED PER CUBIC YARD OF BORROW ASSUMING NO LOSS BY EVAPORATION?

FOR EACH 1 C.F. OF BORROW, WE WANT TO RAISE ω FROM 8% TO 13%.

$\therefore\ \Delta\omega = +5\%$

$$.05 = \frac{W_w}{W_s} = \frac{W_w}{88.89 \text{ pounds}}$$

W_w = 4.44 POUNDS WATER TO BE ADDED PER C.F. BORROW

$$\frac{(4.44 \text{ pounds})\ (27)}{8.33 \text{ pounds/gallon}} = 14.4\ \frac{\text{gallons}}{\text{c.y.}}$$

PROBLEM II-4 (continued)

(C) IF THE COMPACTED FILL BECOMES SATURATED AT CONSTANT VOLUME, WHAT WILL BE THE WATER CONTENT AND MOIST UNIT WEIGHT OF THE SOIL?

$$G_s = 2.66 = \frac{W_s \; / \; V_s}{62.4 \;\; pounds/ft^3} = \frac{112.0 \; / \; V_s}{62.4}$$

$$V_s = 0.6747 \;\; c.f.$$

$$V_v = 1.000 - 0.6747 = 0.3253 \;\; c.f.$$

$$G_w = 1.00 = \frac{W_w}{(V_w) \; (62.4)} = \frac{W_w}{(0.3253) \; (62.4)} \quad or \quad W_w = 20.29 \;\; pounds$$

$$\omega = \frac{W_w}{W_s} = \frac{20.29 \;\; pounds}{112.0 \;\; pounds} = 0.181 = 18.1\%$$

$$\gamma = \frac{112.0 + 20.29}{1 \;\; c.f.} = 132.3 \;\; pounds/ft^3$$

$$W_S = \frac{W_T}{1 + w}$$

$$\gamma_d = \frac{\gamma_T}{1 + w}$$

SINGLE FRICTION PILE IN COHESIVE SOIL

METHOD	SOIL TYPES	MINIMUM SAFETY FACTOR
α METHOD (TOTAL STRESS)	SOFT TO MEDIUM CLAYS	3
β METHOD (EFFECTIVE STRESS)	SOFT TO STIFF CLAYS	3

PROBLEM VII-4 (continued)

PART A: (α METHOD)

SURFACE AREA:

LAYER	PILE SURFACE AREA (A)
1	$12'\pi$ (1 FT) = 37.7 FT^2
2	$18\ \pi$ (1 FT) = 56.5 FT^2
3	$10\ \pi$ (1 FT) = 31.4 FT^2
	ΣA = 125.6 FT^2

REDUCED STRENGTH VALUES:

LAYER	C (psf)	q_U (tsf)	α	$\alpha \cdot c \cdot A$
1	800	0.8	0.88	26.5k
2	1,600	1.6	0.65	58.8K
3	2,000	2.0	0.56	35.2K
				Q_S=120.5 K/pile

PROBLEM VII-4 (continued)

PART B: GROUP CAPACITY

$Q_S = 2\ (B + W)LC_1 \qquad\qquad Q_P = 9C_2BW$

$B = 6\ FT \qquad\qquad W = 8\ FT$

$C_1 = WEIGHTED\ C$

$C_2 = 2{,}000\ psf$

$$Q_S = \frac{2(5 + 7)}{1{,}000}\ x \left[(12 \cdot 800) + (18 \cdot 1{,}600) + (10 \cdot 2{,}000)\right] = 1{,}402\ K$$

$$Q_P = \frac{9\ (2{,}000)}{1{,}000}\ (5)\ (7) = 630\ K$$

$$Q_T = 1{,}402 + 630 = 2{,}032\ k \qquad Q_{ALL} = \frac{2{,}032}{3} = 677\ K$$

$$Q_{ALL}\ FOR\ SINGLE\ PILE = \frac{677}{12} = \frac{56.4\ K}{-}$$

∴ *SINGLE PILE CAPACITY GOVERNS*

–

EFFECTIVE STRESS

CONCEPT:

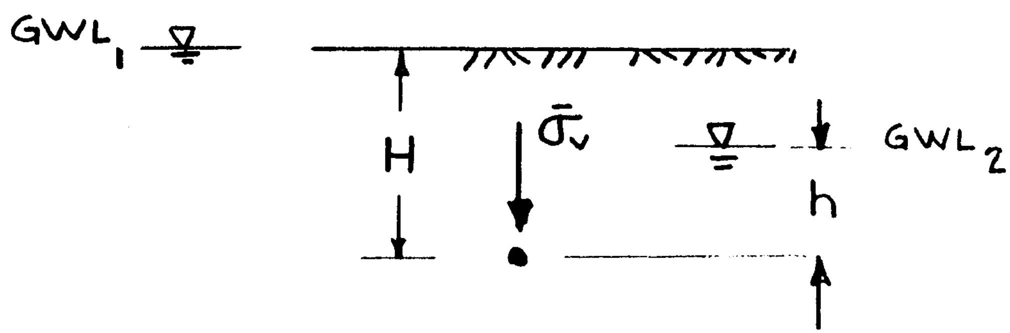
TOTAL STRESS	=	EFFECTIVE STRESS	+	PORE PRESS.
σ	=	$\bar{\sigma}$	+	μ

FOR GEOSTATIC CONDITION (vertical):

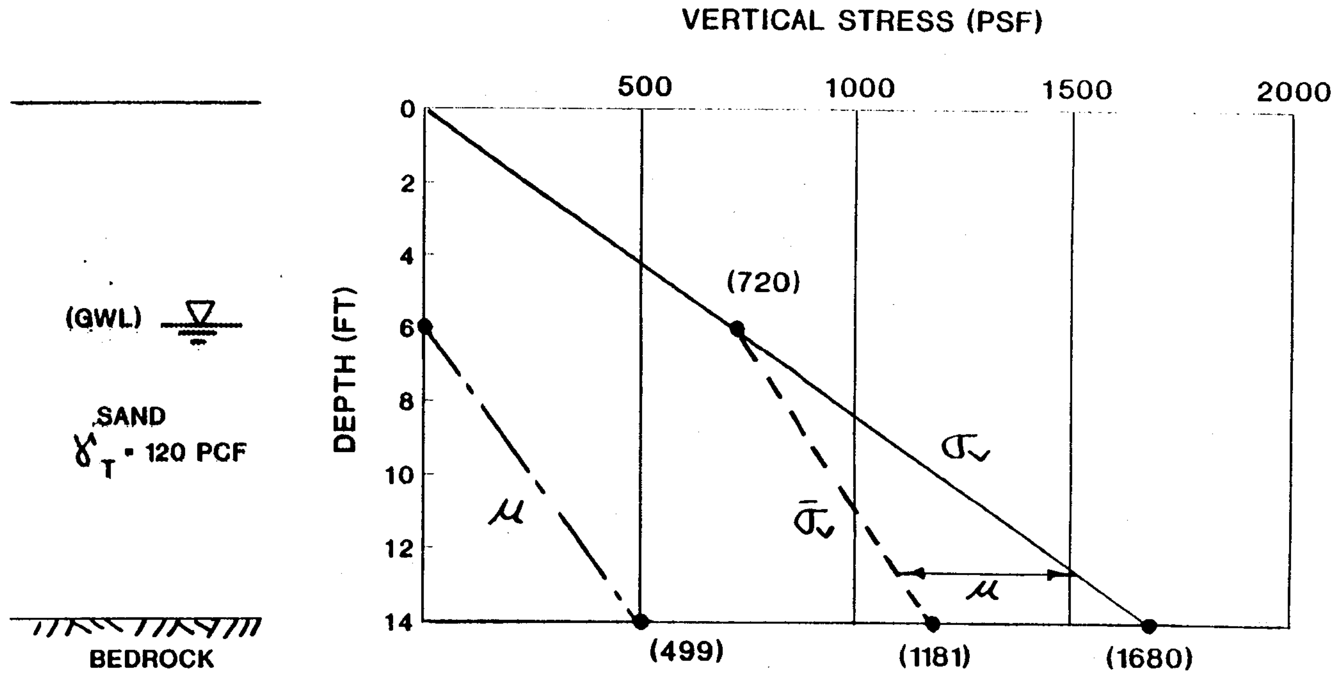
$\sigma_v = \gamma_T H$

$\bar{\sigma}_v = (\gamma_t - \gamma_w)\ H$ (for water at G.S.)

$\bar{\sigma}_v = \gamma_T H - \gamma_w h$ (water below G.S.)



SIMPLE EXAMPLE



CALCULATIONS:

@ 6' -> $\sigma_v = 6 \times 120 = 720$ psf
$\mu = 0$
$\therefore\ \bar{\sigma}_v = 720 - 0 = 720$ psf

@ 14' -> $\sigma_v = 14 \times 120 = 1{,}680$ psf
$\mu = 8 \times 62.4 = 4.99$ psf
$\therefore\ \bar{\sigma}_v = 1{,}680 - 499 = 1{,}181$ psf

NOT SO SIMPLE EXAMPLE

PLOT σ_v, $\bar{\sigma}_v$, and μ FOR:

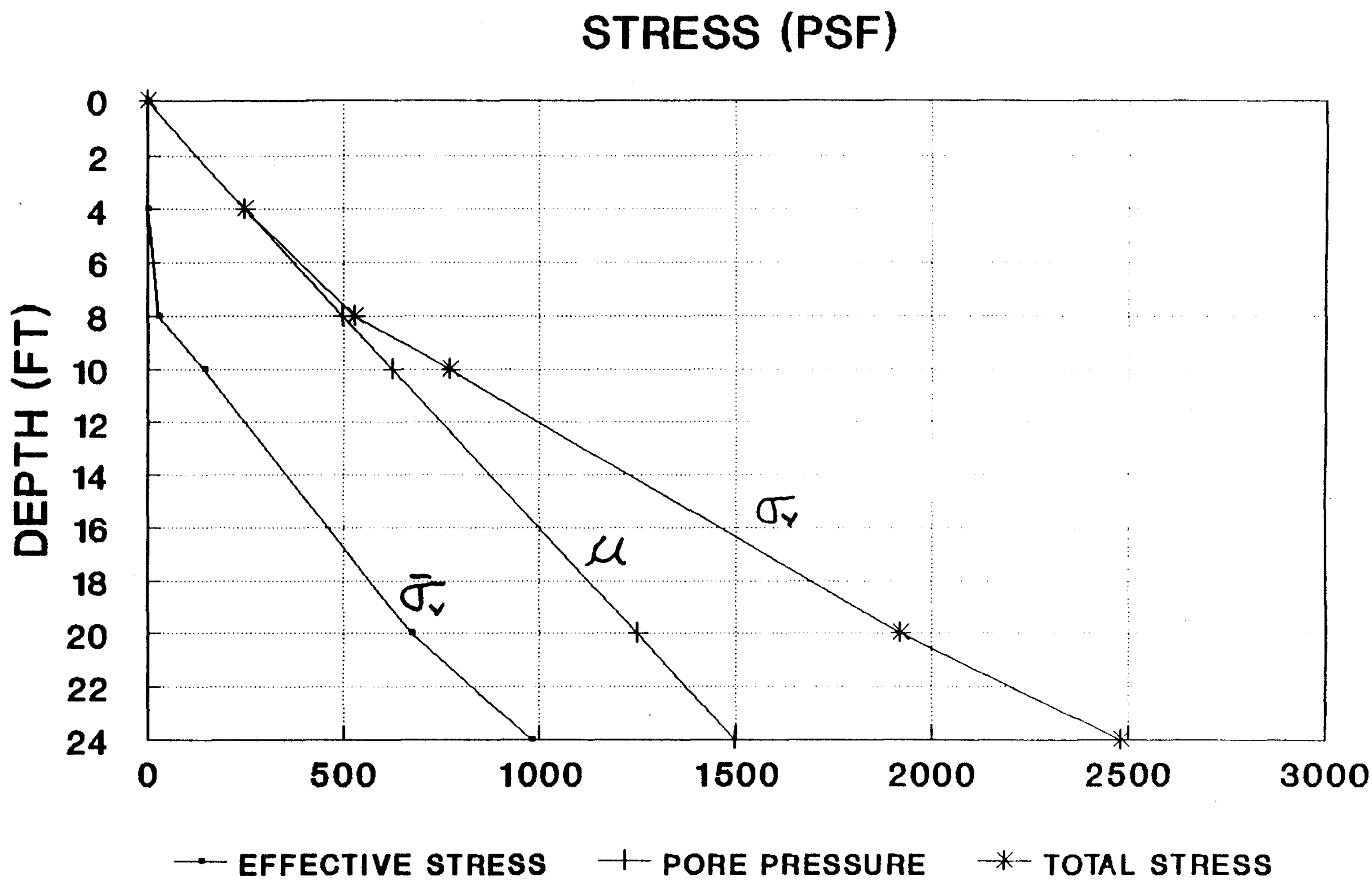
DEPTH (FT)		TOTAL UNIT WT. γ_T
0		
	WATER	62.4 PCF
4		
	PEAT	70 PCF
8		
	SAND	120 PCF
10		
	CLAY	115 PCF
20		
	TILL	140 PCF
24		

NOT SO SIMPLE (continued)

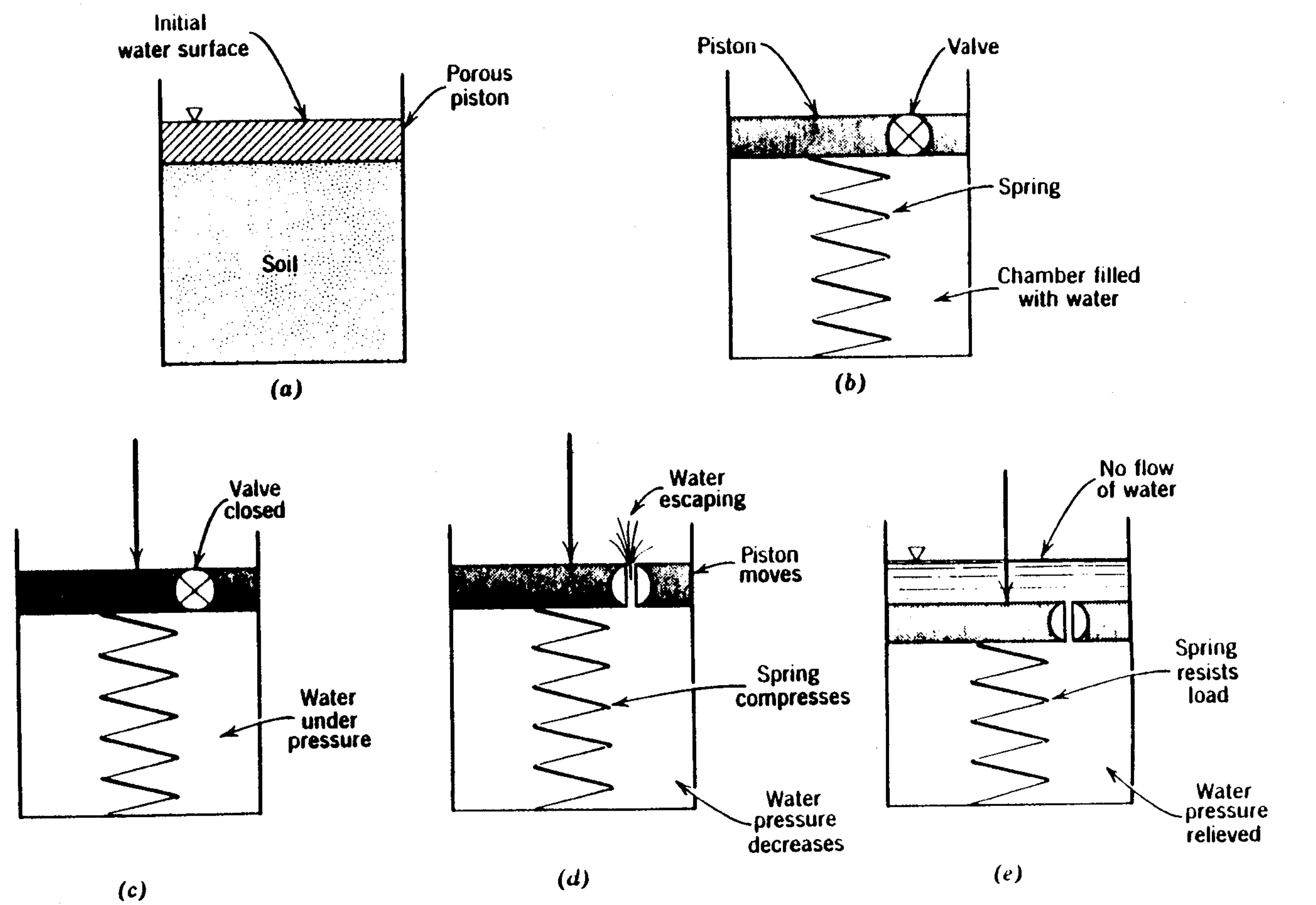
CALCULATIONS:

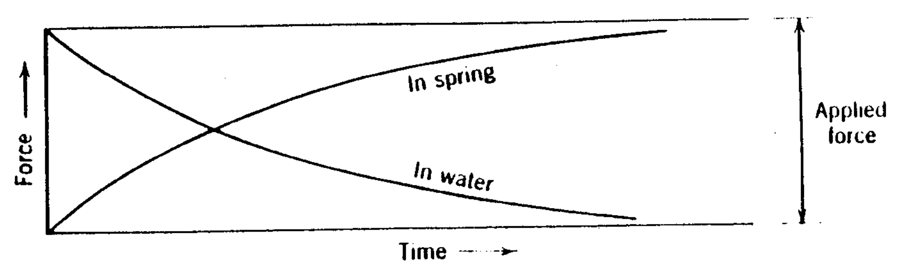
Depth (feet)	H (feet)	γ_T (pcf)	$\Delta\sigma_v$ (psf)	σ_v (psf)	μ_w (psf)	$\bar{\sigma}_v$ (psf)
0				0	0	0
(water)	4	62.4	249			
4				249	249	0
(peat)	4	70	280			
8				529	499	30
(sand)	2	120	240			
10				769	624	145
(clay)	10	115	1,150			
20				1,919	1,248	671
(till)	4	140	560			
24				2,479	1,497	982

NOTE: $\Delta\sigma_v = H \times \gamma_T$ $\sigma_v = \Sigma\Delta\sigma_v$ $\bar{\sigma}_v = \sigma_v - \mu_w$


STRESS (PSF)
DEPTH (FT)
0
2
4
6
8
10
12
14
16
18
20
22
24
0
500
1000
1500
2000
2500
3000
σv
u
σ̄v
EFFECTIVE STRESS
PORE PRESSURE
TOTAL STRESS

CONSOLIDATION

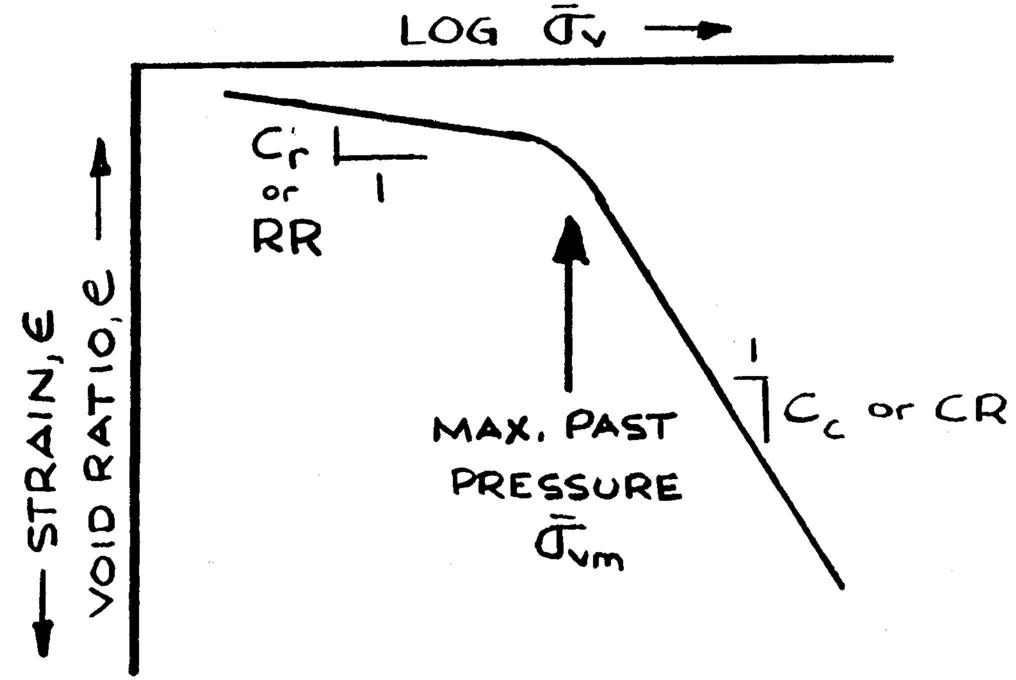




SETTLEMENT
(COMPRESSION)

MAGNITUDE	**TIME**
• $\Delta\bar{\sigma}_v$	• PERMEABILITY
• COMPRESSIBILITY	

CONSOLIDATION PARAMETERS



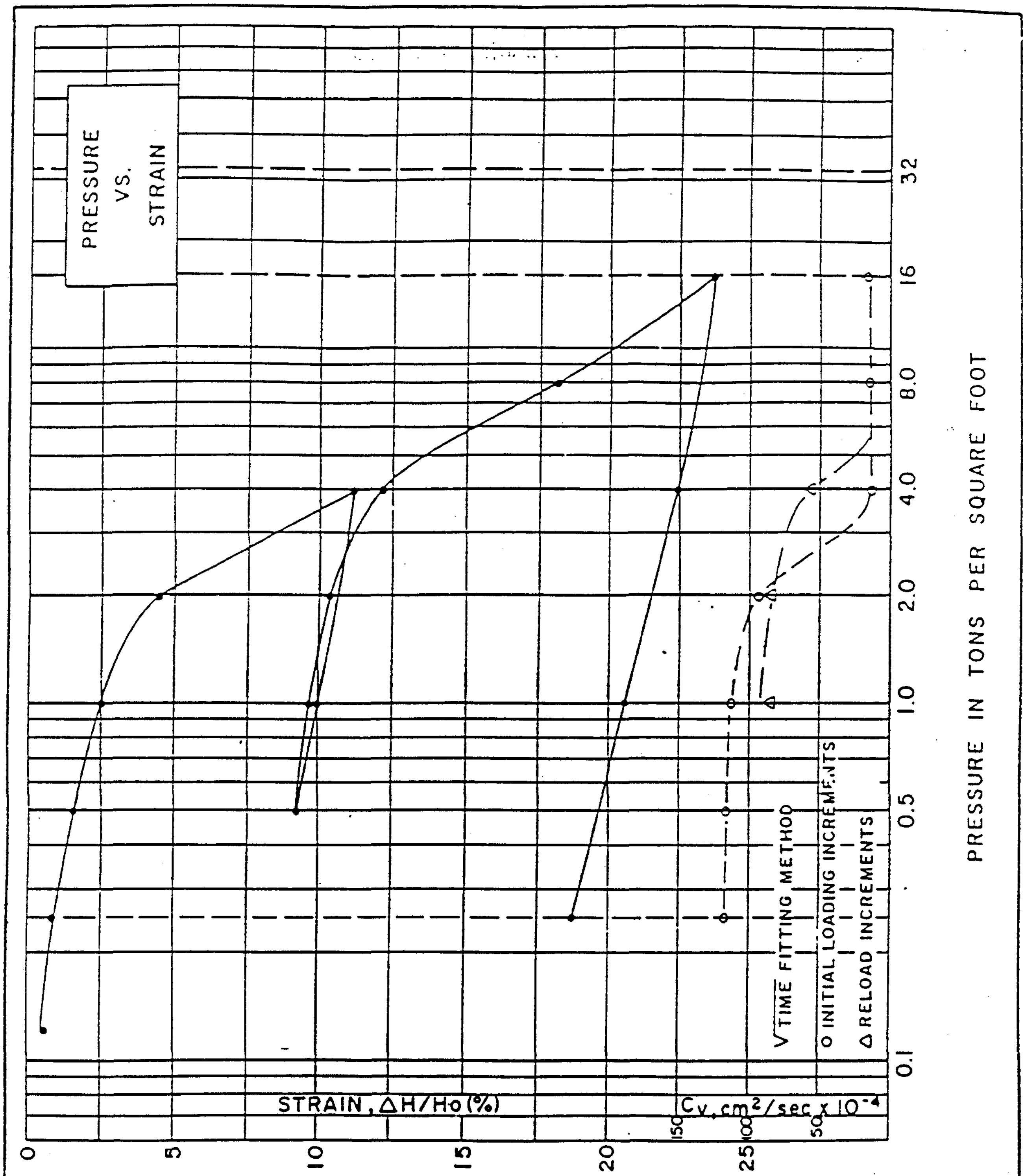
e vs. LOG σ:
- C_c = COMPRESSION INDEX
- C_r = RECOMPRESSION INDEX

E vs. LOG σ:
- CR = COMPRESSION RATIO
- RR = RECOMPRESSION RATIO

NORMALLY vs. OVERCONSOLIDATED

C. C.

STRAIN VS. LOG EFFECTIVE STRESS



SOIL DESCRIPTION: GREY SILTY CLAY

SAMPLE DIAM. 2.50 IN. LIQUID LIMIT 42 % PLASTIC LIMIT 22 % SPECIFIC GRAVITY -

	WATER CONTENT, %	DRY UNIT WEIGHT, pcf	VOID RATIO	SATURATION, %	SAMPLE HEIGHT, INCH
INITIAL	39.9	82.4	-	-	0.800
FINAL	30.2	101.4	-	-	0.650

FREEPORT, MAINE

CONSOLIDATION TEST

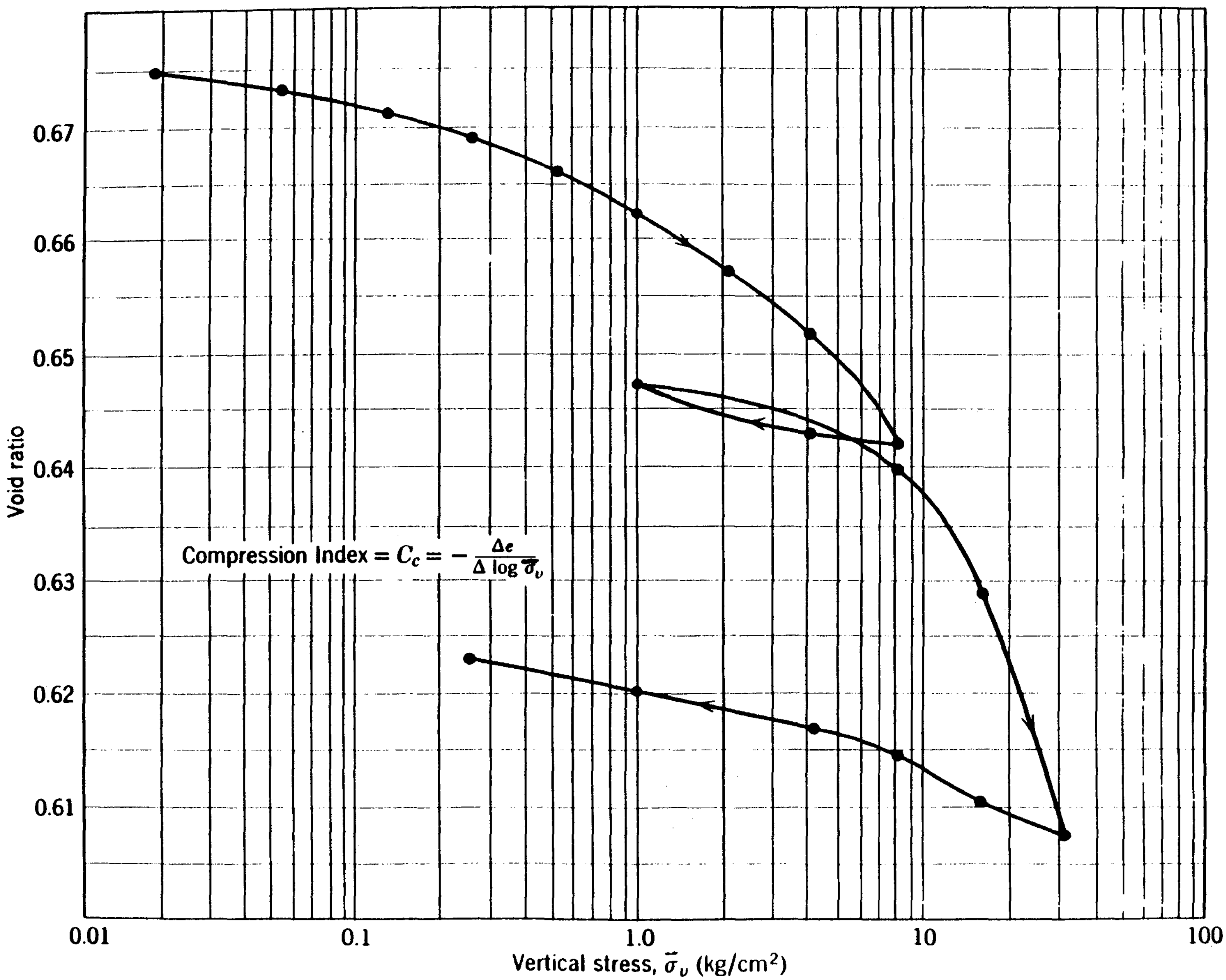
BORING NO. G3U
SAMPLE U1
DEPTH 16.4'-16.6'
TECH.
REVIEWER

TEST SERIES NO. 6
DATE JAN 1981
FILE

GOLDBERG-ZOINO & ASSOCIATES, INC.
Geotechnical-Geohydrological Consultants

FIGURE IV-1

e VS. LOG EFFECTIVE STRESS



$$\frac{\Delta e}{1+e_o} = \Delta \epsilon$$

$$\frac{C_c}{1+e_o} = CR$$

SETTLEMENT CALCULATIONS

$$S = \frac{\Delta e}{1 + e_o} H$$

$$= \underbrace{\frac{C_c}{1 + e_o}}_{CR} H \quad LOG_{10} \frac{\bar{\sigma}_{vo} + \Delta\bar{\sigma}_v}{\bar{\sigma}_{vo}}$$

S	=	SETTLEMENT
Δe	=	CHANGE IN VOID RATIO
e_o	=	INITIAL VOID RATIO
H	=	THICKNESS OF COMPRESSIBLE SOIL
$\bar{\sigma}_{vo}$	=	INITIAL VERTICAL EFFECTIVE STRESS
$\Delta\bar{\sigma}_v$	=	CHANGE VERTICAL EFFECTIVE STRESS

GENERAL EQUATION

$$S = \underbrace{\frac{C_r}{1 + e_o} \; H \; \log \frac{\bar{\sigma}_{vm}}{\bar{\sigma}_{vo}}}_{\text{OVERCONSOLIDATED}} \quad + \quad \underbrace{\frac{C_c}{1 + e_o} \; H \; \log \frac{\bar{\sigma}_{vf}}{\bar{\sigma}_{vm}}}_{\text{NORMALLY CONS.}}$$

C_c = 0.009 (LL-10)

C_r = 1/5 TO 1/10 C_c

EMPIRICAL

PROBLEM IV-1

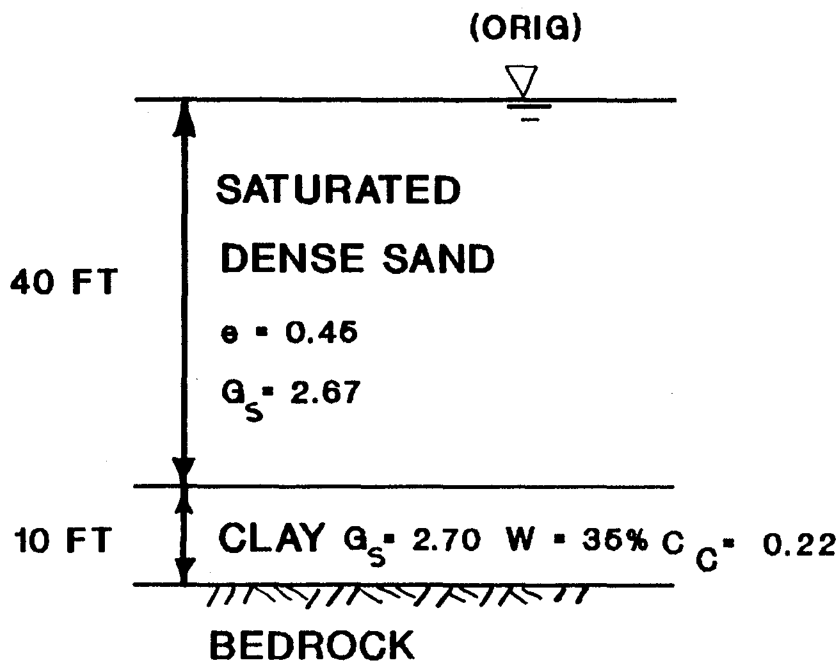
A SOIL PROFILE EXISTING OVER A LARGE AREA CONSISTS OF A DENSE SAND, 40 FEET IN THICKNESS, UNDERLAIN BY A 10-FOOT-THICK LAYER OF NORMALLY CONSOLIDATED CLAY WHICH RESTS ON RELATIVELY INCOMPRESSIBLE BEDROCK. THE PRESENT WATER TABLE IS AT THE GROUND SURFACE, THE TOP OF THE SAND LAYER. IT IS PROPOSED TO PLACE AN EMBANKMENT OF LARGE AREAL EXTENT AND 10 FEET IN THICKNESS ON THE SURFACE OF THE EXISTING GROUND. AT THE SAME TIME, THE EXISTING WATER TABLE IS TO BE LOWERED 20 FEET. THE FILL MATERIAL HAS A MOIST UNIT WEIGHT OF 125 pcf.

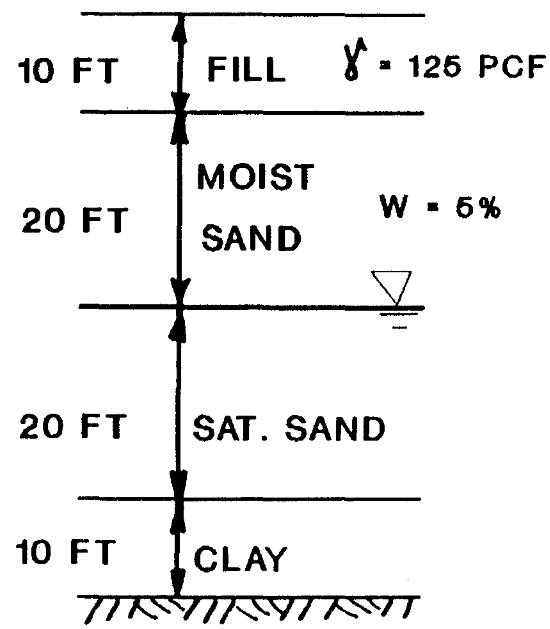
THE SAND HAS A VOID RATIO OF 0.45 AND THE SPECIFIC GRAVITY OF THE SOLIDS IS 2.67. THE SAND IS SATURATED BELOW THE WATER TABLE AND HAS AN AVERAGE WATER CONTENT OF 5% ABOVE THE WATER TABLE.

THE CLAY IS SATURATED, HAS A SPECIFIC GRAVITY OF SOLIDS OF 2.70, AND AN AVERAGE WATER CONTENT OF 35%. THE COMPRESSION INDEX OF THE CLAY HAS BEEN DETERMINED TO BE 0.22.

PROBLEM IV-1 (continued)

ESTIMATE THE TOTAL SETTLEMENT IN INCHES OF THE CLAY LAYER DUE TO THE COMBINED EFFECT OF THE IMPOSED FILL AND THE LOWERED GROUNDWATER LEVEL.

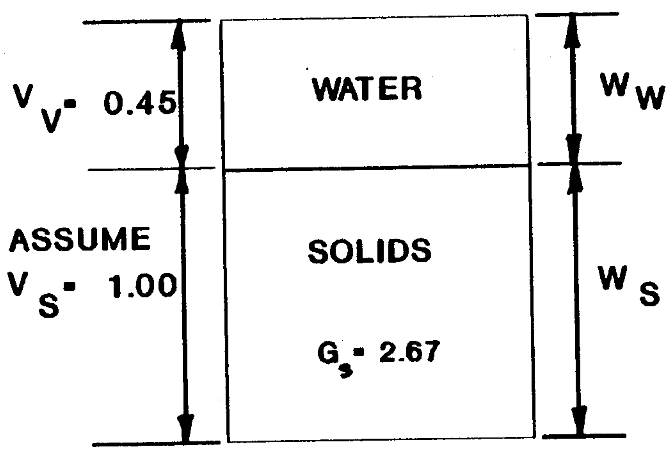




PROBLEM IV-1 (continued)

COMPUTE UNIT WEIGHTS OF EACH SOIL LAYER

SATURATED SAND



$$e = \frac{V_v}{V_s}$$

$$0.45 = \frac{V_v}{1.00}$$

$$V_v = 0.45$$

$$G_s = 2.67 = \frac{W_s}{(V_s)\ 62.4} = \frac{W_s}{(1.00)\ (62.4)}$$

$$W_s = 166.61 \textit{ pounds}$$

$$G_w = 1.00 = \frac{W_w}{(V_w)\ (62.4)} = \frac{W_w}{(0.45 \times 62.4)}$$

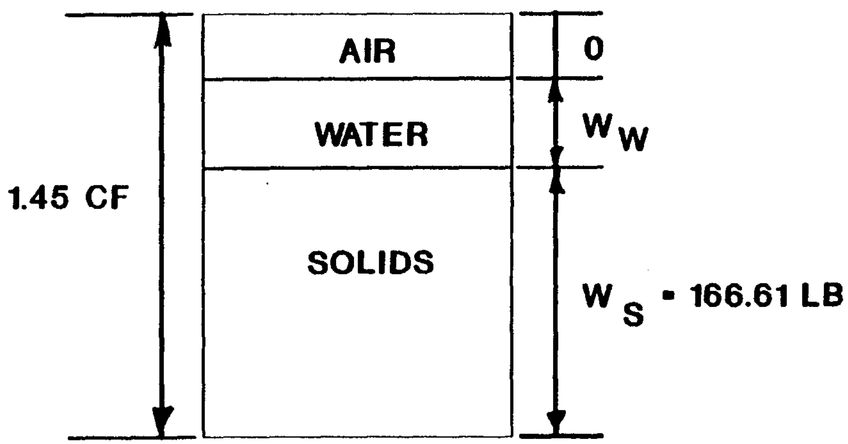
$$W_w = 28.08 \textit{ pounds}$$

$$W_T = 166.61 \textit{ lbs.} + 28.08 \textit{ lbs.} = 194.69 \textit{ lbs.}$$

$$\gamma_{TOT} = \frac{194.69 \textit{ lbs.}}{1.45 \textit{ ft}^3} = 134.3 \textit{ lbs./ft}^3 \textit{ saturated}$$

PROBLEM IV-1 (continued)

MOIST SAND



$$\omega = 0.5 = \frac{W_w}{W_s}$$

$$W_w = (0.5)\ (166.61)$$

$$W_w = 8.33\ pounds$$

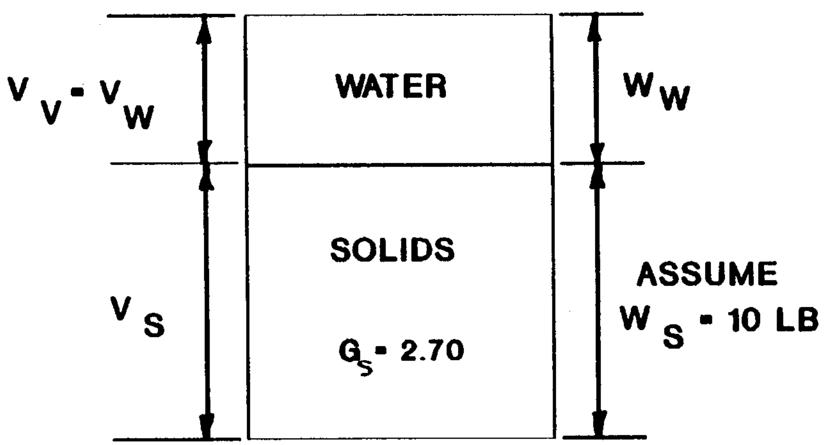
$$W_T = 166.61\ pounds + 8.33$$

$$W_T = 174.94\ pounds$$

$$\gamma_{moist} = \frac{174.94\ pounds}{1.45\ ft^3} = 120.6\ pcf$$

PROBLEM IV-1 (continued)

SATURATED CLAY



W_T = 10 *pounds* +3.5 *pounds* = 13.5 *pounds*

$$\omega = 0.35 = \frac{W_w}{W_s}$$

W_w = (0.35) (10 *pounds*) = 3.50 *pounds*

$$G_s = 2.70 = \frac{W_s}{(V_s)\,(62.4)} = \frac{10 \text{ pounds}}{(V_s)\,(62.4)}$$

$V_s = 0.0594 \ ft^3$

$$G_w = 1.00 = \frac{W_w}{(V_w)\,(62.4)} \quad or \quad V_w = \frac{3.50 \text{ pounds}}{(1.00)\,(62.4)}$$

$V_w = 0.0561 \ ft^3$

$V_T = 0.0594 \ ft^3 + 0.0561 \ ft^3$

$V_T = 0.1155 \ ft^3$

$$\gamma_{tot} = \frac{W_T}{V_T} = \frac{13.5 \text{ pounds}}{0.1155 \text{ c.f.}} = 116.9 \text{ pcf}$$

ALSO $$e_{clay} = \frac{V_{voids}}{V_{solids}} = \frac{0.0561}{0.0594} = 0.944$$

ALT: $Se = \omega G$

PROBLEM IV-1 (continued)

COMPUTE $\bar{\sigma}_{vo}$ AT MID HEIGHT OF CLAY LAYER

- $(40')\ (134.3 - 62.4) = 2{,}876\ psf$
- $(5')\ (116.9 - 62.4) = 273\ psf$

$\bar{\sigma}_{vo} = 3{,}149\ psf$

COMPUTE $\Delta\bar{\sigma}_v$ AT MID HEIGHT OF CLAY LAYER

- (10' Fill)(125 pounds/ft^3) = 1,250 psf

+ increase due to lowering groundwater
20 (120.6) + 20 (134.3) + 5 (116.9) - 25 (62.4) - 3,149 = 974 psf

$\Delta\bar{\sigma}_v = 2{,}224$ psf

PROBLEM IV-1 (continued)

COMPUTE SETTLEMENT OF CLAY LAYER

$$S = \frac{C_c}{1 + e_o} H \quad log \frac{\bar{\sigma}_{vo} + \Delta\bar{\sigma}_v}{\bar{\sigma}_{vo}}$$

$$S = \frac{0.22}{1 + 0.945} (10') \, (12) \quad log \frac{3{,}149 + 2{,}224}{3{,}149}$$

$$S = \frac{0.22}{1.945} (120) \quad log \; 1.70$$

$$S = (13.57) \, (0.23)$$

$$S = 3.12 \; inches$$

RATE OF SETTLEMENT

$$\boxed{t = \frac{T H_d^2}{C_v}}$$

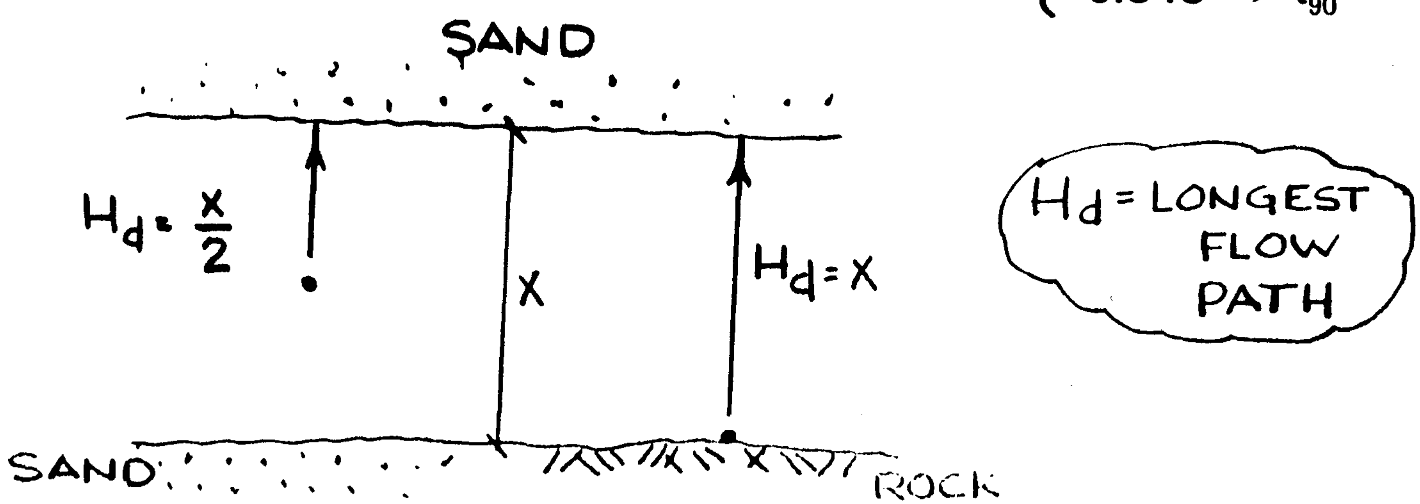
t = TIME

C_v = COEFFICIENT OF CONSOLIDATION

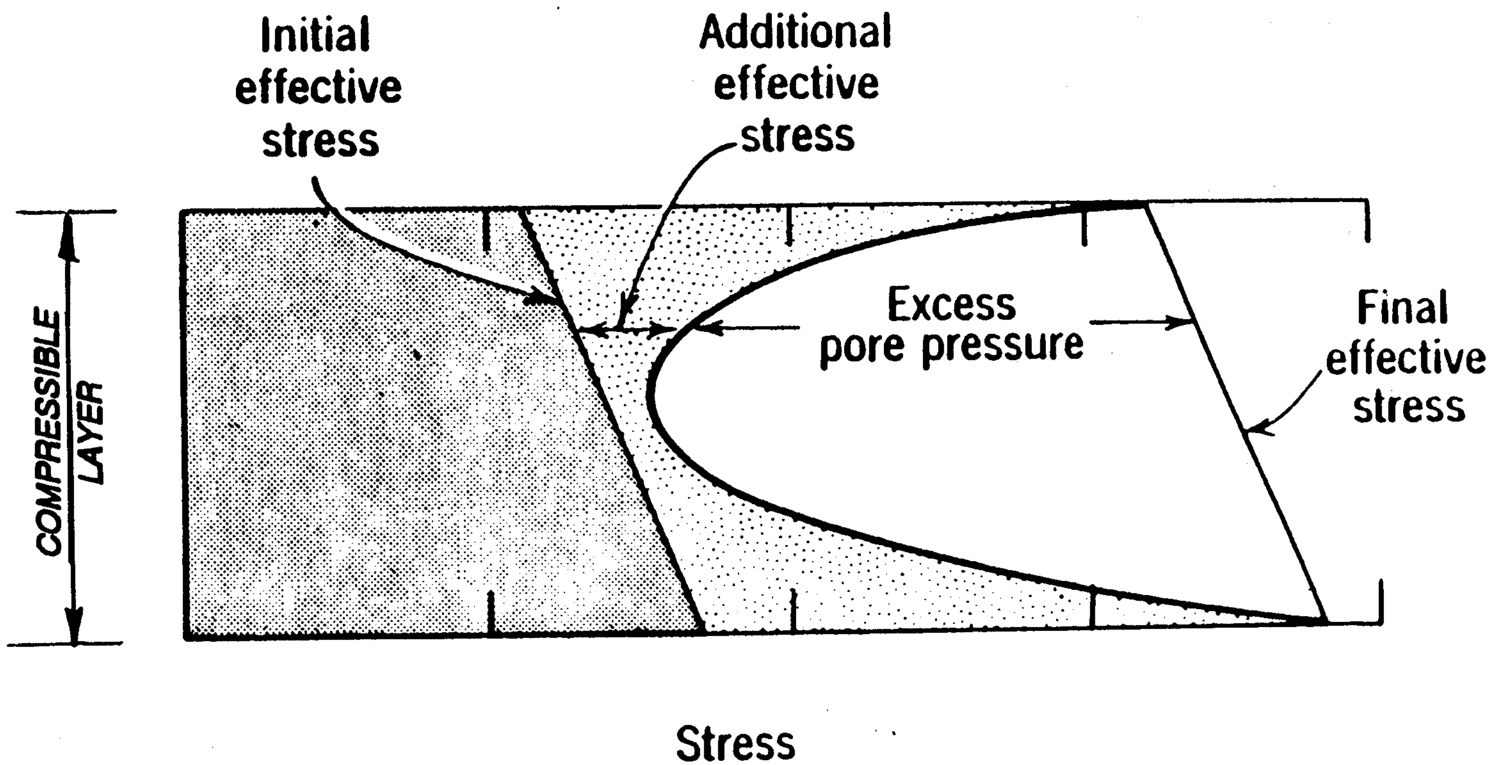
H_d = DRAINAGE PATH

T = TIME FACTOR

$$\begin{cases} 0.197 \rightarrow t_{50} \\ 0.848 \rightarrow t_{90} \end{cases}$$



TIME-DEPENDENT PORE PRESSURE DISSIPATION RESULTS IN INCREASED EFFECTIVE STRESS



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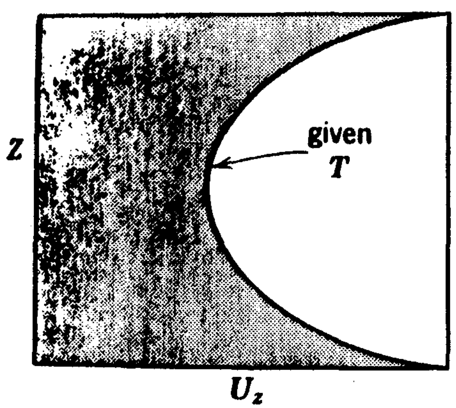
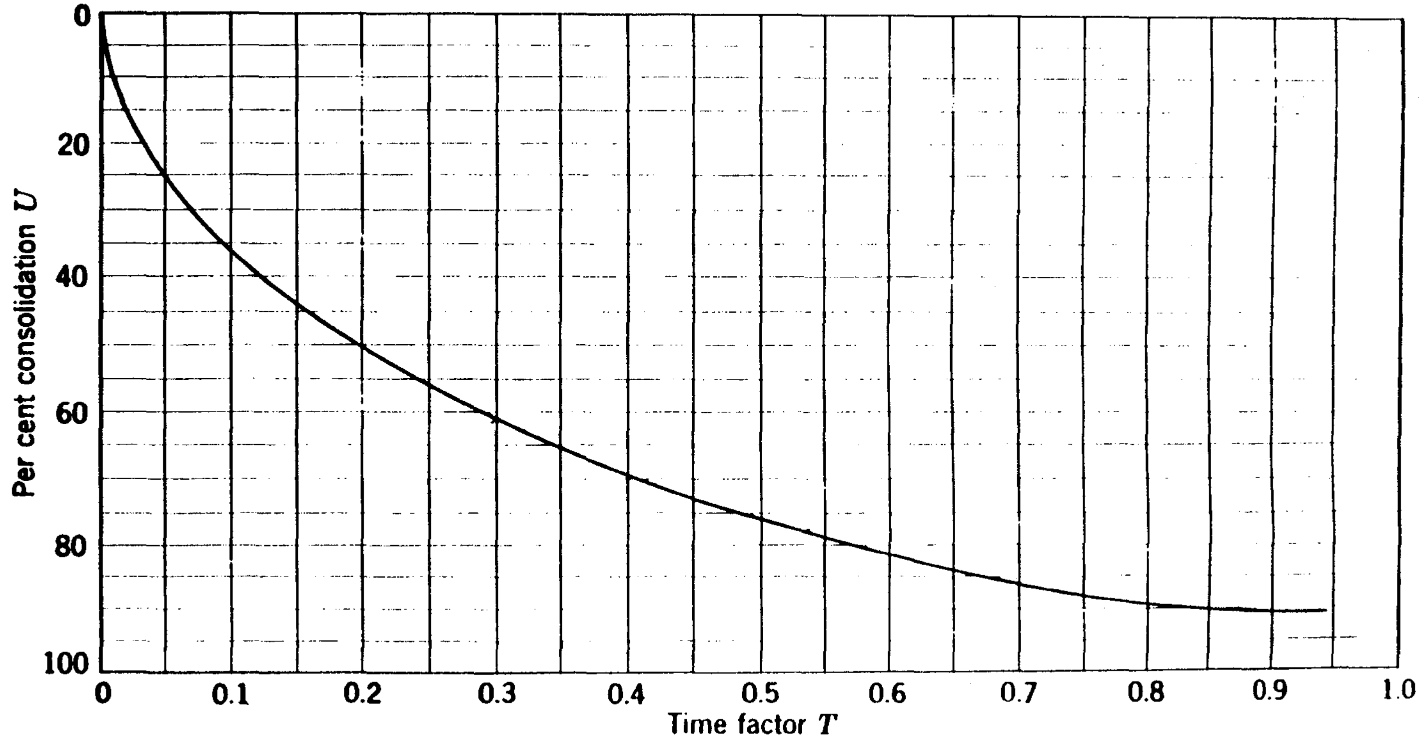
TIME FACTOR VS. DEGREE OF CONSOLIDATION



$$U = \frac{\text{shaded area}}{\text{total area}}$$



EXAMPLE

A. FOR PREVIOUS EXAMPLE, ESTIMATE TIME REQUIRED FOR 50% AND 90% CONSOLIDATION IF $C_v = 0.1$ ft^2/day

$$t = \frac{H_d^2 T}{C_v}$$

	T
$t_{50} \rightarrow$	0.197
$t_{90} \rightarrow$	0.848

$H_d = 10$ *feet (assume rock impervious)*

$$t_{50} = \frac{(10^2)\ (0.197)}{0.10} = 197 \text{ days}$$

$$t_{90} = \frac{(10^2)\ (0.848)}{0.10} = 848 \text{ days}$$

EXAMPLE (continued)

B. WHAT EFFECT ON TIME FOR 90% CONSOL IF CLAY UNDERLAIN BY SAND VS. ROCK? (ASSUME μ IN SAND DECREASED BY 20' ALSO)

$$H_d = 5 \textit{ feet}$$

$$t_{90} = \frac{(5^2)\ (0.848)}{0.10} = 212 \textit{ days}$$

EXAMPLE RETAINING WALL PROBLEM USING COULOMB EQUATION

Given. Retaining wall and backfill as shown in Fig. E13.10-1.

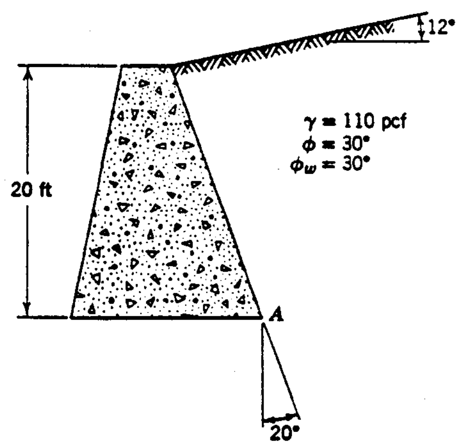


Fig. E13.10-1

Find. Moment of active thrust about point *A*.
Solution Using Eq. 13.12.

$$i = 12^\circ \quad \beta = 110^\circ$$

$$\csc 110^\circ \sin 80^\circ = \frac{\sin 80^\circ}{\sin 70^\circ} = 1.049$$

$$\sqrt{\sin 140^\circ} = 0.803$$

$$\sqrt{\frac{\sin 60^\circ \sin \cancel{28^\circ}\ 18^\circ}{\sin 98^\circ}} = \sqrt{\frac{0.866 \times \cancel{0.470}\ 0.309}{0.990}} = \cancel{0.641}\ 0.520$$

$$P_a = \tfrac{1}{2}(110)(20)^2\left[\frac{1.049}{0.803 + \underset{0.520}{\cancel{0.641}}}\right]^2 = 22{,}000(\underset{0.629}{\cancel{0.528}}) = \overset{13838}{\cancel{11{,}600}} \text{ lb/ft}$$

Normal component of P_a:

$$P_a \cos 30^\circ = \underset{11984}{\cancel{10{,}050}} \text{ lb/ft}$$

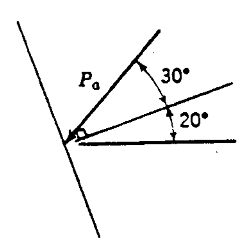


Fig. E13.10-2

P_a acts $\frac{1}{3}$ of way up wall, or at slant distance of 7.1 ft above base (see Fig. E13.10-2).
Moment of P_a about point A = $\overset{11984}{\cancel{10{,}050}} \times 7.1 = \overset{85087}{\cancel{71{,}400}}$ lb-ft/ft.

SOIL MECHANICS

LECTURE 2

- STRENGTH-BEARING CAPACITY
- RETAINING WALLS
- PILES
- FLOW NETS

GENERAL BEARING CAPACITY EQUATIONS

CONTINUOUS FOOTING:

$$q_{ULT} = cN_c + \gamma DN_q + 0.5\gamma BN_\gamma$$

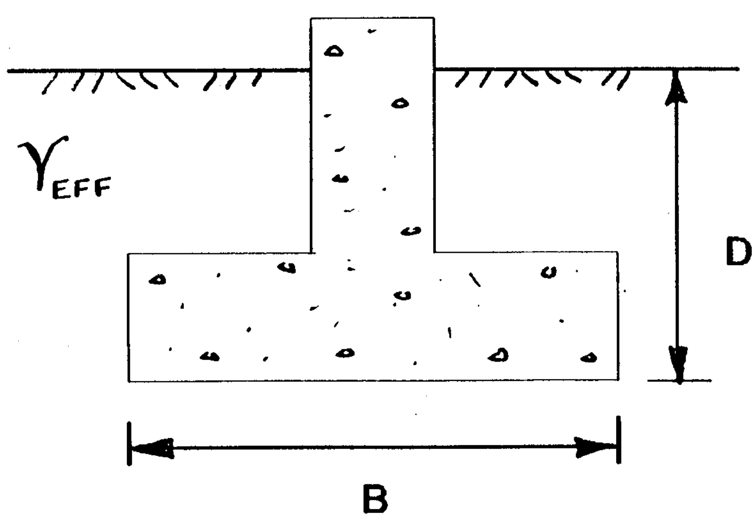
SQUARE OR RECTANGULAR FOOTING:

$$q_{ULT} = cN_c \left(1 + .3\frac{B}{L}\right) + \gamma DN_q + 0.4\gamma BN_\gamma$$

CIRCULAR FOOTING: (R=B/2)

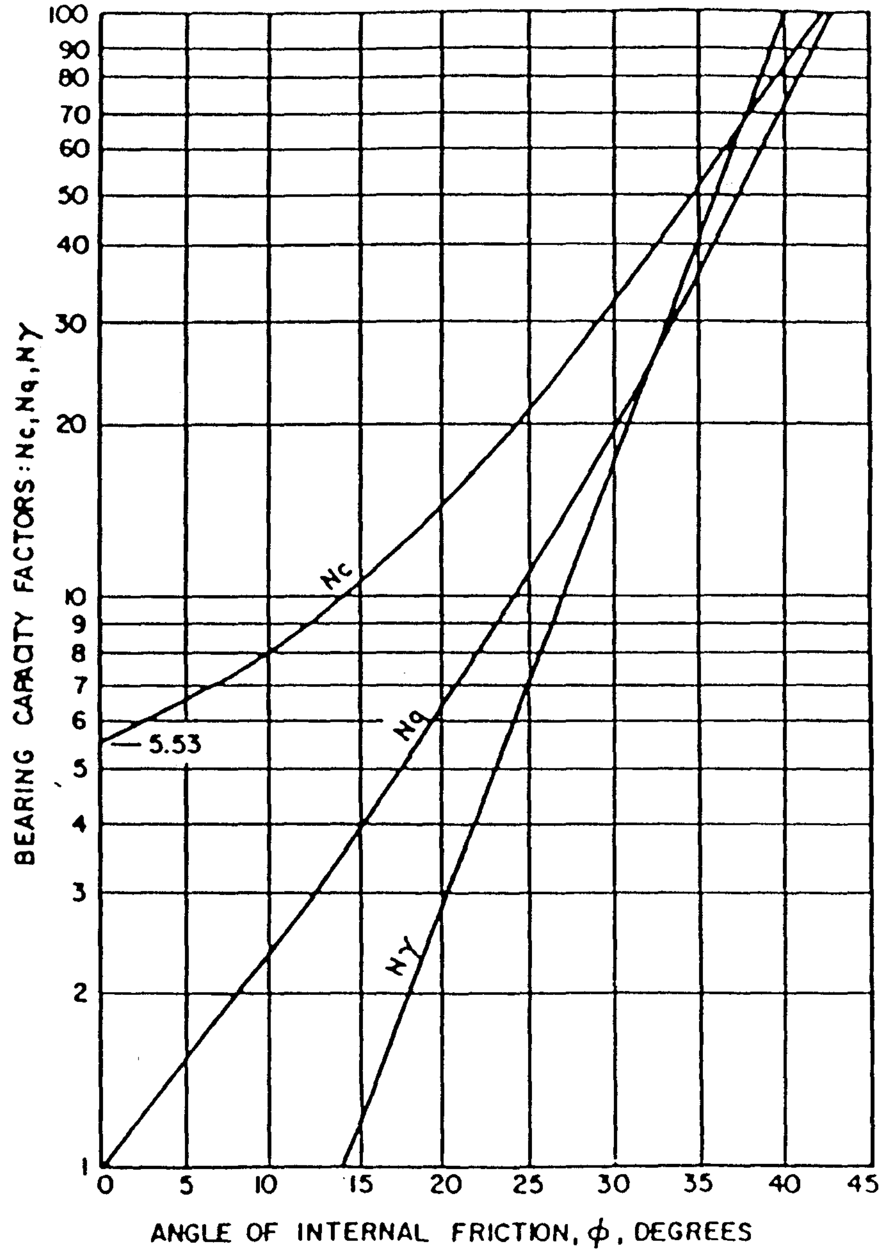
$$q_{ULT} = 1.3\ cN_c + \gamma DN_q + 0.6\gamma RN_\gamma$$

where N_c, N_q, $N\gamma$ ARE BEARING CAPACITY FACTORS



$$Allowable Bearing Capacity = \frac{q_{ULT}}{3}$$

BEARING CAPACITY FACTORS



BEARING CAPACITY EQUATIONS

	FOR COHESIONLESS FOUNDATION SOILS (c=0)	FOR COHESIVE FOUNDATION SOILS (ϕ=0)
CONTINUOUS FOOTING:	$q_{ULT} = \gamma D N_q + 0.5\gamma B N_\gamma$	$q_{ULT} = cN_c + \gamma D$
SQUARE OR RECTANGULAR FOOTING:	$q_{ULT} = \gamma D N_q + 0.4\gamma B N_\gamma$	$q_{ULT} = cN_c \left(1 + .3\frac{B}{L}\right) + \gamma D$
CIRCULAR FOOTING:	$q_{ULT} = \gamma D N_q + 0.6\gamma R N_\gamma$	$q_{ULT} = 1.3cN_c + \gamma D$

ALLOWABLE SOIL PRESSURE FOR FOOTINGS ON SAND

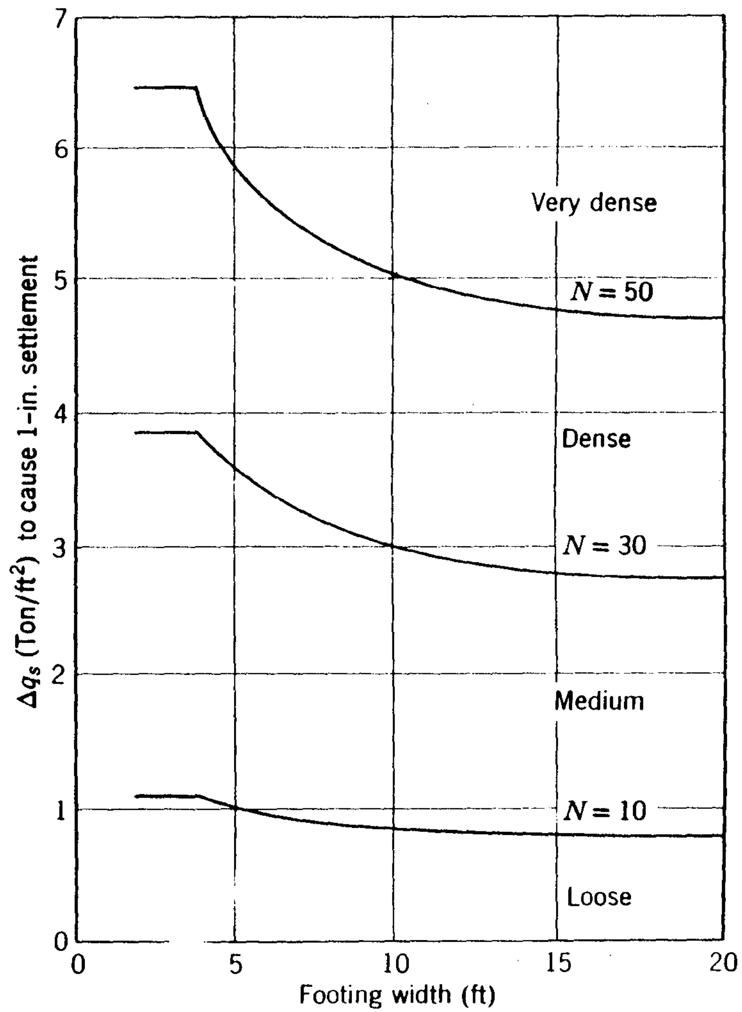
CHART REPRESENTS EMPIRICAL RELATIONSHIP FOR SETTLEMENT NOT EXCEEDING 1 INCH

ALLOWABLE SOIL PRESSURE ASSUMES DEPTH TO WATER TABLE BELOW FOOTING. WHEN WATER TABLE IS AT OR ABOVE FOOTING USE 50% OF CHART VALUE

GENERAL RULE OF THUMB:

$$\boxed{q_{ALL}\ (tsf) = \frac{N}{10}}$$

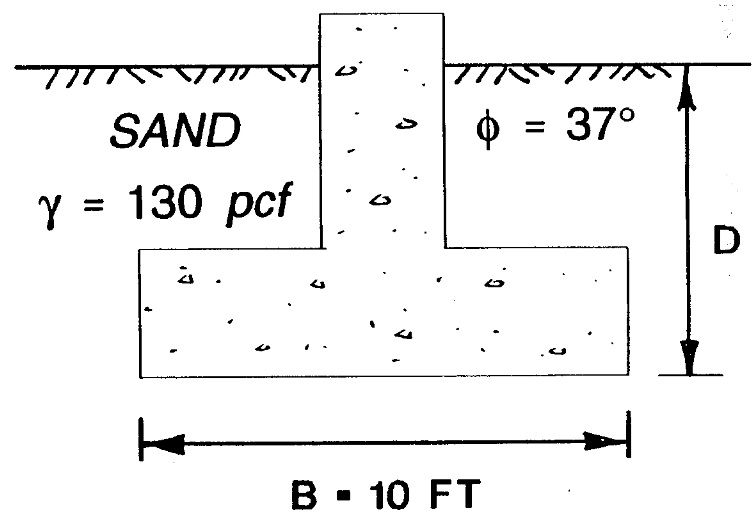
WHERE: N = STANDARD PENETRATION RESISTANCE (SPT) IN BLOWS/FT



PROBLEM V-1

GIVEN: CONTINUOUS ROUGH FOOTING

(A) COMPUTE APPROXIMATE ULTIMATE BEARING CAPACITY FOR DRY SAND



FOR $\phi = 37°$; $N_q = 50$, $N_\gamma = 60$

COHESIONLESS $\therefore\ c = 0$

$$q_{ULT} = cN_C + \gamma DN_q + 0.5\gamma BN_\gamma$$

$$= 0 + \left(\frac{130}{2{,}000}\right)(5)\ (50) + 0.5\left(\frac{130}{2{,}000}\right)(10)\ (60)$$

$$= 0 + 16.2 + 19.5 = 35.7\ tsf$$

PROBLEM V-1 (continued)

(B) COMPUTE BEARING CAPACITY IF BOTTOM OF FOOTING IS 10 FEET BELOW GROUND SURFACE (i.e., D = 10 FT)

$$q_{ULT} = cN_C + \gamma DN_q + 0.5\gamma BN_\gamma$$

$$= 0 + \left(\frac{130}{2,000}\right)(10)\ (50) + 0.5\left(\frac{130}{2,000}\right)(10)\ (60)$$

$$= 0 + 32.5 + 19.5 = 52.0\ tsf$$

PROBLEM V-1 (continued)

(C) WHAT IS EFFECT ON BEARING CAPACITY IN PART A IF GROUNDWATER IS AT GROUND SURFACE?

$\gamma_{EFF} = \gamma_{SAT} - \gamma_w$ *ASSUME* $\gamma = \gamma_{SAT} = 130$ *pcf*

$= 130 - 62.4 = 67.5$ *pcf*

$q_{ULT} = cN_C + \gamma DN_q + 0.5\gamma BN_\gamma$

$= 0 + \left(\frac{67.5}{2{,}000}\right)(5)\ (50) + 0.5\left(\frac{67.5}{2{,}000}\right)(10)\ (60) = 18.6$ *tsf*

PROBLEM V-2

DETERMINE THE MAXIMUM HEIGHT OF FILL FOR A FACTOR OF SAFETY OF 1, THAT CAN BE PLACED AT A SITE WHERE THE SUBSOIL PROFILE CONSISTS OF 150 FEET OF CLAY (q_U = 0.6 tsf, ω = 30%, LL = 20%, PI = 12%) UNDERLAIN BY A DENSE SAND TO CONSIDERABLE DEPTH. ASSUME THE WATER LEVEL IS AT THE GROUND SURFACE AND THAT THE FILL IS RECTANGULAR IN SHAPE, 50 FEET BY 100 FEET.

$\phi = 0$ *FOR CLAY*

$$q_{ULT} = cN_c \left(1 + 0.3\,\frac{B}{L}\right) + \gamma D$$

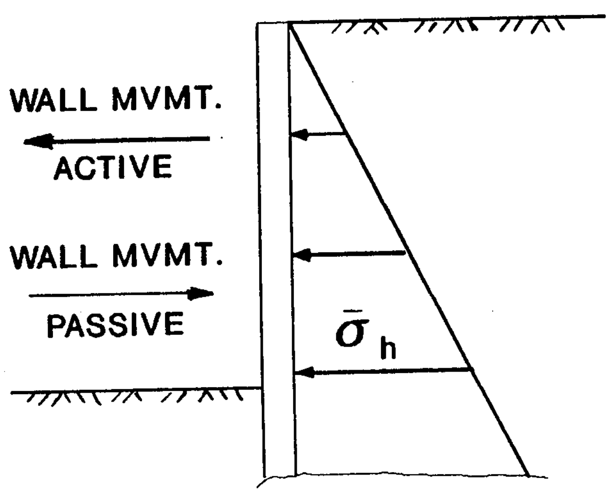
$$= c\,(5.53)\left[1 + 0.3\left(\frac{50}{100}\right)\right] + 0 = 6.35\,c \qquad c = \frac{q_u}{2} = 0.3\ tsf = 600\ psf$$

assume $\gamma_{FILL} = 125\ pcf$

$$MAXIMUM\ HEIGHT\ OF\ FILL = \frac{q_{ULT}}{\gamma_{FILL}} = \frac{6.35\ (600)}{125} = 30.5\ ft$$

LATERAL EARTH PRESSURE

LATERAL EARTH PRESSURE IS A PRODUCT OF THE VERTICAL STRESS AND A COEFFICIENT OF LATERAL EARTH PRESSURE K. THREE GENERAL CASES EXIST:



AT REST K_o

NO WALL MOVEMENT

$$\bar{\sigma}_h = K_o \bar{\sigma}_v$$

SAND & GRAVEL

$K_o = 0.35$ *to* 0.60

CLAY & SILT

$K_o = 0.45$ *to* 0.75

ACTIVE $K_A = \dfrac{1 - \sin\phi}{1 + \sin\phi}$

$$\bar{\sigma}_h = K_A \bar{\sigma}_v - 2c\tan(45 - \phi/2)$$

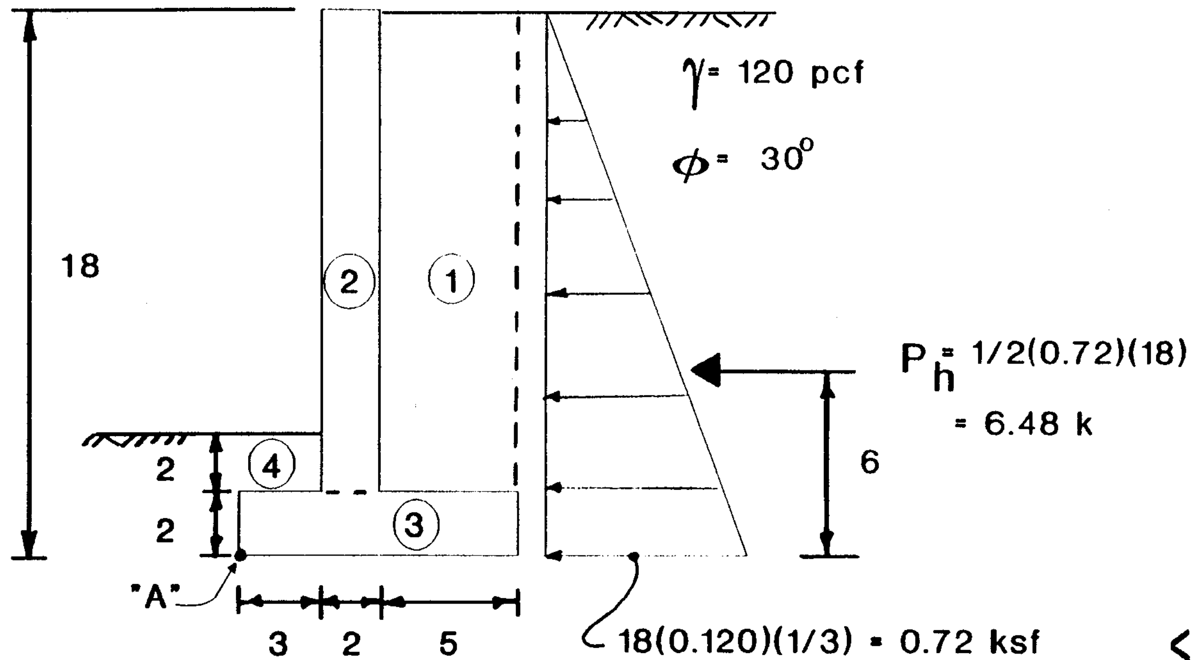
PASSIVE $K_p = \dfrac{1 + \sin\phi}{1 - \sin\phi}$

$$\bar{\sigma}_h = K_p \bar{\sigma}_v + 2c\tan(45 + \phi/2)$$

THE ABOVE IS BASED UPON RANKINE EARTH PRESSURE THEORY FOR LEVEL BACKFILL.

PROBLEM V-3

RETAINING WALL:



STABILITY CALCULATIONS: Sum moments about point "A"

AREA		FORCE (kips)	ARM (ft)	MOMENT (ft-k)
1	16 x 5 = 80 x 0.120	= 9.60	7.5	72.0
2	16 x 2 = 32 x 0.150	= 4.80	4.0	19.2
3	10 x 2 = 20 x 0.150	= 3.00	5.0	15.0
4	2 x 3 = 6 x 0.120	= 0.72	1.5	1.1
		ΣV = 18.12		107.3
P_h x 6 = 38.8 ft-k				-38.8
			ΣM	= 68.5

PROBLEM V-3 (continued)

- FACTOR OF SAFETY AGAINST OVERTURNING: (F.S. $\geq$ 2)

$$F.S. = \frac{RESISTING\ MOMENT}{OVERTURNING\ MOMENT} = \frac{107.3}{38.8} = 2.77 > 2.0\ (O.K.)$$

- LOCATION OF RESULTANT: (MIDDLE THIRD)

$$FROM\ POINT\ "A" \quad \frac{\sum M_A}{\sum V} = \frac{68.5}{18.12} = 3.78\ ft$$

$$THEN\ e = \frac{10}{2} - 3.78 = 1.22' < \frac{10'}{6}\ (O.K.)$$

- SOIL PRESSURE AT BASE:

$$q = \frac{\sum V}{B}\left(\frac{1 \pm 6e}{B}\right)$$

$$AT\ TOE \quad q_{max} = \frac{18.12}{10}\left(1 + \frac{6 \times 1.22}{10}\right) = 1.812\ (1 + 0.732) = 3.14^{ksf}$$

$$AT\ HEEL \quad q \quad = 1.812\ (1 - 0.732) = 0.49^{ksf}$$

PROBLEM V-3 (continued)

- BEARING CAPACITY:

$\phi = 30° \quad \gamma = 120\ pcf$

$$NET\ q_{ULT} = \gamma D\,(N_q - 1) + \frac{1}{2}B\gamma N_\gamma = (0.120)\,(4)\,(20 - 1) + \frac{1}{2}\,(10)\,(0.120)\,(17) = 19.31^{ksf}$$

$$F.S. = \frac{19.31}{3.14} = 6.16 > 3.0\ (O.K.)$$

- SLIDING: (F.S. $\gtrsim$ 1.5)

SHEAR RESISTANCE AVAILABLE ALONG BASE

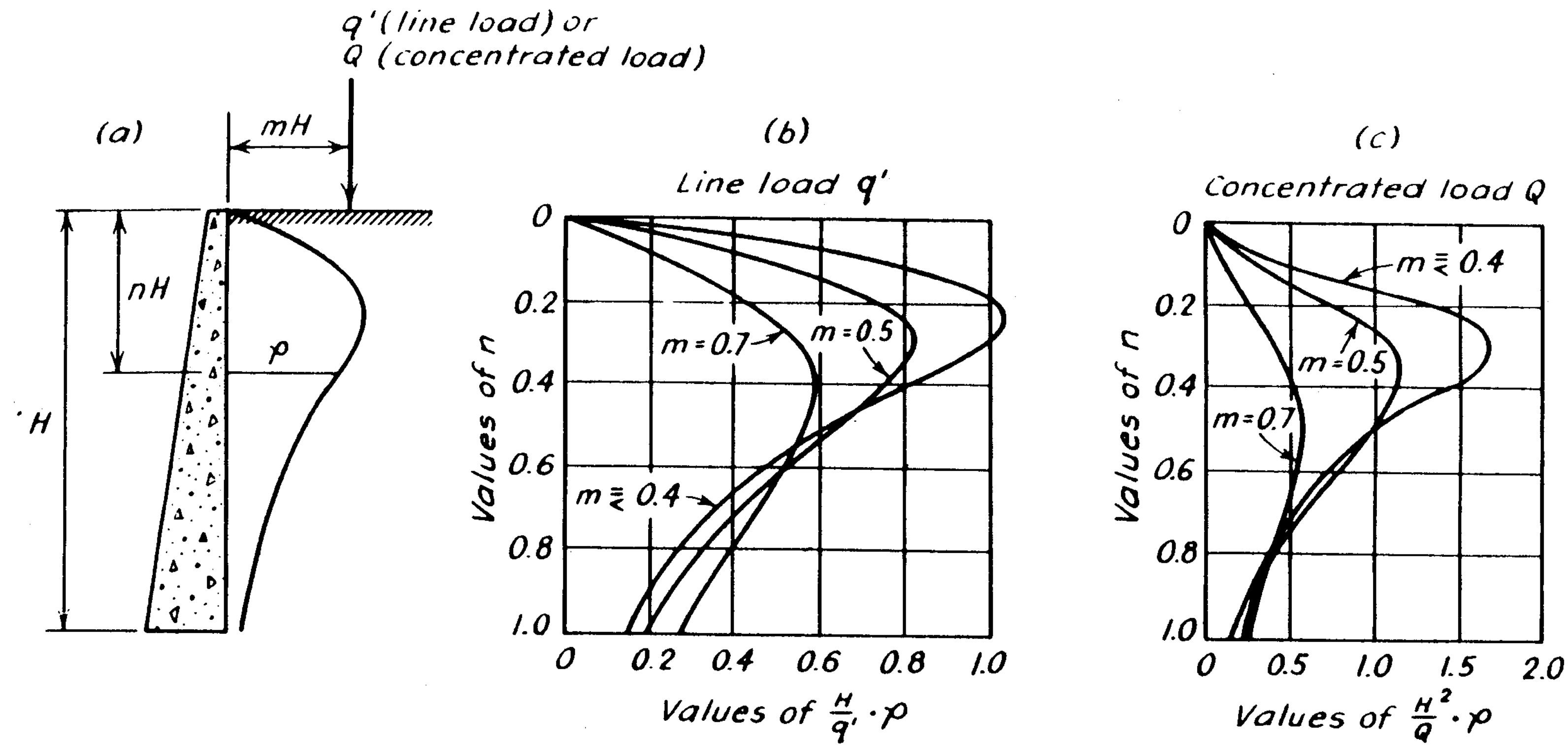
$$S = \sum V \tan\phi = 18.12\ tan 30° = 10.45^{K}$$

PASSIVE FORCE @ TOE : $P_P = \frac{1}{2}\,(0.120)\,(3.0)\,(4)^2 = 2.88^{K}$

$$MIN.\ F.S. = 10.45 \div 6.48 = 1.61\ (O.K.)$$

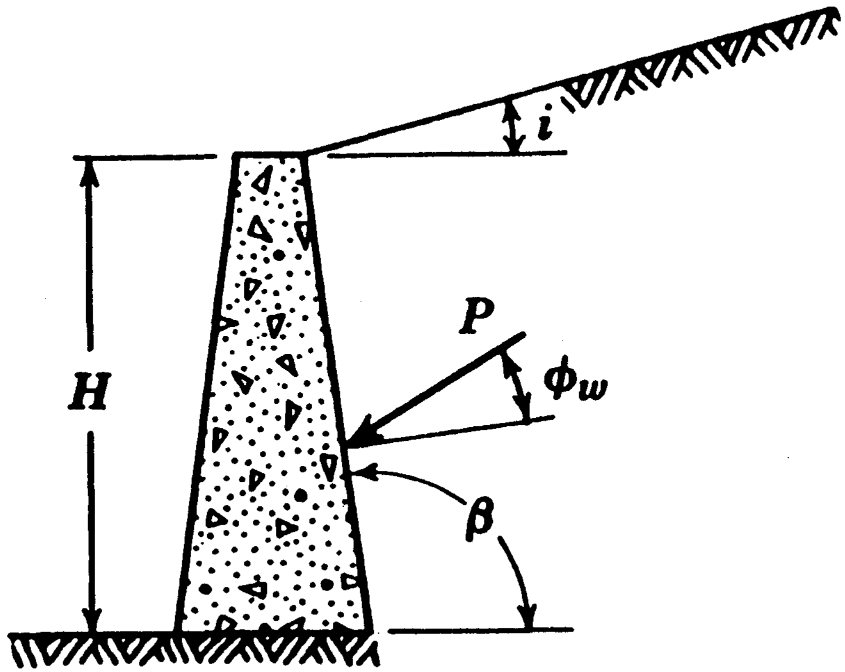
$$MAX.\ F.S. = (10.45 + 2.88) \div 6.48 = 2.06\ (O.K.)$$

LATERAL WALL PRESSURES DUE TO LINE LOADS AND POINT LOADS



COULOMB EQUATION

FOR INCLINED WALL:



Coulomb equation for sloping backfill and wall friction:

$$P_a = \tfrac{1}{2}\gamma H^2 \left\{ \frac{\csc\beta \sin(\beta - \phi)}{\sqrt{\sin(\beta + \phi_w)} + \sqrt{\dfrac{\sin(\phi + \phi_w)\sin(\phi - i)}{\sin(\beta - i)}}} \right\}^2$$

EXAMPLE RETAINING WALL PROBLEM USING COULOMB EQUATION

Given. Retaining wall and backfill as shown in Fig. E13.10-1.

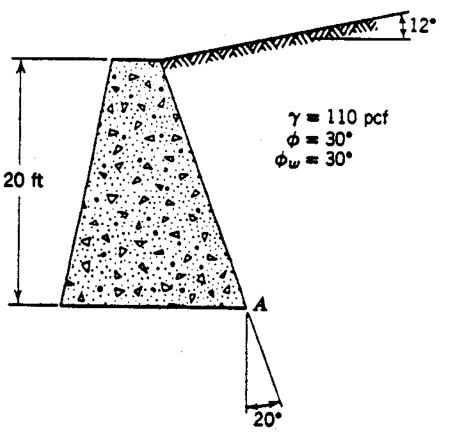


Fig. E13.10-1

Find. Moment of active thrust about point A.
Solution Using Eq. 13.12.

$$i = 12^\circ \quad \beta = 110^\circ$$

$$\csc 110^\circ \sin 80^\circ = \frac{\sin 80^\circ}{\sin 70^\circ} = 1.049$$

$$\sqrt{\sin 140^\circ} = 0.803$$

$$\sqrt{\frac{\sin 60^\circ \sin 28^\circ}{\sin 98^\circ}} = \sqrt{\frac{0.866 \times 0.470}{0.990}} = 0.641$$

$$P_a = \tfrac{1}{2}(110)(20)^2 \left[\frac{1.049}{0.803 + 0.641}\right]^2 = 22{,}000(0.528) = 11{,}600 \text{ lb/ft}$$

Normal component of P_a:

$$P_a \cos 30^\circ = 10{,}050 \text{ lb/ft}$$

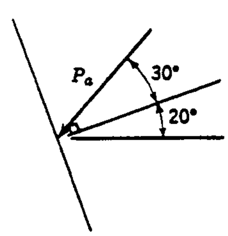


Fig. E13.10-2

P_a acts $\frac{1}{3}$ of way up wall, or at slant distance of 7.1 ft above base (see Fig. E13.10-2).
Moment of P_a about point $A = 10{,}050 \times 7.1 = 71{,}400$ lb-ft/ft.

SINGLE FRICTION PILE IN COHESIVE SOIL

METHOD	SOIL TYPES	MINIMUM SAFETY FACTOR
α METHOD (TOTAL STRESS)	SOFT TO MEDIUM CLAYS	3
β METHOD (EFFECTIVE STRESS)	SOFT TO STIFF CLAYS	3

SINGLE FRICTION PILE IN COHESIVE SOIL
(α METHOD)

SIDE FRICTION:

$$Q_S = \alpha\ C\ \pi\ d\ L$$

END BEARING:

$$Q_P = 9\ C\ A_P$$

$$Q_{ULT} = Q_S + Q_P$$

$$\boxed{Q_{ALLOWABLE} = \frac{Q_{ULT}}{FS = 3}}$$

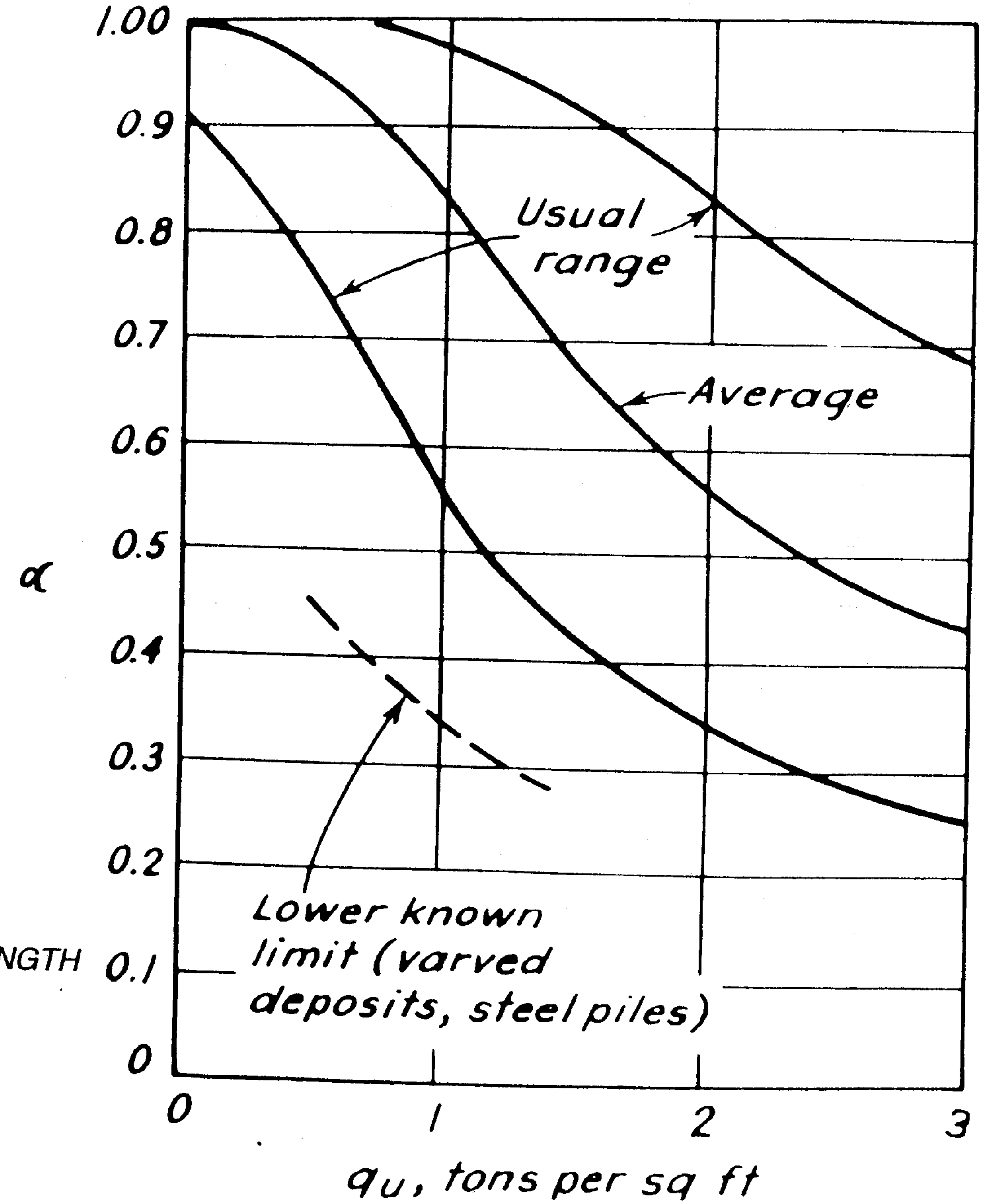
C = UNDRAINED SHEAR STRENGTH

d = PILE DIAMETER

L = LENGTHOFEMBEDMENT

α = REDUCTION COEFFICIENT

A_P = AREA OF PILE TIP



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SINGLE FRICTION PILE IN COHESIVE SOIL
(β METHOD)

SIDE FRICTION:

$$Q_S = \beta \bar{\sigma}_{V_{AVG}}$$

β = *SKIN FRICTION FACTOR*

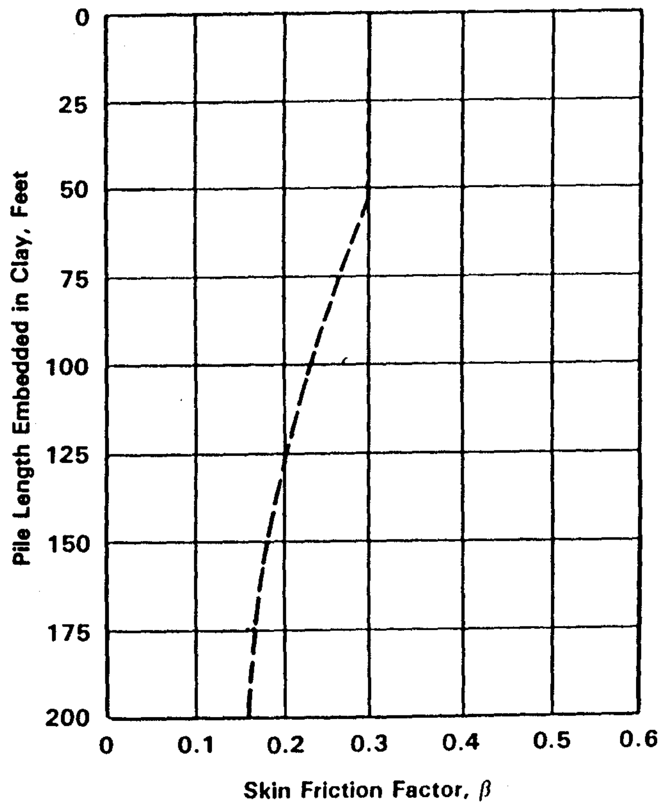
$\bar{\sigma}_{V_{AVG}}$ = *AVERAGE EFFECTIVE VERTICAL STRESS IN BEARING STRATA ALONG PILE SHAFT*

END BEARING:

$$Q_P = 9\ C\ A_P$$

$$Q_{ULT} = Q_S + Q_P$$

$$\boxed{Q_{ALLOWABLE} = \frac{Q_{ULT}}{FS = 3}}$$



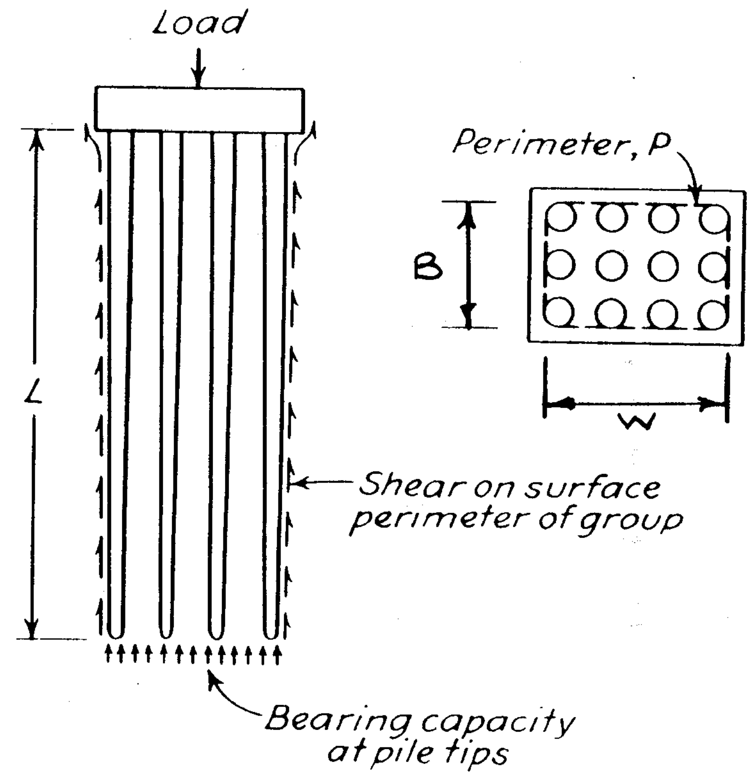
VALUES OF β FOR DRIVEN PILES IN SOFT AND MEDIUM CLAYS (Cu < 2,000 PSF)(AFTER MEYERHOF, 1976)

GROUP CAPACITY IN COHESIVE SOIL

$$Q_S = 2\,(B + W)\,LC_1$$

$$Q_P = 9C_2BW$$

$$Q_{ULT} = \frac{Qs + Qp}{3}$$



B = WIDTH OF GROUP

W = LENGTH OF GROUP

C_1 = AVERAGE UNDRAINED SHEAR STRENGTH ALONG L

C_2 = UNDRAINED SHEAR STRENGTH AT TIP OF PILES

L = LENGTH OF EMBEDMENT

PROBLEM VII-1

END BEARING PILES:

A PILE WAS DRIVEN WITH A 3,000-POUND HAMMER WITH A FREE FALL OF 20 FEET. IF THE PILE SANK 6 INCHES UNDER THE LAST FIVE BLOWS, WHAT IS ITS SAFE BEARING VALUE?

$$\boxed{Q_{ALL} = \frac{2WH}{S + 1}}$$

WHERE:

Q_{ALL} = SAFELOAD (POUNDS)

W = WEIGHT OF HAMMER (POUNDS)

H = HEIGHT OF FALL (FEET)

S = PENETRATION OF PILE UNDER LAST BLOWS OF RAM (INCHES)

C = 1 FOR DROP HAMMER

$$S = \frac{6}{5} = 1.2 \text{ INCHES}$$

$$Q_{ALL} = \frac{2 \times 3{,}000 \times 20}{1.2 + 1} = 54{,}500 \text{ LBS.}$$

$$Q_{ALL} = 27 \text{ TONS}$$

PROBLEM VII-4

GIVEN A GROUP OF FRICTION PILES AND A SOIL PROFILE AS SHOWN BELOW:

A. CALCULATE THE ALLOWABLE LOAD OF THE PILE GROUP TREATING EACH PILE INDIVIDUALLY

B. CALCULATE THE ALLOWABLE LOAD OF THE PILE GROUP TREATING THE PILES AS A COMPOSITE GROUP USING A REASONABLE FACTOR OF SAFETY

SOIL PROFILE

ELEV. 0

LAYER (1) C = 800 psf

ELEV. -12

LAYER (2) C = 1,600 psf

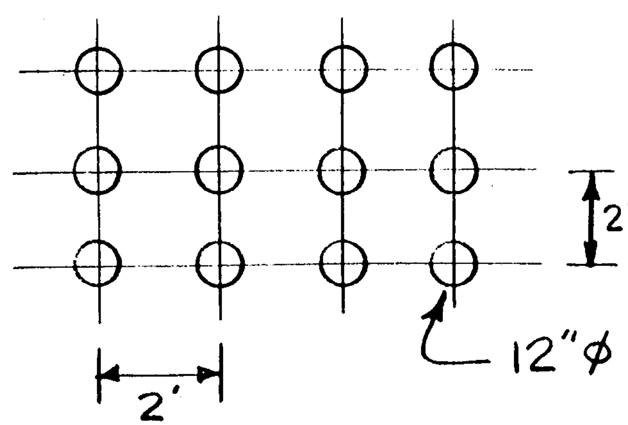
ELEV. -30

LAYER (3) C = 2,000 psf

ELEV. -40

ASSUMPTIONS

ASSUME CLAY IS SATURATED. SOIL BENEATH LAYER (3) HAS SAME COHESION AND NO ROCK IN VICINITY.



PROBLEM VII-4 (continued)

PART A: (α METHOD)

SURFACE AREA:

LAYER	PILE SOURCE AREA (A)
1	$12'\pi$ (1 FT) = 37.7 FT^2
2	$18\ \pi$ (1 FT) = 56.5 FT^2
3	$10\ \pi$ (1 FT) = 31.4 FT^2
	ΣA = 125.6 FT^2

REDUCED STRENGTH VALUES:

LAYER	C (psf)	q_U (tsf)	α	$\alpha \cdot c \cdot A$
1	800	0.8	0.88	26.5k
2	1,600	1.6	0.65	58.8K
3	2,000	2.0	0.56	35.2K
				Q_S=120.5 K/pile

PROBLEM VII-4 (continued)

$$Q_P = 9CA_P \qquad A_P = \frac{\pi(1)^2}{4} = 0.79\ ft^2$$

$$Q_P = \frac{9(2{,}000\ psf)\ (0.79\ FT^2)}{1{,}000} = 5.0\ K$$

$$Q_T = Q_S + Q_P = 120.5 + 5.0 = 125.5\ K$$

$$Q_{ALL} = \frac{Q_T}{(FS = 3)} = \frac{125.5}{3} = \underline{41.8\ K/PILE}$$

PROBLEM VII-4 (continued)

PART A: (β METHOD)

$$ASSUME\ \gamma_T = 120\ pcf$$

$$\bar{\sigma}_{V_{AVG}} = 20\ (120 - 62.4) = 1{,}152\ psf$$

$$Q_S = \beta\bar{\sigma}_{V_{AVG}}\Sigma A = 0.3\ \frac{(1{,}152\ psf)\ (125.6\ FT^2)}{1{,}000} = 44.4\ K/pile$$

$$Q_P = 5.0 \qquad Q_T = 44.4 + 5.0 = 49.4\ K/pile$$

$$Q_{ALL} = \frac{49.5}{3} = \underline{\underline{16.5\ K/pile}}$$

PROBLEM VII-4 (continued)

PART B: GROUP CAPACITY

$Q_S = 2\ (B + W)LC_1$ $\qquad$ $Q_P = 9C_2BW$

$B = 6\ FT$ $\qquad$ $W = 8\ FT$
$C_1 = WEIGHTED\ C$
$C_2 = 2{,}000\ psf$

$$Q_S = \frac{2(5 + 7)}{1{,}000}\ x\left[(12 \cdot 800) + (18 \cdot 1{,}600) + 10 \cdot 2{,}000)\right] = 1{,}402\ K$$

$$Q_P = \frac{9\ (2{,}000)}{1{,}000}\ (5)\ (7) = 630\ K$$

$$Q_T = 1{,}402 + 630 = 2{,}032\ k \qquad Q_{ALL} = \frac{2{,}032}{3} = 677\ K$$

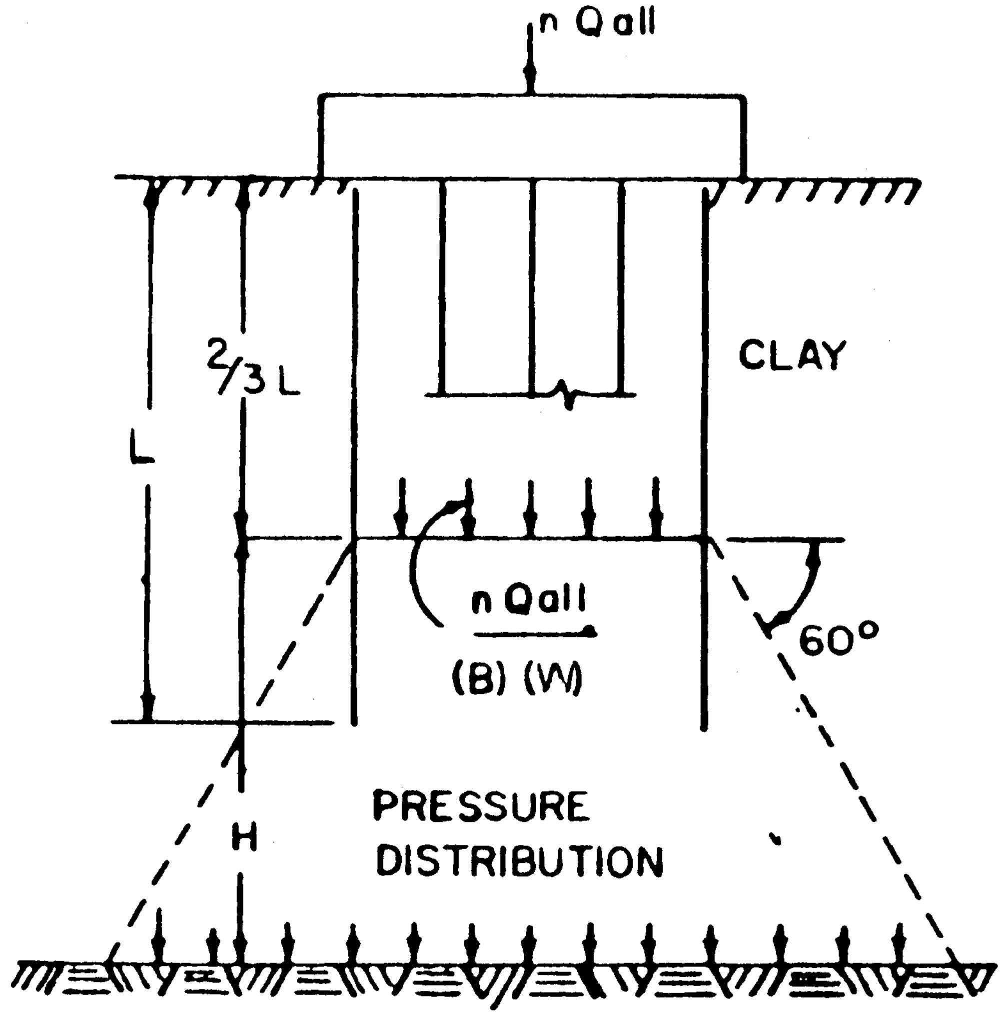
$$Q_{ALL}\ FOR\ SINGLE\ PILE = \frac{677}{12} = \frac{56.4\ K}{-}$$

$\therefore$ <u>*SINGLE PILE CAPACITY GOVERNS*</u>

IMPORTANT NOTE:

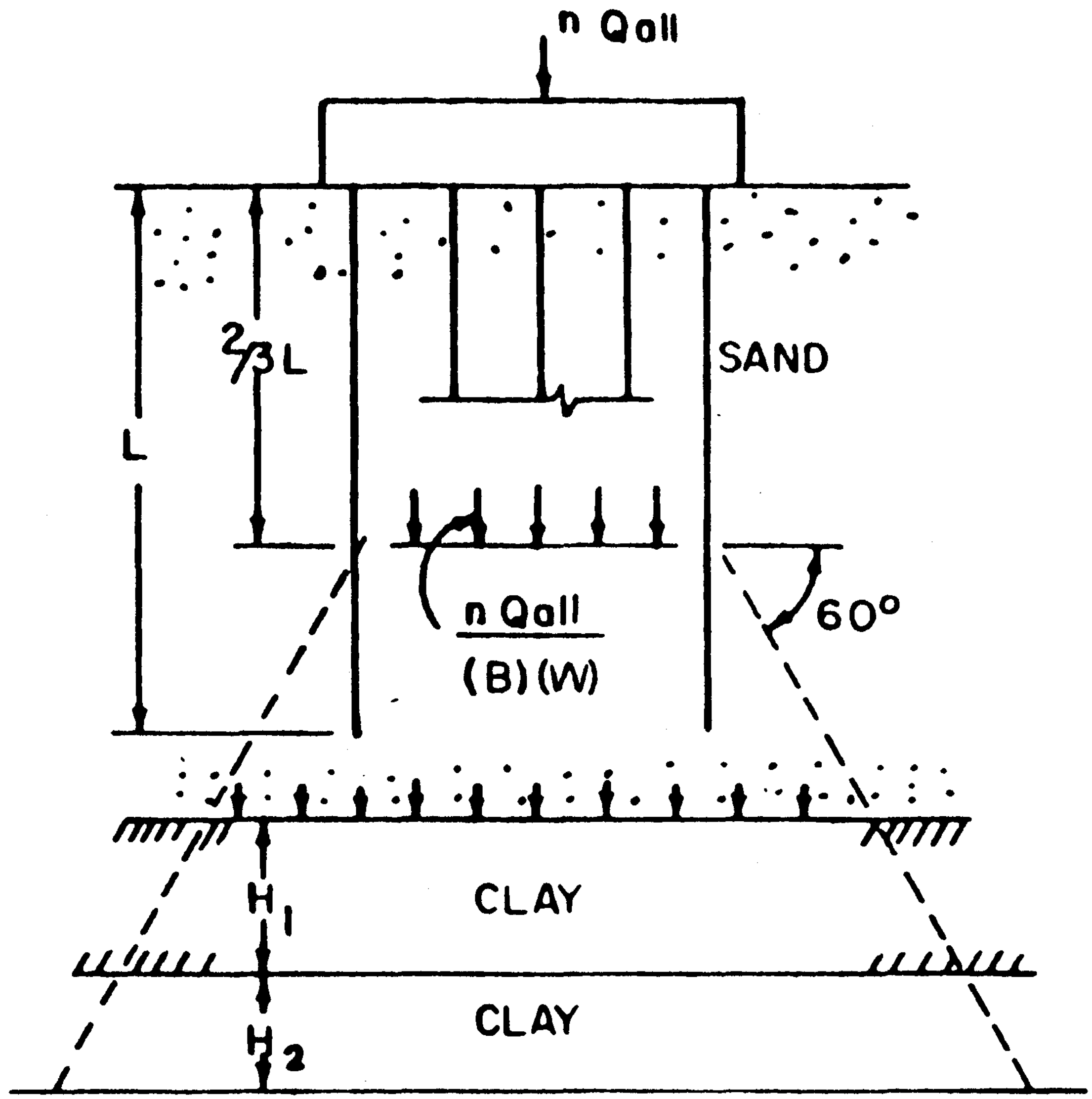
PILE CAPACITIES DO NOT
CONSIDER SETTLEMENT
WHICH MAY LIMIT CAPACITY

SETTLEMENT OF FRICTION PILES IN CLAY



PLAN AREA TO OUTSIDE OF PILE GROUP = BxW
SETTLEMENT OF PILE GROUP = COMPRESSION OF LAYER H UNDER PRESSURE DISTRIBUTION SHOWN.

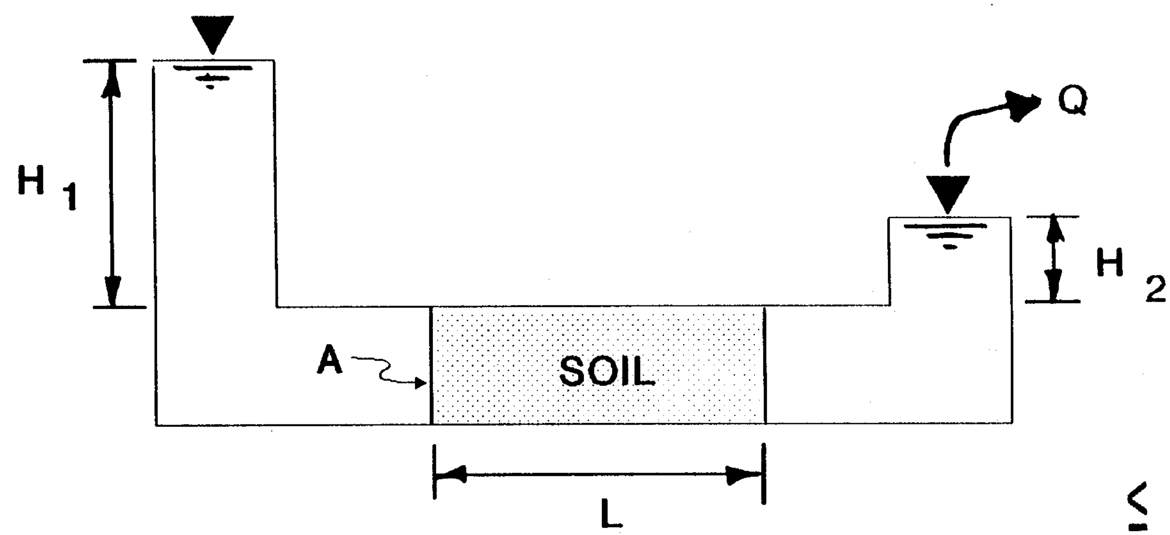
SETTLEMENT OF FRICTION PILES IN SAND UNDERLAIN BY CLAY



SETTLEMENT OF PILE GROUP = COMPRESSION OF LAYERS H_1 AND H_2 UNDER PRESSURE DISTRIBUTION SHOWN. n Qall IS LIMITED BY BEARING CAPACITY OF CLAY LAYERS.

PERMEABILITY

$$\boxed{Q = k \cdot i \cdot A \cdot t}$$



Q = FLOW QUANTITY

k = SOIL PERMEABILITY

A = AREA

i = HYDRAULIC GRADIENT

t = TIME

$$i = \frac{\Delta H}{L}$$

SIMPLE FLOW PROBLEM

DURING A CONSTANT HEAD PERMEABILITY TEST ON A SAMPLE OF SAND, 150 cc OF WATER WAS COLLECTED IN 2 MIN. THE SAMPLE HAD A LENGTH OF 10 cm AND A DIAMETER OF 5 cm. THE HEAD WAS MAINTAINED AT 20 cm. COMPUTE COEFFICIENT OF PERMEABILITY IN cm^2/sec.

$$AREA;\ A = \frac{\pi D^2}{4} = \frac{\pi (5)^2}{4} = 19.63\ cm^2$$

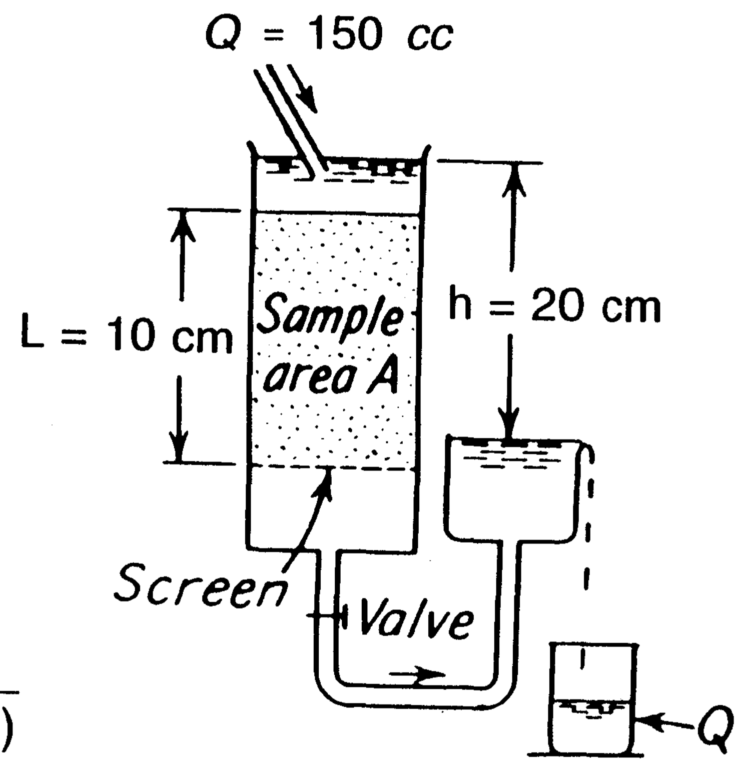
$$i = \frac{\Delta h}{L} = \frac{20}{10} = 2.0$$

$$Q = k \cdot i \cdot A \cdot t$$

$$k = \frac{Q}{i \cdot A \cdot t}$$

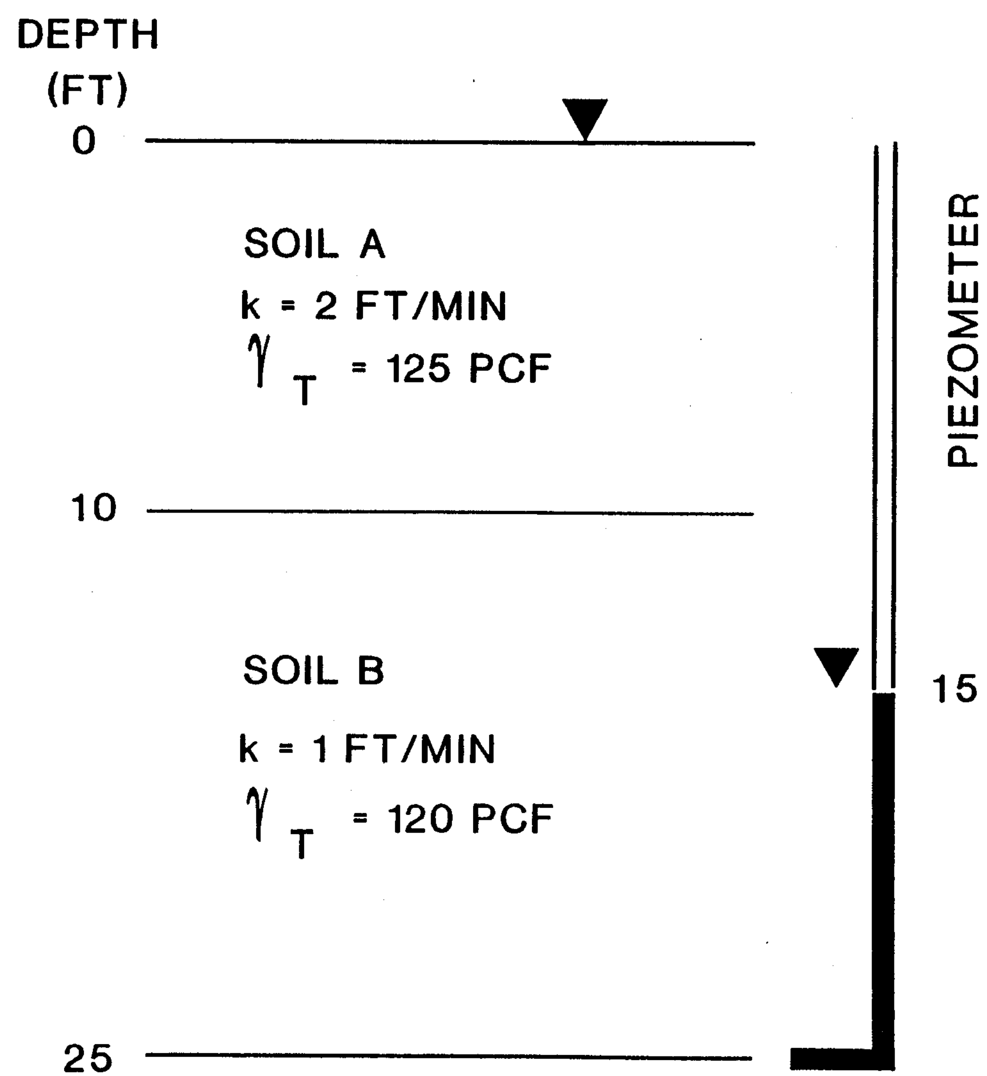
$$k = \frac{150\ cc}{(2.0)\ (19.63\ cm^2)\ (2\ \text{min})\ (60\ \text{sec/min})}$$

$$= 3.2\ x\ 10^{-2}\ cm^2/\text{sec}$$



EXAMPLE PROBLEM: FLOW AND EFFECTIVE STRESS

IN THE PROFILE SHOWN, STEADY-STATE VERTICAL SEEPAGE IS OCCURRING. COMPUTE EFFECTIVE STRESSES AT DEPTHS OF 0, 10 AND 25 FEET.



EXAMPLE PROBLEM (continued)

COMPUTE TOTAL STRESSES:

$$\sigma_V \text{ @ } 10 \ FT = 125\ (10) = 1{,}250 \ psf$$

$$\sigma_V \text{ @ } 25 \ FT = 1{,}250 + 15\ (120) = 3{,}050 \ psf$$

EXAMPLE PROBLEM (continued)

<u>DETERMINE PORE PRESSURES</u>:

HYDROSTATIC CONDITIONS DO NOT EXIST

$$Q_A = Q_B = k\ i\ A$$

FOR 1 *FT*^{2} *AREA*; $(ki)_A = (ki)_B$

$$i_A = \frac{\Delta\ h_A}{10}\ ;\ \ i_B = \frac{\Delta h_B}{15}$$

$$\frac{\Delta h_A}{10} = \frac{\Delta h_B}{15}\left(\frac{k_B}{k_A}\right) = \frac{\Delta h_B}{15}\left(\frac{1}{2}\right)$$

$$\Delta h_A = \frac{\Delta h_B}{3}$$

$$\Delta h_A + \Delta h_B = 15\ FT$$

SUBSTITUTING FOR Δh_A; $\dfrac{\Delta h_B}{3} + \Delta h_B = 15$

$\Delta h_B = 11.3\ FT \qquad \therefore \qquad \Delta h_A = 15 - 11.3 = 3.7\ FT$

EXAMPLE PROBLEM (continued)

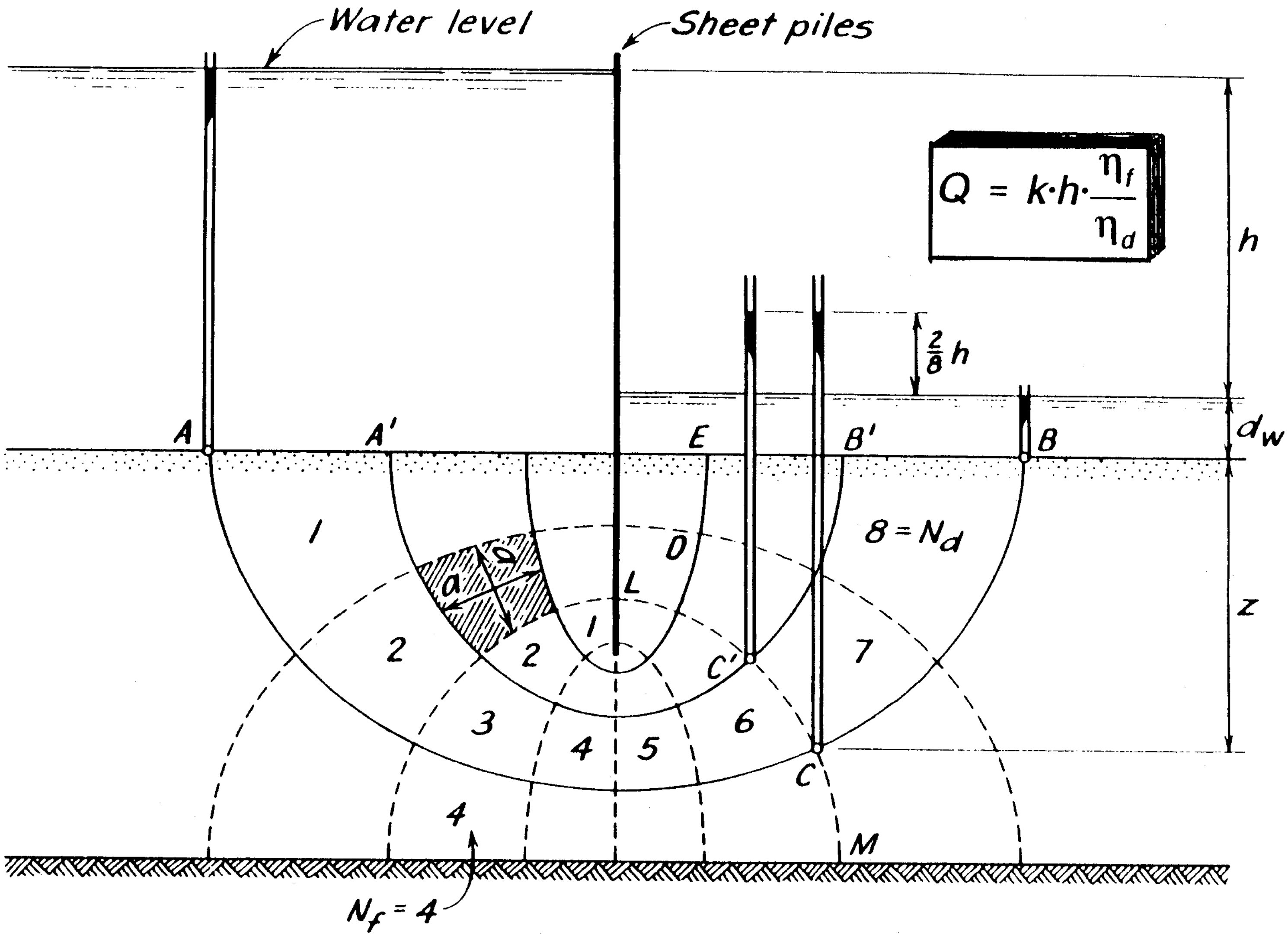
PIEZOMETRIC LEVEL @ 10' = 3.7 FT $h_p = (10 - 3.7)\gamma_w = 393$ psf

PIEZOMETRIC LEVEL @ 25' = 15 FT $h_p = (25 - 15)\gamma_w = 624$ psf

COMPUTE EFFECTIVE STRESSES:

DEPTH (FEET)	σ_v (psf)	μ(psf)	$\bar{\sigma}_v = \sigma_v - \mu$ (psf)
0	0	0	0
10	1,250	393	857
25	3,050	624	2,426

SEEPAGE RATE FROM FLOW NETS



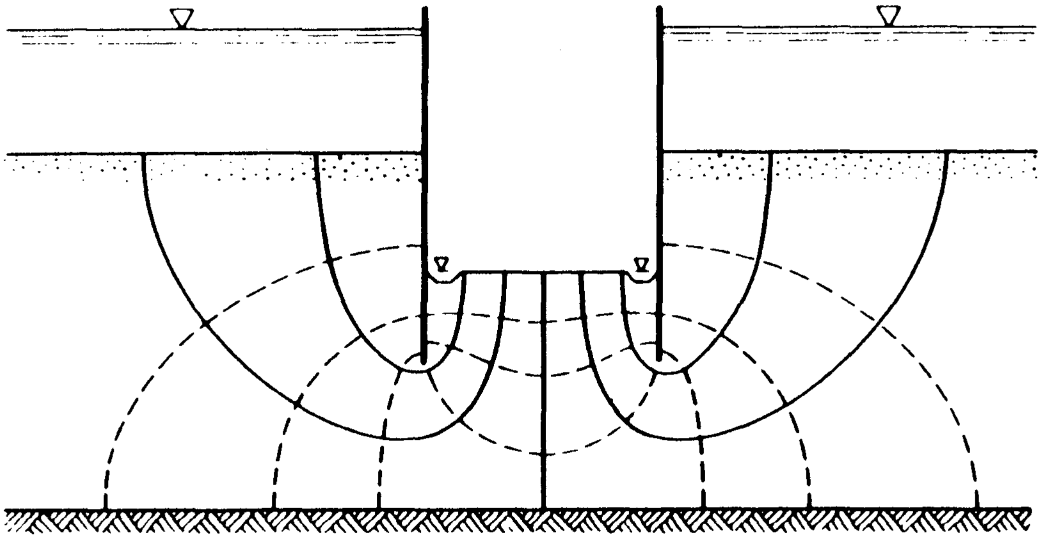
Q = SEEPAGE RATE

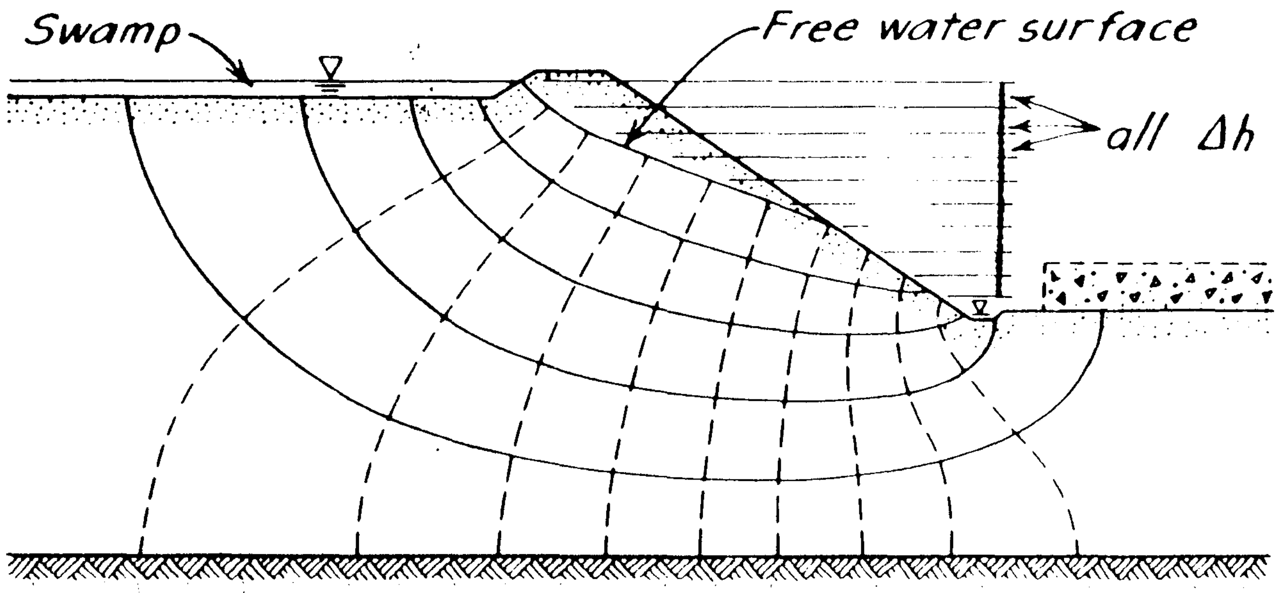
k = COEFFICIENT OF PERMEABILITY

η_f = NUMBER OF FLOW CHANNELS

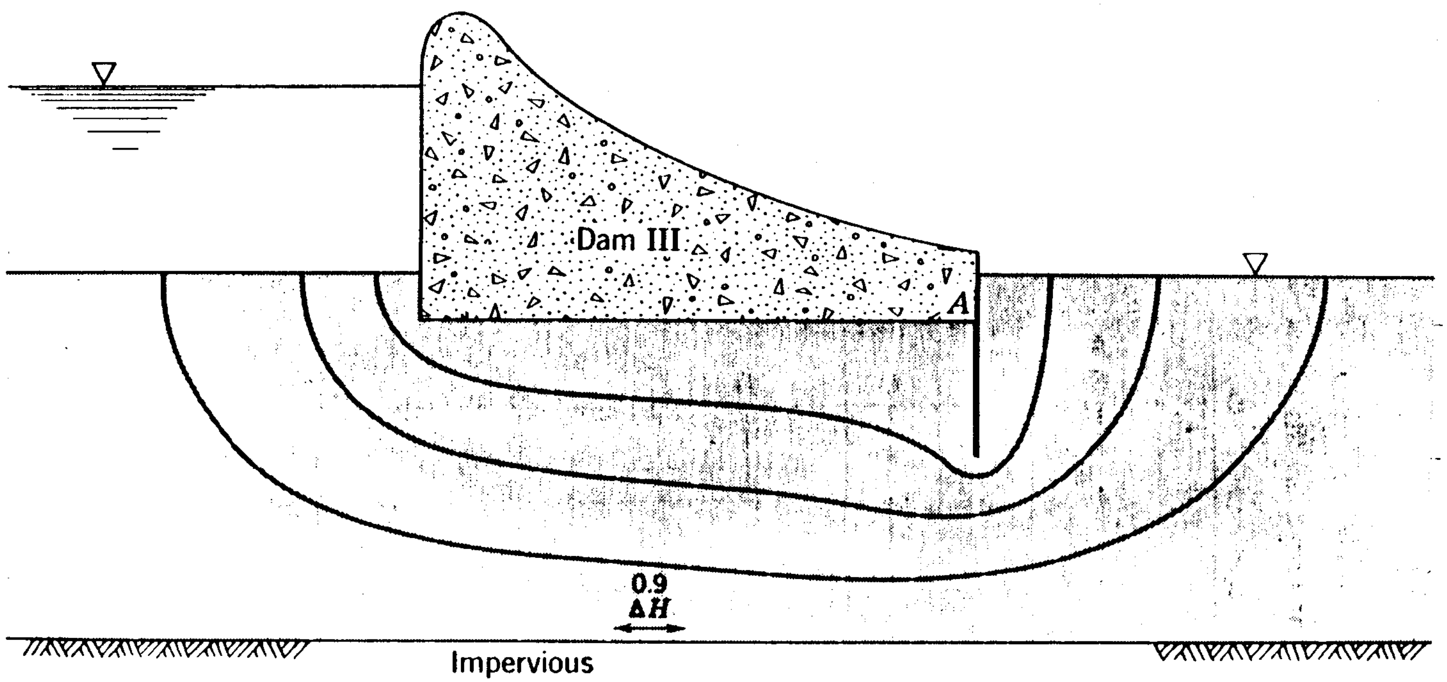
η_d = NUMBER OF EQUIPOTENTIAL DROPS

SAMPLE FLOW NET

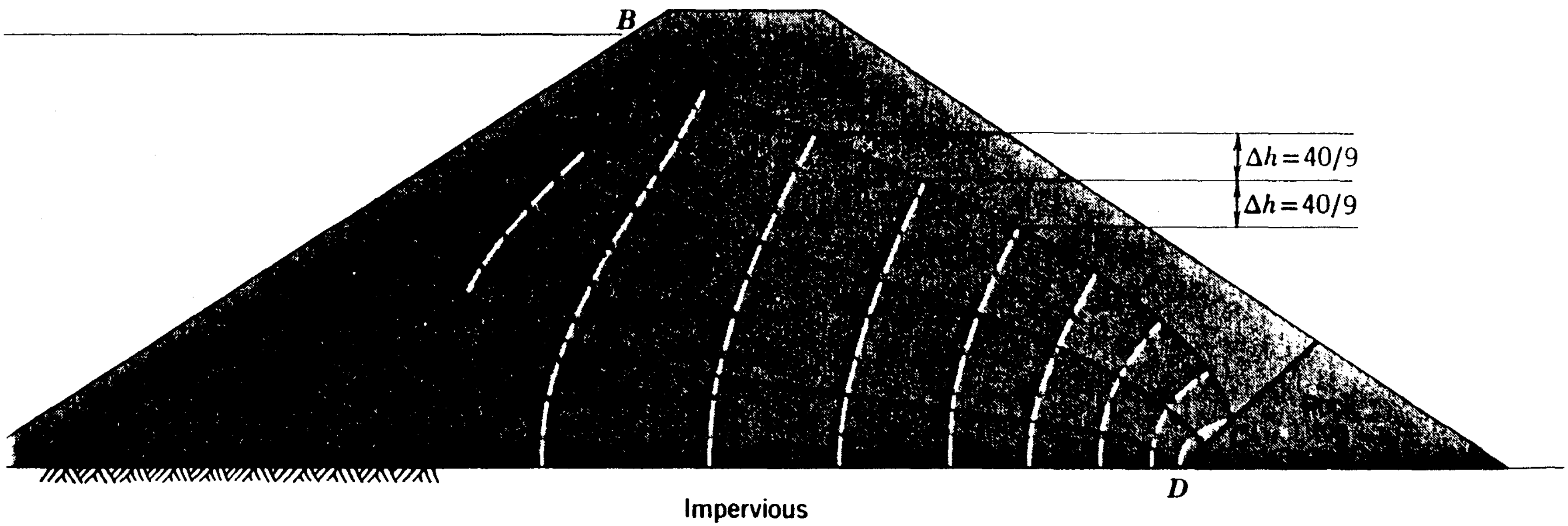




SAMPLE FLOW NET



SAMPLE FLOW NET



EXAMPLE PROBLEM: FLOW NETS

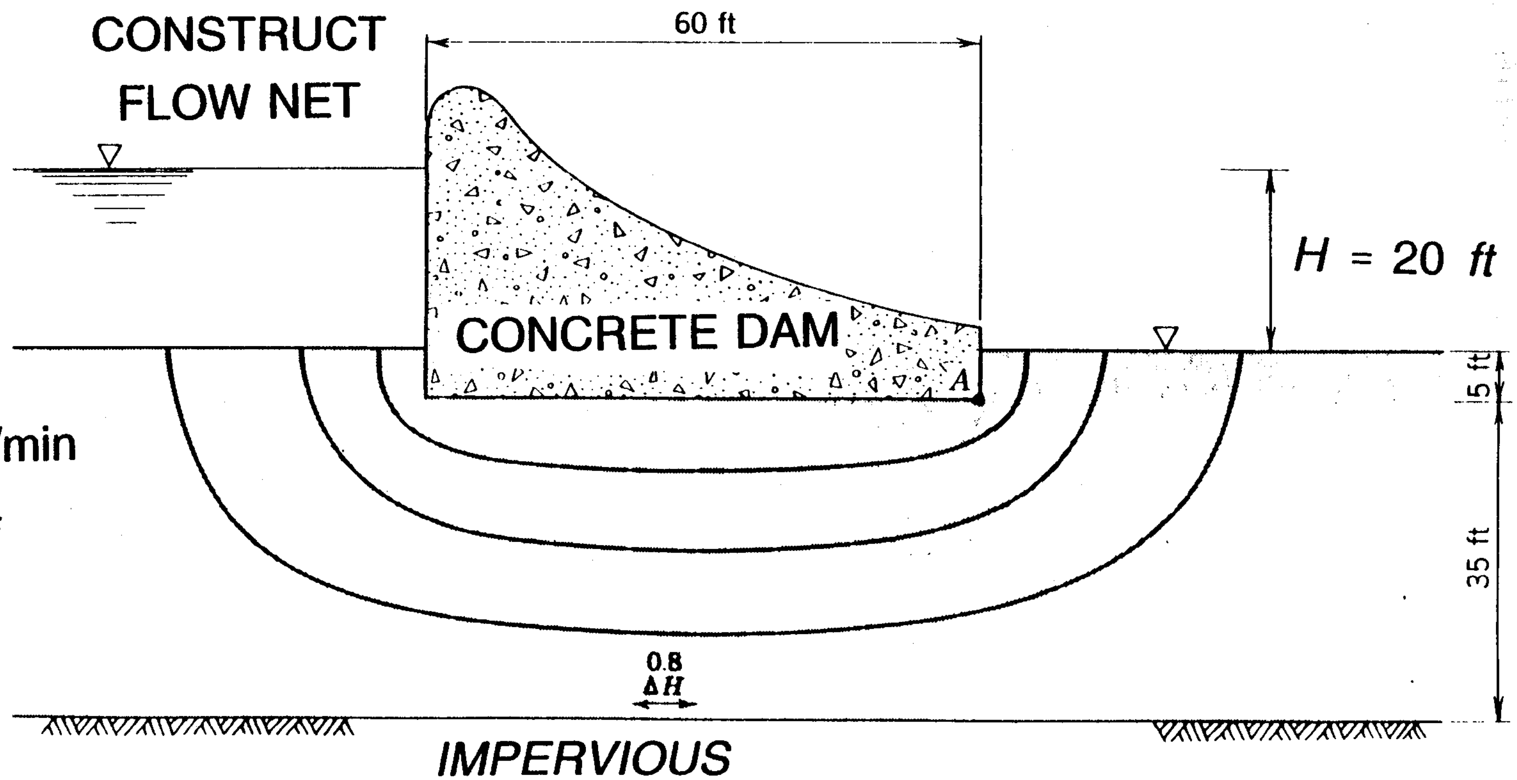
FOR THE FOLLOWING CONDITION, DETERMINE:

(A) SEEPAGE RATE BELOW DAM

(B) UPLIFT PRESSURE AT POINT "A"

(C) IF "QUICK CONDITION" EXISTS AT TOE OF DAM

CONSTRUCT FLOW NET



$k = 10^{-4}\ ft^2/min$

$\gamma_{SAT} = 122\ pcf$

EXAMPLE PROBLEM: FLOW NETS (continued)

PART A:

$$Q = k \cdot H \cdot \frac{\eta_f}{\eta_d} \qquad \textit{FROM FLOW NET:} \quad \eta_f = 4,\ \eta_d = 12$$

$$Q = (10^{-4})\ (20) \left(\frac{4}{12}\right) = 6.7\ x\ 10^{-3}\ cfm/foot$$

PART B:

$$\eta_d = 10.5$$

$$\textit{TOTAL HEAD @ "A"} = 20 - \frac{10.5}{12}(20) = 2.5\ ft$$

$$\textit{TOTAL HEAD} = \textit{ELEVATION HEAD} + \textit{PRESSURE HEAD}$$

$$\therefore\ 2.5 = -5.0 + h_p$$

$$h_p = 7.5\ ft$$

$$\textit{UPLIFT PRESSURE} = 7.5\ (62.4) = 437\ psf$$

EXAMPLE PROBLEM: FLOW NETS (continued)

PART C:

"QUICK CONDITION" WILL OCCUR IF $\bar{\sigma}_v = 0$

- CHECK POINT AGAINST VERTICAL FACE OF TOE OF DAM

$$h_T = 20 - \left(\frac{11}{12}\right)(20) = 1.7\ \mathit{ft}\ @\ 2.5\ \mathit{ft}\ \mathit{BELOW\ TAILWATER}$$

$$h_p = 1.7 + 2.5 = 4.2\ \mathit{ft} = 262\ \mathit{psf}$$

$$\sigma_v = 2.5\ (122) = 305\ \mathit{psf}$$

$$\bar{\sigma}_v = 305 - 262 = 43\ \mathit{psf} > 0$$

$\therefore$ *NO QUICK CONDITION EXISTS*

ANISOTROPIC FLOW

TRANSFORMED SECTION

- *REDUCE HORIZONTAL DISTANCE BY* $\sqrt{\dfrac{k_v}{k_h}}$
- *USE* $k_e = \sqrt{k_v \cdot k_H}$

FOR FLOW ONLY

VII. Surveying

Instructor: Harry Parker

Tape

11

Massachusetts General Laws, Chapter 112: Registration of Certain Professions and Occupations

Section 60A: Definitions

Section 60C: Submission of evidence of education and experience; examinations; exemption from written examination; rules and regulations. (see 250 CMR 6.00)

Section 60E: Display of certificate of registration, etc.

Section 61: Suspension, revocation or cancellation of certificate, registration, etc.

81D: Definitions, e.g.

- Board
- Professional engineer
- Practice of engineering
- Land surveyor
- Practice of land surveying

***

Relationship of engineering to surveying, from Ch. 112, S. 81D

Practice of engineering: "... nor shall it include the practice of land surveying, except that a registered professional engineer *qualified in the branch of civil engineering* may perform land surveying incidental to his engineering work for the locating or relocating of any of the fixed works embraced within the practice of civil engineering excluding property line determination." (Emphasis added)

Parabolic Vertical Curves

Definitions

PVI =	Point of intersection of tangents
PVC =	Point of vertical curvature, beginning of curve
PVT =	Point of vertical tangency, end of curve
g_1 , g_2 =	rate of grade expressed in %, with proper sign
A =	(g_1 - g_2) algebraic difference of rates of grade expressed in %
L =	Length of curve in stations (L is measured on horizontal plane)
e =	Middle ordinate, expressed in feet
d =	Corrections (offsets) from grade line to curve, expressed in feet
x =	Distance in stations from PVC or PVT to point on curve

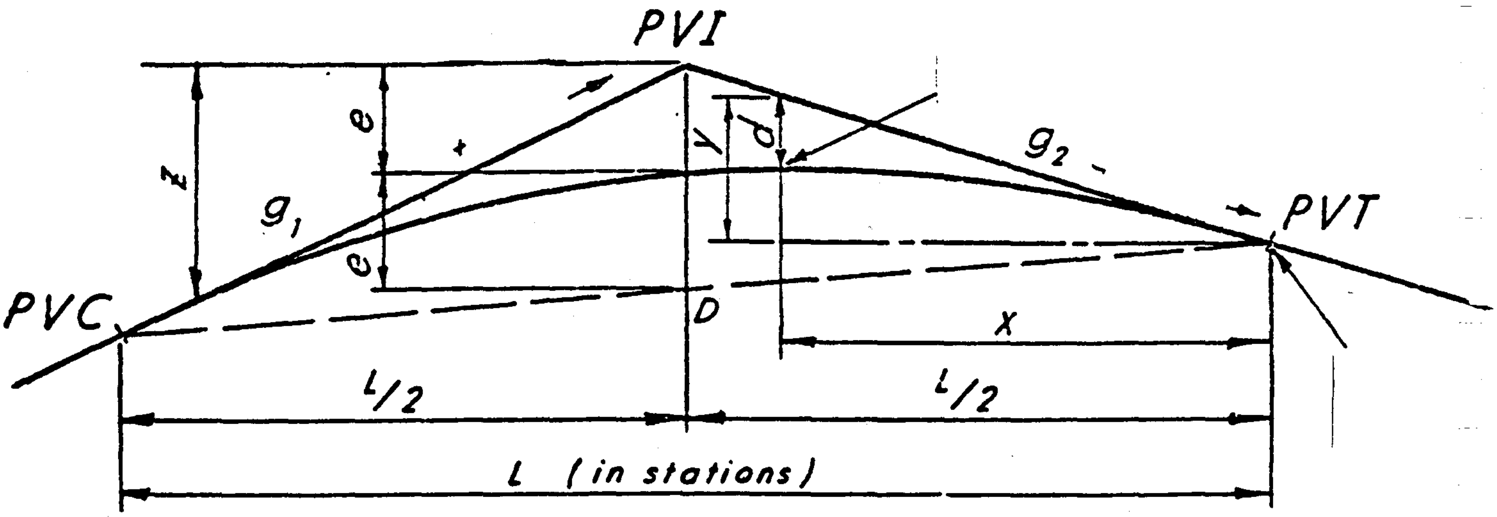
Equations

$$e = \frac{L(g_1 - g_2)}{8} = \frac{AL}{8} \text{ , or}$$

$$e = 1/2 \left(\text{Elev. Point I} - \frac{\text{Elev. at PVC} + \text{Elev. at PVT}}{2}\right)$$

$$d = \frac{Ax^2}{2L}$$

Parabolic Vertical Curves



Sight Distance Computations

Definitions

S = Non-passing sight distance, feet
h_1 = Height of eye above roadway, feet
h_2 = Height of object above roadway, feet

Equations

When S is less than L,

$$L = \frac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2}$$

When S is greater than L,

$$L = \frac{200(\sqrt{2h_1} + \sqrt{2h_2})^2}{A}$$

MINIMUM SIGHT DISTANCES

Design Speed mph	Non-Passing Sight Distance	Passing Sight Distance	
		2 Lane Highway	3 Lane Highway
40	275	1300	-
50	350	1700	1200
60	475	2000	1400
70	600	2300	1600

Parabolic Vertical Curves

Question 1.
Compute the length of vertical curve required for a highway between the two profile grade shown on the diagram. Assume that elevation 648.0 is desired at the summit.

Given: $g_1 = +4.0\%$, $g_2 = -2.0\%$

$A = g_1 - g_2 = +4 - (-2) = 6$

The high point (summit) or the low point (sag) is always on the the lesser gradient side of the PVI.

$x = \frac{g_2 L}{A} = \frac{2L}{6} = \frac{L}{3}$ stations

$d = \frac{Ax^2}{2L} = \frac{6(L/3)^2}{2L} = \frac{L}{3}$ feet

$\frac{y}{x} = g_2$, or $y = g_2 x = \frac{2L}{3}$

Elevation at PVT $= 652.6 - \frac{g_2 L}{2} = 652.6 - L$

Then, high point on summit minus elevation at PVT + d = y

$$648 - (652.6 - L) + \frac{L}{3} = \frac{2L}{3}$$

L = 6.9 stations, say 700 feet

Parabolic Vertical Curves

Question 1.

Figure 1.

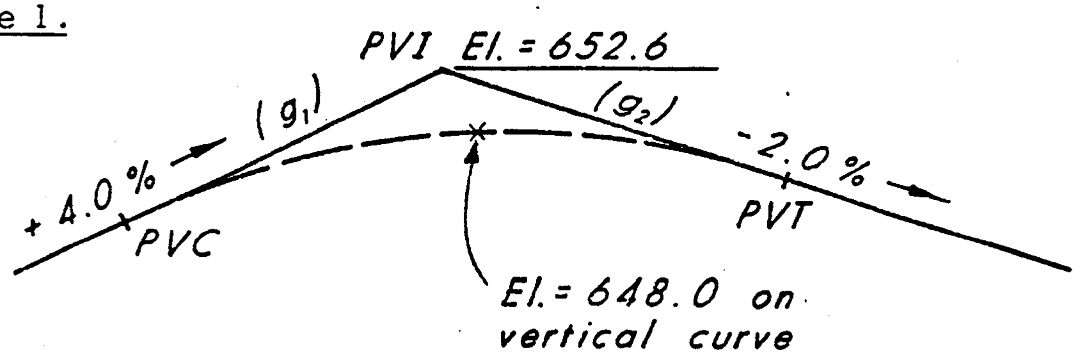
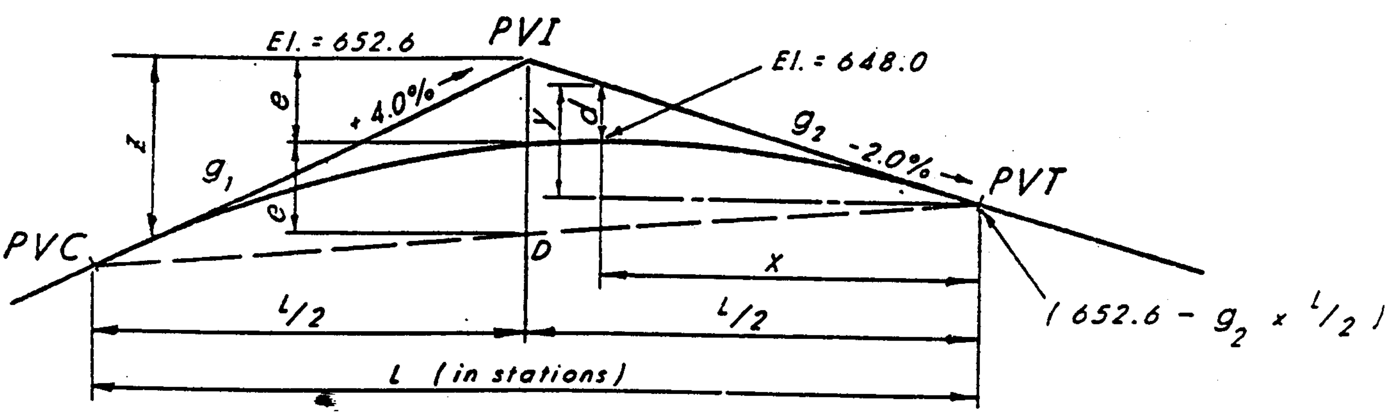
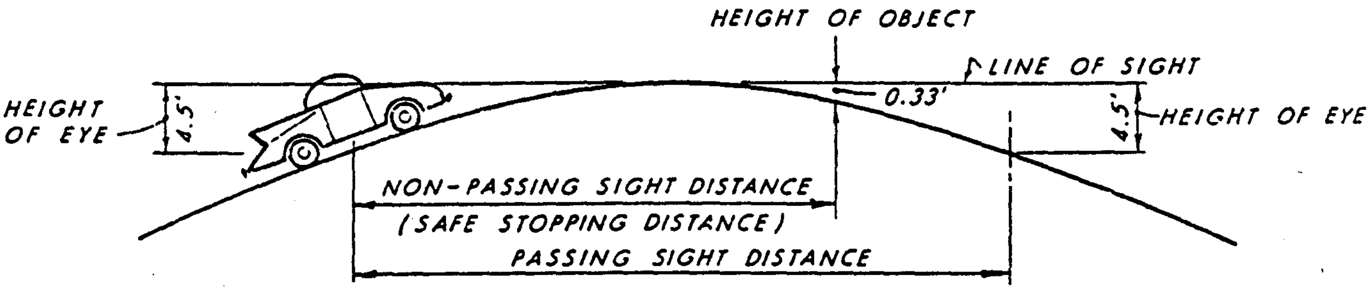


Figure 2.



Parabolic Vertical Curves

Question 2.
On a thruway highway what would your recommendation be for length of vertical curve, non-passing over a summit with a 2.8% ascending grade and a 3.5% descending grade. Grades intersect at station 25 + 50 at elevation 130. feet.



Assume 70 MPH design speed. Solve for condition where sight distance is less than curve length, therefor:

S = 600 (from table)

$$A = g_1 - g_2$$

$$= 2.8 - (-3.5) = 6.3$$

$$L = \frac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2}$$

$$= \frac{6.3\,(600)^2}{100(\sqrt{2 \times 4.5} + \sqrt{2 \times 0.333})^2} = 1557 \text{ feet}$$

Parabolic Vertical Curves

Question 2a.

Refering to figure 2 (Question 1)

PVI at elevation 130.
$g_1 = +2.8\%$, $g_2 = -3.5\%$
PVC at station 18+00
L = 1500'

Compute elevation of curve at each station:

$$\text{El. at PVC} = (130.) - \left(g_1 \frac{xL}{2}\right) = (130.) - \left(2.8 \frac{x15}{2}\right) = 109.00$$

$$\text{El. at PVT} = (130.) - \left(g_2 \frac{xL}{2}\right) = (130.) - \left(3.5 \times \frac{15}{2}\right) = 103.75$$

El. at points to left of PVI (on tangent):

$$\text{Sta. 20+50 El.} = (130.) - (y) = (130) - \left(g_1 \frac{500}{100}\right) =$$

$$(130.) - 2.8(5) = 116.00$$

Tangent offset, d, at horizontal distance x, measured from PVC to station 20+50:

$$d = \frac{Ax^2}{2L} = \frac{6.3(2.5)^2}{2(15)} = 1.31 \text{ feet}$$

El. of curve at station 20+50 = 116.00 - 1.31 = <u>114.69</u>

Etc.

Parabolic Vertical Curves

Question 1.

Figure 1.

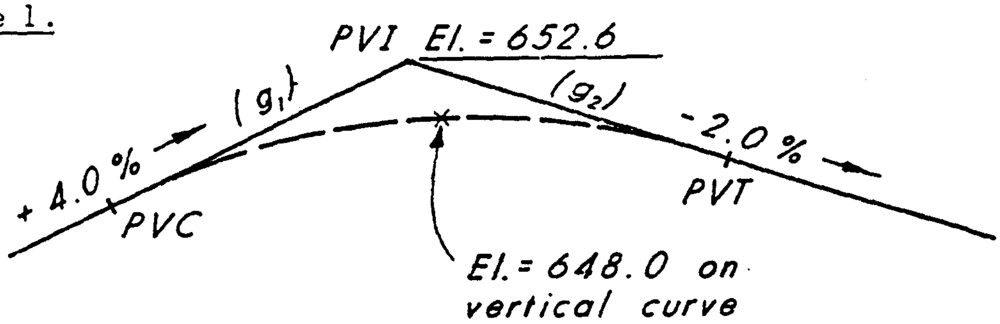
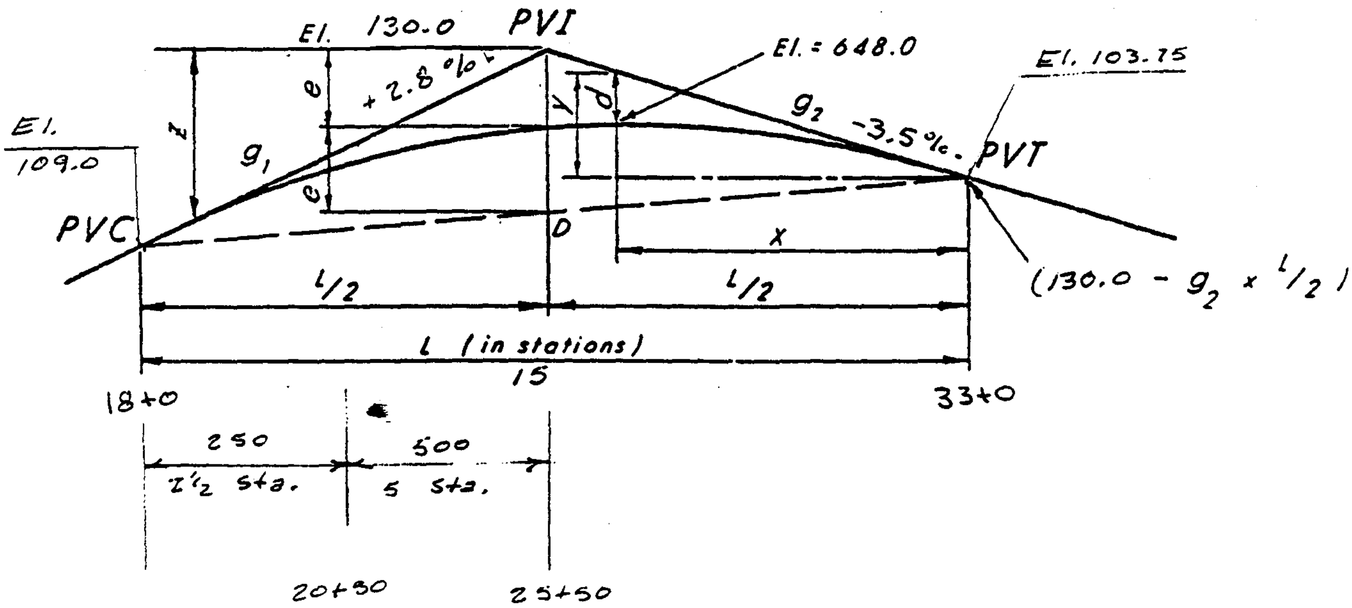


Figure 2.



Parabolic Vertical Curves

Question 3.

Given:
Secondary road to pass under bridge
Length of vertical curve = 800 feet
Approach grade (g_1) = -4.0%
Station of PVC = 11+00
Lowest point of bridge overpass = 100.0 feet
Lowest point of bridge overpass is at station 16+0
Clearance under overpass to be 14 feet
Permissable passing sight distance = 1200 feet

Solve:
A. Required grade of tangent from PVI to PVT
B. Station and elevation of PVI
C. Will profile shown satisfy conditions?

Low point of sag curve to right of PVI (by inspection), therefore g_2 will be lower magnitude than g_1.
From PVT, x = 3 stations, L = 8 stations.
Formula to find sag or high point: $x = \frac{(\text{lesser grade})(L)}{A} = \frac{g_2 L}{A}$

$A = g_1 - g_2 = -4 - g = -(4 + g_2)$ Used as positive integer

$$x = \frac{g_2 L}{4 + g_2}$$

$$3 = \frac{g_2(8)}{4 + g_2}$$

$g_2 = +2.4$ Answer A.

Parabolic Vertical Curves

Question 3.

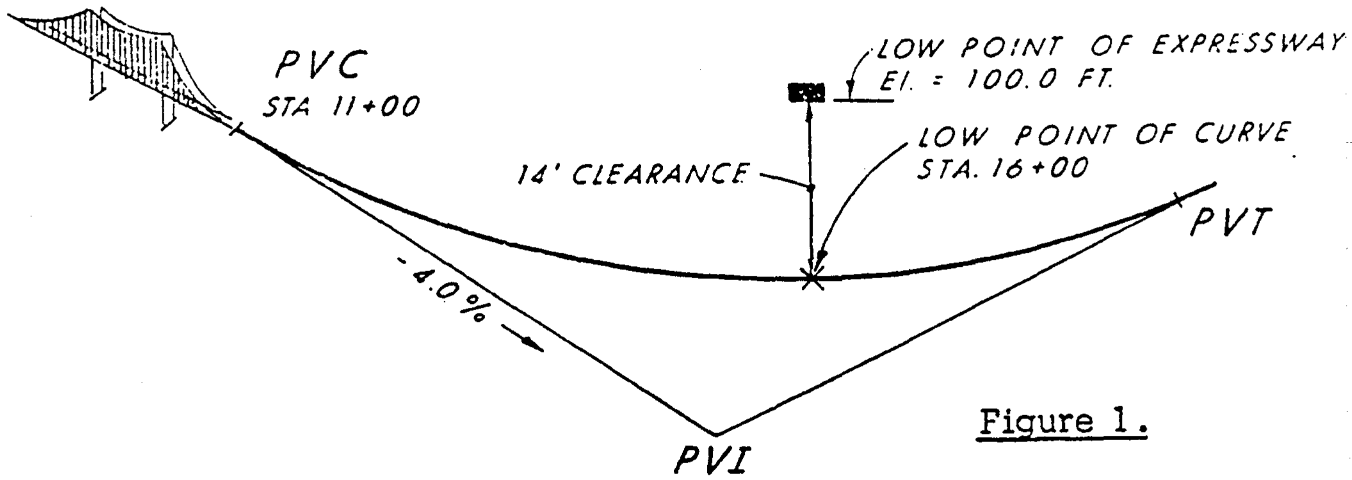
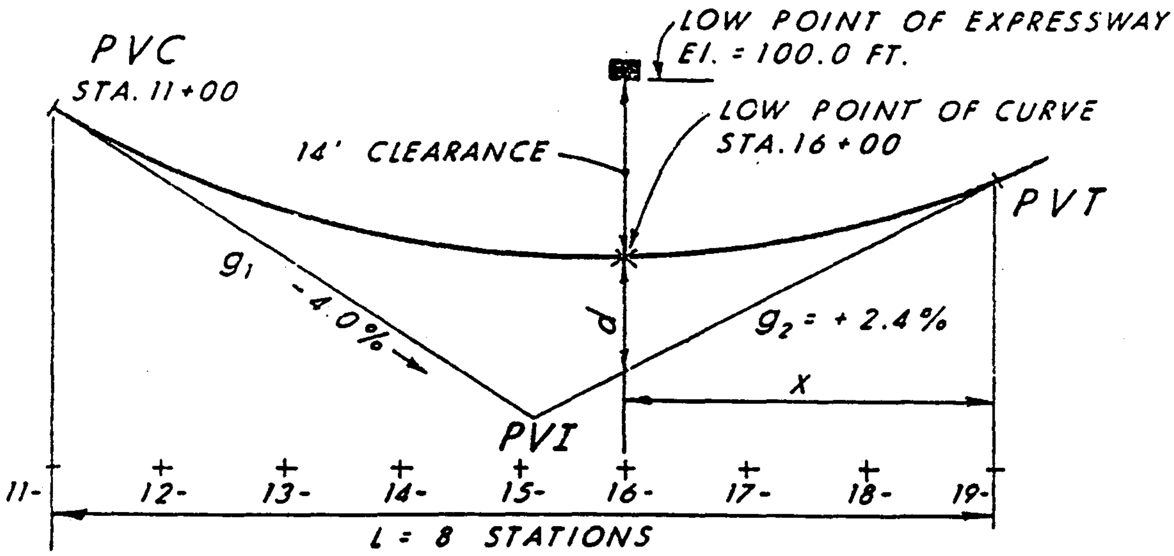
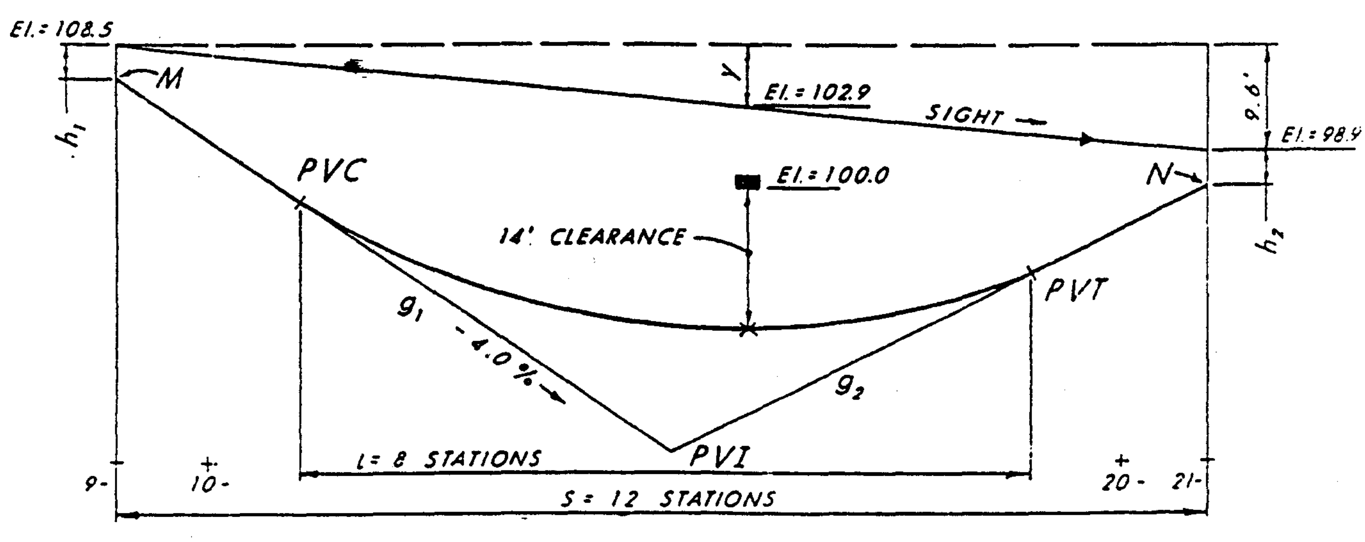


Figure 1.





Parabolic Vertical Curves

Question 3. (2)

By inspection (Figure 1.) PVI is at station 15+00 Answer B.
Elevation at low point of sag curve = 100 - 14 = 86.0 Answer B

Offset from tangent to low point:

$$d = \frac{Ax^2}{2L} = \frac{6.4(3)^2}{2(8)} = 3.6$$

El. on tangent at same station = 86.0 - 3.6 = 82.4
El. at PVI = 82.4 - (g x1) = 82.4 - 2.4 = 80.0 Answer B.

El. of PVT = 80.0 + g_2(400) = 80 + (2.4 x 4) = 89.6
El. of PVC = 80.0 + g_1(400) = 80 + (4 x 4) = 96.0

Check 1200 foot sight distance, from 200 feet prior to PVC (point M) to 200 feet beyond PVT (point N).

g_1 (2 sta.)

El. at point M = 96.0 + 4(2) = 104.0
El. at point N = 89.6 + 2.4(2) = 94.4

g_2 (2 sta)

El. at eye level at point M 104.0 + 4.5 = 108.5
El. of object at point N = 94.4 + 4.5 = 98.9

El. of line of sight at station 16+0 (overpass):

(sta 9+to sta 16+)

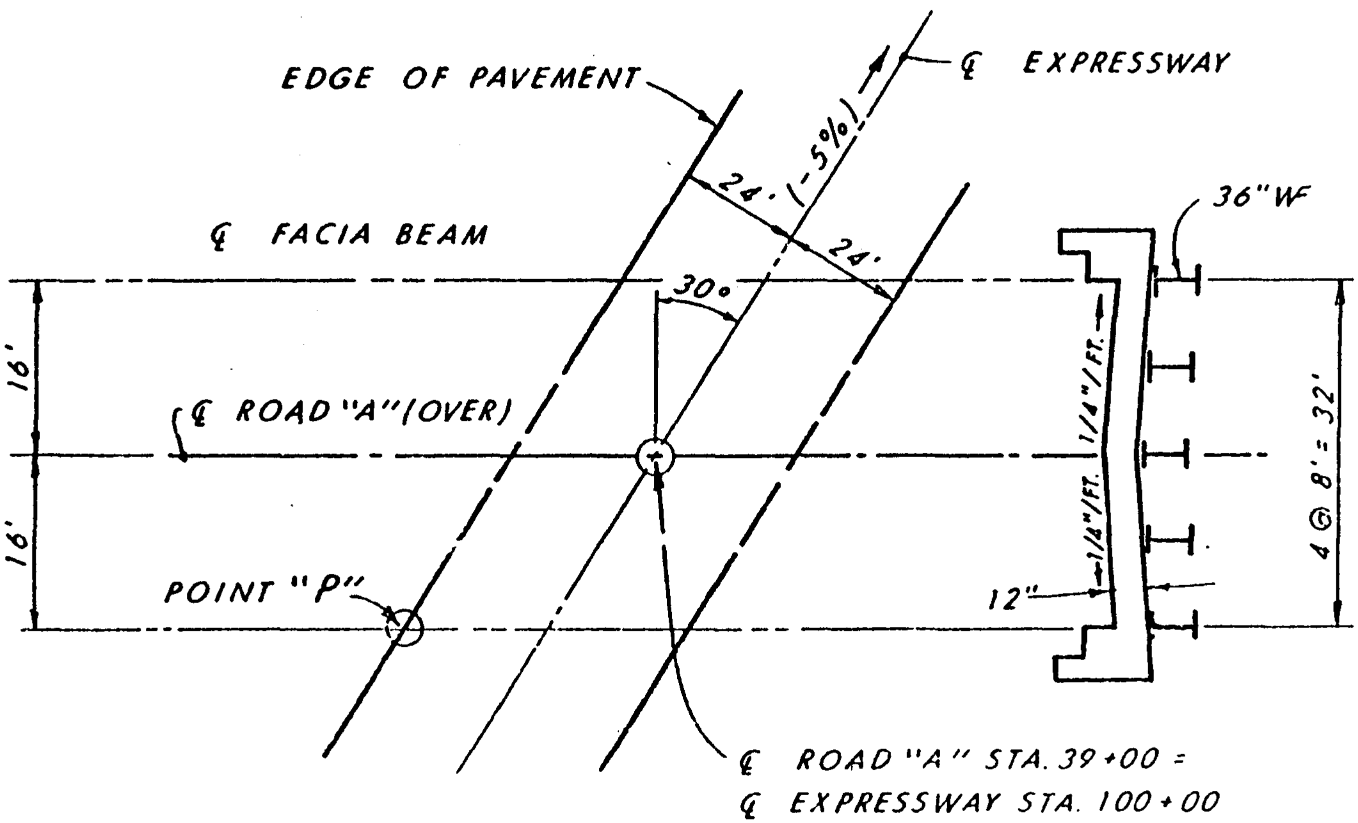
$$y = \frac{700}{1200}(108.5 - 98.9) = 5.6$$

El. = 108.5 - 5.6 = 102.9 Not a clear line of sight.

Answer C.

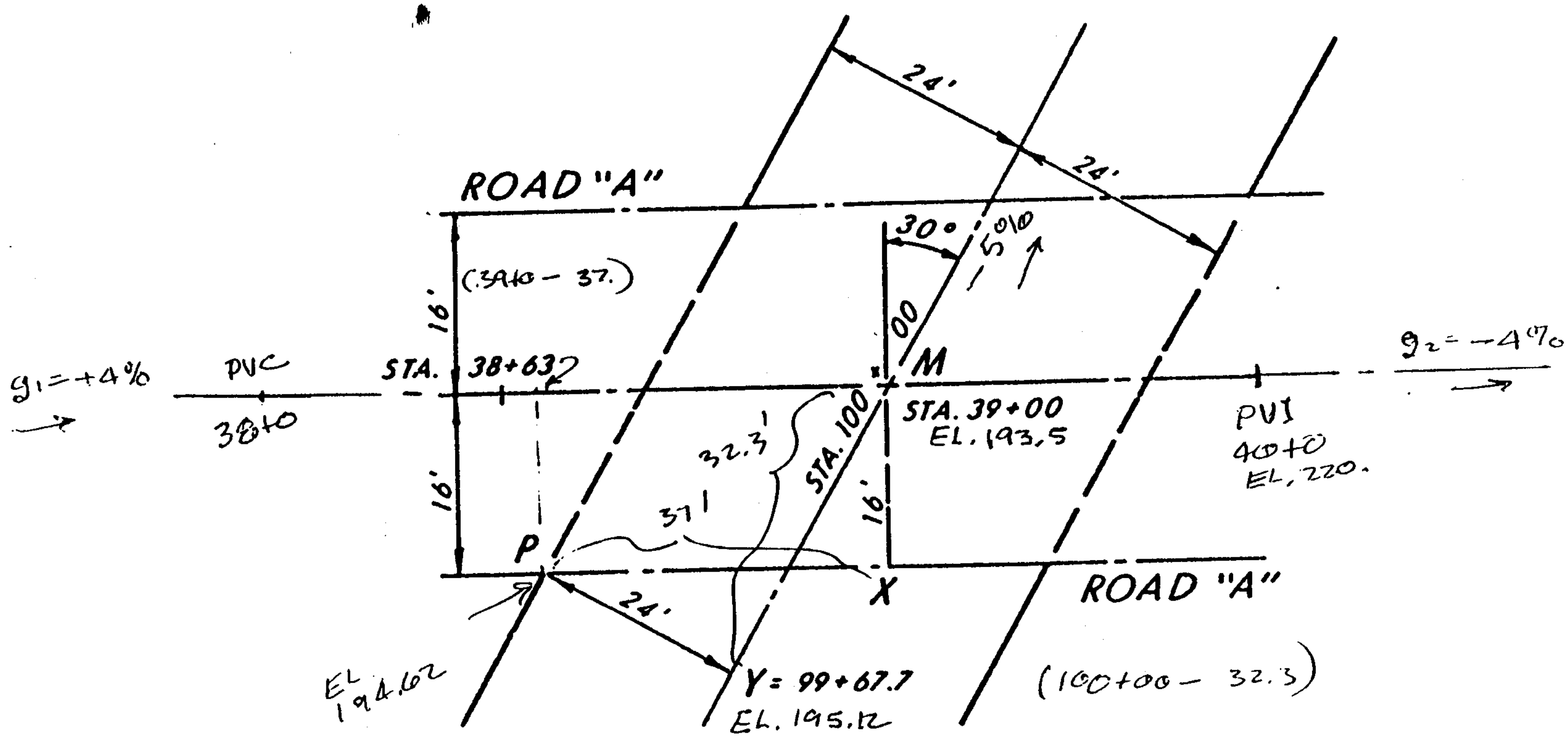
above obstruction

QUESTION 4. A GRADE SEPARATION STRUCTURE FOR ROAD A IS TO BE BUILT OVER AN EXPRESSWAY AS SHOWN BELOW. THE EXPRESSWAY IS ON A -5% GRADE AND THE ELEVATION OF THE EXPRESSWAY AT THE INTERSECTION STATION 100+00 IS 193.5. THE TRANSVERSE SLOPE OF BOTH ROADS IS $\frac{1}{4}$ INCH PER FOOT. ROAD A IS ON A VERTICAL CURVE HAVING THE FOLLOWING DATA: PVI STATION: 40+00, PVI ELEVATION: 220 FEET, GRADES: +4% to -4%, LENGTH OF CURVE: 400 FEET. FIND THE CLEARANCE AT THE CRITICAL POINT P.



By trigonometry: PX = 37' and, MY = 32.3'
Then: P Station on Road A = 38 + 63, P Station on Expressway = 99 + 67.7
elevation of Expressway at Station 99 + 67.7: 193.5 + (.05 x 32.3) = 195.12
Then: Elevation at P

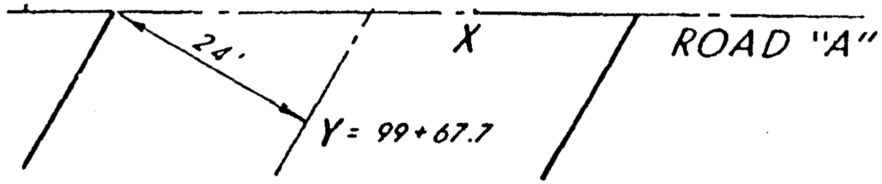
$$195.12 - \frac{\frac{1}{4}(24)}{12} = 194.62$$



Elevation on Vertical Curve of Road A at Station 38 + 63 (center line):

$$\text{Tangent Offset} = \frac{A\,x^2}{2\,L} = \frac{8\,(.63^2)}{2\,(4)} = .4$$

Tangent Elevation: 220 - (.04 x 137) = 214.52



Elevation on Vertical Curve of Road A at Station 38 + 63 (center line):

$$\text{Tangent Offset} = \frac{A\,x^2}{2\,L} = \frac{8\,(.63^2)}{2\,(4)} = .4$$

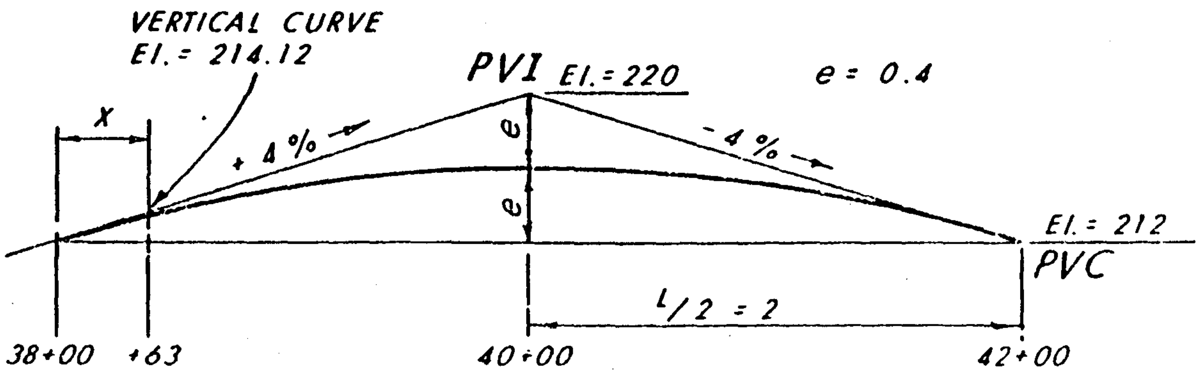
Tangent Elevation: $220 - (.04 \times 137) = 214.52$

Elevation of Vertical Curve: $214.52 - .4 = 214.12$

36WF   Slab

$$\text{Elevation at P: } 214.12 - \frac{(\frac{1}{4} \times 16) + 36 + 12}{12} = 209.79$$

Then: Clearance at critical point P = 209.79 - 194.62 = 15.17 feet.....Answer



Parabolic Vertical Curves

Question 5.

The summit of a four-lane highway has an ascending grade of 3.7% and a descending grade of 2.9%, intersecting at station 42+00 at a PVI elevation of 170.0.

A. For 60 MPH design speed what is recommended stopping sight distance?

B. Determine minimum length of vertical curve to provide the recommended sight distance.

C. Compute elevations on vertical curve at PVC, PVT, Midpoint and quarter-point stations.

Parabolic Vertical Curves

Question 5. (cont.)

A. From "Minimum Sight Distance" table, minimum stopping distance is 475 feet.

B. Design speed = 60 MPH, h_1 = 4.5 feet, h_2 = 0.333 feet.
Use formula $L = \dfrac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2}$

$A = g_1 - g_2 = 3.7 - (-2.9) = 6.6$

$L = \dfrac{6.6(475)^2}{100(\sqrt{2x4.5} + \sqrt{2x.333})^2} = $ 1020, say 1000 feet.

C. L = 1000 feet, or 10 stations
El. at Pvc $= 170.00 - \dfrac{g_1 L}{2} = 170. - \dfrac{3.7\,(10)}{2} = 151.5$

El. at PVT $= 170.00 - \dfrac{g_2 L}{2} = 170. - \dfrac{2.9\,(10)}{2} = 155.5$

Etc.

CIRCULAR CURVES

Definitions

PC =	Point of curvature, beginning of curve
PI =	Point of intersection of tangents
PT =	Point of tangency, end of curve
R =	Radius of curve
D =	Degree of curve *
I =	Deflection angle between tangents at PI; also central angle
T =	Tangent distance, from PC to PI, PI to PT
L =	Length of curve from PC to PT (measured on 100-foot chords for chord definition, on arc for arc definition)
C =	Length of chord, from PC to PT
E =	External distance, from PI to mid-point on curve
M =	Mid-ordinate, distance from mid-point of curve to mid-point of long chord

* Degree of curve: Central angle subtended by arc of 100 feet, arc definition. Central angle subtended by 100 foot chord, chord definition.

Equations

$R = 5{,}729.578/D$ exact for arc definition

$\quad = 50/\sin D/2$ exact for chord definition

$T = R \tan I/2$

$M = R(1 - \cos I/2)$

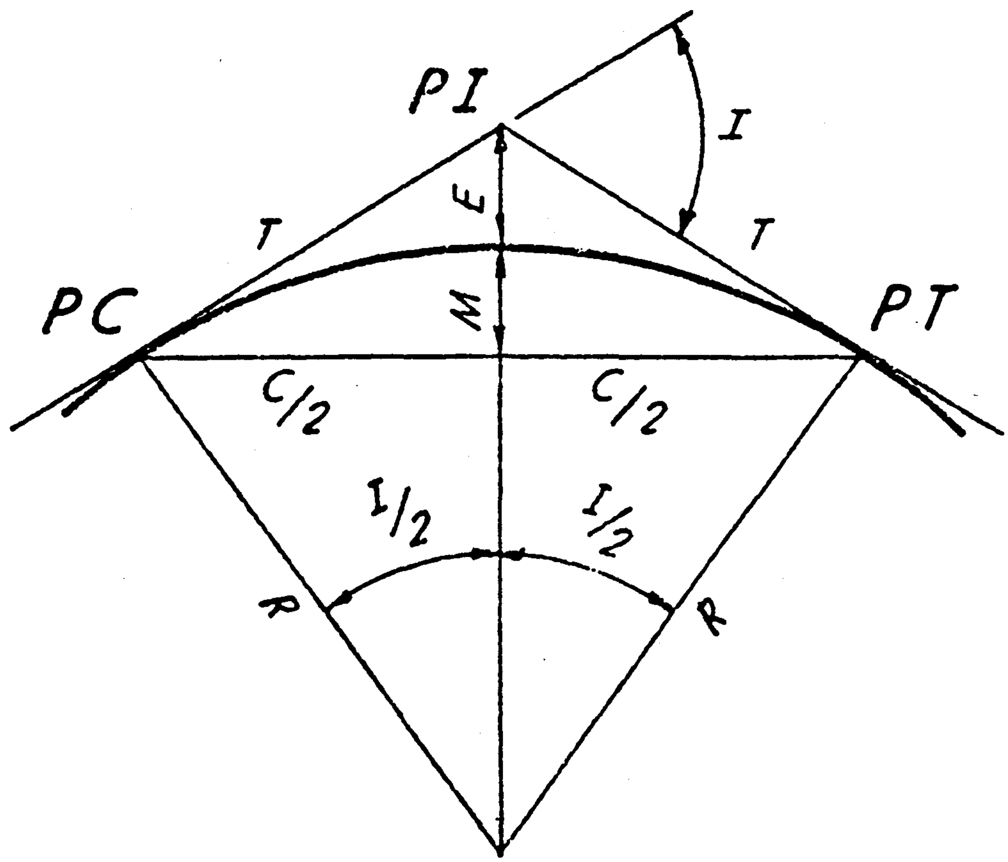
$C = 2R \sin I/2$

$L = 100\, I/D$

$$E = R\,\frac{(1 - \cos I/2)}{\cos I/2}$$

Note: 5729.578 = 1 radian = central angle of curve equal to R

CIRCULAR CURVE



Question 1.

Two tangents for the centerline of a highway project intersect at station 354+50, with a deflection angle of 40° 45'. Design a horizontal curve for this intersection, locating the external secant of the centerline of the highway at 50 feet from the point of intersection of the tangents, using a degree of curve to the nearest one-half degree.

Given: $E = 50.00$, $I = 40^\circ 45'$

$$R = \frac{E \cos I/2}{1 - \cos I/2} = \frac{50 \times .93743}{1 - .93743} = 749.09, \text{ or } \underline{750.00}$$

$$D = \frac{5729.578}{750.00} = 7.64, \text{ or } \underline{7^\circ 38'}$$

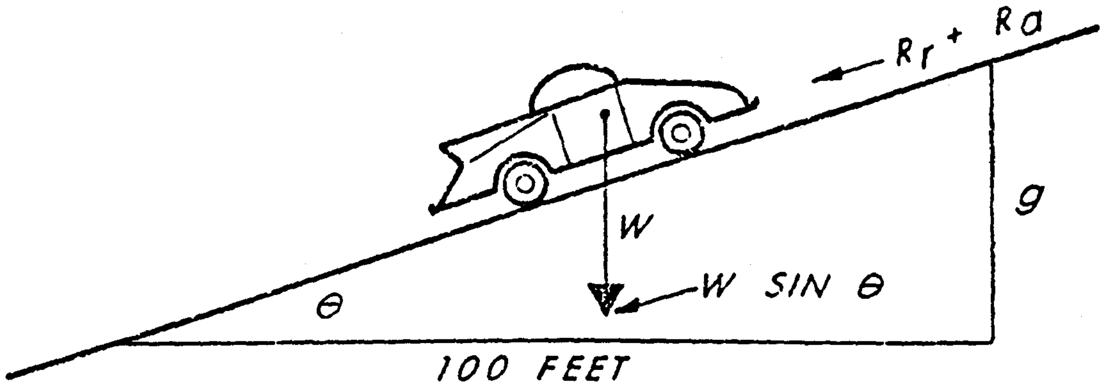
$$T = R \tan I/2 = 750.00 \times .37073 = \underline{278.05 \text{ feet}}$$

$$L = \frac{100\, I}{D} = \frac{100 \times 40\ 45'}{7^\circ 38'} = \frac{100 \times 40.75}{7.64} = \underline{533.38 \text{ feet}}$$

PI: 354 + 50
PC = 35450 - 278.05 = 35171.95, or <u>351 + 71.95</u>
PT = 35171.95 + 533.38 = 35705.33, or <u>357 + 5.33</u>

Miscellaneous Highway Design

Question 1.



Miscellaneous Highway Design

Question 1.

Vehicle weight, W = 2500 pounds
Frontal area, A = 20 square feet
Max. tractive force of 270 pounds at 30 MPH
Gravel road, rolling resistance = 45 pounds per ton vehicle weight
K = 0.0018

Q. What grade can car climb in still air, maintaining 30 MPH speed.?

$R_a = KAV^2 = 0.0018(20)(30)^2 = 32.4$ lbs. (air resistance)

$R_r = \frac{45 \times 2500}{2000} = 56.25$ lbs. (rolling resistance)

Tractive force $= 270 = R_r + R_a \pm R_g$

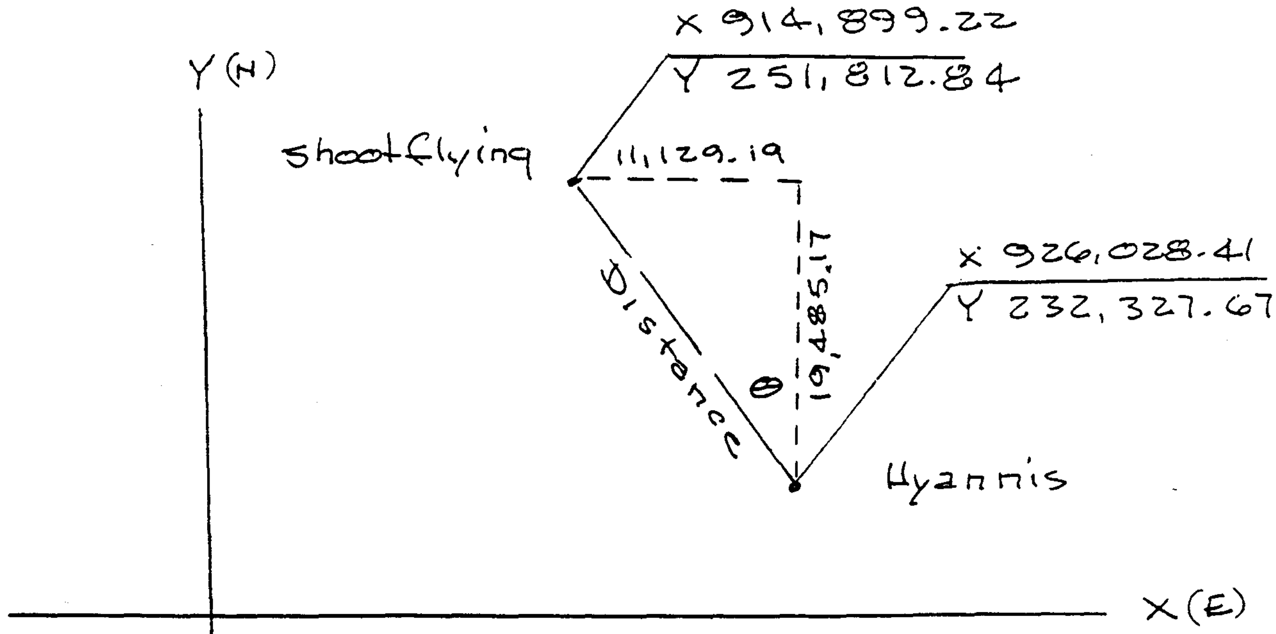
(R_g is grade resistance; positive on up-grade)

$R_g = 270 - R_r - R_a = 270 - 56.25 - 32.4 = 181.35$ lbs. (grade resistance)

$R_g = W(\sin\theta) = \frac{W(g)}{100} = 181.35$ $\left(\sin\theta = \frac{g}{100}\right)$

$g = \frac{R_g(100)}{W} = \frac{181.35(100)}{2500} = 7.25\%$

Question 4.



$$\tan \theta = \frac{11{,}129.19}{19{,}485.17} = 0.5711621$$

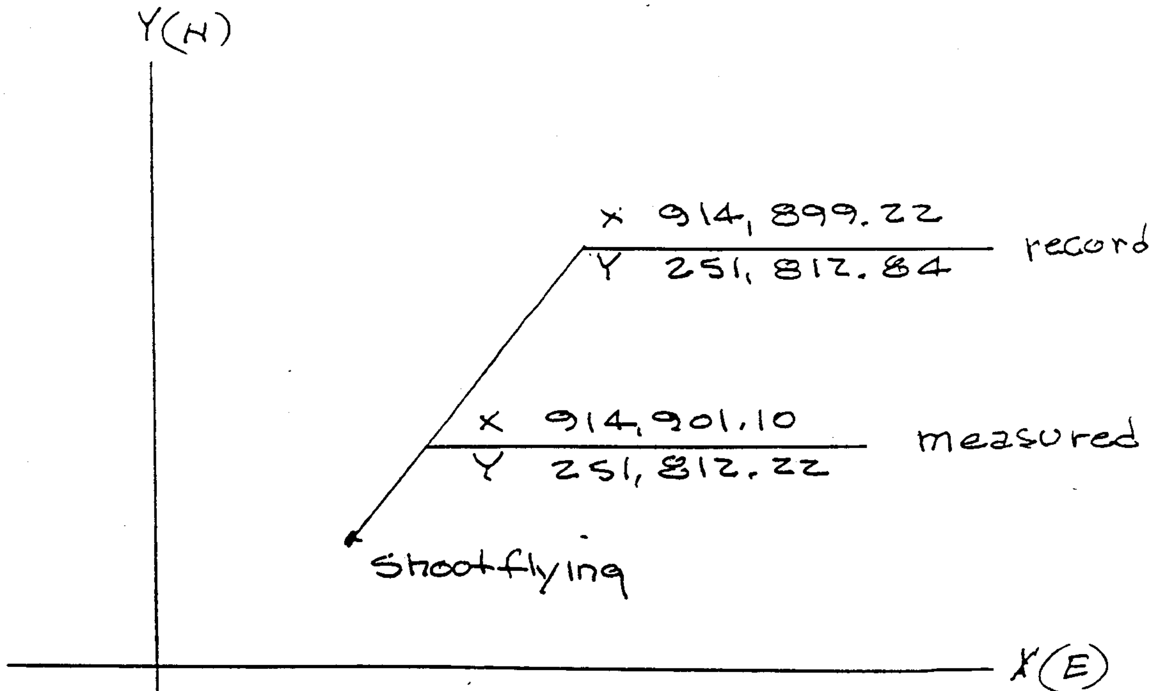
$$\theta = 29^\circ\ 44'\ 00.1''$$

$$\cos \theta = \frac{19{,}485.17}{D}$$

$$D = 22{,}439.49$$

$$\left.\begin{array}{l} \\ \text{Grid bearing} = N\ 29^\circ 44' 00.1''\ W \end{array}\right\}\ a.$$

Question 4. (cont)



$$\text{Linear error} = \sqrt{(\Delta x)^2 + (\Delta Y)^2}$$

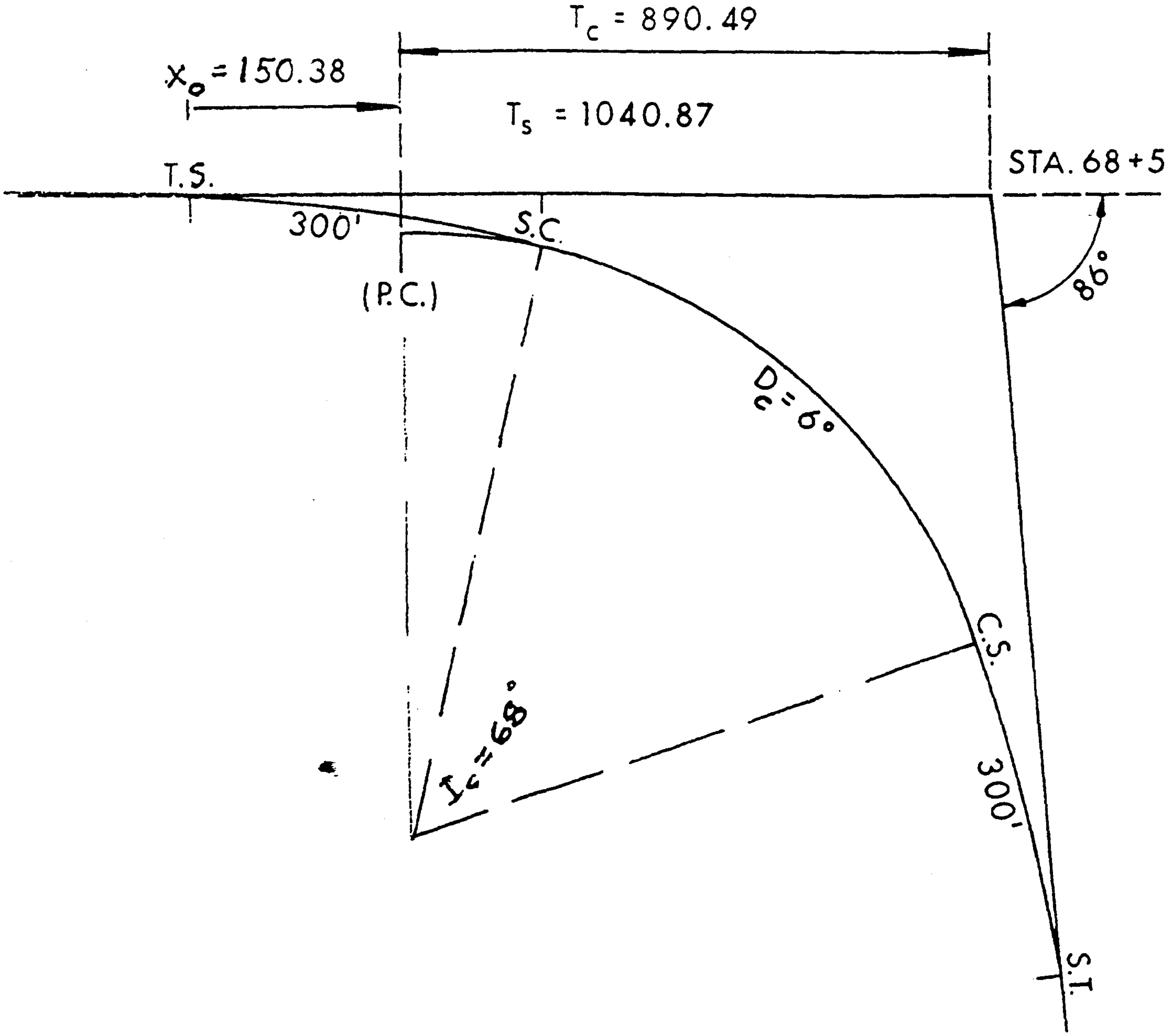
$$= \sqrt{(1.88)^2 + (.62)^2}$$

$$= 1.98$$

$$\text{Closure} = \frac{1.98}{26,910.12} = \frac{1}{13,591}$$

Question 7: Spiral Curve

Given: spiral curve with an intersection of tangents angle = 86°; length of spiral = 300 feet; station of P.I. = 68 + 50; degree of curve = 6°. Determine stations of T.S., S.C., C.S., and S.T.



Question 7: Spiral Curve (2)

Definitions

T.S.	Tangent to spiral
S.C.	Spiral to curve
C.S.	Curve to spiral
S.T.	Spiral to tangent
I	Total central angle; angle of tangents
I_s	Spiral angle
I_c	Angle of circular curve
D_c	Degree of curve (circular curve)
R_c	Radius of circular curve
L_c	Length of circular curve
Y_c	Offset from tangent to S.C.
X_c	Spiral chord
X_o	tangent distance from T.S. to beginning of L.C.
L.C.	Long curve, from T.S. to point of curvature of circular curve of central angle equal to Sc (spiral angle)
T_c	Tangent of circular curve
T_s	Tangent of spiral

Question 7: Spiral Curve (3)

$$I_s = \frac{L_s D_c}{200} = \frac{300(6)}{200} = 9$$

$$I_c = I - 2I_s = 86 - 18 = 68$$

$$R_c = \frac{5729.578}{D_c} = \frac{5729.578}{6} = 954.93$$

$$L_c = \frac{I_c(100)}{D_c} = \frac{68(100)}{6} = 1133.33$$

$$Y_c = \frac{(L_s)^2}{D_c R_c} = \frac{(300)^2}{6(954.93)} = 15.71$$

(offset from tangent

$$X_c = \frac{Y_c}{\cot i_c} \qquad (i_c = 1/3\ I_s)$$

$$= \frac{15.71}{\cot 3} = 299.76$$

$$X_o = X_c - R_c(\sin I_s)$$

$$= 299.76 - 954.93(\sin 9) = 150.38$$

$$T_c = R_c(\tan I/2) = 954.93\ (\tan 43) = 890.49$$

$$T_s = X_o + T_c = 150.38 + 890.49 = 1040.87$$

Question 7: Spiral Curve (4)

P.I. =	68 + 50.00
	- 1040.87
T.S. =	58 + 09.13
	+ 300.00
S.C. =	61 + 09.13
	+1133.33
C.S. =	72 + 42.46
	+300.00
S.T. =	75 + 42.46

** N O T E S **

** N O T E S **

** N O T E S **

VIII. Economics

Instructor: Richard Pike

Tape

12

ENGINEERING ECONOMIC DECISION ANALYSIS

The following material is a review of the economic analysis of engineering projects taking the time value of money into account.

Ways of moving money around in time, or accounting for the time value of money are presented. Means of handling single sums as well as various types of uniform series are reviewed.

The present worth, future worth, annual worth, internal rate of return, and benefit/cost ratio methods of analysis are examined. These methods are presented as ways to determine the financial viability of an individual engineering project and also as a way to choose the best of several alternative engineering projects.

The incremental approach to the internal rate of return and benefit/cost ratio methods of analysis is stressed when using these methods to choose the best of several alternative projects.

The annual worth method of analysis is demonstrated as a means of analyzing projects with different lives assuming that using the least common multiple of lives is an acceptable way of arriving at a common life.

The capitalized cost and capital recovery cost methods of analysis are reviewed as they pertain to the determination of the financial viability of projects with perpetual or infinitely long lives.

Various methods of financing projects are explored and the determination of the principal and interest portions of a direct reduction loan payment are reviewed.

Methods of determining an effective annual interest rate are presented and means of adjusting interest rates to accomodate cash flows occuring more often than annually are explored.

A short set of interest tables are provided which will allow the solution of most engineering economic decision analysis problems.

TIME VALUE OF MONEY

To illustrate the concept of the time value of money we will assume that \$100 is deposited into an account that pays interest at a rate of 10% per year. The growth of the original investment is shown for a three year period below.

EOY	ACCOUNT BALANCE	+	INTEREST		
0	\$100				
1	100	+	\$10	=	\$110
2	110	+	11	=	121
3	121	+	12.10	=	133.10

You can see that the original deposit of \$100 grows to \$133.10 over the three year period.

The formula for finding the future vvalue of a present sum is shown below where F equals the future value, P equals the present value, i equals the interest rate per interest period, and n equals the number of interest periods.

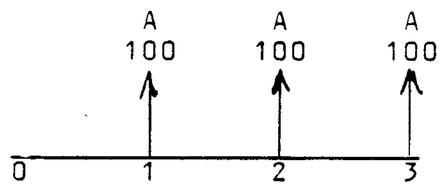
$$F = P(1+i)^n$$

$$F = \$100(1+0.10)^3 = \$133.10$$

The formula for finding the present value of a future sum is shown below.

$$P = F(1+i)^{-n}$$

A uniform series of cash flows is defined as a series of equal cash flows which occur over consecutive time periods. A cash flow diagram and a cash flow profile of a $100 uniform series for three years are shown below.



EOY	CF
0	0
1	100
2	100
3	100

The present value at time zero of the cash flow shown above could be determined by treating each value individually and using the formula $P = F(1+i)^{-n}$.

$$P_o = 100(1+i)^{-1} + 100(1+i)^{-2} + 100(1+i)^{-3}$$

The formulae for finding the present value of a uniform series and an equivalent uniform series for a present value are shown below.

$$P = A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] \qquad A = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

The present value at time zero of the cash flow shown above could also be determined by treating each value as part of a uniform sereis and using the formula shown above.

$$P_o = \$100\left[\frac{(1+i)^3-1}{i(1+i)^3}\right]$$

For any given interest rate, the present value will be the same.

Interest tables are provided which show the calculations for various combinations of interest rates (i) and interest periods (n). The column designations are shown below.

(F/Pi,n) reads find F given P at i% for n periods.

(P/Fi,n) reads find P given F at i% for n periods.

(P/Ai,n) reads find P given A at i% over n periods.

(A/Pi,n) reads find A given P at i% over n periods.

(F/Ai,n) reads find F given A at i% over n periods.

(A/Fi,n) reads find A given F at i% over n periods.

To find the present value of the cash flow diagramed on the previous page you could use either of the procedures shown below.

P_o = 100(P/Fi,1) + 100(P/Fi,2) + 100(P/Fi,3)

or

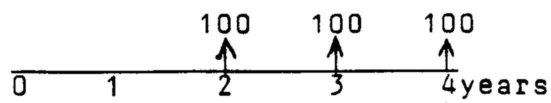
P_o = 100(P/Ai,3)

If you assume i equals 10% per year, then:

P_o = 100(P/A10,3) = 100(2.487) = 248.70

It is important to note that the formulae dealing with a uniform series were developed based on the fact that the beginning point of a uniform series occurs one time period prior to the first instance of the series and that the ending point of a uniform series occurs at the final instance of the series.

In the illustration shown below note that the factor 100(P/Ai,n) gives a single value at the beginning of the series, or at t=1. This value must then be multiplied by (P/Fi,1) to move the value from t=1 to t=0.



$$P_o = 100(P/Ai,3)(P/Fi,1)$$

The formulae for finding the future value of a uniform series and the equivalent uniform series of some future value are shown below.

$$F = A\left[\frac{(1+i)^n-1}{i}\right] \qquad A = F\left[\frac{i}{(1+i)^n-1}\right]$$

LOAN CALCULATIONS

There are three common types of loans which we must be aware of when engineering projects are being financed. In addition to the amount of the loan repayment, we should be interested in the amount of the interest portion of each payment as the interest portion is tax deductible.

In an "Interest Only Balloon Note", only interest payments are made on a periodic basis until the loan is due. When the loan is due a balloon payment consisting of the full principal amount plus the interest due for the final payment period is made.

A second type of loan requires equal principal payments to be made each payment period plus the interest due on the unpaid principal balance at the beginning of the payment period.

A third common type of loan is known as a Direct Reduction Loan. By using the uniform series formulae we are able to determine the payments and the associated principal and interest amounts for a direct reduction loan.

To find the periodic payment for a direct reduction loan the following formula is used.

$$A = P(A/Pi,n)$$

If you were to borrow $1,000 for a period of thirty years with annual payments and an interest rate of 8% per year, the annual payment would be found as shown below.

$$A = 1000(A/P8,30) = 1000(0.0888) = \$88.80 \text{ per year}$$

It is important to remember that the interest portion of the loan payment is a tax deductible expense. To find the amount of the interest, we would first find the principal portion by multiplying the periodic payment by the factor (P/Fi,n) where n equals the number of payments left in the life of the loan at the beginning of the payment period in question. For the loan discussed above the calculations for the principal portions of the first and last payments are shown below.

For the first payment: Principal = 88.80(P/F8,30) = $8.83

For the second payment: Principal = 88.80(P/F8,1) = $82.22

You can see from the preceeding calculations that the principal portion of a direct reduction loan decreases over time. The interest portion of the loan payment is equal to the total periodic payment less the principal portion

For the loan in question, the interest portion of the first payment is $88.80 - $8.83 = $79.97. The interest portion of the final payment is $88.80 - $82.22 = $6.58.

To further illustrate the effect that time can have on the value of money let us determine how much the cost of tuition might be for a college education beginning twenty years from now. We will assume the cost of college tuition will increase by 10% per year over the next twenty years.

The cost per $1000 of todays tuition cost will be 1000(F/P10,20) = $6,728.00

The annual payment necessary to accumulate the required funds twenty years from now is shown below for various interest rates.

Interest rate = 4% per year: A = $6728(A/F4,20) = $226.06 per year per thousand dollars of today's tuition cost.

Interest rate = 10% per year: A = $6728(A/F10,20) = $117.74 per year per thousand dollars of today's tuition cost.

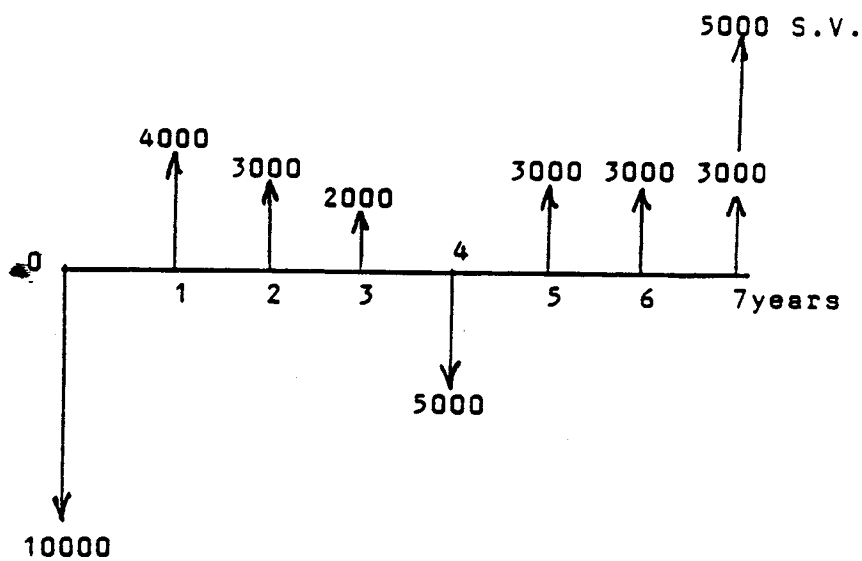
Interest rate = 30% per year: A = $6728(A/F30,20) = $10.70 per year per thousand dollars of today's tuition cost.

Bear in mind these values are for the first year cost only.

PROJECT ANALYSIS

An engineering project with a cash flow profile which may be considered typical will now be analyzed taking the time value of money into account. The time value of money will be considered to be the minimum attractive rate of return to the company. Five methods of analysis will be discussed.

TYPICAL CASH FLOW DIAGRAM



ASSUME i=10%

PRESENT WORTH

PW = -10000+4000(P/F10,1)+3000(P/F10,2)+2000(P/F10,3)
-5000(P/F10,4)+3000(P/A10,3)(P/F10,4)+5000(P/F10,7)
= 1865

FUTURE WORTH

FW = -10000(F/P10,7)+4000(F/P10,6)+3000(F/P10,5)+2000(P/F10,4)
-5000(F/P10,3)+3000(F/A10,3)+5000 = 3628

ANNUAL WORTH

AW = P(A/P10,7) = 1865(A/P10,7) = 384

or

AW = F(A/F10,7) = 3628(A/F10,7) = 384

INTERNAL RATE OF RETURN (IRR or ROR)

PW = 0 = -10000+4000(P/Fi,1)+3000(P/Fi,2)+2000(P/Fi,3)
-5000(P/Fi,4)+3000(P/Ai,3)(P/Fi,4)+5000(P/Fi,7)

at i=10%; PW = 1865
at i=12%; PW = 1049
at i=15%; PW = -3
therefore 12% < IRR < 15%
IRR = 14.99% by interpolation.

BENEFIT COST RATIO (B/C)

$$B/C = \frac{[4000(P/F10,1)+3000(P/F10,2)+2000(P/F10,3) +3000(P/A10,3)(P/F10,4)+5000(P/F10,7)}{10000+5000(P/F10,4)}$$

B/C = 1.14

A present worth, future worth, or annual worth which is equal to zero or is positive indicates a desirable project. A negative present worth, future worth, or annual worth indicates the project is not desirable. An internal rate of return which is equal to or greater than the minimum attractive rate of return (MARR) indicates a desirable project while an internal rate of return which is less than the minimum attractive rate of return indicates the project is not desirable. A benefit cost ratio which is equal to or greater than (1) indicates a desirable project while a benefit cost ratio which is less than (1) indicates the project is not desirable.

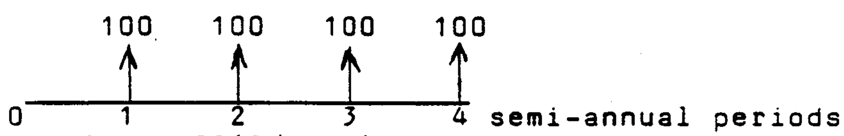
INTEREST COMPOUNDED MORE OFTEN THAN ANNUALLY

$$i_{eff} = (1+\frac{r}{m})^m-1 = (1+i)^m-1 = e^r-1$$

i_{eff} = effective annual interest rate

r = nominal annual interest rate
m = number of compounding periods per year
i = interest rate per interest period.

Find the present worth of the cash flow shown in the diagram below. The interest rate is 12% compounded quarterly



$PW = 100(P/Ai,4)$

Note that i must be the interest rate per semi-annual period.

$$i_{eff} = (1+\frac{0.12}{4})^4-1 = 0.1233 = (1+i)^2-1$$

i = 6.09% per semi-annual period

$PW = 100(P/A6.09,4)$

$$PW = 100\left[\frac{(1+0.0609)^4-1}{0.0609(1+0.0609)^4}\right]$$

$PW = 346$

CHOOSING AMONG ALTERNATIVE INVESTMENTS

When there is more than one project to be concerned with and we are being asked to choose the best among several alternative projects some adjustments must be made for the internal rate of return and benefit cost ratio analysis.

RANKING ALTERNATIVE INVESTMENT OPPORTUNITIES

PRESENT WORTH, OR ANNUAL WORTH, OR FUTURE WORTH

vs

INTERNAL RATE OF RETURN OR BENEFIT/COST RATIO

To illustrate the analysis we will consider the two projects with the cash flows profiled below.

EOY	CF(A)	CF(B)
0	-50000	-15000
1-20	6000	1850

MARR = 10%

PRESENT WORTH

$PW_{(A)} = -50000+6000(P/A10,20) = 1082$

$PW_{(B)} = -15000+1850(P/A10,20) = 750$

INTERNAL RATE OF RETURN

$PW_{(A)} = 0 = -50000+6000(P/Ai,20)$
$i = 10.35\%$

$PW_{(B)} = 0 = -15000+1850(P/Ai,20)$
$i = 10.78\%$

	PW	IRR
PROJECT A	1082	10.35%
PROJECT B	750	10.78%

WHICH IS BEST?

As can be seen from the previous example, the present worth analysis shows project A to be the best and the internal rate of return analysis shows project B to be the best. Because project A is actually the best we must make adjustments to the internal rate of return analysis to ensure we will make the right choice when choosing among alternatives. The same would hold true for the benefit/cost ratio analysis. We will adjust the analysis to use an incremental approach.

INCREMENTAL APPROACH TO IRR

The first step is to order the projects by initial cost, from lowest initial cost to highest initial cost.

In this case the order becomes B,A

The second step is to determine the incremental cash flow between the two lowest cost alternatives.

EOY	CF(A)	CF(B)	CF(A-B)
0	-50000	-15000	-35000
1-20	6000	1850	4150

$$PW_{(A-B)} = 0 = -35000+4150(P/Ai,20)$$

assume i = MARR = 10%

then PW = 331

A positive PW indicates the IRR $>$ MARR.
Therefore the incremental investment should be made.
Therefore we will eliminate the project with the lower cost or CF(B).
There are no other projects under consideration therefore we will choose the project that is left, or CF(A).

If there had been more than the two alternatives we would have then analyzed the incremental cash flow between CF(A) and the next higher cost alternative in the same manner. A negative PW would indicate the IRR $<$ MARR which would mean the incremental investment should not be made and the project with the higher cost would be eliminated.

DEALING WITH CHOOSING AMONG ALTERNATIVES WITH UNEQUAL LIVES

Ideally we should be analyzing projects with equal lives, but occasionally that is not possible. When projects have different lives a common method of analysis is the Annual Worth. The basic assumption under this condition is that there is a common life equal to the least common multiple of the lives involved, and that the cash flows of the projects repeat themselves until the least common multiple is reached.
The example shown below deals with two projects, one having a ten year life and the other having a five year life.

EOY	CF(A)	EOY	CF(B)
0	-10000	0	-5000
1-10	2000	1-5	1500

$AW_{(A)}$ = -10000(A/P10,10)+2000 = 3627

$AW_{(B)}$ = -5000(A/P10,5)+1500 = 2819

The above example shows the annual worths using the original lives of the alternatives.

The least common multiple of a five year life and a ten year life is ten years. The cash flow profile for project B over a ten year life with cash flows repeating is shown below. You can see that the annual worth of project B is the same using the five year life as above or the ten year life as below.

EOY	CF(B)
0	-5000
1-5	1500
5	-5000
6-10	1500

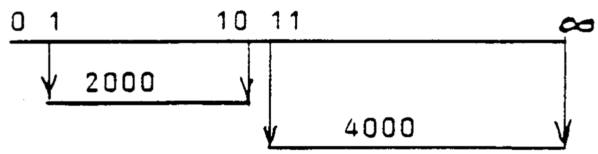
$AW_{(B)}$ = -5000(A/P10,10)+1500-15000(P/F10,5)(A/P10,10)

$AW_{(B)}$ = 2819

PROJECTS WITH PERPETUAL LIVES

$$\text{CAPITALIZED COST} = P = \frac{A}{i}$$

$$\text{CAPITAL RECOVERY COST} = A = Pi$$

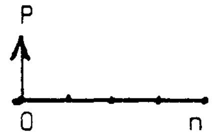
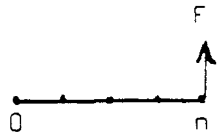
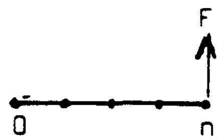
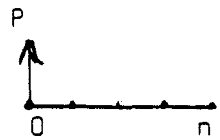
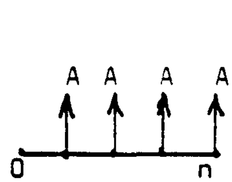
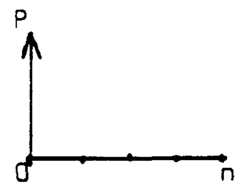
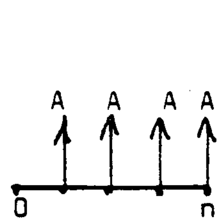
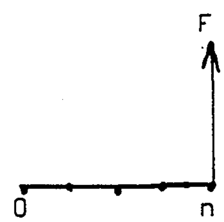
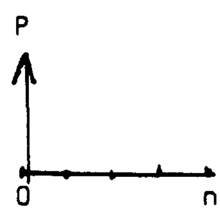
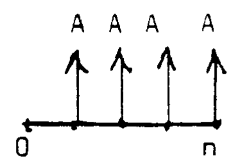
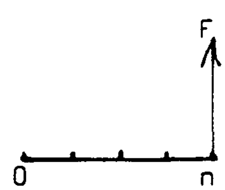
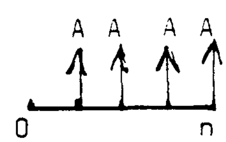


$$P = 5000 + \frac{4000}{0.10} - 2000(P/A10,10) = 32710$$

$$A = 5000(0.10) + 4000 - 2000(P/A10,10)(0.10) = 3271$$

P is a present value at time zero.

A is a uniform series over an infinitely long time.

MULTIPLY THIS	BY	THIS	TO CONVERT THIS	TO THIS
P	x	(F/Pi,n) $(1+i)^{n}$		
F	x	(P/Fi,n) $(1+i)^{-n}$		
A	x	(P/Ai,n) $\frac{(1+i)^{n}-1}{i(1+i)^{n}}$		
A	x	(F/Ai,n) $\frac{(1+i)^{n}-1}{i}$		
P	x	(A/Pi,n) $\frac{i(1+i)^{n}}{(1+i)^{n}-1}$		
F	x	(A/Fi,n) $\frac{i}{(1+i)^{n}-1}$		

1% Compound Interest Factors 1%

	SINGLE PAYMENT		UNIFORM PAYMENT SERIES				GRADIENT SERIES	
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth
n	Find F Given P F/P	Find P Given F P/F	Find A Given F A/F	Find A Given P A/P	Find F Given A F/A	Find P Given A P/A	Find A Given G A/G	Find P Given G P/G
1	1.010	.9901	1.0000	1.0100	1.000	.990	0	0
2	1.020	.9803	.4975	.5075	2.010	1.970	.498	.980
3	1.030	.9706	.3300	.3400	3.030	2.941	.993	2.921
4	1.041	.9610	.2463	.2563	4.060	3.902	1.488	5.804
5	1.051	.9515	.1960	.2060	5.101	4.853	1.980	9.610
6	1.062	.9420	.1625	.1725	6.152	5.795	2.471	14.321
7	1.072	.9327	.1386	.1486	7.214	6.728	2.960	19.917
8	1.083	.9235	.1207	.1307	8.286	7.652	3.448	26.381
9	1.094	.9143	.1067	.1167	9.369	8.566	3.934	33.696
10	1.105	.9053	.0956	.1056	10.462	9.471	4.418	41.843
11	1.116	.8963	.0865	.0965	11.567	10.368	4.901	50.807
12	1.127	.8874	.0788	.0888	12.683	11.255	5.381	60.569
13	1.138	.8787	.0724	.0824	13.809	12.134	5.861	71.113
14	1.149	.8700	.0669	.0769	14.947	13.004	6.338	82.422
15	1.161	.8613	.0621	.0721	16.097	13.865	6.814	94.481
16	1.173	.8528	.0579	.0679	17.258	14.718	7.289	107.273
17	1.184	.8444	.0543	.0643	18.430	15.562	7.761	120.783
18	1.196	.8360	.0510	.0610	19.615	16.398	8.232	134.996
19	1.208	.8277	.0481	.0581	20.811	17.226	8.702	149.895
20	1.220	.8195	.0454	.0554	22.019	18.046	9.169	165.466
21	1.232	.8114	.0430	.0530	23.239	18.857	9.635	181.695
22	1.245	.8034	.0409	.0509	24.472	19.660	10.100	198.566
23	1.257	.7954	.0389	.0489	25.716	20.456	10.563	216.066
24	1.270	.7876	.0371	.0471	26.973	21.243	11.024	234.180
25	1.282	.7798	.0354	.0454	28.243	22.023	11.483	252.894
26	1.295	.7720	.0339	.0439	29.526	22.795	11.941	272.196
27	1.308	.7644	.0324	.0424	30.821	23.560	12.397	292.070
28	1.321	.7568	.0311	.0411	32.129	24.316	12.852	312.505
29	1.335	.7493	.0299	.0399	33.450	25.066	13.304	333.486
30	1.348	.7419	.0287	.0387	34.785	25.808	13.756	355.002
36	1.431	.6989	.0232	.0332	43.077	30.108	16.428	494.621
40	1.489	.6717	.0205	.0305	48.886	32.835	18.178	596.856
48	1.612	.6203	.0163	.0263	61.223	37.974	21.598	820.146
50	1.645	.6080	.0155	.0255	64.463	39.196	22.436	879.418
52	1.678	.5961	.0148	.0248	67.769	40.394	23.269	939.918
60	1.817	.5504	.0122	.0222	81.670	44.955	26.533	1192.806
70	2.007	.4983	.0099	.0199	100.676	50.169	30.470	1528.647
72	2.047	.4885	.0096	.0196	104.710	51.150	31.239	1597.867
80	2.217	.4511	.0082	.0182	121.672	54.888	34.249	1879.877
84	2.307	.4335	.0077	.0177	130.672	56.648	35.717	2023.315
90	2.449	.4084	.0069	.0169	144.863	59.161	37.872	2240.567
96	2.599	.3847	.0063	.0163	159.927	61.528	39.973	2459.430
100	2.705	.3697	.0059	.0159	170.481	63.029	41.343	2605.776
104	2.815	.3553	.0055	.0155	181.464	64.471	42.688	2752.182
120	3.300	.3030	.0043	.0143	230.039	69.701	47.835	3334.115
240	10.893	.0918	.0010	.0110	989.255	90.819	75.739	6878.602
360	35.950	.0278	.0003	.0103	3494.694	97.218	89.699	8720.432
480	118.648	.0084	.0001	.0101	11764.773	99.157	95.920	9511.158

1½% Compound Interest Factors 1½%

	SINGLE PAYMENT		UNIFORM PAYMENT SERIES				GRADIENT SERIES	
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given G *A/G*	Find *P* Given G *P/G*
1	1.015	.9852	1.0000	1.0150	1.000	.985	0	0
2	1.030	.9707	.4963	.5113	2.015	1.956	.496	.971
3	1.046	.9563	.3284	.3434	3.045	2.912	.990	2.883
4	1.061	.9422	.2444	.2594	4.091	3.854	1.481	5.710
5	1.077	.9283	.1941	.2091	5.152	4.783	1.970	9.423
6	1.093	.9145	.1605	.1755	6.230	5.697	2.457	13.996
7	1.110	.9010	.1366	.1516	7.323	6.598	2.940	19.402
8	1.126	.8877	.1186	.1336	8.433	7.486	3.422	25.616
9	1.143	.8746	.1046	.1196	9.559	8.361	3.901	32.612
10	1.161	.8617	.0934	.1084	10.703	9.222	4.377	40.367
11	1.178	.8489	.0843	.0993	11.863	10.071	4.851	48.857
12	1.196	.8364	.0767	.0917	13.041	10.908	5.323	58.057
13	1.214	.8240	.0702	.0852	14.237	11.732	5.792	67.945
14	1.232	.8118	.0647	.0797	15.450	12.543	6.258	78.499
15	1.250	.7999	.0599	.0749	16.682	13.343	6.722	89.697
16	1.269	.7880	.0558	.0708	17.932	14.131	7.184	101.518
17	1.288	.7764	.0521	.0671	19.201	14.908	7.643	113.940
18	1.307	.7649	.0488	.0638	20.489	15.673	8.100	126.943
19	1.327	.7536	.0459	.0609	21.797	16.426	8.554	140.508
20	1.347	.7425	.0432	.0582	23.124	17.169	9.006	154.615
21	1.367	.7315	.0409	.0559	24.471	17.900	9.455	169.245
22	1.388	.7207	.0387	.0537	25.838	18.621	9.902	184.380
23	1.408	.7100	.0367	.0517	27.225	19.331	10.346	200.001
24	1.430	.6995	.0349	.0499	28.634	20.030	10.788	216.090
25	1.451	.6892	.0333	.0483	30.063	20.720	11.228	232.631
26	1.473	.6790	.0317	.0467	31.514	21.399	11.665	249.607
27	1.495	.6690	.0303	.0453	32.987	22.068	12.099	267.000
28	1.517	.6591	.0290	.0440	34.481	22.727	12.531	284.796
29	1.540	.6494	.0278	.0428	35.999	23.376	12.961	302.978
30	1.563	.6398	.0266	.0416	37.539	24.016	13.388	321.531
36	1.709	.5851	.0212	.0362	47.276	27.661	15.901	439.830
40	1.814	.5513	.0184	.0334	54.268	29.916	17.528	524.357
48	2.043	.4894	.0144	.0294	69.565	34.043	20.667	703.546
50	2.105	.4750	.0136	.0286	73.683	35.000	21.428	749.964
52	2.169	.4611	.0128	.0278	77.925	35.929	22.179	796.877
60	2.443	.4093	.0104	.0254	96.215	39.380	25.093	988.167
70	2.835	.3527	.0082	.0232	122.364	43.155	28.529	1231.166
72	2.921	.3423	.0078	.0228	128.077	43.845	29.189	1279.794
80	3.291	.3039	.0065	.0215	152.711	46.407	31.742	1473.074
84	3.493	.2863	.0060	.0210	166.173	47.579	32.967	1568.514
90	3.819	.2619	.0053	.0203	187.930	49.210	34.740	1709.544
96	4.176	.2395	.0047	.0197	211.720	50.702	36.438	1847.473
100	4.432	.2256	.0044	.0194	228.803	51.625	37.530	1937.451
104	4.704	.2126	.0040	.0190	246.934	52.494	38.589	2025.705
120	5.969	.1675	.0030	.0180	331.288	55.498	42.519	2359.711
240	35.633	.0281	.0004	.0154	2308.854	64.796	59.737	3870.691
360	212.704	.0047	.0001	.0151	14113.586	66.353	64.966	4310.716
480	1269.698	.0008		.0150	84579.837	66.614	66.288	4415.741

2% Compound Interest Factors 2%

	SINGLE PAYMENT		UNIFORM PAYMENT SERIES				GRADIENT SERIES	
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given G *A/G*	Find *P* Given G *P/G*
1	1.020	.9804	1.0000	1.0200	1.000	.980	0	0
2	1.040	.9612	.4950	.5150	2.020	1.942	.495	.961
3	1.061	.9423	.3268	.3468	3.060	2.884	.987	2.846
4	1.082	.9238	.2426	.2626	4.122	3.808	1.475	5.617
5	1.104	.9057	.1922	.2122	5.204	4.713	1.960	9.240
6	1.126	.8880	.1585	.1785	6.308	5.601	2.442	13.680
7	1.149	.8706	.1345	.1545	7.434	6.472	2.921	18.903
8	1.172	.8535	.1165	.1365	8.583	7.325	3.396	24.878
9	1.195	.8368	.1025	.1225	9.755	8.162	3.868	31.572
10	1.219	.8203	.0913	.1113	10.950	8.983	4.337	38.955
11	1.243	.8043	.0822	.1022	12.169	9.787	4.802	46.998
12	1.268	.7885	.0746	.0946	13.412	10.575	5.264	55.671
13	1.294	.7730	.0681	.0881	14.680	11.348	5.723	64.948
14	1.319	.7579	.0626	.0826	15.974	12.106	6.179	74.800
15	1.346	.7430	.0578	.0778	17.293	12.849	6.631	85.202
16	1.373	.7284	.0537	.0737	18.639	13.578	7.080	96.129
17	1.400	.7142	.0500	.0700	20.012	14.292	7.526	107.555
18	1.428	.7002	.0467	.0667	21.412	14.992	7.968	119.458
19	1.457	.6864	.0438	.0638	22.841	15.678	8.407	131.814
20	1.486	.6730	.0412	.0612	24.297	16.351	8.843	144.600
21	1.516	.6598	.0388	.0588	25.783	17.011	9.276	157.796
22	1.546	.6468	.0366	.0566	27.299	17.658	9.705	171.379
23	1.577	.6342	.0347	.0547	28.845	18.292	10.132	185.331
24	1.608	.6217	.0329	.0529	30.422	18.914	10.555	199.630
25	1.641	.6095	.0312	.0512	32.030	19.523	10.974	214.259
26	1.673	.5976	.0297	.0497	33.671	20.121	11.391	229.199
27	1.707	.5859	.0283	.0483	35.344	20.707	11.804	244.431
28	1.741	.5744	.0270	.0470	37.051	21.281	12.214	259.939
29	1.776	.5631	.0258	.0458	38.792	21.844	12.621	275.706
30	1.811	.5521	.0246	.0446	40.568	22.396	13.025	291.716
36	2.040	.4902	.0192	.0392	51.994	25.489	15.381	392.040
40	2.208	.4529	.0166	.0366	60.402	27.355	16.889	461.993
48	2.587	.3865	.0126	.0326	79.354	30.673	19.756	605.966
50	2.692	.3715	.0118	.0318	84.579	31.424	20.442	642.361
52	2.800	.3571	.0111	.0311	90.016	32.145	21.116	678.785
60	3.281	.3048	.0088	.0288	114.052	34.761	23.696	823.698
70	4.000	.2500	.0067	.0267	149.978	37.499	26.663	999.834
72	4.161	.2403	.0063	.0263	158.057	37.984	27.223	1034.056
80	4.875	.2051	.0052	.0252	193.772	39.745	29.357	1166.787
84	5.277	.1895	.0047	.0247	213.867	40.526	30.362	1230.419
90	5.943	.1683	.0040	.0240	247.157	41.587	31.793	1322.170
96	6.693	.1494	.0035	.0235	284.647	42.529	33.137	1409.297
100	7.245	.1380	.0032	.0232	312.232	43.098	33.986	1464.753
104	7.842	.1275	.0029	.0229	342.092	43.624	34.799	1518.087
120	10.765	.0929	.0020	.0220	488.258	45.355	37.711	1710.416
240	115.889	.0086	.0002	.0202	5744.437	49.569	47.911	2374.880
360	1247.561	.0008		.0200	62328.056	49.960	49.711	2483.568
480	13430.199	.0001		.0200	671459.945	49.996	49.964	2498.027

5% Compound Interest Factors 5%

	SINGLE PAYMENT		UNIFORM PAYMENT SERIES				GRADIENT SERIES	
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given G *A/G*	Find *P* Given G *P/G*
1	1.050	.9524	1.0000	1.0500	1.000	.952	0	0
2	1.102	.9070	.4878	.5378	2.050	1.859	.488	.907
3	1.158	.8638	.3172	.3672	3.152	2.723	.967	2.635
4	1.216	.8227	.2320	.2820	4.310	3.546	1.439	5.103
5	1.276	.7835	.1810	.2310	5.526	4.329	1.903	8.237
6	1.340	.7462	.1470	.1970	6.802	5.076	2.358	11.968
7	1.407	.7107	.1228	.1728	8.142	5.786	2.805	16.232
8	1.477	.6768	.1047	.1547	9.549	6.463	3.245	20.970
9	1.551	.6446	.0907	.1407	11.027	7.108	3.676	26.127
10	1.629	.6139	.0795	.1295	12.578	7.722	4.099	31.652
11	1.710	.5847	.0704	.1204	14.207	8.306	4.514	37.499
12	1.796	.5568	.0628	.1128	15.917	8.863	4.922	43.624
13	1.886	.5303	.0565	.1065	17.713	9.394	5.322	49.988
14	1.980	.5051	.0510	.1010	19.599	9.899	5.713	56.554
15	2.079	.4810	.0463	.0963	21.579	10.380	6.097	63.288
16	2.183	.4581	.0423	.0923	23.657	10.838	6.474	70.160
17	2.292	.4363	.0387	.0887	25.840	11.274	6.842	77.140
18	2.407	.4155	.0355	.0855	28.132	11.690	7.203	84.204
19	2.527	.3957	.0327	.0827	30.539	12.085	7.557	91.328
20	2.653	.3769	.0302	.0802	33.066	12.462	7.903	98.488
21	2.786	.3589	.0280	.0780	35.719	12.821	8.242	105.667
22	2.925	.3418	.0260	.0760	38.505	13.163	8.573	112.846
23	3.072	.3256	.0241	.0741	41.430	13.489	8.897	120.009
24	3.225	.3101	.0225	.0725	44.502	13.799	9.214	127.140
25	3.386	.2953	.0210	.0710	47.727	14.094	9.524	134.228
26	3.556	.2812	.0196	.0696	51.113	14.375	9.827	141.259
27	3.733	.2678	.0183	.0683	54.669	14.643	10.122	148.223
28	3.920	.2551	.0171	.0671	58.403	14.898	10.411	155.110
29	4.116	.2429	.0160	.0660	62.323	15.141	10.694	161.913
30	4.322	.2314	.0151	.0651	66.439	15.372	10.969	168.623
31	4.538	.2204	.0141	.0641	70.761	15.593	11.238	175.233
32	4.765	.2099	.0133	.0633	75.299	15.803	11.501	181.739
33	5.003	.1999	.0125	.0625	80.064	16.003	11.757	188.135
34	5.253	.1904	.0118	.0618	85.067	16.193	12.006	194.417
35	5.516	.1813	.0111	.0611	90.320	16.374	12.250	200.581
40	7.040	.1420	.0083	.0583	120.800	17.159	13.377	229.545
45	8.985	.1113	.0063	.0563	159.700	17.774	14.364	255.315
50	11.467	.0872	.0048	.0548	209.348	18.256	15.223	277.915
55	14.636	.0683	.0037	.0537	272.713	18.633	15.966	297.510
60	18.679	.0535	.0028	.0528	353.584	18.929	16.606	314.343
65	23.840	.0419	.0022	.0522	456.798	19.161	17.154	328.691
70	30.426	.0329	.0017	.0517	588.529	19.343	17.621	340.841
75	38.833	.0258	.0013	.0513	756.654	19.485	18.018	351.072
80	49.561	.0202	.0010	.0510	971.229	19.596	18.353	359.646
85	63.254	.0158	.0008	.0508	1245.087	19.684	18.635	366.801
90	80.730	.0124	.0006	.0506	1594.607	19.752	18.871	372.749
95	103.035	.0097	.0005	.0505	2040.694	19.806	19.069	377.677
100	131.501	.0076	.0004	.0504	2610.025	19.848	19.234	381.749

10% Compound Interest Factors 10%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given *G* *A/G*	Find *P* Given *G* *P/G*	*n*
1	1.100	.9091	1.0000	1.1000	1.000	0.909	0	0	1
2	1.210	.8264	.4762	.5762	2.100	1.736	0.476	0.826	2
3	1.331	.7513	.3021	.4021	3.310	2.487	0.937	2.329	3
4	1.464	.6830	.2155	.3155	4.641	3.170	1.381	4.378	4
5	1.611	.6209	.1638	.2638	6.105	3.791	1.810	6.862	5
6	1.772	.5645	.1296	.2296	7.716	4.355	2.224	9.684	6
7	1.949	.5132	.1054	.2054	9.487	4.868	2.622	12.763	7
8	2.144	.4665	.0874	.1874	11.436	5.335	3.004	16.029	8
9	2.358	.4241	.0736	.1736	13.579	5.759	3.372	19.421	9
10	2.594	.3855	.0627	.1627	15.937	6.145	3.725	22.891	10
11	2.853	.3505	.0540	.1540	18.531	6.495	4.064	26.396	11
12	3.138	.3186	.0468	.1468	21.384	6.814	4.388	29.901	12
13	3.452	.2897	.0408	.1408	24.523	7.103	4.699	33.377	13
14	3.797	.2633	.0357	.1357	27.975	7.367	4.996	36.801	14
15	4.177	.2394	.0315	.1315	31.772	7.606	5.279	40.152	15
16	4.595	.2176	.0278	.1278	35.950	7.824	5.549	43.416	16
17	5.054	.1978	.0247	.1247	40.545	8.022	5.807	46.582	17
18	5.560	.1799	.0219	.1219	45.599	8.201	6.053	49.640	18
19	6.116	.1635	.0195	.1195	51.159	8.365	6.286	52.583	19
20	6.728	.1486	.0175	.1175	57.275	8.514	6.508	55.407	20
21	7.400	.1351	.0156	.1156	64.003	8.649	6.719	58.110	21
22	8.140	.1228	.0140	.1140	71.403	8.772	6.919	60.689	22
23	8.954	.1117	.0126	.1126	79.543	8.883	7.108	63.146	23
24	9.850	.1015	.0113	.1113	88.497	8.985	7.288	65.481	24
25	10.835	.0923	.0102	.1102	98.347	9.077	7.458	67.696	25
26	11.918	.0839	.00916	.1092	109.182	9.161	7.619	69.794	26
27	13.110	.0763	.00826	.1083	121.100	9.237	7.770	71.777	27
28	14.421	.0693	.00745	.1075	134.210	9.307	7.914	73.650	28
29	15.863	.0630	.00673	.1067	148.631	9.370	8.049	75.415	29
30	17.449	.0573	.00608	.1061	164.494	9.427	8.176	77.077	30
31	19.194	.0521	.00550	.1055	181.944	9.479	8.296	78.640	31
32	21.114	.0474	.00497	.1050	201.138	9.526	8.409	80.108	32
33	23.225	.0431	.00450	.1045	222.252	9.569	8.515	81.486	33
34	25.548	.0391	.00407	.1041	245.477	9.609	8.615	82.777	34
35	28.102	.0356	.00369	.1037	271.025	9.644	8.709	83.987	35
40	45.259	.0221	.00226	.1023	442.593	9.779	9.096	88.953	40
45	72.891	.0137	.00139	.1014	718.905	9.863	9.374	92.454	45
50	117.391	.00852	.00086	.1009	1 163.9	9.915	9.570	94.889	50
55	189.059	.00529	.00053	.1005	1 880.6	9.947	9.708	96.562	55
60	304.482	.00328	.00033	.1003	3 034.8	9.967	9.802	97.701	60
65	490.371	.00204	.00020	.1002	4 893.7	9.980	9.867	98.471	65
70	789.748	.00127	.00013	.1001	7 887.5	9.987	9.911	98.987	70
75	1 271.9	.00079	.00008	.1001	12 709.0	9.992	9.941	99.332	75
80	2 048.4	.00049	.00005	.1000	20 474.0	9.995	9.961	99.561	80
85	3 299.0	.00030	.00003	.1000	32 979.7	9.997	9.974	99.712	85
90	5 313.0	.00019	.00002	.1000	53 120.3	9.998	9.983	99.812	90
95	8 556.7	.00012	.00001	.1000	85 556.9	9.999	9.989	99.877	95
100	13 780.6	.00007	.00001	.1000	137 796.3	9.999	9.993	99.920	100

12% Compound Interest Factors 12%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth	
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given *G* *A/G*	Find *P* Given *G* *P/G*	*n*
1	1.120	.8929	1.0000	1.1200	1.000	0.893	0	0	1
2	1.254	.7972	.4717	.5917	2.120	1.690	0.472	0.797	2
3	1.405	.7118	.2963	.4163	3.374	2.402	0.925	2.221	3
4	1.574	.6355	.2092	.3292	4.779	3.037	1.359	4.127	4
5	1.762	.5674	.1574	.2774	6.353	3.605	1.775	6.397	5
6	1.974	.5066	.1232	.2432	8.115	4.111	2.172	8.930	6
7	2.211	.4523	.0991	.2191	10.089	4.564	2.551	11.644	7
8	2.476	.4039	.0813	.2013	12.300	4.968	2.913	14.471	8
9	2.773	.3606	.0677	.1877	14.776	5.328	3.257	17.356	9
10	3.106	.3220	.0570	.1770	17.549	5.650	3.585	20.254	10
11	3.479	.2875	.0484	.1684	20.655	5.938	3.895	23.129	11
12	3.896	.2567	.0414	.1614	24.133	6.194	4.190	25.952	12
13	4.363	.2292	.0357	.1557	28.029	6.424	4.468	28.702	13
14	4.887	.2046	.0309	.1509	32.393	6.628	4.732	31.362	14
15	5.474	.1827	.0268	.1468	37.280	6.811	4.980	33.920	15
16	6.130	.1631	.0234	.1434	42.753	6.974	5.215	36.367	16
17	6.866	.1456	.0205	.1405	48.884	7.120	5.435	38.697	17
18	7.690	.1300	.0179	.1379	55.750	7.250	5.643	40.908	18
19	8.613	.1161	.0158	.1358	63.440	7.366	5.838	42.998	19
20	9.646	.1037	.0139	.1339	72.052	7.469	6.020	44.968	20
21	10.804	.0926	.0122	.1322	81.699	7.562	6.191	46.819	21
22	12.100	.0826	.0108	.1308	92.503	7.645	6.351	48.554	22
23	13.552	.0738	.00956	.1296	104.603	7.718	6.501	50.178	23
24	15.179	.0659	.00846	.1285	118.155	7.784	6.641	51.693	24
25	17.000	.0588	.00750	.1275	133.334	7.843	6.771	53.105	25
26	19.040	.0525	.00665	.1267	150.334	7.896	6.892	54.418	26
27	21.325	.0469	.00590	.1259	169.374	7.943	7.005	55.637	27
28	23.884	.0419	.00524	.1252	190.699	7.984	7.110	56.767	28
29	26.750	.0374	.00466	.1247	214.583	8.022	7.207	57.814	29
30	29.960	.0334	.00414	.1241	241.333	8.055	7.297	58.782	30
31	33.555	.0298	.00369	.1237	271.293	8.085	7.381	59.676	31
32	37.582	.0266	.00328	.1233	304.848	8.112	7.459	60.501	32
33	42.092	.0238	.00292	.1229	342.429	8.135	7.530	61.261	33
34	47.143	.0212	.00260	.1226	384.521	8.157	7.596	61.961	34
35	52.800	.0189	.00232	.1223	431.663	8.176	7.658	62.605	35
40	93.051	.0107	.00130	.1213	767.091	8.244	7.899	65.116	40
45	163.988	.00610	.00074	.1207	1 358.2	8.283	8.057	66.734	45
50	289.002	.00346	.00042	.1204	2 400.0	8.304	8.160	67.762	50
55	509.321	.00196	.00024	.1202	4 236.0	8.317	8.225	68.408	55
60	897.597	.00111	.00013	.1201	7 471.6	8.324	8.266	68.810	60
65	1 581.9	.00063	.00008	.1201	13 173.9	8.328	8.292	69.058	65
70	2 787.8	.00036	.00004	.1200	23 223.3	8.330	8.308	69.210	70
75	4 913.1	.00020	.00002	.1200	40 933.8	8.332	8.318	69.303	75
80	8 658.5	.00012	.00001	.1200	72 145.7	8.332	8.324	69.359	80
85	15 259.2	.00007	.00001	.1200	127 151.7	8.333	8.328	69.393	85
90	26 891.9	.00004		.1200	224 091.1	8.333	8.330	69.414	90
95	47 392.8	.00002		.1200	394 931.4	8.333	8.331	69.426	95
100	83 522.3	.00001		.1200	696 010.5	8.333	8.332	69.434	100

18% Compound Interest Factors 18%

	SINGLE PAYMENT		UNIFORM PAYMENT SERIES				GRADIENT SERIES	
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth
n	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given G *A/G*	Find *P* Given G *P/G*
1	1.180	.8475	1.0000	1.1800	1.000	.847	0	0
2	1.392	.7182	.4587	.6387	2.180	1.566	.459	.718
3	1.643	.6086	.2799	.4599	3.572	2.174	.890	1.935
4	1.939	.5158	.1917	.3717	5.215	2.690	1.295	3.483
5	2.288	.4371	.1398	.3198	7.154	3.127	1.673	5.231
6	2.700	.3704	.1059	.2859	9.442	3.498	2.025	7.083
7	3.185	.3139	.0824	.2624	12.142	3.812	2.353	8.967
8	3.759	.2660	.0652	.2452	15.327	4.078	2.656	10.829
9	4.435	.2255	.0524	.2324	19.086	4.303	2.936	12.633
10	5.234	.1911	.0425	.2225	23.521	4.494	3.194	14.352
11	6.176	.1619	.0348	.2148	28.755	4.656	3.430	15.972
12	7.288	.1372	.0286	.2086	34.931	4.793	3.647	17.481
13	8.599	.1163	.0237	.2037	42.219	4.910	3.845	18.877
14	10.147	.0985	.0197	.1997	50.818	5.008	4.025	20.158
15	11.974	.0835	.0164	.1964	60.965	5.092	4.189	21.327
16	14.129	.0708	.0137	.1937	72.939	5.162	4.337	22.389
17	16.672	.0600	.0115	.1915	87.068	5.222	4.471	23.348
18	19.673	.0508	.0096	.1896	103.740	5.273	4.592	24.212
19	23.214	.0431	.0081	.1881	123.414	5.316	4.700	24.988
20	27.393	.0365	.0068	.1868	146.628	5.353	4.798	25.681
21	32.324	.0309	.0057	.1857	174.021	5.384	4.885	26.300
22	38.142	.0262	.0048	.1848	206.345	5.410	4.963	26.851
23	45.008	.0222	.0041	.1841	244.487	5.432	5.033	27.339
24	53.109	.0188	.0035	.1835	289.494	5.451	5.095	27.772
25	62.669	.0160	.0029	.1829	342.603	5.467	5.150	28.155
26	73.949	.0135	.0025	.1825	405.272	5.480	5.199	28.494
27	87.260	.0115	.0021	.1821	479.221	5.492	5.243	28.791
28	102.967	.0097	.0018	.1818	566.481	5.502	5.281	29.054
29	121.501	.0082	.0015	.1815	669.447	5.510	5.315	29.284
30	143.371	.0070	.0013	.1813	790.948	5.517	5.345	29.486
31	169.177	.0059	.0011	.1811	934.319	5.523	5.371	29.664
32	199.629	.0050	.0009	.1809	1103.496	5.528	5.394	29.819
33	235.563	.0042	.0008	.1808	1303.125	5.532	5.415	29.955
34	277.964	.0036	.0006	.1806	1538.688	5.536	5.433	30.074
35	327.997	.0030	.0006	.1806	1816.652	5.539	5.449	30.177
40	750.378	.0013	.0002	.1802	4163.213	5.548	5.502	30.527
45	1716.684	.0006	.0001	.1801	9531.577	5.552	5.529	30.701
50	3927.357	.0003		.1800	21813.094	5.554	5.543	30.786
55	8984.841	.0001		.1800	49910.228	5.555	5.549	30.827
60	20555.140			.1800	114189.666	5.555	5.553	30.846
65	47025.181			.1800	261245.449	5.555	5.554	30.856
70	107582.222			.1800	597673.458	5.556	5.555	30.860
75	246122.064			.1800	1367339.243	5.556	5.555	30.862
80	563067.660			.1800	3128148.114	5.556	5.555	30.863
85	1288162.408			.1800	7156452.266	5.556	5.555	30.864

25% Compound Interest Factors 25%

	SINGLE PAYMENT		UNIFORM PAYMENT SERIES				GRADIENT SERIES	
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth
	Find *F* Given *P* *F/P*	Find *P* Given *F* *P/F*	Find *A* Given *F* *A/F*	Find *A* Given *P* *A/P*	Find *F* Given *A* *F/A*	Find *P* Given *A* *P/A*	Find *A* Given *G* *A/G*	Find *P* Given *G* *P/G*
n								
1	1.250	.8000	1.0000	1.2500	1.000	.800	0	0
2	1.563	.6400	.4444	.6944	2.250	1.440	.444	.640
3	1.953	.5120	.2623	.5123	3.813	1.952	.852	1.664
4	2.441	.4096	.1734	.4234	5.766	2.362	1.225	2.893
5	3.052	.3277	.1218	.3718	8.207	2.689	1.563	4.204,
6	3.815	.2621	.0888	.3388	11.259	2.951	1.868	5.514
7	4.768	.2097	.0663	.3163	15.073	3.161	2.142	6.773
8	5.960	.1678	.0504	.3004	19.842	3.329	2.387	7.947
9	7.451	.1342	.0388	.2888	25.802	3.463	2.605	9.021
10	9.313	.1074	.0301	.2801	33.253	3.571	2.797	9.987
11	11.642	.0859	.0235	.2735	42.566	3.656	2.966	10.864
12	14.552	.0687	.0184	.2684	54.208	3.725	3.115	11.602
13	18.190	.0550	.0145	.2645	68.760	3.780	3.244	12.262
14	22.737	.0440	.0115	.2615	86.949	3.824	3.356	12.833
15	28.422	.0352	.0091	.2591	109.687	3.859	3.453	13.326
16	35.527	.0281	.0072	.2572	138.109	3.887	3.537	13.748
17	44.409	.0225	.0058	.2558	173.636	3.910	3.608	14.108
18	55.511	.0180	.0046	.2546	218.045	3.928	3.670	14.415
19	69.389	.0144	.0037	.2537	273.556	3.942	3.722	14.674
20	86.736	.0115	.0029	.2529	342.945	3.954	3.767	14.893
21	108.420	.0092	.0023	.2523	429.681	3.963	3.805	15.078
22	135.525	.0074	.0019	.2519	538.101	3.970	3.836	15.233
23	169.407	.0059	.0015	.2515	673.626	3.976	3.863	15.362
24	211.758	.0047	.0012	.2512	843.033	3.981	3.886	15.471
25	264.698	.0038	.0009	.2509	1054.791	3.985	3.905	15.562
26	330.872	.0030	.0008	.2508	1319.489	3.988	3.921	15.637
27	413.590	.0024	.0006	.2506	1650.361	3.990	3.935	15.700
28	516.988	.0019	.0005	.2505	2063.952	3.992	3.946	15.752
29	646.235	.0015	.0004	.2504	2580.939	3.994	3.955	15.796
30	807.794	.0012	.0003	.2503	3227.174	3.995	3.963	15.832
31	1009.742	.0010	.0002	.2502	4034.968	3.996	3.969	15.861
32	1262.177	.0008	.0002	.2502	5044.710	3.997	3.975	15.886
33	1577.722	.0006	.0002	.2502	6306.887	3.997	3.979	15.906
34	1972.152	.0005	.0001	.2501	7884.609	3.998	3.983	15.923
35	2465.190	.0004	.0001	.2501	9856.761	3.998	3.986	15.937
40	7523.164	.0001		.2500	30088.655	3.999	3.995	15.977
45	22958.874			.2500	91831.496	4.000	3.998	15.991
50	70064.923			.2500	280255.693	4.000	3.999	15.997
55	213821.177			.2500	855280.707	4.000	4.000	15.999
60	652530.447			.2500	2610117.787	4.000	4.000	16.000