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General Report No. 2

A MAGNITUDE-FREQUENCY-AREA RELATION FOR FLOODS IN ARIZONA

A STUDY TO ADVANCE THE METHODOLOGY OF ASSESSING THE VULNERABILITY OF BRIDGES TO FLOODS FOR THE ARIZONA DEPARTMENT OF TRANSPORTATION

by

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The Engineering Experiment Station
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January 1980

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If the vulnerability of a bridge to floods is to be assessed, it is necessary to estimate the floods which may occur at that bridge site in the nominal remaining useful life of the bridge. If the calculations show that the bridge would be destroyed or severely damaged in a 10-year flood, it should be considered to be highly vulnerable. Although there is a chance that a 10-year flood will not occur next year, or the year after, or in the next 25 years, there is also a chance that it will occur next year, or several times in 25 years - destroying not only the existing bridge but also its replacements (unless someone changes the design).

It is readily apparent that a bridge with a nominal life of 25 years which can only withstand a 10-yr flood is just sitting there waiting for a disaster to happen. It is not as obvious whether a bridge which can withstand a 50-yr or 100-yr flood is or is not an acceptable risk. There is a good chance that the "rare" flood will never happen during the life of the bridge, but there is also a good chance that it will. From an economic standpoint, the determining factor is whether the cost of "insuring" that the bridge will probably not fail exceeds the product of the loss if the bridge fails and the probability of failure in its lifetime.

Since it is not possible to design for a flood of infinite magnitude, it is useful to define a "maximum" flood. Three distinctions are considered here. The maximum <u>possible</u> flood is a flood of such magnitude that anything greater cannot be accepted as possible when storm intensity

and runoff are considered. Such a flood would be considered to have zero probability. The maximum probable flood is a flood of such magnitude that it is possible but very unlikely, that is, its probability is very small but not zero. Different persons would assign different probability values in their definitions. Both of these concepts suggest very large floods for which there is really no evidence. The maximum expected flood is a flood of a magnitude that has happened in the general region and has been "measured". This is the concept adopted here. The term "maximum" is intended to suggest that designing for a greater flood cannot be justified by the economics at a particular bridge site. Any additional expenditure should be considered to be in competition with other safety features of the highway system.

It is desirable, then, to obtain a relation which will provide reasonable estimates of the magnitude of the maximum expected flood and the magnitude of floods of various frequencies for any bridge site in the State of Arizona. The accuracy of the estimate of the 10-yr flood is not of very great importance because a bridge which would fall down in a 20-yr flood would not be acceptable, and a bridge which would fail in a 5-yr flood would be even less acceptable. Much the same argument can be made about the estimate for the "rare" 50-year or 100-year flood except in cases where the cost of making the bridge safe is about the same as the probable loss—then an accurate estimate is greatly to be desired. Because most, if not all, bridges should be designed for the maximum expected flood just on the basis of the risk of loss of the bridge, this is the flood for which an accurate estimate is most needed. Unfortunately, it is also the most difficult to estimate, for what is the maximum flood magnitude that can be expected without stretching the imagination?

Since other simple flood prediction methods for the State of Arizona seem to reject or ignore the concept of a maximum flood, a fresh look at the flood experience of the State appears to be appropriate. The United States Geological Survey (USGS) has compiled about 3500 station years of record for 143 streamflow gaging stations around the State of Arizona (1). The length of record for each site varies from 6 to 69 years. The records are taken from the period 1911 to 1975. Not all the records are continuous.

This investigation used these USGS records plus records for 11 stations on the Walnut Gulch watershed of the Southwest Watershed Research Center near Tombstone. Three records for the Colorado River mainstem have not been considered because of its unique nature. Also, 38 isolated historic peaks have been deleted from the frequency analysis to avoid biasing the rare events.

Flood records for Arizona can hardly be considered adequate. This statement is not meant to be critical of the USGS. There is an enormous area to cover, most of which was economically insignificant until recent years. Also, annual water yield, its distribution within the year and variation between years tended to have greater interest to water users than flood peaks. Therefore it should be assumed that the USGS has done as well as it could with its available financial and manpower budgets. There are not enough station years of record for accurate flood peak prediction. There will not be enough in our lifetime.

The numerical values of peak flows are open to question. The problem of estimating a flow from a gaged depth is not simple. Its solution involves correct application of the principles of hydraulics and sediment transport, and good judgment concerning what was probably happening in the

neighborhood of the gaging station at the time of the event. Few, or perhaps none, of the greatest floods were measured with a current meter and recording depth gage. Seldom was a competent hydraulic engineer present to actually observe the peak flow—or even any part of the event. It is reasonable to expect that the person estimating the flow from the depth made conservative assumptions that tend to make the flow estimate larger. The larger the flood, the greater is the overestimation of the flood. This would seem to be true especially for the isolated historic peaks. Also, crest gage readings tend to be too high because waves or presence of brush can raise the high water mark, which adds some fraction of the velocity head to the depth. Large floods can wipe out a gaging station or its recording equipment, and someone must estimate the flow from high water marks. Thus estimates of flow get duly recorded in the books to be accepted as true by all who follow.

The methodology employed here is intentionally unconventional, possibly unique, and is intended to present a different view of the problem of flood prediction that could lead to new understanding and insight. It is based on the entire recorded flood experience of Arizona. Figure 1 shows 3604 station years of record of maximum annual flows for 151 gaging stations throughout the state. Thirty-eight isolated historic peaks are not shown because they would bias the analysis in favor of larger flows. Since this is a raster plot made by computer, some points may be lost due to the presence of grid lines. Each point may be compared with the corresponding information in the USGS Statistical Summaries of Arizona Steamflow Data. Even though this information may be questioned for the reasons stated above, all data presumably based on gaging measurements is accepted as is. Figure

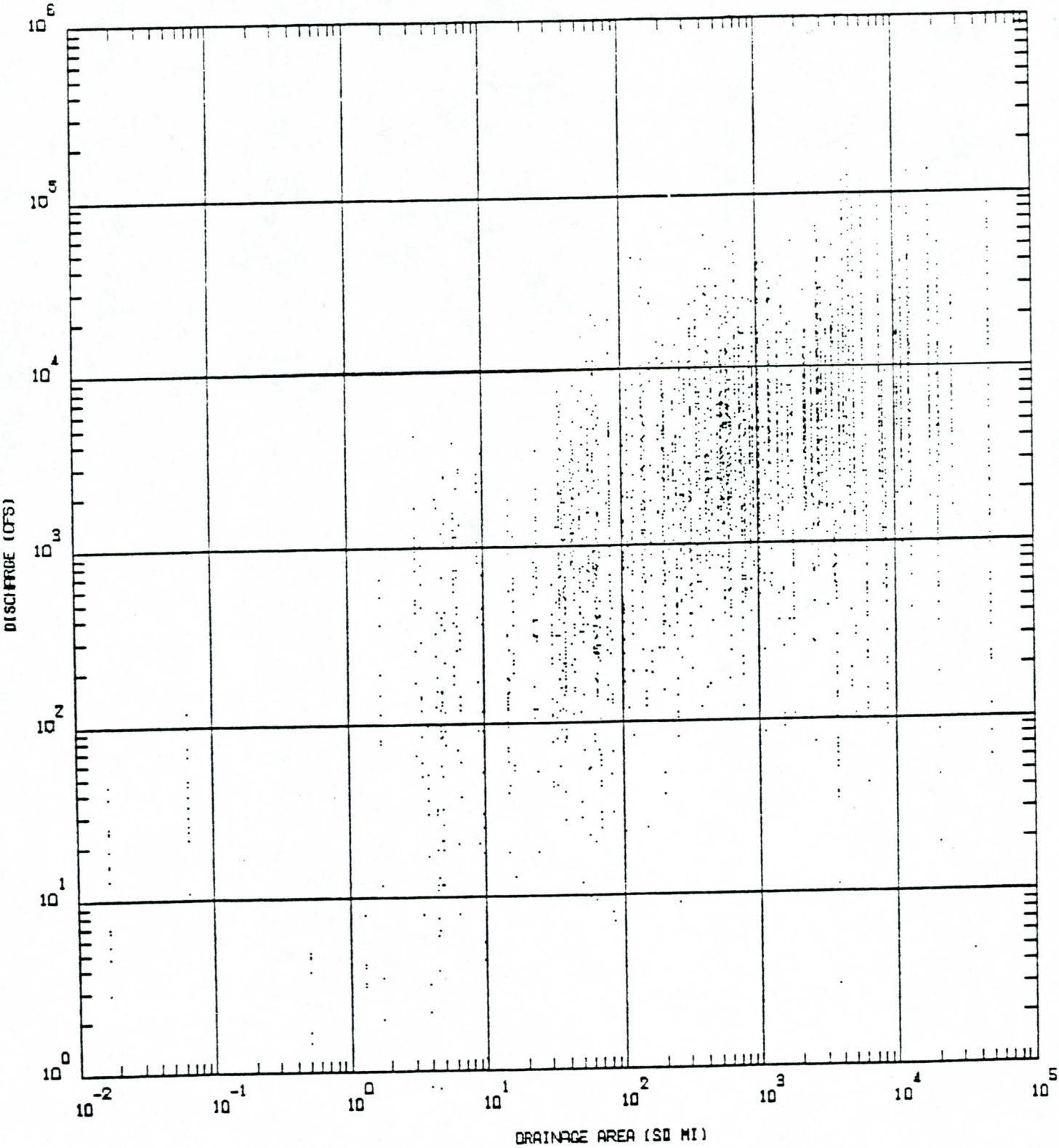
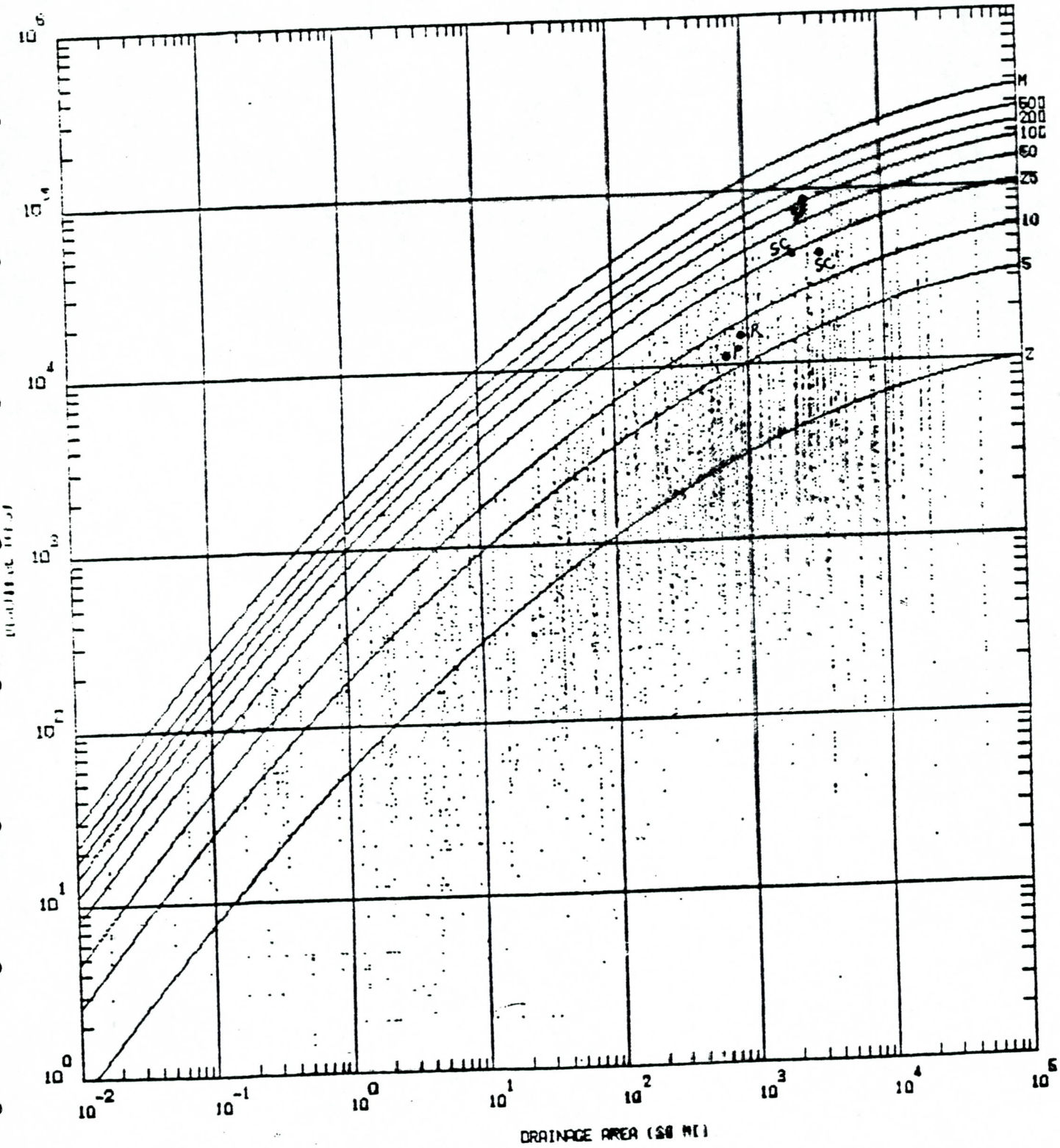


FIG. 1. 3604 PEAK ANNUAL FLOWS FOR 151 GAGED SITES IN ARIZONA. TO 1976



4910 PEAK ANNUAL FLOWS FOR 210 SITES IN ARIZONA. TO 1981

SF : SAN FRANCISCO R AT CLIFTON
SC : SANTA CRUZ R AT CONGRESS ST AND AT MARANA
R : RILLITO R NEAR ORACLE RD
P : PANTANO WASH NEAR VAIL

} 10-83 EVENT

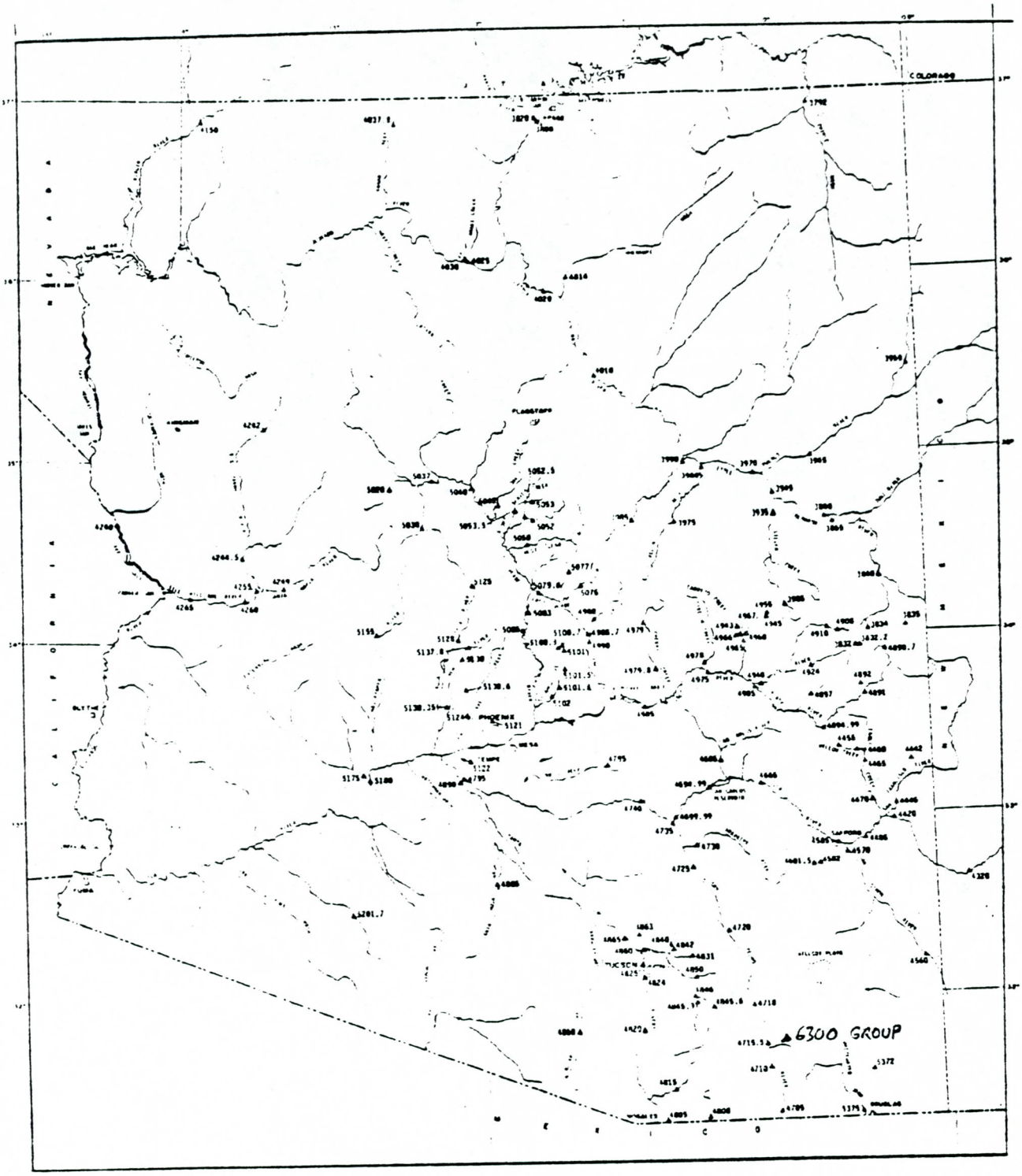


FIG. 2. LOCATION OF GAGING STATIONS.

2 shows the location of the gaging stations. It is evident that large regions are under-represented.

There appears to be an upper envelope to the 3604 points in Figure 1. This reinforces the concept of maximum expected flood stated above. For this data a parabolic shape in log Q vs log A coordinates seems reasonable:

$$\log Q_{max} = \log C_1 + (C_2 + C_3 \log A) \log A \qquad (1)$$

or

$$Q_{max} = C_1 A^{(C_2 + C_3 \log A)} \quad . \qquad (2)$$

To evaluate the constants C_1, C_2, C_3, three points may be chosen to lie on the envelope:

A (mi^2)	Q (cfs)
1	2000
100	41000
10000	243000

The constants are readily determined to give

$$Q_{max} = 2000\ A^{(0.789 - 0.067 \log A)} \qquad (3)$$

where Q is in cubic feet per second and A is the watershed area in square miles upstream from the site. This curve is labeled M in Figure 6. It happens to lie above all 38 of the isolated historic peaks which were ignored. The use of this curve for watersheds less than 1 mi^2 is questionable, with only four gaged sites in the state used here. However, the M curve at A =0.01 mi^2 is nearly tangent to a straight line representing 5 in/hr rainfall assuming all the rain runs off. Thus this simple formula, Eq. (3), appears to be not unreasonable over the tremendous range $0.01 < A < 100{,}000\ mi^2$. More and better data may lead to a revision of

the constants in the equation in the future. A convincing demonstration of regionalization, or other breakdown of the total flood experience, may lead to the use of a family of curves, rather than a single curve. But for now, Eq. (3) seems adequate.

Inspection of Figure 1 leads to the observation that the 3604 data points may be distributed in approximately the same way for all $A > 1$ mi^2. Then any curve Q_T representing the flood with return interval T yr would have the same shape as Q_{max}, or

$$Q_T = 2000\ U\ A^{(0.789\ -\ 0.067\ \log A)} \tag{4}$$

in which $U = Q_T / Q_{max}$ and is yet to be determined. U is a factor less than unity related to the probability of occurrence (or return interval T). This relation can be determined empirically in the following manner. Several curves may be arbitrarily defined, say

$$Q_i = 2000\ (1 - i/80)\ A^{(0.789\ -\ 0.067\ \log A)} \tag{5}$$

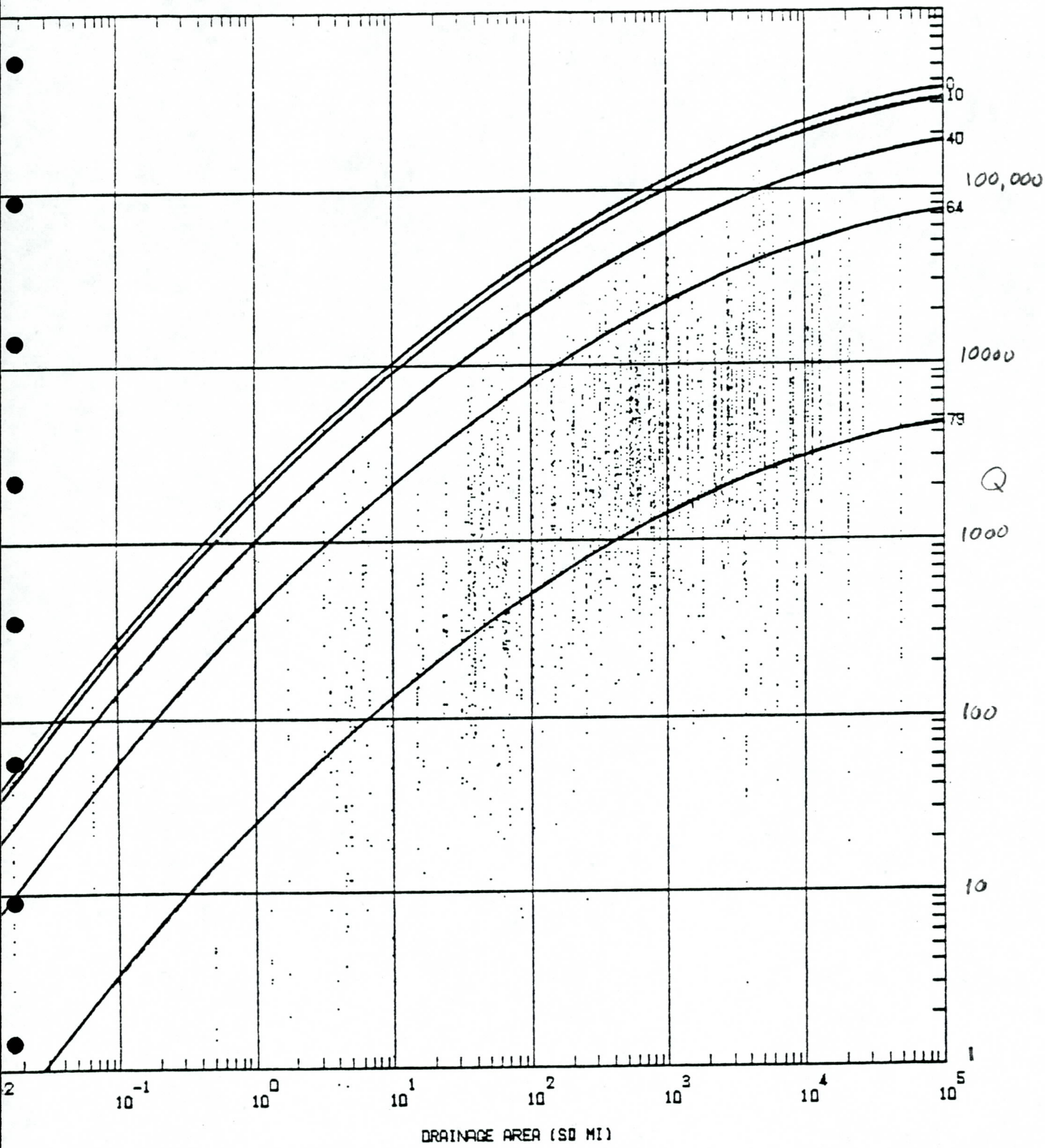
where

$$i = 0,1,2,\ \ldots,\ 80\ .$$

Some of these are shown in Figure 3. The data points falling below each curve are counted. This is readily accomplished with a short computer program. If there is a total of n points, and there are n_i points below the Q_i curve, the exceedance probability is

$$p_i = 1/T = 1 - n_i/n$$

and the Q_i curve is a Q_T curve where T is the return interval for the flow Q_i. The results of this computation are shown in Figure 4 which uses arithmetic (linear) scales, and Figure 5 which uses standard log-probability



3. $Q_i = 2000\ (1-i/80)\ A^{(.789-.067 \log A)}$ for $i = 0, 10, 40, 64, 79$.

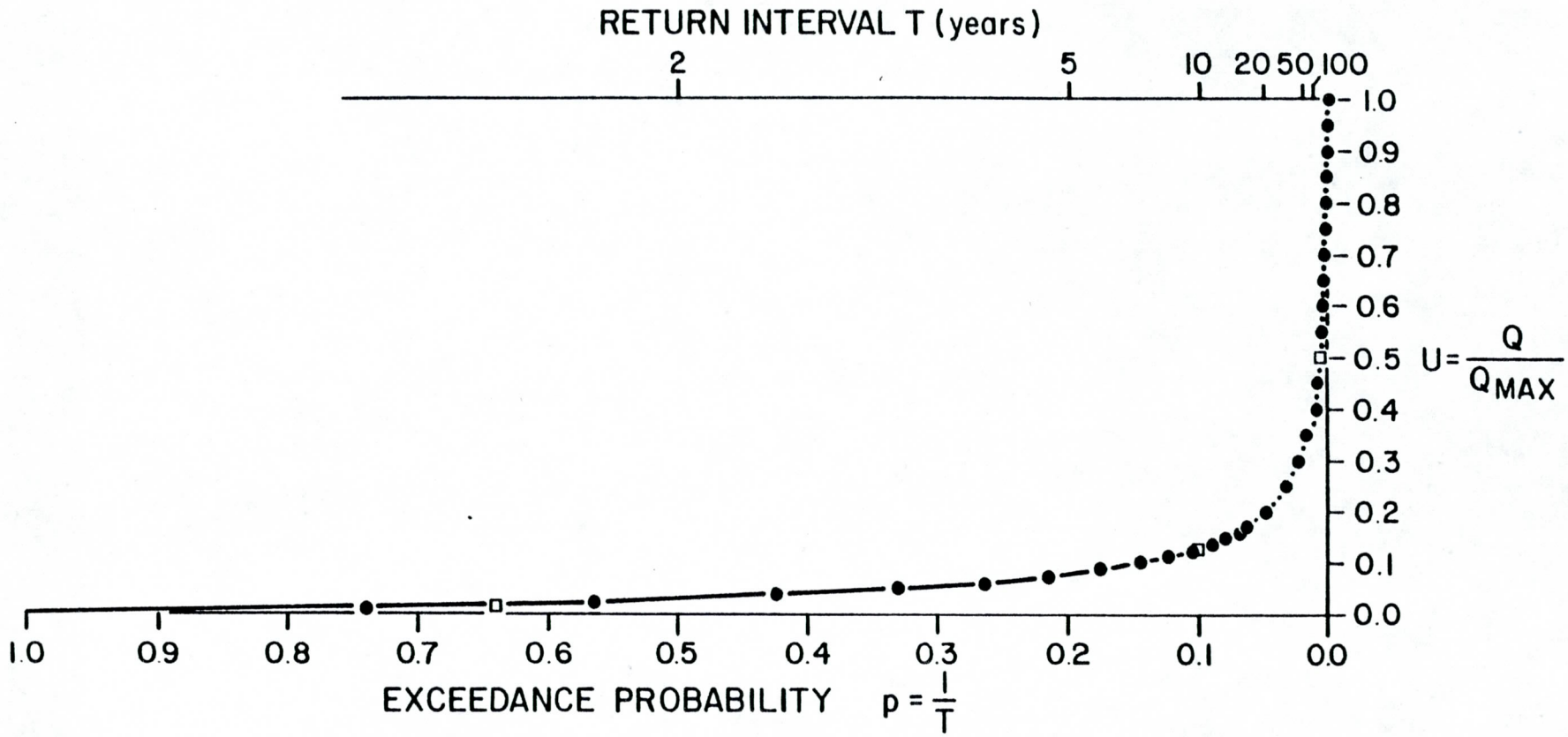


FIG. 4. MAGNITUDE-FREQUENCY CURVE, ARITHMETIC SCALES.

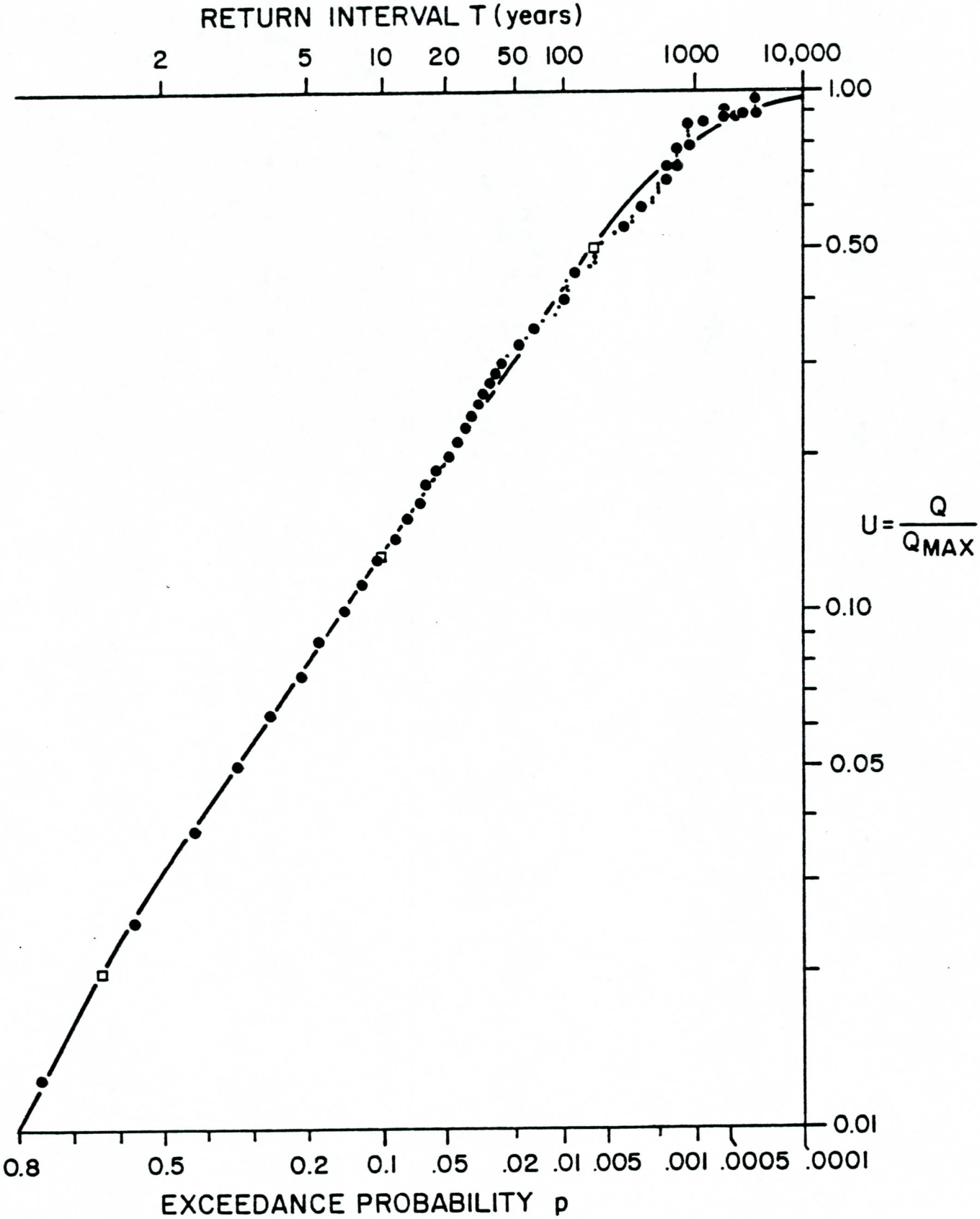


FIG. 5. MAGNITUDE-FREQUENCY CURVE, LOG-PROBABILITY SCALES.

scales. The portion of the curve below $U = Q_T/Q_{max} = 0.35$ represents about 98% of the data and a return interval of about 50 years. The remaining 2%, or about 70 events for Arizona, must be used to estimate the frequency of rare large floods. Of course the arithmetic scale of Figure 4 is inadequate for this purpose, but it adds an interesting perspective to the problem of flood prediction.

There is a definite bend at the upper end of the curve in Figure 5, which reinforces the conviction that there is a maximum flow that can be reasonably expected. This is contrary to the implications of the conventional log-normal, log-Pearson III, or Gumbel curves, which extrapolate to infinitely large flows. But there is no reason why the natural process of flooding must follow a straight line on log-probability paper or any particular mathematical or statistical curve. It is interesting to observe that a log-normal straight-line extrapolation in Figure 5 would plot as a tail in Figure 4 so thin it could hardly be resolved. For this reason it is ludicrous to expect the given data to provide a good estimate of events rarer than the 100-yr flood. Another fault of the log-normal, log-Pearson III, or Gumbel distributions is in the conventional method of using all data points for a gaging station to define the parameters of the distribution. This practice allows the smaller flows to exert undue influence on the tail region of large flows. It should be noted that the large events often come from a different type of storm than the small events, and that the watershed response to large and small storms is different.

It is not necessary to find a mathematical expression for the curve in Figure 4 or Figure 5. All significant properties can be determined

directly from the plot. However, for convenience an expression will be developed. The curve in Figure 4 suggests a function of type

$$p = \frac{1 + C_1U - (1 + C_1)U^2}{1 + C_2U + C_3U^2} \tag{7}$$

where $U = Q/Q_{max}$. It can be seen that when $Q = 0$, $p = 1$; and when $Q = Q_{max}$, $p = 0$. To evaluate the constants C_1, C_2, C_3, three points may be chosen to lie on the curve, such as

U	p
0.02	0.64
0.128	0.10
0.5	0.006

These points are marked as squares in Figures 4 and 5. The three points define three linear equations in C_1, C_2, and C_3. The solution gives

$$p = \frac{1}{T} = \frac{1 - 0.83U - 0.17U^2}{1 + 20.6U + 313U^2} \tag{8}$$

The inverse of this expression is

$$U = \frac{\sqrt{1.3689T^2 + 1285.516T - 827.64} - 0.83T - 20.6}{0.34T + 626.} \tag{9}$$

where

$$1 \leq T < \infty \quad .$$

Some of these digits are not significant, but they are retained to provide an exact inverse. From this expression the following table of flood magnitude vs. return interval can be prepared:

T (yr)	$U = Q/Q_{max}$
2	0.031
5	0.080
10	0.129
25	0.217
50	0.306
100	0.413
200	0.535
500	0.699
∞	1.000

Note that $Q_{100}/Q_{10} = 3.2$, $Q_{50}/Q_{10} = 2.4$, $Q_{100}/Q_{50} = 1.4$, $Q_{max}/Q_{50} = 3.3$, and $Q_{max}/Q_{100} = 2.4$. Probably only one digit is significant, with the second digit being suggestive. These values of U are used to generate the Q_T curves in Figure 6. Either Eqs. (4) and (9) or Figure 6 may be used to estimate the flood for any return interval and for any bridge site in the State. This estimate will be acceptable for at least half the sites which have records. For the others the estimate will be too high or too low for various reasons. However, the estimate for large rare floods will not be too low, by reason of the method used here to prepare the Q_T curves. Some reasons why the estimate may be (or seem to be) too high are:

1. Large infiltration capacity of the watershed or the stream bed.
2. Storage of water in reservoirs constructed upstream.
3. Heavy vegetative cover as in the forests.
4. Persistent but unexplained lack of the type of storms that produce large peak flows.
5. Systematic error in recording or computing flow at the gaging station.

Some reasons why the estimate may be (or seem to be) too low are:

1. Low infiltration capacity of the watershed or the stream bed.

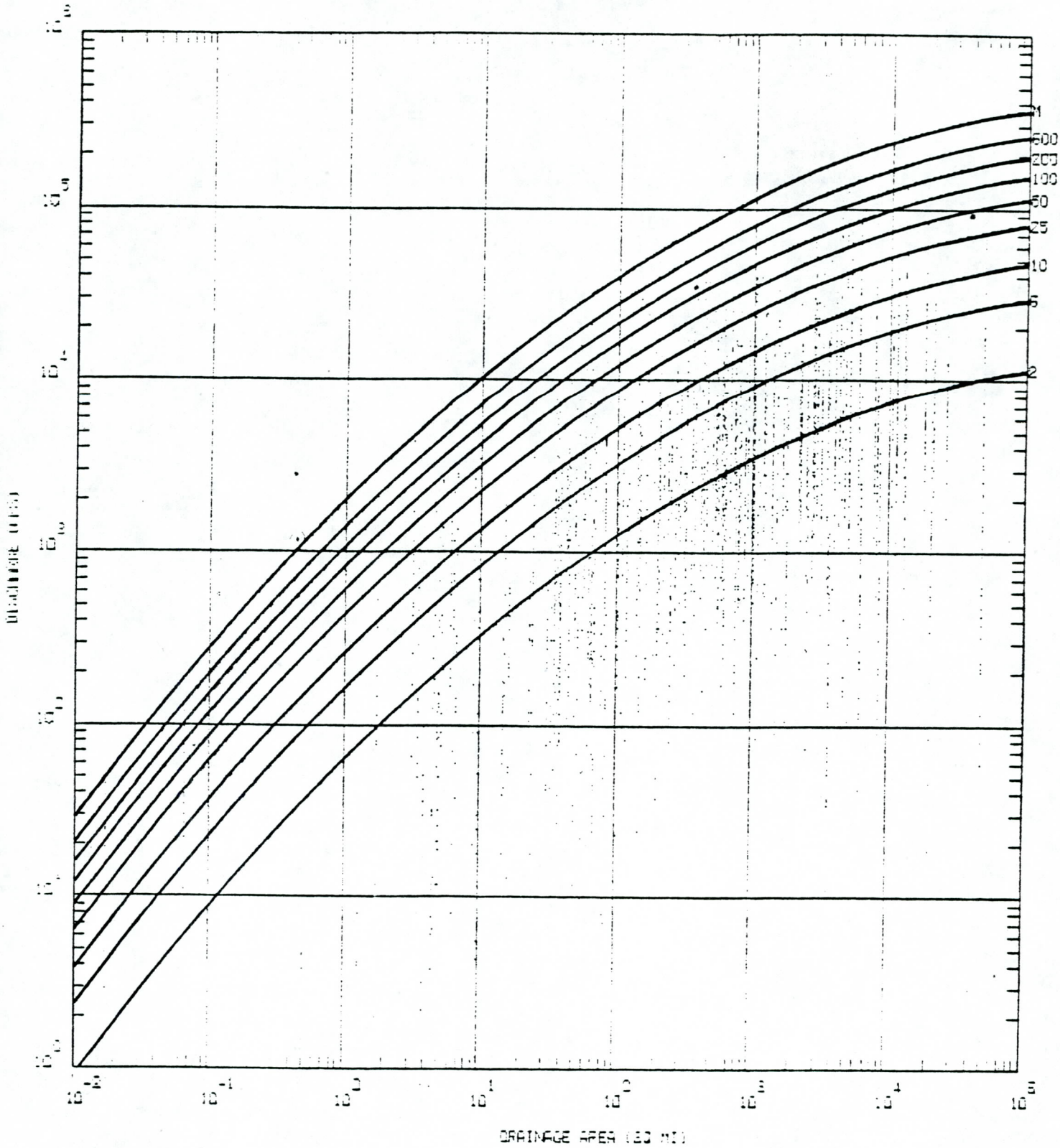


FIG. 6. MAGNITUDE-DRAINAGE AREA CURVES FOR VARIOUS RETURN INTERVALS (YR.).

2. Low retention capacity of the watershed due to extensive bedrock exposure, urbanization, etc.

3. Persistent large amounts of precipitation.

4. Equal concentration times from tributaries.

5. Systematic error in recording or computing flow at the gaging station.

The discrepancy between recorded data at a site and the predicted magnitude-frequency relation obtained from the curves of Figure 6 could also be wholly or in part a matter of chance. To test this possibility a fictitious record having the same length as the actual record was generated for each gaging station using a random number subroutine (RANF in CDC Fortran) and Eqs. (4) and (9). The generated data is plotted in Figure 7 and one must look closely to see any differences from Figure 1, the real data. This test makes Eqs. (4) and (9) look good.

Figures 8 and 9 show another way to plot the data of Figures 1 and 7. Here the data is plotted as normalized discharges Q/Q_{max} (Q_{max} as predicted from Eq. (3) vs. return interval $T = (n + 1)/m$ on log-probability paper; n is the number of years of record, and m is the rank of each event. The scatter of the real data (Figure 8) is twice as much as for the generated data (Figure 9). The proposed relation does not look as good in this test; only half the scatter band can be attributed to the randomness of the flood events.

The proposed magnitude-frequency-area relation is usable in its present form. Improvement of the relation should concentrate on the maximum expected flood. How good are the measurements of the largest floods? Is the curve high or low? Is the meteorology of large storms different in different regions of the state? Is watershed response different? In the

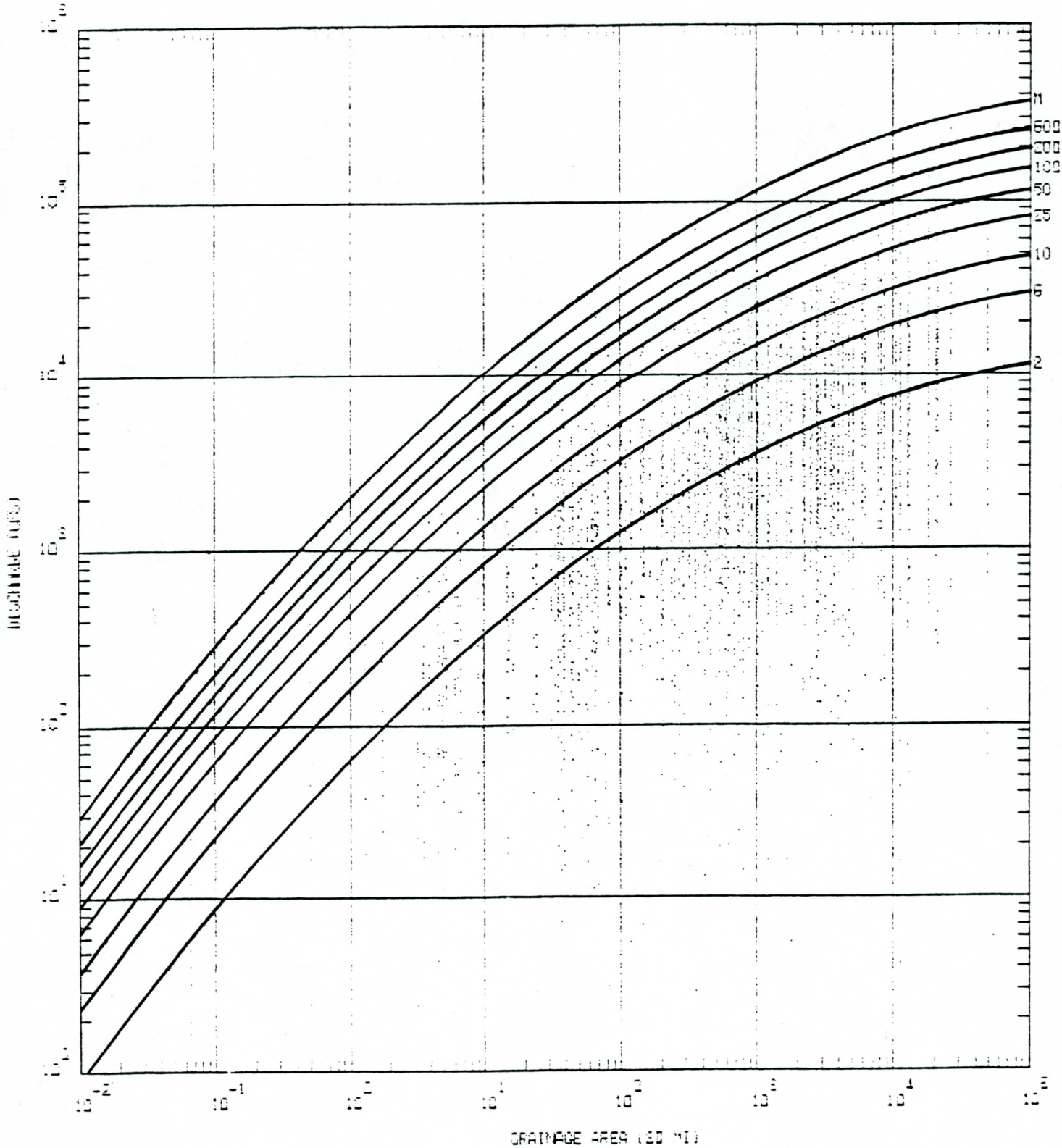


FIG. 7. COMPUTER SIMULATED PEAK ANNUAL FLOWS FOR 151 GAGED SITES USING RANF AND EQS. 4 & 9.

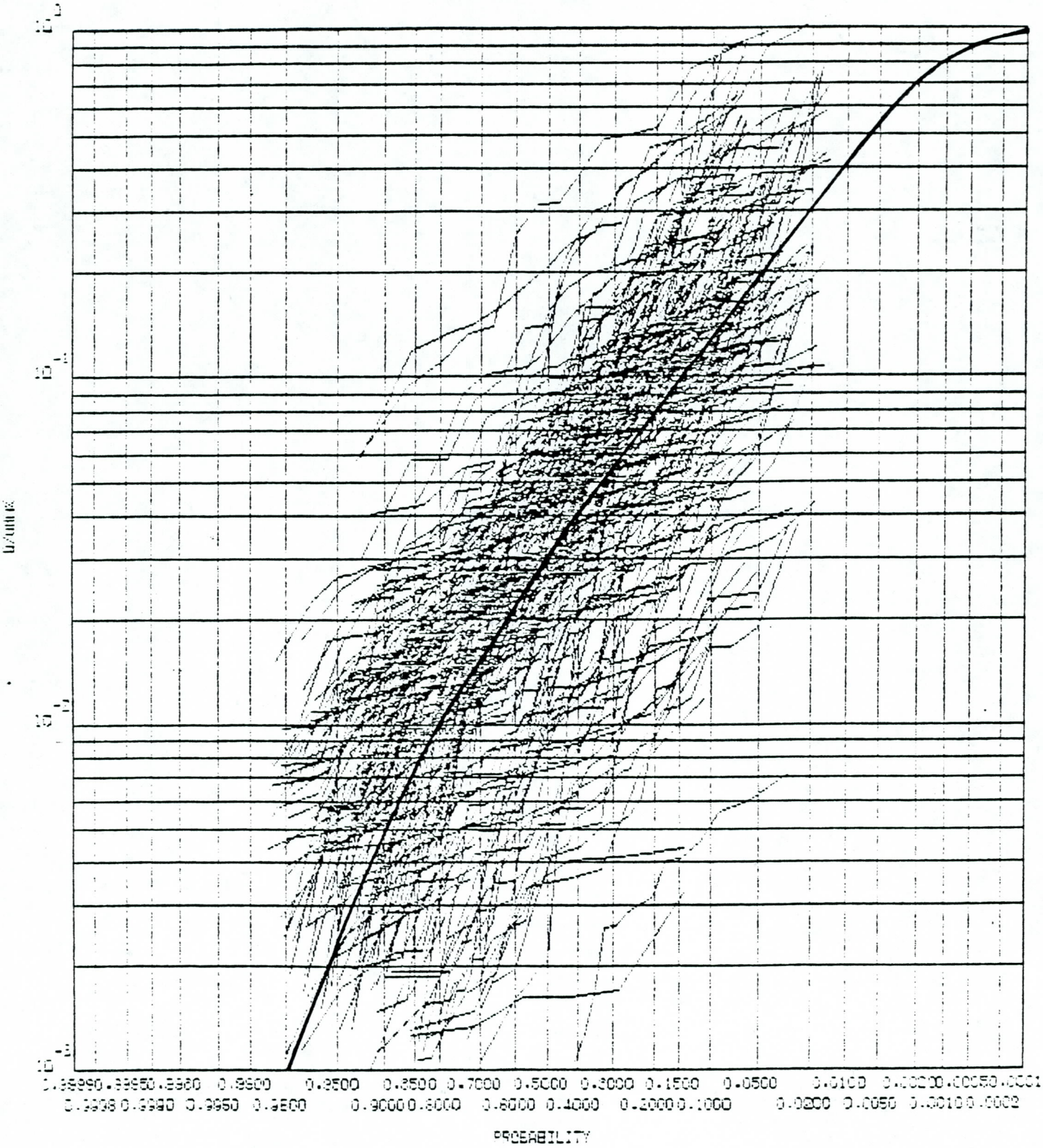


FIG. 8. MAGNITUDE-RETURN INTERVAL PLOTS FOR 151 GAGED SITES; ACTUAL RECORDS. CURVE REPRESENTS EQ. 9.

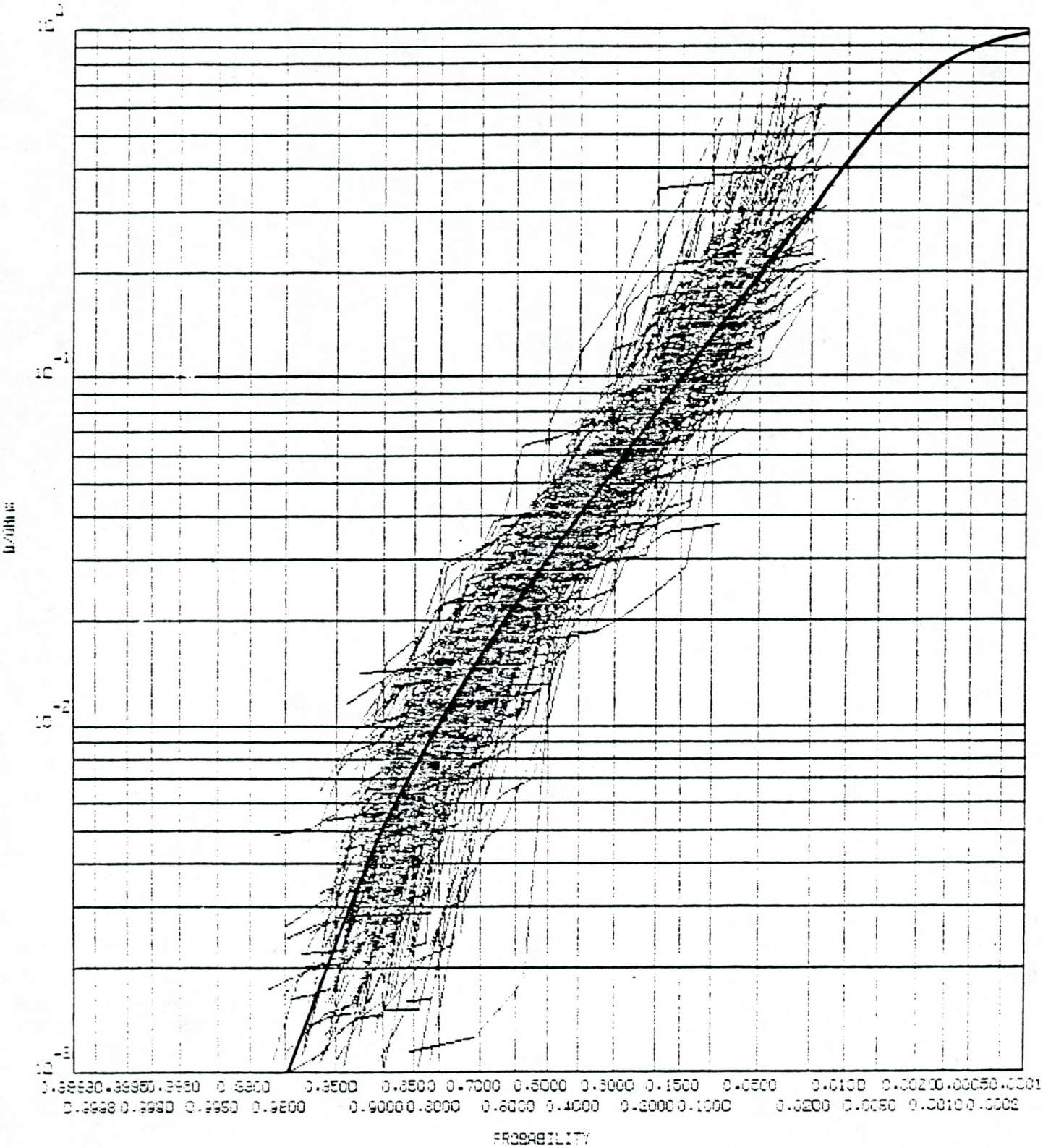


FIG. 9. MAGNITUDE-RETURN INTERVAL PLOTS FOR 151 GAGED SITES; SIMULATED RECORDS. CURVE REPRESENTS EQ. 9.

course of investigating maximum expected floods, the rare floods would be better understood, and a more reliable and precise relation should be evolved.

Figure 10 shows how the ratio of the larger recorded peak flows to the predicted values for similar return intervals for the 151 gaged sites varies over the state. Although there is some suggestion of regionalized effect, there are low ratios mixed with high ratios and vice versa, and high and low clusters in neighboring apparently similar regions. Figure 11 indicates whether the larger peak flows are primarily summer or winter events, or a combination. As would be expected, winter floods are more important in the high elevations, but they do not dominate or explain the suggestion of regionalization in Figure 10.

In the Appendix are plots of recorded peak annual flows for the 151 gaged sites along with the curve from Eqs. (4) and (9). Many of the plots include log-Pearson III values taken from Roeske (2).

Roeske states that the flood frequency equations used in his report are considered to be the best that can be derived at the present time. However, there are a number of questionable aspects to his equations.

Records obtained from the recently placed crest stage gages are too short to be reliable. The gage records themselves may be unreliable because of waves, brush, change of stream channel during flood, and sediment deposit or scour during flood. For example, the Matthai enveloping curve reported by Roeske is too high to be applicable to Arizona. Investigation of the 8-18-71 Bronco Creek (p 72, Roeske) event suggests that the stated flow is high by a factor of about 5. In general, the computation of flow from a crest stage gage record or high water marks (see General Report No. 1) can be seriously in error.

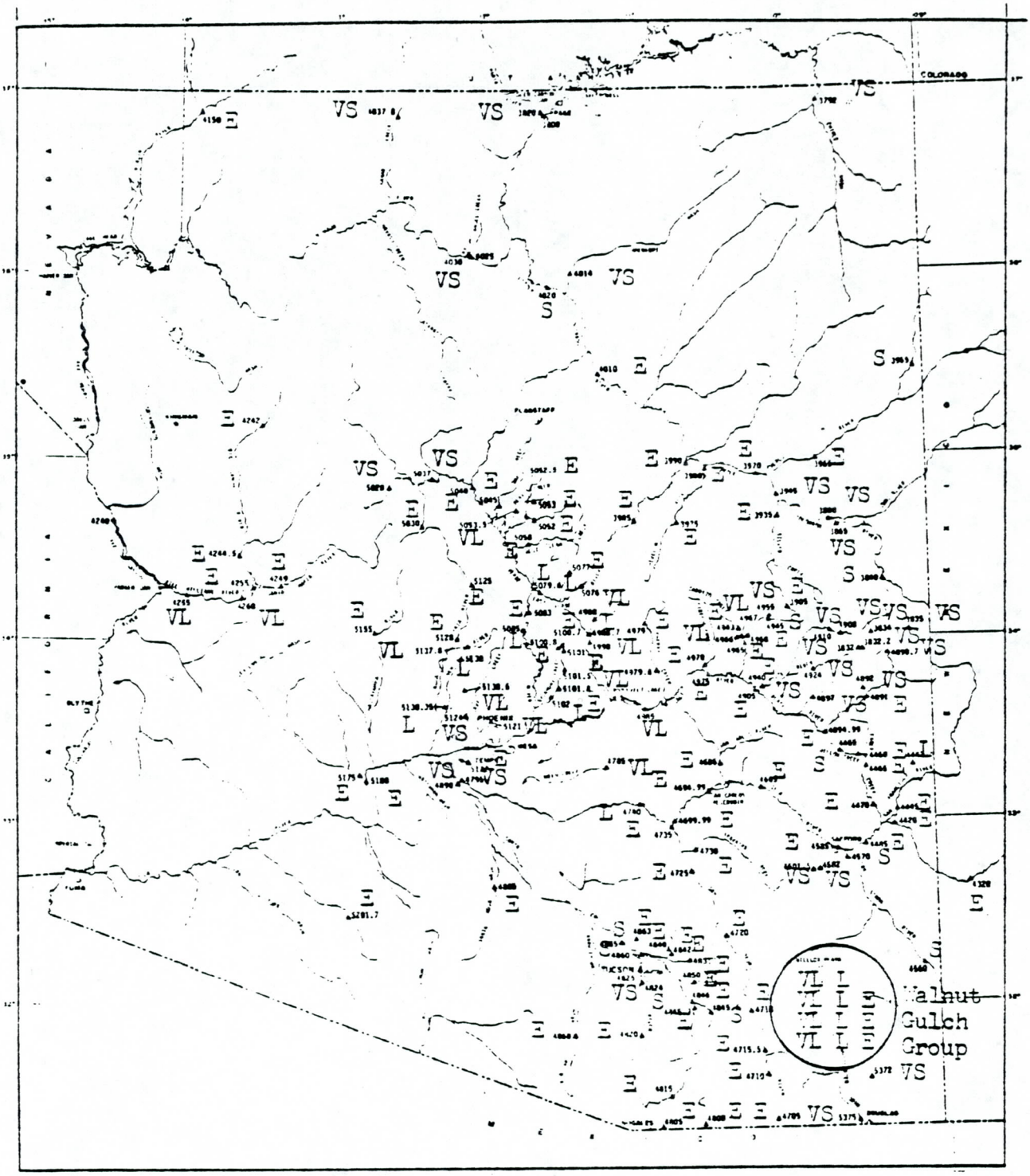


FIG. 10. RATIO OF LARGER RECORDED PEAK FLOWS TO PREDICTED VALUES AT 151 GAGED SITES. VL = VERY LARGE; L = LARGE; E = APPROXIMATELY EQUAL; S = SMALL; VS = VERY SMALL.

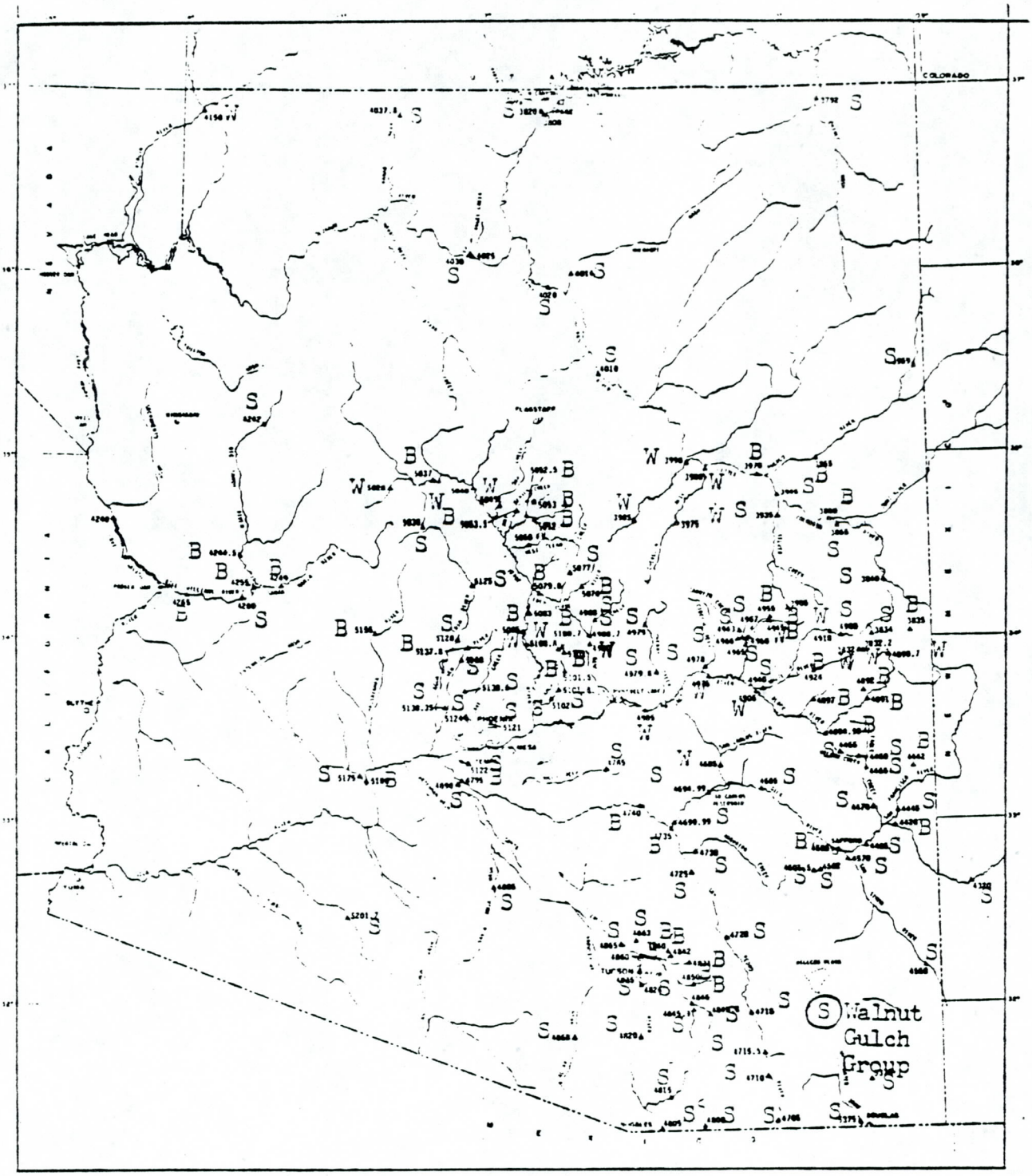


FIG. 11. SEASONAL OCCURRENCE OF LARGER RECORDED PEAK FLOWS AT 151 GAGED SITES. S = LARGER PEAK FLOWS OCCUR PRIMARILY IN SUMMER (JUNE-OCTOBER); W = LARGER PEAK FLOWS OCCUR PRIMARILY IN WINTER (NOVEMBER-MAY); B = LARGER PEAK FLOWS OCCUR BOTH SUMMER AND WINTER WITH ROUGHLY EQUAL OCCURRENCE.

The regression equations of Roeske tend to overestimate flows for $A < 1\ mi^2$ and $A > 1000\ mi^2$ because of the straight line graph in log Q - log A coordinates.

Any differences in Roeske's regions 1,2,3,4,5 are not statistically significant enough, in view of the shortage and quality of data, to justify different equations for these regions. The high elevation (HE) regions do seem to have smaller peak flows. However, most HE site records are short and could have low values because of a lack of normally expected conditions for heavy runoff in recent years. Changes in watershed conditions, say by a fire, could drastically increase runoff. In region 3, the flow is expressed as proportional to elevation to a negative power and precipitation to a positive power, both powers being near unity. Since precipitation tends to increase with elevation, the net effect is probably negligible. In region 4, the flow is expressed as proportional to elevation to the 2.25 to 7.04 power. This leads to the dubious result that Q_{500} at 7000 ft would be 3 times as great as at 6000 ft. Leaving elevation out of the expression would be better. Although annual precipitation is greater at higher elevations, short-term summer storm intensities tend to be lower so the net effect of elevation on peak runoff may be negligible.

Some of Roeske's peak flow predictions are very low or very high. For example, 09379200 Chinle Wash, has a stated Q_{500} of 23,700 cfs (after regional weighting) for a drainage area of 3660 mi^2. Ignoring the undetermined effect of the Many Farms Reservoir, this is equivalent to 6.48 cfs per square mile or 0.010 in/hr rainfall runoff. For comparison, Eq. 3 of this report gives Q_{max} = 182,000 cfs or seven times as much as Roeske's Q_{500}. A straight line extrapolation of the larger flows recorded gives a Q_{500} estimate of about 100,000 cfs. or four times as much.

On the other hand, 09512100 Indian Bend Wash has a stated Q_{500} of 247,000 cfs for a drainage area of 139 mi^2 (1780 cfs per square mile or 2.8 in/hr rainfall runoff), while Eq. 3 gives Q_{max} = 48,000 cfs or one-fifth as much as Roeske's Q_{500}. The concentration time would be about 2 hours for this watershed, and if the runoff was half the rainfall, a rainfall intensity of 5.6 in/hr would be implied. Osborn (3) has found the central core of thunderstorms to be less than half the area of a watershed of such size and has never found as much as 3 inches of rain in a half hour recorded at any rain gage in the state. The implied rainfall intensity over the entire watershed for a period of two hours is hardly imaginable. Since much of the watershed has been cut off by the detention dike associated with the Granite Reef Aqueduct, Q_{max} should really be 32,000 cfs for a drainage area of 62 mi^2.

Thus Roeske's extreme results could be due to anomalous or erroneous figures, or to application of a mindless log-Pearson III computer program.

Eqs. (4) and (9) or Figure 12 in this report will give a peak flood prediction that is not too low for the large return period flows, and thus it is conservative even though the upper tail of the distribution is finite, i.e. bounded by the envelope of statewide recorded flood experience. Some of the predictions, particularly in the high elevation regions, may be too high, and may be adjusted downward using good judgment.

In determining flood magnitudes for various return intervals at a site, the records for gaging stations in the region and the corresponding plots in the Appendix should be inspected. Further study of the regional streams and the site in question should be made to determine the most likely frequency distribution for the site. Care should be exercised to determine

any unusual features of the site or region that could affect the distribution function, including possible changes in watershed characteristics such as through urbanization or a forest fire. In this manner the data available today may be used to predict peak flows for various return intervals that are reasonable and adequate for engineering design. Figure 12, which shows the 2-, 10-, 50-, 100-yr and maximum expected curves, can be read with as much precision as is justified.

In general, it will be found that a bridge should be designed (or modified) for the maximum expected flood. The savings possible through designing for a lesser flood should then be compared with the cost of possible loss of the bridge. Some small, nagging uncertainty about whether the bridge is completely, unquestionably safe may remain, but flood magnitudes estimated as proposed herein should be the least of the uncertainty. The possible hydraulics of the flow, the sediment movement, and the geometry of the future situation will be more of a problem.

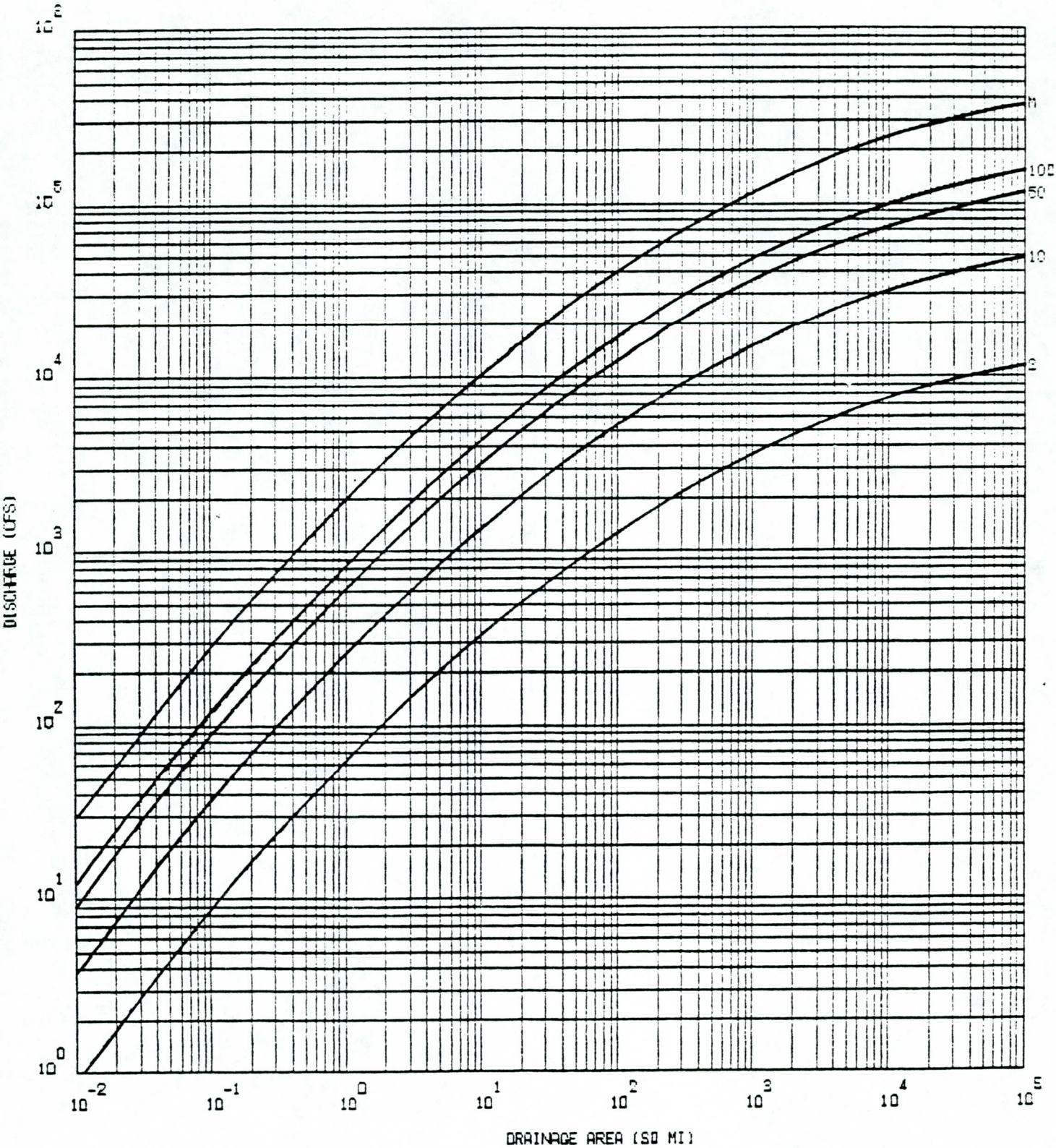


FIG. 12. MAGNITUDE-DRAINAGE AREA CURVES FOR VARIOUS RETURN INTERVALS (YR.); TAKEN FROM FIG. 6.

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