A THEORETICALLY DERIVED SEDIMENT TRANSPORT EQUATION FOR SANDBED CHANNELS IN ARID REGIONS

By Michael E. Zeller, P.E.[1], and William T. Fullerton[2]

ABSTRACT: A theoretically derived sediment transport equation is presented which can be used to quantitatively assess geometric changes in sandbed channels in arid regions. A design example demonstrating the practical application of the equation is also presented.

INTRODUCTION

A major consideration in designing erosion-control measures, hydraulic structures, and channelization schemes for urbanized drainage systems and waterways in alluvial material is the sediment transport in the system. Designs which do not properly take sediment transport into consideration can develop many problems. Channels that are too steep or constrictive can initiate headcuts which will work their way up the system, causing severe erosion, bank failure, and possible loss of structures. Further downstream, this excess sediment can be deposited, reducing the flow area of channels and increasing flood stages. Likewise, a channel which is built at a gradient that is too flat will accumulate material, and adjacent land will suffer from increased flood stages.

Drop structures are often required to reduce gradients of channelized systems in order to prevent degradation. They can be designed most efficiently and effectively when the sediment transport characteristics of the system are known and considered. In addition, the information on equilibrium slopes behind the grade-control structures is often important in determining the spacing of controls. With the sediment transport properties of the system known, the optimum slope (equilibrium slope) for a channel can be determined. From this background, and given appropriate economic considerations, the optimum spacing and height of the drop structures can also be evaluated.

Due to the dynamic nature of the system, when designing structures that will alter the waterways in alluvial material, analysis of the sediment transport is especially critical. Many such channels in arid regions are located in sandy material that is on steep slopes. These two factors combine to produce large transport rates. Therefore, when systems are altered in such a way so as to create imbalances between sediment transport capacities and supplies, the resulting changes in the system are rapid and often adverse. Specifically, the result can be the failure or uselessness of a project with a planned 100-year design life within a period of less than 10 years.

[1]Vice President, Simons, Li & Associates, Inc., Tucson, Arizona.
[2]Senior Hydraulic Engineer, Simons, Li & Assoc., Inc., Fort Collins, Colorado.
(Presented at the Daryl B. Simons *Symposium on Erosion and Sedimentation*, July, 1983)

A THEORETICALLY DERIVED SEDIMENT TRANSPORT EQUATION FOR SANDBED CHANNELS IN ARID REGIONS

By Michael E. Zeller, P.E.[1], and William T. Fullerton[2]

ABSTRACT: A theoretically derived sediment transport equation is presented which can be used to quantitatively assess geometric changes in sandbed channels in arid regions. A design example demonstrating the practical application of the equation is also presented.

INTRODUCTION

A major consideration in designing erosion-control measures, hydraulic structures, and channelization schemes for urbanized drainage systems and waterways in alluvial material is the sediment transport in the system. Designs which do not properly take sediment transport into consideration can develop many problems. Channels that are too steep or constrictive can initiate headcuts which will work their way up the system, causing severe erosion, bank failure, and possible loss of structures. Further downstream, this excess sediment can be deposited, reducing the flow area of channels and increasing flood stages. Likewise, a channel which is built at a gradient that is too flat will accumulate material, and adjacent land will suffer from increased flood stages.

Drop structures are often required to reduce gradients of channelized systems in order to prevent degradation. They can be designed most efficiently and effectively when the sediment transport characteristics of the system are known and considered. In addition, the information on equilibrium slopes behind the grade-control structures is often important in determining the spacing of controls. With the sediment transport properties of the system known, the optimum slope (equilibrium slope) for a channel can be determined. From this background, and given appropriate economic considerations, the optimum spacing and height of the drop structures can also be evaluated.

Due to the dynamic nature of the system, when designing structures that will alter the waterways in alluvial material, analysis of the sediment transport is especially critical. Many such channels in arid regions are located in sandy material that is on steep slopes. These two factors combine to produce large transport rates. Therefore, when systems are altered in such a way so as to create imbalances between sediment transport capacities and supplies, the resulting changes in the system are rapid and often adverse. Specifically, the result can be the failure or uselessness of a project with a planned 100-year design life within a period of less than 10 years.

[1]Vice President, Simons, Li & Associates, Inc., Tucson, Arizona.
[2]Senior Hydraulic Engineer, Simons, Li & Assoc., Inc., Fort Collins, Colorado.
(Presented at the Daryl B. Simons *Symposium on Erosion and Sedimentation*, July, 1983)

Unfortunately, the best methods to predict the sediment transport characteristics of a waterway are difficult, time consuming, and require specialized knowledge of the subject for proper application. This makes them impractical for use by the design engineer, and thus the sediment transport is either ignored or determined by inappropriate methods. Due to the rarity of the flow events in arid regions, investigation of the transport rate using the measured data is not feasible. Therefore, a theoretical approach is needed. This points to the need for a method of analysis that is better suited to the design engineer's needs. Application of such a methodology must be both accurate and efficient.

This paper presents a method for analysis of sediment transport which meets the above criteria. The determination of sediment transport as presented in this paper is based upon an easily applied power relationship between the dependent variable, sediment transport rate, and the five independent variables of velocity, depth, Manning's "n", median sediment diameter, and sediment gradation. The equation was developed using a sophisticated technique for analyzing the sediment transport capacity of channels. The method used consists of a determination of the transport capacity using a combination of the Meyer-Peter, Muller bed-load transport equation (1)* and Einstein's integration of the suspended bed-material load (2). The transport capacity was determined for a variety of flow conditions likely to occur within sandbed channels in arid regions. The sediment transport capacity was then regressed against the flow conditions to obtain the power relationship. This process was repeated for a variety of sediment size distributions that can be encountered in arid regions.

GENERATION OF THE SEDIMENT TRANSPORT EQUATION

The basic procedure for developing the relationship for sediment transport was outlined in the introduction. This section presents the specific steps in the process.

Establishment of Typical Parameter Ranges to be Found In Arid Regions.--The procedure outlined previously could be used to develop transport equations for the entire range of parameters that is physically possible. However, in order to produce the most accurate relationships, it was necessary to determine the range of independent variables applicable to the study area. The variables investigated were the velocity (V), median sediment diameter (D_{50}), depth (Y), Froude number (F), slope (S), discharge per unit width (q), sediment gradation coefficient (G_s), and hydraulic roughness (Manning's "n"). The median diameter, D_{50}, of the sediment and its gradation coefficient ($G_s = 1/2[D_{50}/D_{16} + D_{84}/D_{50}]$) describe the sediment, and the other variables characterize the flow conditions.

The practical range for these variables was determined from data obtained in and around the vicinity of Tucson and Pima County, Arizona. A large portion of the information used was associated with several studies which were conducted by Simons, Li & Associates, Inc., in that area (3,4,5). Table 1 lists typical values for sediment properties at sample locations in and around the Tucson and Pima County, Arizona, region.

*Numbers in parentheses refer to corresponding references at the end of this paper.

Table 1. **Sampled Grain-Size Distribution In and Around the Vicinity of Tucson and Pima County, Arizona.**

Sample Location	D_{16}	D_{50}	D_{84}	G
1	0.75	2.30	10.00	3.70
2	0.40	1.15	4.10	3.20
3	0.65	2.10	10.00	4.00
4	0.90	2.30	9.00	3.30
6	0.30	0.90	2.80	3.10
7	0.42	1.10	2.10	2.30
8	0.37	1.10	2.40	2.60
9	0.18	0.95	4.10	4.80
10	0.38	1.05	2.00	2.30

In addition, the grain-size analysis was plotted and the results showed that the distribution in all cases could be approximated very well by log-normal distributions. This means that by using the median grain diameter and the gradation coefficient of a distribution, a distribution which nearly matches it can be produced mathematically. The study undertaken in developing this paper considers this as a property of sediments in arid regions. By choosing several values for (median size) and G_s (gradation coefficient) to represent the range of expected sediment distributions, sediment transport equations are developed for each of the combinations. The range of D_{50} used was from 0.5 mm to 10.0 mm, while G_s varied from 2.0 to 5.0.

From the Tucson and Pima County data previously mentioned, the range of hydraulic conditions was assessed. The velocities ranged from 3 to 30 fps. Slopes from 0.001 to 0.040 were employed. In arid regions, slopes on the order of 0.01 and less are predominantly found on larger river systems. Greater slopes, on the other hand, are predominantly found on the smaller tributaries and on urban channels.

The resistance to flow is influenced greatly by the occurrence of bed forms in the sandy channel beds of ephemeral streams in arid regions. Because of relatively high velocities and steep channels, major flows are in the upper regime, with anti-dunes present. However, the trend normally is for high resistance at low flows, due to formation of dunes in the lower regime; but, as these dunes plume out into a plane bed with sediment movement at higher discharge, the resistance greatly decreases. As the discharge becomes even larger, anti-dunes begin to form and the resistance correspondingly begins to increase. The resistance again becomes rather high when the anti-dunes become violent and start breaking. Similar relationships can be expected for all sandbed channels in arid regions.

A reasonable range of unit-width discharges was selected ranging from 10 to 200 cubic feet per second per foot of width (cfs/ft.). Much of the data provided did not include depths. However, by controlling the range of the velocities and Froude number, the limits on depth were established. Velocity, depth, Manning's "n", median sediment diameter, and sediment gradation were chosen as the five independent variables in the sediment transport equation developed. Since the combination of flow-condition variables listed total to six, and only three equations are present to relate them (Manning's, continuity, and the Froude number definition), three of the variables must be given in order to calculate the velocity and depth. The three chosen variables were slope, Manning's "n", and discharge per unit width. These variables were alternatively varied among their specific ranges. The velocity and depth were then determined and checked against their accepted ranges. If they did not conform, the calculated flow conditions were discarded. The valid flow conditions were then used to calculate the sediment transport capacities to be used in the regression analysis.

Methodology.--Velocity and depth of flow are needed to calculate the sediment transport capacity. To calculate the hydraulic parameters of velocity and depth, the wide-channel version of the Manning equation was used (6). For this case, the equation for depth is:

$$Y = \left(\frac{q \times n}{1.486 \times S_f^{1/2}}\right)^{3/5} \qquad \text{(I)}$$

In Equation I, q is the discharge per unit width, n is the Manning resistance parameter, and S is the friction slope (which is assumed equal to the bed slope). The velocity can then be calculated from the equation:

$$V = q/Y \qquad \text{(II)}$$

In addition, the Froude number is defined as:

$$F = \frac{V}{(gY)^{1/2}} \qquad \text{(III)}$$

In Equation III, "g" is the acceleration due to gravity. The parameters "q" "S_f", and "n" are chosen using the scheme described in the previous section. The computed values of "V" and "F" are then compared with an accepted range to determine whether the hydraulic conditions should be rejected. If they are accepted, the transport capacity is then calculated using the procedure outlined in the following paragraphs.

Sediment Transport.--The amount of material transported in a channel reach is the result of the interaction of two processes. The first is the transport capacity of the reach. This is determined in part by the hydraulic conditions which are a direct result of the water discharge, channel configuration, and channel resistance. The other major factor is the sediment sizes present.

Smaller particles can be transported at larger rates than larger particles under the same flow conditions. The second process is the supply of sediment entering the reach. This is determined by the nature of (a) the channel and watershed above the study reach, (b) the development to which the channel and watershed have already been subjected, and (c) the development to which the channel and watershed may be subjected in the future.

For the purpose of this paper, the sediment transport rate of a channel is assumed to be equal to its sediment transport capacity. This is an acceptable approximation for sandbed channels in arid regions, since coarse material is usually not present—which is evidence of a general lack of armoring potential along such streams.

Transport of the bed-material load of a channel is divided into two regions. The sediment moving in a layer close to the bed is referred to as the bed load. The sediment which is carried in the remaining upper portion of the flow is referred to as suspended load. The total bed-material load is the sum of the two. The turbulent mixing process and the action of gravity on the sediment particles cause a continual transfer between the two regions. There is no distinct line between the regions. The definitions are made in order to aid in the mathematical description of the process. A third type of load, the wash load, is also defined. It is made up of fine particles which are not present in appreciable quantities in the bed, and will not settle out.

Sediments of different sizes will experience different rates of transport. Therefore, the transport capacities for a range of sediment sizes are determined and totaled in order to produce an acceptable determination of total transport capacity. The total transport capacity for a channel section is:

$$Q_s = T \times \Sigma P_i (q_{bi} + q_{si}) \qquad \text{(IV)}$$

In Equation IV, "T" is the top width of the channel, "P_i" is the fraction of one sediment size, "q_{bi}" is the bed-load transport rate per unit width for the ith size, and "q_{si}" is the suspended-load transport rate per unit width for the i^{th} sediment size.

The bed load is determined using the Meyer-Peter, Muller equation, which is a simple and commonly used bed-load transport equation. It was adopted for this study because of its applicability to the conditions which generally exist in arid regions. The equation is:

$$q_b = \frac{12.85}{\sqrt{\rho \gamma_s}} (\tau_o - \tau_c)^{1.5} \qquad \text{(V)}$$

In Equation V,

$$\tau_c = \varsigma_s (\gamma_o - \gamma_c) D_s \qquad \text{(VI)}$$

In Equations V and VI, "q_b" is the bed-load transport rate, in volume per unit width for a specific size of sediment, "τ_c" is the critical tractive force necessary to initiate particle motion, "ρ" is the density of water, "γ_s" is the specific weight of sediment, "γ" is the specific weight of water, "D_s"

is the size of sediment, and "ζ_s" is a constant dependent on flow conditions, often referred to as Shields' parameter (7).

For the flow conditions in the study area, "ζ_s" is approximately 0.047. The boundary shear stress acting on the grain is:

$$\tau_o = \frac{1}{8}\rho f_o V^2 \qquad \text{(VII)}$$

In Equation VII, "ρ" is the density of the flowing water, and f_o is the Darcy-Weisbach friction factor. A friction factor was adopted which corresponds to a Manning's "n" which is smaller than the overall Manning's "n" used to determine velocity and depth. The reason for the reduction is that only a portion of the overall resistance, a combination of particle resistance and a portion of the bedform resistance, is effective in promoting sediment transport. The remainder of the overall resistance results primarily from form resistance caused by the non-effective bedform resistance, resistance due to variations in the channel geometry, and resistance due to encroachment of vegetal growth.

The suspended load is determined by using a modified solution developed by Einstein. This method relies upon an integration of the sediment concentration profile as a function of depth. The nature of the profile is assumed to be in equilibrium, and therefore, the rate at which sediment is transported upward due to turbulence and the concentration gradient is exactly equal to the rate at which gravity is transporting sediment downward. If the sediment concentration is known at one point, then the entire concentration is determined. The point of known concentration is assumed to be the upper limit of the bed-load layer. The resulting equation is:

$$q_s = \frac{q_b}{11.6}\frac{G^{w-1}}{(1-G)^w}[(\frac{V}{U_*} + 2.5)I_1 + 2.5I_2] \qquad \text{(VIII)}$$

In Equation VIII, "q_s" is the suspended load, "q_b" is the bed load, "G" is the relative depth of the bed layer, "U_*" is the shear velocity, "V" is the mean velocity of flow, "I_1" and "I_2" are the Einstein integrals, and "w" is a dimensionless parameter given by the relation:

$$w = \frac{V_s}{U_*} \qquad \text{(IX)}$$

In Equation IX, "V_s" is the fall velocity of the sediment particle and "U_*" is the Von Karman constant (assumed 0.4).

The "I_1" and "I_2" parameters are integrals which cannot be evaluated directly. One must either use tables or numerical techniques. In the computer routine used to determine transport capacity for development of the results provided in this paper, these integrals are evaluated using a numerical technique.

It is normally assumed that the bed layer is equal to twice the sediment size. However, for small sediment sizes this assumption yields erroneously high transport rates. In this study the bed-layer thickness was calibrated to produce sediment transport rates that are consistent with measurements obtained from the arid regions located in and around the vicinity of Tucson and Pima County, Arizona (8).

SEDIMENT TRANSPORT REGRESSION EQUATION

A single relationship, defining the bed-material transport rate in volume per unit width, was derived from the computed results of the sediment transport capacity determination. The resulting equation is:

$$q_s = 0.0064 \frac{n^{1.77} V^{4.32} G_s^{0.45}}{Y^{0.30} D_{50}^{0.61}} \qquad \text{(X)}$$

In Equation X, "q_s" is the bed-material transport rate, in units of cubic feet per second per foot of width (cfs/ft.), and all other parameters are as previously defined within this paper. Note that D_{50} is in millimeters. All other variables are in the ft.-lb.-sec. system of units.

Equation X shows the high level of dependence that sediment transport rates have with respect to velocity. However, while the dependence on the other hydraulic parameters is less important, it can be seen that they still play a critical role in determining rates of sediment transport.

When applying Equation X, care should be taken so that the range of parameters being used are not out of the range used to develop the equation. The procedure outlined in the next section should be used to determine if the application of Equation X to a particular set of conditions is valid.

RANGE OF APPLICABILITY OF SEDIMENT TRANSPORT EQUATION

Since the sediment transport equation presented was developed from a specific range of parameters, it should not be utilized for situations which are far beyond this range. Table 2 lists the range of parameters used in the development of Equation X. When using the equation to determine sediment transport rates, the conditions should be checked against Table 2. If any conditions are not within or near the ranges outlined in Table 2, the equation should not be applied. For the range of parameters used, Equation 10 provides results within ten percent of the theoretically computed values. If application outside of the range of parameters is contemplated, the validity of the equation should be checked considering physical significance against the sensitivity of parameters.

Sediment transport capacity equations are valid for particle sizes which fall primarily within the sand and gravel range. For particle sizes less than sand (0.0625 mm), sediment transport capacity is controlled by sediment supply from the watershed and erosion of bank material. Transport capacity rarely controls for silts and clays.

A sorting of sediment particles takes place during a storm event in which the larger particles become dispersed with the smaller sizes. Sediment transport capacity equations neglect this sorting process, and thus for larger particle sizes the equations tend to overpredict transport because with the additional weight of the smaller sizes a greater shear stress is required for movement.

Table 2. Range of Parameters Examined

Parameter	Range
Froude Number	Unlimited
Depth	1 - 20 ft.
Velocity	3 - 30 fps
Manning's "n"	0.018 - 0.035
Bed Slope	0.001 - 0.040 ft./ft.
Unit Discharge	10 - 200 cfs/ft.
Particle Size	0.5 mm $\leq D_{50} \leq$ 10 mm
Gradation Coefficient	$2 \leq G_s \leq 5$

There are several other checks that should be made in order to insure that Equation X is applicable to a given problem. The equation is based on the assumption that all the sediment sizes present can be moved by the flow. If this is not true, armoring will take place. Equation X is not applicable when armoring occurs. This can be determined using Shields criteria for critical shear stress (VII). The bed shear stress is given by:

$$\tau_o = \gamma RS \qquad \text{(XI)}$$

In Equation XI, "γ" is the unit weight of water, "R" is the hydraulic radius, and "S" is the bed slope. The diameter of the largest particles moving is then:

$$D = \tau_o / [(\gamma_s - \gamma)0.047] \qquad \text{(XII)}$$

In Equation XII, "D" is the diameter of the sediment, "γ_s" is the unit weight of sediment, and 0.047 is the recommended value of Shields' parameter (9). All units are in feet, pounds and seconds. When no sediment of the computed size, or larger, is present in significant quantities, Equation X is applicable.

Equation X was developed for sandbed channels. Therefore, It does not apply to conditions when the bed material has cohesion. Equation X would overpredict transport rates in a cohesive channel.

PRACTICAL APPLICATION OF THE SEDIMENT TRANSPORT EQUATION TO DETERMINE EQUILIBRIUM CHANNEL SLOPES

The concept of an equilibrium slope is useful in that its calculation will provide an understanding of the long-term effects such measures as constricting the channel or reducing incoming sediment supply will have on the profile of the channel. The sediment transport equation previously determined (i.e., Equation X) can be used to obtain an estimate of the equilibrium channel slope. The following paragraphs describe the procedure and illustrate its use.

Procedure for Determining Equilibrium Channel Slopes.--The equilibrium channel slope is defined as the slope at which the sediment transport capacity of the channel is equal to the incoming sediment supply. Under this condition, the channel neither aggrades nor degrades. When the present slope of a channel is greater than the equilibrium slope, the channel will degrade in order to reach its equilibrium slope. The degradation will not be uniform along the entire channel reach. More accurately, the channel can be viewed as pivoting around a downstream control point. This point is one which is relatively unaffected by erosion or deposition. Examples of channel features which can act as control points are grade-control structures, confluences with other channels, and rock outcrops in channel beds.

Calculation of the equilibrium slope about which the channel will pivot is accomplished using the definition of a channel in equilibrium. That is:

$$Q_{s\ in} = Q_{s\ out} \qquad \text{(XIII)}$$

In Equation XIII, "$Q_{s\ in}$" represents the supply rate of sediment into the channel and "$Q_{s\ out}$" represents the sediment transport rate out of the channel.

The procedure starts by determining the sediment transport rate into the channel. The upstream sediment supply should be determined from a section of channel upstream of the reach in question. The supply reach must be close to its equilibrium condition. The best choice is a section of the natural channel, upstream of the channelized section, which has not been disturbed by man s activities. Another choice would be an upstream channelized section which has been in existence for many years and has not experienced a recent change in profile or cross section.

Once the supply reach has been selected, it is necessary to calculate the sediment supply rate. The first step in this process is the selection of a reference or "dominant" discharge for which the sediment supply is to be determined. The concept of the "dominant" discharge is that it best represents the discharge which will determine the long-term response of the channel. A good choice in many cases is the mean-annual flood (i.e., an event with an approximate recurrence interval of 2 years).

The hydraulic conditions are first calculated for the "dominant" discharge. If they fail within the applicable range of Equation X, then the sediment transport rate is calculated.

The solution for the equilibrium slope is obtained by a trial-and-error procedure. The calculation of the sediment transport rate proceeds in the same manner as for the supply reach. For the first calculation, the slope is assumed to be the design slope. After calculating the sediment transport rate, it is compared with the upstream supply. If the supply is greater, then a steeper slope is chosen. If the supply is less than the downstream transport capacity, a flatter slope is chosen. The sediment transport rate for the downstream channel is recalculated using the revised estimate of slope. This procedure is repeatedly performed until the supply and transport rates are nearly equal, at which time the equilibrium slope has been determined.

Design Example.--The following is an example of the procedure by which the equilibrium slope of a channel can be calculated:

An upstream channelized section has been in existence for many years and has not changed significantly. It has been proposed that the channelization be carried out the remainder of the distance to the main river because of a proposed development along the unchannelized portion. Since the slope for the downstream section was greater, the designer of the channel decided to make a more confined channel than the upstream channel because the steeper slope could result in faster velocities, and thus a smaller channel could handle the same amount of water and be cheaper to construct.

The pertinent information for checking the channel response is:

Parameter	Upstream Channel	New Downstream Channel
Dominant Discharge (mean annual flood)	500 cfs	500 cfs
Channel shape	trapezoidal	trapezoidal
Channel slope	0.01 ft./ft.	0.02 ft./ft.
Mean sediment diameter	D_{50} =2 mm	D_{50} =2 mm
Gradation coefficient	$G_s = 3$	$G_s = 3$
Channel resistance (n)	0.025	0.025
Channel base width	20 ft.	10 ft.
Side slope ratio	2H:1V	2H:1V

The first step is the computation of the sediment supply from the upstream channel. The flow conditions are computed assuming normal depth (the channel is steep). The sediment transport equation is:

$$q_s = 0.0064 \frac{n^{1.77} V^{4.32} G_s^{0.45}}{Y^{0.30} D_{50}^{0.61}} \qquad (X)$$

The hydraulic conditions are:

$Y = 2.27$ ft.;

$A = 55.62$ ft.^{2};

$V = 8.96$ ft./sec.;

$T_w = 29.10$ ft.;

$Y_h = 1.92$ ft.;

$F = 1.14$.

The variable "T_w" is the top width of flow, and the variable "Y_h" the hydraulic depth of flow. Since the sediment transport equations were developed for a unit-width channel, it is felt that the top width of flow, "T_w", offers the best representation of the average channel characteristics when used with the hydraulic depth, "T_h". All sediment transport rates in this example are calculated using this combination of flow parameters.

The unit sediment transport rate for thC preceding conditions is:

$$q_s = 0.107 \text{ cfs/ft.} \qquad \text{(XIV)}$$

The total sediment transport rate (Q_s) is obtained by multiplying the rate per unit width by unit width by the top width of flow:

$$Q_s = 3.12 \text{ cfs} \qquad \text{(XV)}$$

The next step is determination of the equilibrium slope for the downstream channel with a sediment supply rate of 3.12 cfs. This requires a trial-and-error procedure by which a given slope is chosen to compute the flow conditions and from the flow conditions the sediment transport rate is calculated. When the computed rate is equal to the supply rate, the equilibrium slope has been found. For an initial guess, the design slope of 0.02 ft./ft. was chosen. Table 3 presents the results of the calculations. The conditions used, including incipient motion, fall within the range of the sediment transport equation. All sediment sizes smaller than 79 mm will be moving; and Since none of the sediment present is this large, armoring will not be a problem. The resulting equilibrium slope is 0.0094 ft./ft. This is substantially less than the proposed 0.02 ft./ft. design slope. Over the 1000 feet of channel length, using the main river as a control and assuming no sediment is supplied by the channel banks, a headcut of 10.6 feet would develop! As one possible solution to prevent such an occurrence, a wider channel could be chosen which could maintain equilibrium at a two-percent slope, if such a slope is desired. Purely from a qualitative standpoint, this design channel would have to be wider than the upstream channel in order to maintain a steeper slope.

Table 3. Equilibrium Slope Calculations

Slope	Thalweg Depth (ft.)	Area (ft.^{2})	Velocity (ft./sec.)	Top Width (ft.)	Hydraulic Depth (ft.)	Froude Number (N/A)	q_s (cfs)	Q_s (cfs)
0.020	2.61	39.72	12.59	20.44	1.94	1.59	0.46	9.40
0.015	2.82	44.03	11.36	21.27	2.07	1.39	0.29	6.17
0.013	2.92	46.35	10.79	21.70	2.14	1.30	0.23	4.99
0.011	3.06	49.22	10.16	22.22	2.22	1.20	0.18	4.00
0.010	3.13	50.94	9.82	22.53	2.26	1.15	0.15	3.38
0.009	3.22	52.92	9.45	22.88	2.31	1.09	0.13	2.97

Therefore: $\underline{S = 0.0094}$ ft./ft. = Final Slope (interpolating).

Check incipient notion:

$f_o = 116.5n^2R_h^{-1/3} = 0.056$ (R_h = hydraulic radius).

$\tau_o = 1/8 \rho f_o V^2 = 1.25$ lbs./ft.^{2}

$D = \tau_o / [0.047(\gamma_s - \gamma)] = 0.26$ ft. $= 79$ mm. (no armoring).

The latter conclusion is drawn from the fact that if the downstream channel is of the same size as the upstream channel, its velocity would he higher. Therefore, the sediment transport rate would be higher. In order for the steeper channel to have an equal transport rate, it is necessary to reduce its velocity. If the slope is to remain constant, then the channel must be widened. It should be cautioned though that channels with unreasonably high width-to-depth ratios will develop low-flow channels which are more constrictive. The low-flow channel will then have a flatter equilibrium slope than predicted by the analysis. By using Lacey's formulas for computing stable channels (10), indications are that the design channel width-to-depth ratio should be less than 7.0 times the velocity of the "dominant" discharge in order to preclude the development of such low-flow channels.

In addition, the Sides of the channel should be checked to see if they can withstand the predicted velocities. If not, a suitable form of bank protection should be included in the design.

REFERENCES

1. Meyer-Peter, E., and Muller, R., 1948, "Formulas for bed-load transport," Proceedings, III Cong, IAHR, Stockholm, pp. 39-64.

2. Einstein, H.A., 1950, "The bed-load function for sediment transportation in open channel flows," U.S. Dept. of Agriculture, Soil Conservation Service, T.B. No. 1026.

3. Simons, Li & Associates, Inc., 1982, "Sediment Transport Analysis of Rillito River and Tributaries for the Tucson Urban Study."

4, Simons, Li & Associates, Inc., 1981, "Hydraulic and Geomorphic Analysis for Rio Nuevo Development Project, Santa Cruz River, Tucson, Arizona."

5. Simons, Li & Associates, Inc., 1981, "Theoretically-Derived Sediment Transport Equations for Pima County, Arizona."

6. Chow, V.T., 1959, <u>Open-Channel Hydraulics</u>, McGraw-Hill, New York.

7. Shields, A., 1936, "Anwendung der Aehnlichkeitsmechanik und Turbulenz forschung auf die Geschiebewegung," Mitteilung Preussiscben Versuchanstalt Wasser, Erd, Schiffbau, Berlin, No. 26 (in German).

8. Laney, R.L., 1972, "Chemical Quality of the Water in the Tucson Basin, Arizona," U.S. Geological Survey Water-Supply Paper 1939-D.

9. Gessler, J., 1971, "Beginning and ceasing of sediment motion, <u>River Mechanics</u>, edited by H.W. Shen, Chapter 7, Fort Collins, Colorado, 22p.

10. Simons, D.B., and Senturk, F., 1977, <u>Sediment Transport Technology</u>, Water Resources Publications, Fort Collins, Colorado.