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III.
Structural Analysis and Design

Instructor: Peter Branagan

Tapes

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PREFACE

The subject matter that is presented as a review of structural analysis and design has been selected and prepared with the professional engineer examination as a criterion. A comprehensive review of the extremely broad field of both areas is neither reasonable nor feasible. It is felt that as a review, the material will serve to prompt one's memory and direct the review process along the general paths covered in both classrooms and office practice.

No attempt has been made to discuss derivation of formulas, or development of analytical or design processes, since these must be presumed as prerequisites for the examination.

The format is a problem solving presentation and attempts to address the areas which have appeared in past exams. It is hoped that the material will accomplish its goal of stimulating the memory banks and supplying a measure of confidence to the applicant's approach to the coming exam.

The writer expresses thanks for their help to the following: Professors Kenneth Leet and Leroy Cahoon of the Department of Civil Engineering, Northeastern University, for permission to use material from their publications; to Doctor Ronald Sharpin for his kind encouragement, and for giving me the opportunity to present this material; and to my wife, Theresa, who patiently supported the project and expertly typed much of the material.

James Regan

Introduction:

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The structural section of the PE refresher course consists of two sections. The first section reviews various "hand" methods of analysis that may be useful in determining the design parameters of a problem. The second section reviews the design of a structure's members using materials that are commonly encountered: concrete, steel, wood, and masonry.

The first section includes analysis methods for simple structures. It considers the Cable theorem, and the determination of deflections by both the Moment Area Method and the Law of Virtual Work. Analysis of Indeterminate structures by the Flexibility Method and by a Moment Distribution Method are reviewed. Frame sidesway by the moment distribution method is also considered. Finally the rigidity of masonry walls with openings is analyzed for determining the distribution of lateral forces to the individual piers.

The second section reviews structural design for imposed forces and moments . Structural steel members are reviewed to resist axial, flexural, and combined forces per, AISC, Manual of Steel Construction, 9th ed. Reinforced concrete design by the ACI 318 Building Code is reviewed for beams, shear, and column design. The Specification for Wood Construction, NFPA,is used to review wood members for axial, flexural, and shear forces. Masonry members are designed by ACI 531 Building Code Requirements for Concrete Masonry Structures to resist axial and shear forces. Composite construction of wood beams with steel side plates, and concrete slabs on steel sections are included. Finally, seismic code requirements are reviewed.

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STRUCTURAL

ANALYSIS

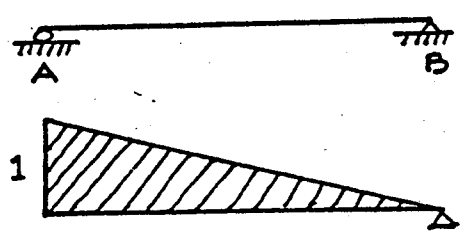
INFLUENCE LINES

An influence line is a curve, the ordinates of which give the values of some particular function (shear, moment, reaction, bar force, etc.) in a fixed element (member section, support, bar in a truss, etc.) due to a unit load acting at the point corresponding to the particular ordinate being considered.

Influence lines for statically determinate structures are straight lines. To draw such diagrams it is only necessary to compute the ordinates at a few control points and connect their terminals with straight lines.

For statically indeterminate structures the influence lines are curved and their construction involves an analysis which may require considerable time. The Meuller-Breslau principle provides a simple method to draw approximate shapes of these influence lines. This principle may be stated as follows:

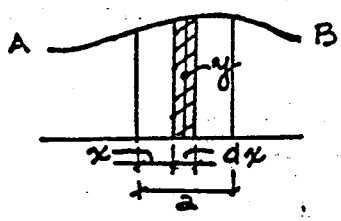
> The ordinates of the influence line for any stress element of any structure are proportional to those of the deflection curve which is obtained by removing the restraint corresponding to the stress resultant and introducing in its place a corresponding deflection.



The reaction restraint at A is removed and an upward deflection (corresponds to an upward reaction) is introduced. The area enclosed between the original and the final positions of the beam is the influence diagram for the reaction at A.

USE OF INFLUENCE LINES

1. Influence lines show where to place a load on a structure to maximize the value of the function for which the influence line is drawn.
2. The value of the function due to a concentrated load equals the product of the magnitude of the load and the ordinate of the influence line at the point of application of the load.
3. For maximum uniform loads place the load over those portions of the curve for which the ordinates have the same sign.
4. The value of a function due to a uniformly distributed load equals the product of the intensity of the load and the net area under that portion of the influence line which is under consideration.



AB is the influence line for a given function (shear, reaction, moment, etc.) on a beam which has a uniform load of w pounds per foot.

The portion of load in distance dx is treated as a concentrated load and is equal to wdx.

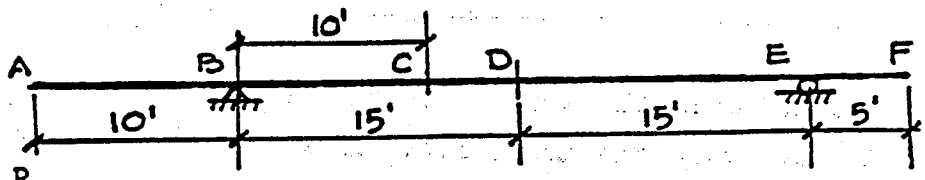
By Rule 4, the value of function F due to this differential load is dF = wdx y.

$$\therefore\ F = \int_0^a dF = \int_0^a w\,dx\ y = w\int_0^a y\,dx$$

which is the same as saying, w(the uniform load per foot) multiplied by the area under the influence curve corresponding to the portion of the structure being considered, in this case, distance a.

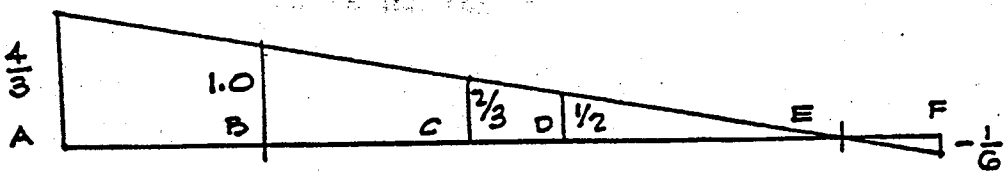
EXAMPLE

1. Draw the influence lines for the following functions: Reaction at B, Reaction at E, Shear at D, Moment at B, Moment at C.
2. Use the influence line to determine the reaction at point B due to a uniform dead load of 200 lb per ft.
3. What is the maximum positive and maximum negative moment produced at point C by a concentrated load of 50 kips?



I.L. R_B

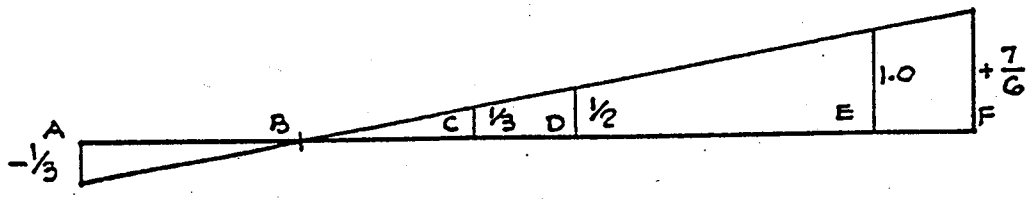
Place a unit load at B, the reaction at B is 1; place a unit load at E, the reaction at B is 0. We now have two points on the influence line, draw a straight line through these points and extend this line to intersect perpendiculars from A and F, This sloping line is the influence line for the reaction at B.



By proportion, when the unit load is at A, the reaction at B is 4/3; and when the unit load is at F, the reaction at B is -1/6.

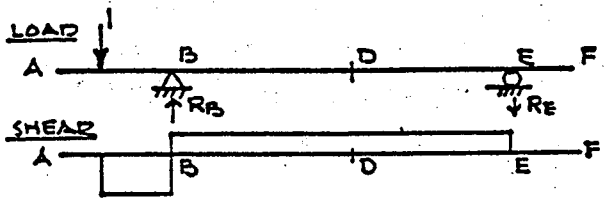
I.L. R_E

Place a unit load at E, the reaction at E is 1, place a unit load at B, the reaction at E is 0. Again, we have two points on a straight line, and connecting these points in the same way as for the influence line for the reaction at B we can easily draw the influence line for the reaction at E.

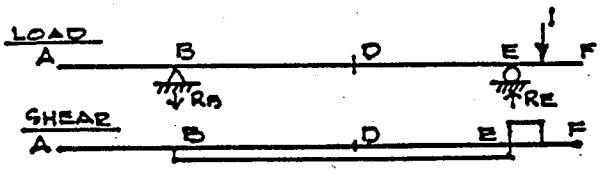


From the diagram, when the unit load is at A, the reaction at E is -1/3; and when the unit load is at F, the reaction at E is +7/6.

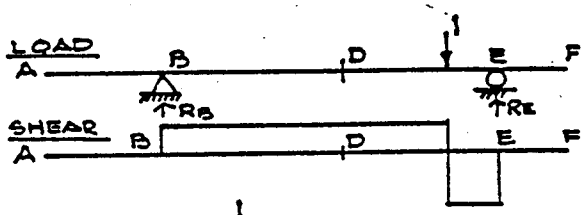
I.L. Shear at D.



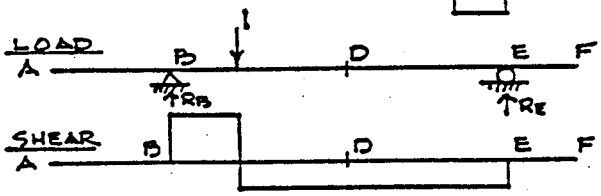
A unit load at any point between A & B will induce a shear at D equal to the negative of the reaction at E.



Placing a unit load at any point between E & F will cause a shear at D equal to the reaction at B.

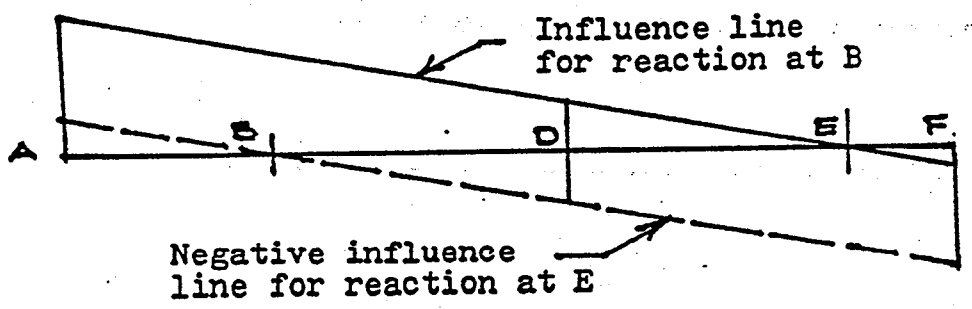


For a unit load at any point between D and E the shear at D is equal to the reaction at B.

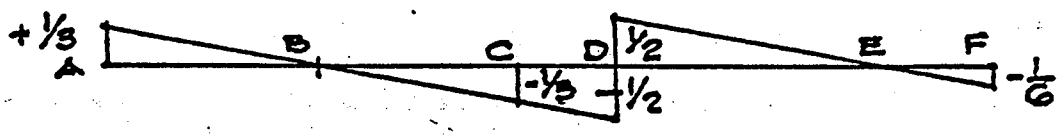


If the unit load is placed at any point between B and D the shear at D is equal to the negative of the reaction at E.

So, to draw the influence line for the shear at D, draw the influence line for the reaction at B and add to it the negative influence line for the reaction at E. Then draw at D a perpendicular line which intersects both of the influence lines.

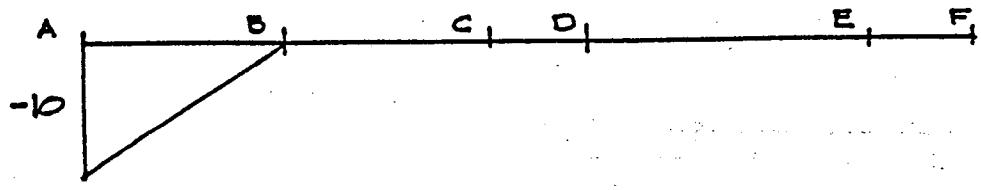


The final influence line for the shear at D is the left-hand portion of the lower influence line and the right-hand portion of the upper influence line, thusly



I.L. for the Moment at B

Place the unit load at A and take moments about point B;
1 x 10 counter-clockwise is negative, and $M_B = -10$.
For a unit load at B the moment at B = 0. At any other point to the right of B the application of the unit load does not induce any moment at B. Then the influence line for the moment at B is



I.L. for the Moment at C

If the unit load is at A the reaction at B, from the influence line for the reaction at B, is equal to 4/3. and taking moments about C we have

$\curvearrowright +$ $\quad -1 \times 20 + 4/3 \times 10 = -20/3$

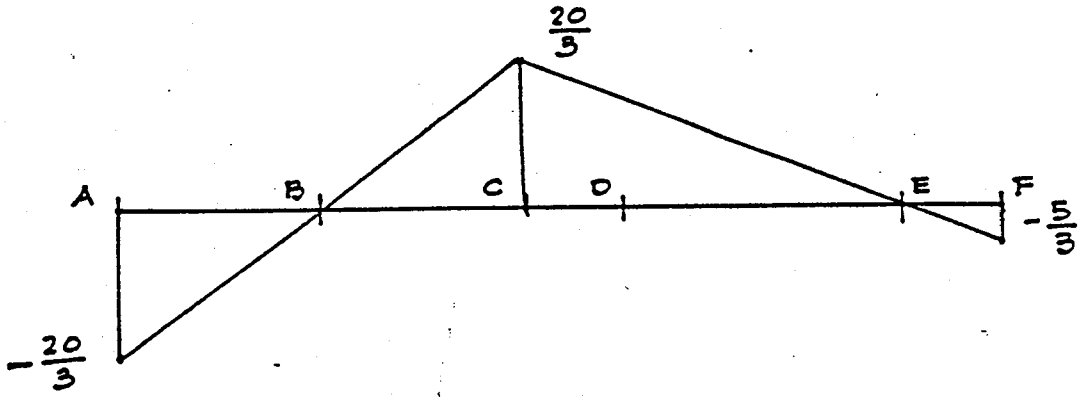
When the unit load is at C the reaction at B is 2/3 and taking moments about C

$\curvearrowleft +$ $\quad 2/3 \times 10 = +20/3$

Placing the unit load at F the reaction at E = 7/6 and summing moments about C

$\curvearrowright +$ $\quad -1 \times 25 + 7/6 \times 20 = -5/3$

When the unit load is at B or E there is no moment induced at C. The final influence line is drawn by connecting the control points with straight lines.



The reaction at B due to a uniform load of 200 lbs/ft by Rule 4 is

$$200 \times \text{Area enclosed by the influence line.}$$
$$= 200\left[\tfrac{1}{2}(4/3 \times 40 - 1/6 \times 5)\right]$$
$$= 15750/3 = 5250^{\#}$$

The maximum reaction at B occurs when the positive portion of the influence line is loaded.

$$200 \times 1/2 \times 4/3 \times 40 = 16000/3 = 5333^{\#}$$

The maximum positive moment at C produced by a concentrated load of 50^k is obtained by placing the 50^k at the point of maximum positive value for the influence line for moment at C, this is at point C itself and the ordinate value is + 20/3. Therefore, the maximum positive moment is

$$20/3 \times 50 = 333.3^{ft\text{-}k}$$

The maximum negative moment at C is produced by placing the 50^k load at the point of maximum negative value of the influence line for moment at C. This location is at point A, and the maximum negative moment is

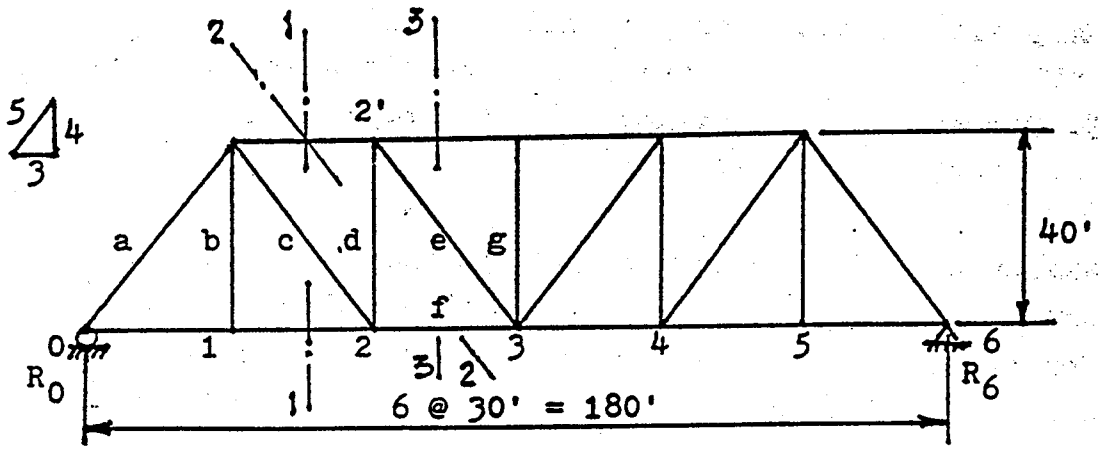
$$50 \times (-20/3) = -333.3^{ft\text{-}k}$$

Influence lines for bar forces in a truss are drawn for a unit load moving along the truss, the load is usually placed on the bottom chord. Since the truss is determinate, the influence lines are straight lines between the panel points.

The evaluation is done by either the method of sections or the method of joints. In using the method of sections it is best to use the free body diagram which does not contain the unit load.

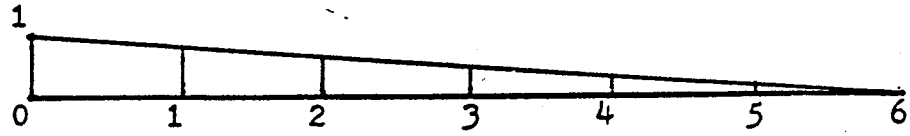
EXAMPLE

Given a truss having parallel top and bottom chords.
Draw the influence lines for the forces in bars a thru g.

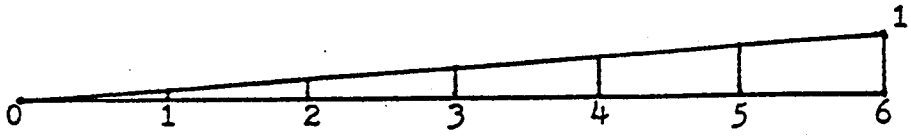


Draw the influence lines for the reactions R_0 and R_6.

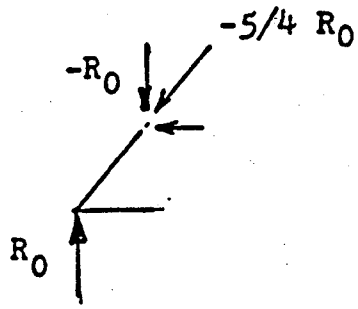
I.L. for the reaction R_0



I.L. for the reaction R_6



I.L. for the force in bar a.



Using the method of joints and isolating joint 0 as a free body, by inspection: when the unit load is at point 0 the bar force in a is equal to zero. If the unit load is at any point from 1 to 6 then

$$S_a = -5/4\ R_0$$

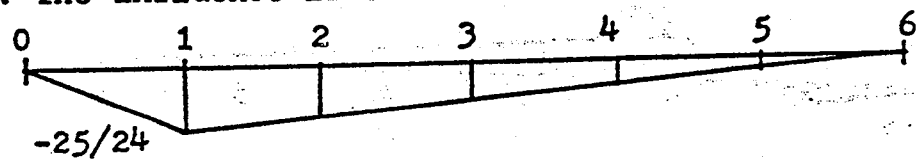
and is minus because the bar is in compression.

When the unit load is at point 6 the force in bar a is = 0.

A unit load at point 1 will induce a force in bar a =

$$S_a = -5/4 \times 5/6 = -25/24$$

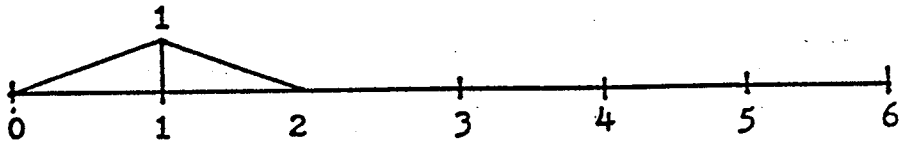
∴ The influence line for the force in bar a is



I.L. for the force in bar b.

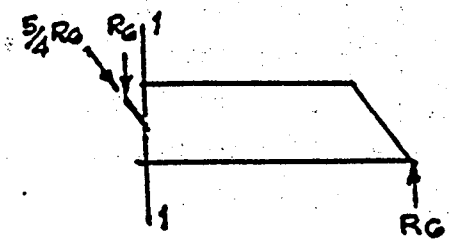
When the unit load is at 0, $S_b = 0$; when the unit load is at point 1, $S_b = 1$; and when the unit load is at points 2 thru 6 the force in bar b is = 0.

The influence line for the force in bar b is then



<u>NOTE</u> In general, in the free body diagram above, and in those to follow, only the forces related to the pertinent discussion are shown.

I.L. for the force in bar c.

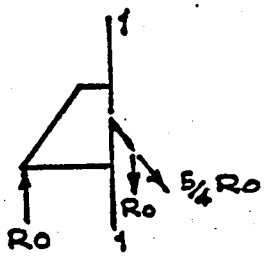


Cutting section at 1-1 and using the right-hand free body, if the unit load moves from point 0 to point 1, the force in bar c is

$$S_c = -5/4\ R_6 \text{ compression.}$$

If the load is at point 1 then

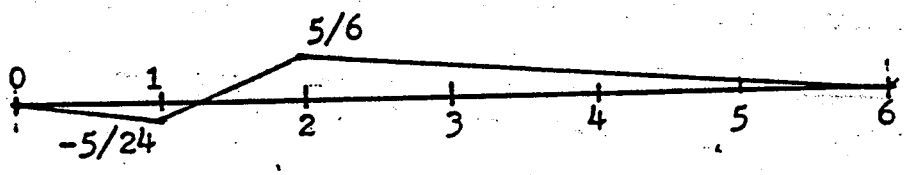
$$S_c = -5/4 \times 1/6 = -\ 5/24$$



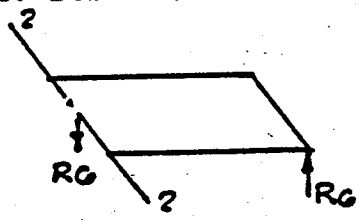
Using the left-hand free body, as the load moves from point 2 to point 6, the force in bar c is $5/4\ R_0$ tension. When the unit load is at point 2

$$S_c = 5/4 \times 2/3 = 10/12 = 5/6$$

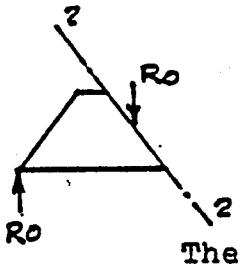
∴ The influence line for the force in bar c is



I.L. for the force in bar d.



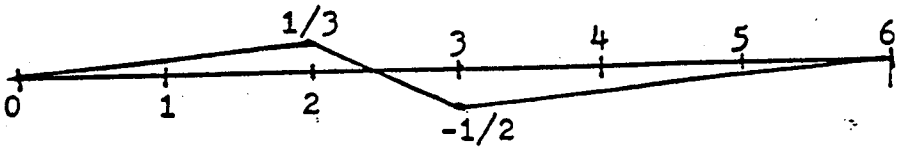
Cutting section at 2-2 and using the right-hand free body, as the unit load moves from 0 to 2 the bar force in d is $+\ R_6$ tension. With the unit load at 2 the force in bar d is +1/3.



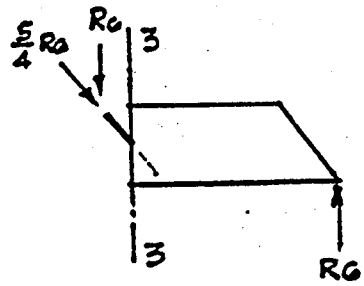
Using the left-hand free body, the bar force in d is equal to $-R_0$ compression. If the unit load is at 3,

$$-R_0 = -1/2$$

The influence line for the bar force in d is



I.L. for the force in bar e.

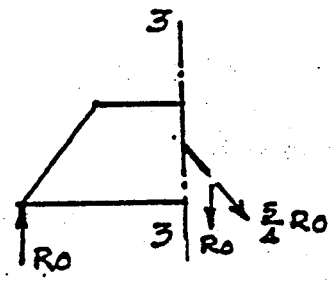


Cutting section 3-3 and using the right-hand free body, if the unit load moves from point 0 to point 2, the force in bar e is

$$S_e = -5/4\ R_6$$

If the unit load is at point 2 then

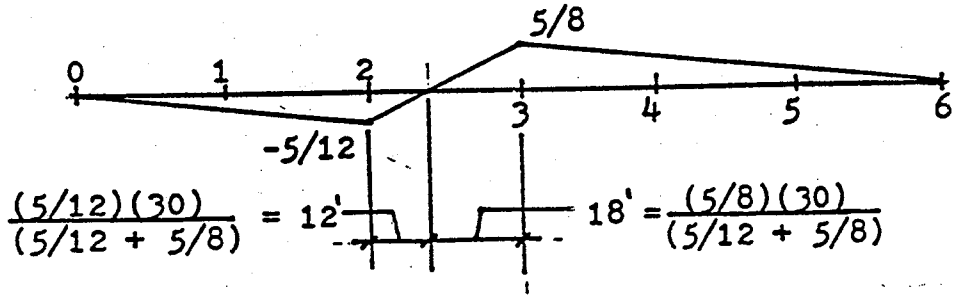
$$S_e = -5/4 \times 1/3 = -5/12$$



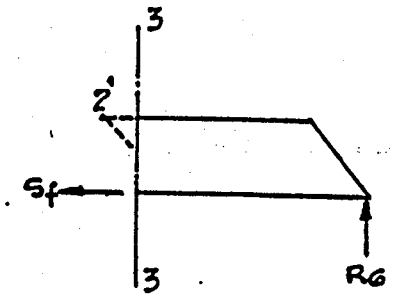
Using the left-hand free body, as the load moves from point 3 to point 6, the force in bar e is 5/4 R_0 tension. When the unit load is at point 3

$$S_e = 5/4 \times 1/2 = +5/8$$

∴ The influence line for the force in bar e is



I.L. for the force in bar f.



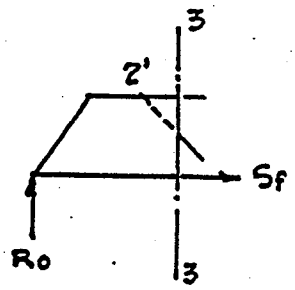
Cutting section 3-3 and using the right-hand free body, take moments about point 2' and consider the unit load as moving from point 2 to point 6.

$\circlearrowleft + \quad -120\ R_6 + 40\ S_f = 0$

$S_f = 3\ R_6$ tension

When the unit load is at point 2,

$R_6 = 1/3$ and $S_f = 3 \times 1/3 = 1$



Using the left-hand free body, take moments about 2' again, but now consider the unit load to be moving from point 0 to point 2.

$\circlearrowleft + \quad 60\ R_0 - 40\ S_f = 0$

$S_f = 3/2\ R_0$ tension

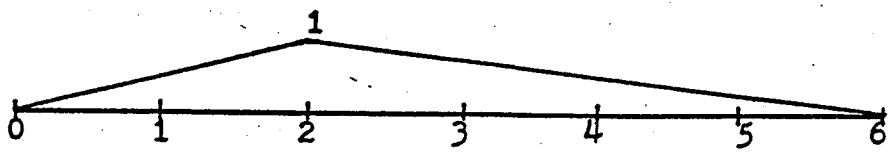
When the unit load is at point 2,

$R_0 = 2/3$ and $S_f = 3/2 \times 2/3 = 1$

If the unit load is at points 0 or 6

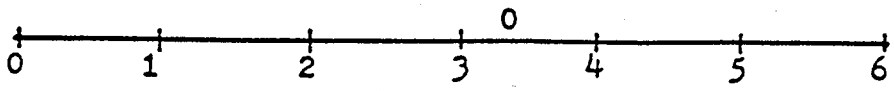
$S_f = 0$

∴ The influence line for the force in bar f is



I.L. for the force in bar g.

When the unit load is placed anywhere along the bottom chord of the truss there is no stress induced in bar g; therefore, the influence line for the force in bar g is



<u>EXAMPLE</u> Use of truss influence lines.

a. Determine the force in bar e due to dead weight of the truss of 0.6^k per foot.

b. What is the maximum tension in this bar if a uniform live load of 1^k per foot and a concentrated load of 8^k moves along the bottom chord of the truss?

c. What is the maximum compression induced in bar e by these live loads?

d. Find the total tensile and compressive forces in bar e.

Solution:

a. Since the dead load applies throughout the length of the truss, by rule 4, the force in bar e due to the dead load is

$$S_e = 1/2(-5/12)(72)(0.6) + 1/2(5/8)(108)(0.6)$$
$$S_e = -9.00 + 20.25 = 11.25 \text{ tension}$$

b. For the live load tension in bar e, use the positive portion of the influence line.

$$S_e = 1/2(5/8)(108)(1) + (5/8)(8)$$
$$S_e = 33.75 + 5.00 = 38.75$$

c. For the live load compression in bar e, use the negative portion of the influence line.

$$S_e = 1/2 \times (-5/12)(72)(1) - (5/12)(8)$$
$$S_e = -15.00 - 3.33 = -18.33$$

d. The maximum tension in bar e is

$$38.75 + 11.25 = +50^k$$

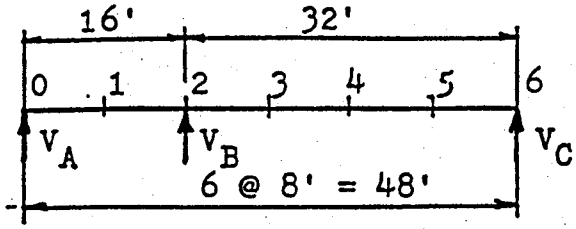
The maximum compression in bar e is

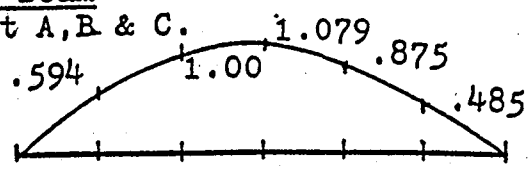
$$-18.33 + 11.25 = -7.08^k$$

It is apparent from these calculations, that a reversal of stress occurs in bar e as the live load moves across the truss. This is important in the design of the member and its connections because stress reversals affect metal fa[illegible] [illegible]ould become a critical consideration.

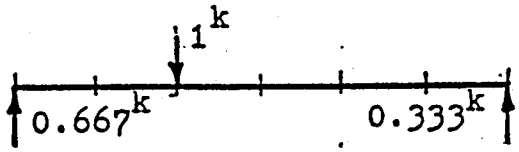
Influence Lines for Indeterminate Beam

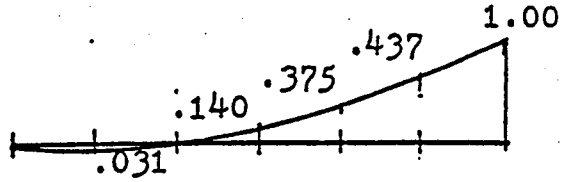
Find the I.L. for the reactions at A, B & C.



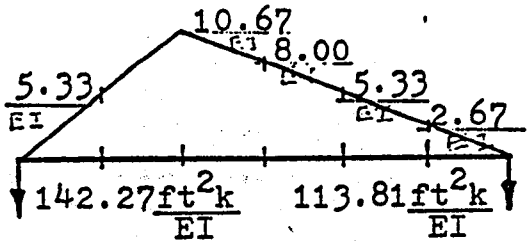


I.L. for V_B

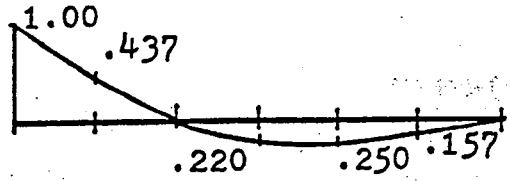




- I.L. for V_C



Conjugate Beam loaded with M/EI diagram.



I.L. for V_A

Solution:

Remove V_B and place a unit load at B (point 2), compute the deflections at points 1 through 5.

Using the conjugate beam method, cut sections at points 1 through 5 and compute the moment on the conjugate beam at the respective points.

$$\delta_1=(142.27)(8)-\tfrac{1}{2}(8)(5.33)(2.67)=1081$$
$$\delta_2=(142.27)(16)-\tfrac{1}{2}(16)(10.67)(5.33)=1820$$
$$\delta_3=(113.81)(24)-\tfrac{1}{2}(24)(8.00)(8.00)=1963$$
$$\delta_4=(113.81)(16)-\tfrac{1}{2}(16)(5.33)(5.33)=1593$$
$$\delta_5=(113.81)(8)-\tfrac{1}{2}(8)(2.67)(2.67)=882$$

$$V_B = \frac{\delta_B}{\delta_{bb}}$$

Since δ_{bb} (the deflection at B due to a unit load at B) is the same as δ_2, dividing each of the point deflections above by δ_2 will give the value of the ordinate of the influence line at the corresponding point.

The ordinate at point 1 is:

$$\frac{1081}{1820} = 0.594$$

at point 2:

$$\frac{1820}{1820} = 1.00$$

at point 3:

$$\frac{1963}{1820} = 1.079 \quad \text{etc.}$$

To find the influence line for the reaction at C, use the influence line for the reaction at B to obtain values for V_B at the various points along the beam. Applying a specific value of V_B at B and a unit load at the related point (determined from the IL for V_B), the reaction at C is determined by statics. Then using the influence lines for the reactions at B and C, the reaction at A is readily found.

e.g. For point 1, place a unit load at point 1, the reaction at B, from the influence line for V_B, is 0.594. Apply this at B and take moments about point O and solve for V_C.

For the reaction at A, simply sum the forces in vertical direction.

Repeat this process for each of the points along the beam.

Load point 1 and sum moments about O.

$$\circlearrowleft + \quad 1 \times 8 - 0.594 \times 16 + 48V_C = 0$$

$$V_C = 0.031$$

Summing forces in the vertical direction:

$$+\uparrow \quad -1.00 + 0.0594 - 0.031 + V_A = 0$$

$$V_A = 0.437$$

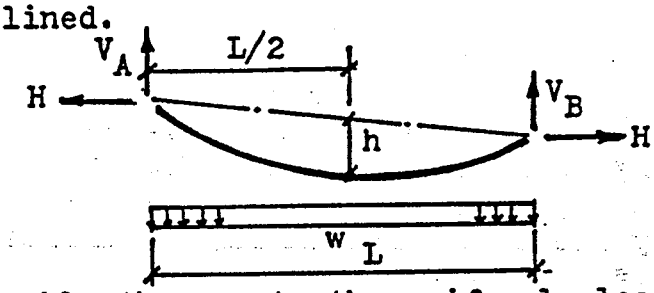
Load point 2.

$$\circlearrowleft + \quad 1.00 \times 16 - 1.00 \times 16 + 48V_C = 0$$

$$V_C = 0 \quad \text{etc.}$$

CABLES

General Cable Theorem: At any point on a cable acted upon by vertical loads, the product of the horizontal component of cable tension and the vertical distance from that point to the cable chord equals the bending moment which would occur at that section if the loads carried by the cable were acting on an end-supported beam of the same span. The cable theorem holds whether the cable chord is horizontal or inclined.



Applying the cable theorem to the uniformly loaded cable, at mid-span

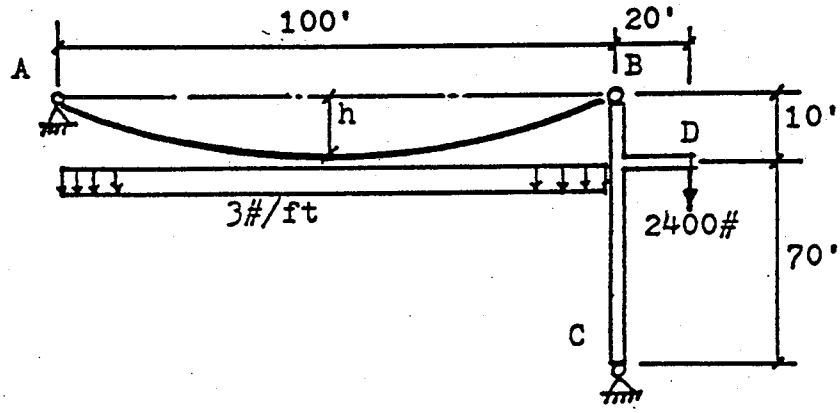
$$H \cdot h = wL^2/8$$

EXAMPLE

For the given structure:

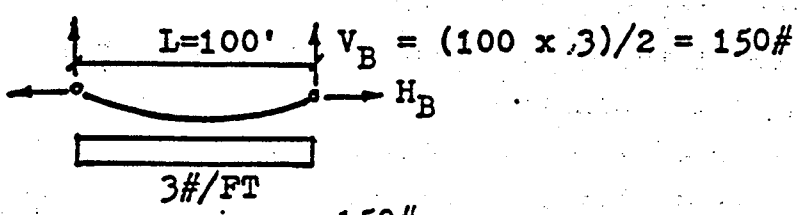
a. Find h at mid-span.

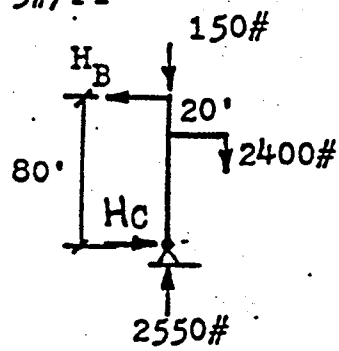
b. Determine the maximum cable tension.



Solution:

a.





$M_C = 0$ (+)

$(-H_B \times 80)+(20 \times 2400)=0$

$48000/80 = H_B = 600\#$

$\therefore h = wL^2/8H$

$= (3 \times 100^2)/(8 \times 600)$

$= 30000/(8 \times 600)$

$= 50/8 = 6.25'$ Ans.

b. $T_{max.} = \sqrt{V_B^2 + H_B^2} = \sqrt{(150)^2 + (600)^2}$

$= \sqrt{382500} = 618.5\#$ Ans.

Also, using the expression for T_{max} developed in strength of materials texts,

$T_{max.} = H\left[1+16n^2\right]^{1/2}$

where $n = h/L = 6.25/100 = .0625$

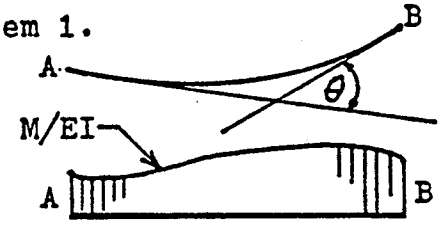
$T_{max.} = 600\left[1+16(.0625)^2\right]^{1/2} = 618.5\#$

DEFLECTIONS

MOMENT AREA METHOD: (Based on the following two theorems)

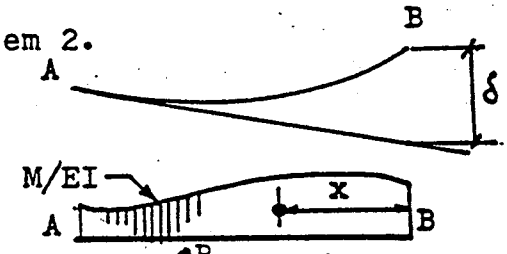
1. The change in slope of the tangents to the elastic curve between two points, A and B on the elastic curve, is equal to the area under the M/EI curve between these two points.
2. The deflection of a point B on the elastic curve from the tangent to this curve at point A is equal to the static moment about an axis through point B of the area under the M/EI curve between the points A and B.

Theorem 1.



$$\theta = \int_A^B M dx/EI$$

Theorem 2.

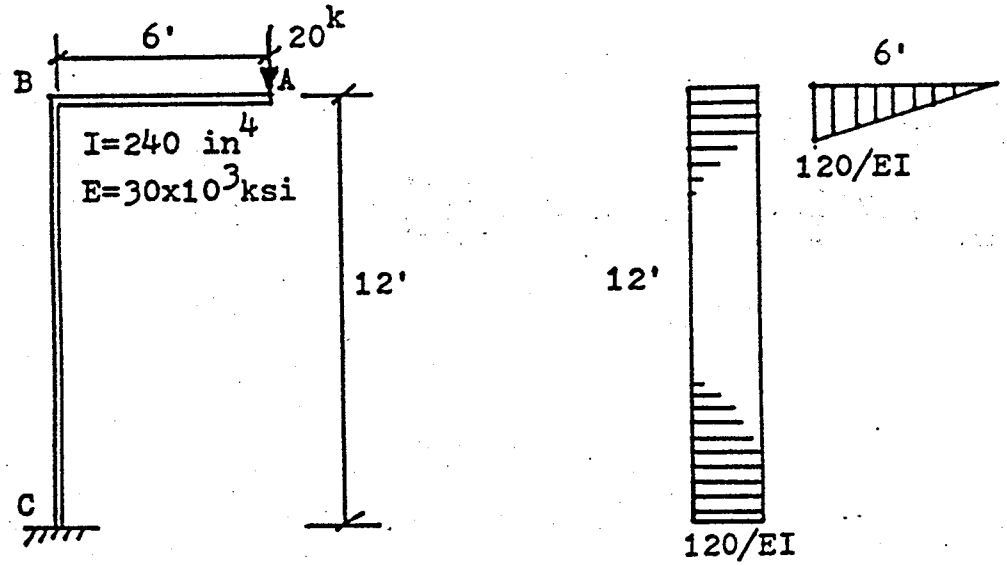


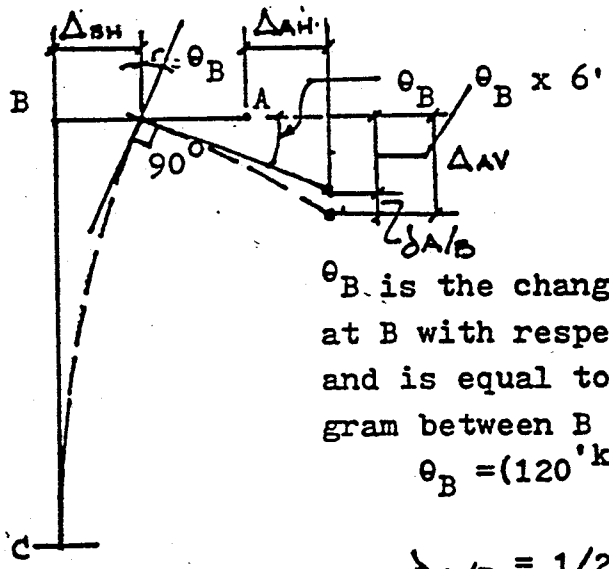
$$\delta = \int_A^B M x dx/EI$$

The areas and the moments of the areas of the bending moment diagrams may be found by integration, but in most cases the moment diagram may be broken up into a series of relativly simple geometric shapes. Knowing the areas and centroids of these shapes, the moments of the areas equal the centroidal distances from the axis of moments multiplied by the respective areas.

EXAMPLE

Find the vertical and horizontal deflection of point A due to bending strains. (Use the moment-area method)





θ_B is the change in slope of the tangent at B with respect to the tangent at C and is equal to the area of the M/EI diagram between B and C.

$$\theta_B = (120^{'k} x 12')/EI = \frac{(1440^{ft^2k} \; x 144^{in^2})}{EI \; ft^2}$$

$$\delta_{A/B} = 1/2 \; x \; \frac{120^{'k}}{EI} x \; 6' \; x \; 2/3 \; x \; 6' x \frac{1728 in^3}{ft^3}$$

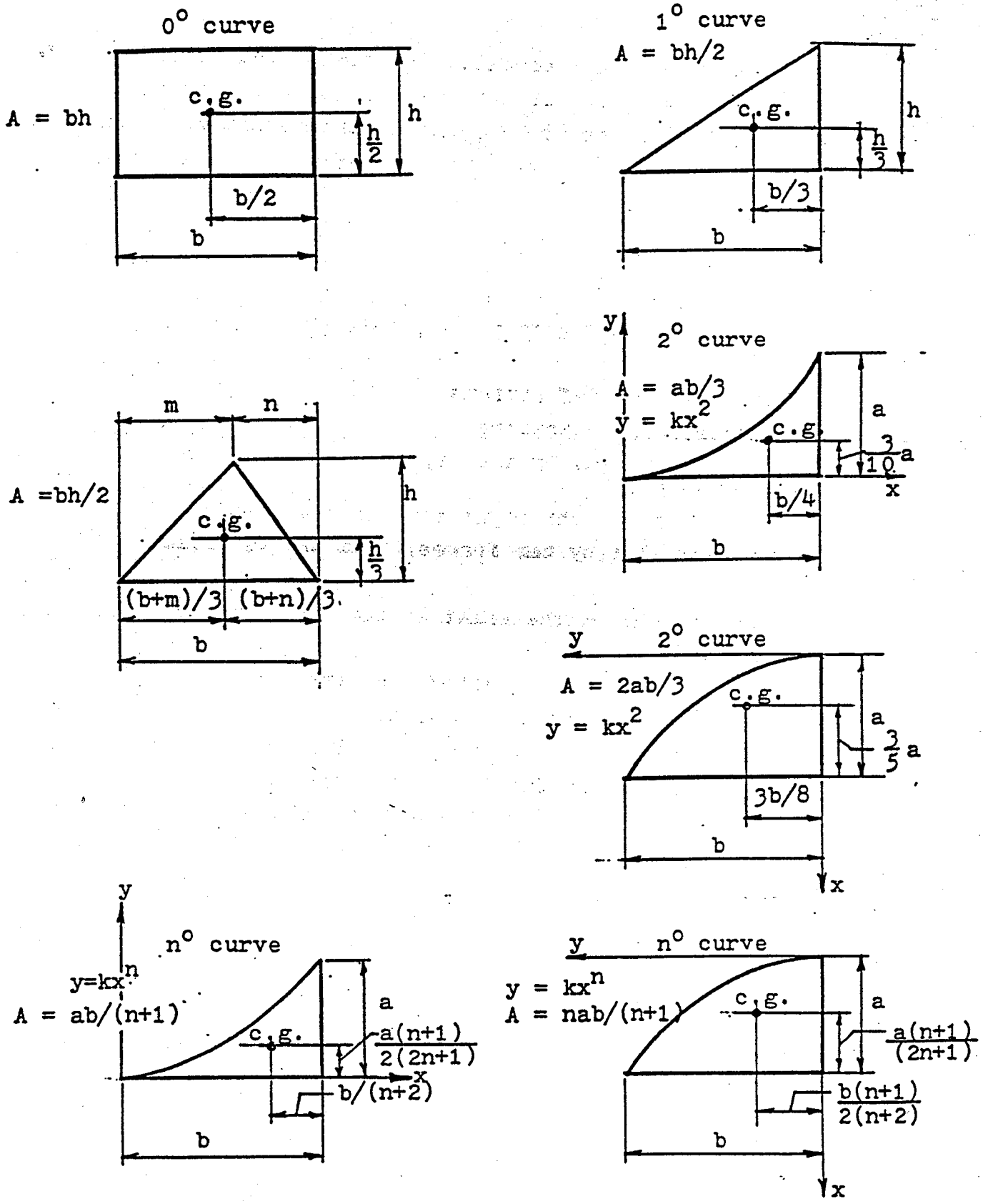
$$= \frac{1440 \; ft^3k}{EI} \; x \; \frac{1728 \; in^3}{ft^3}$$

$$\Delta_{AV} = (\theta_B \times 6') + \delta_{A/B}$$

$$\Delta_{AV} = \frac{1440}{EI} \times 144 \times 6' \times \frac{12''}{ft} + \frac{1440}{EI} \times 1728 = 2.4'' \text{ Ans.}$$

$$\Delta_{BH} = \Delta_{AH} = \delta_{B/C}$$

$$\delta_{B/C} = \frac{120^{ft\ k}}{EI} \times 12' \times 6' \times \frac{1728^{in^3}}{ft^3} = 2.07'' \text{ Ans.}$$



GEOMETRIC DATA FOR VARIOUS

DEFLECTIONS (cont.)

LAW OF VIRTUAL WORK:

If a structure is in equilibrium under a set of forces and if the structure is given a virtual displacement consistent with the constraints of the structure, then the external virtual work done is equal to the internal virtual work done.

$$W_{(external)} = W_{(internal)}$$

The general equation is:

$$\begin{matrix} 1 \times \Delta \\ 1 \times \alpha \end{matrix} + W_R = \sum \frac{F_Q F_P L}{AE} + \int_0^L M_Q M_P \frac{ds}{EI}$$

$1 \times \Delta$ is used for deflections.

$1 \times \alpha$ is used for rotations.

W_R is the work done by the supports, this is due to Q forces only.

The first term on the right side of the equation is the work done by bar forces; that is, by axial forces.

The second term on the right is the work done by flexural forces.

Note: We have neglected the work done by shear and torsion.

F_Q = Forces in frame due to a unit load Q.

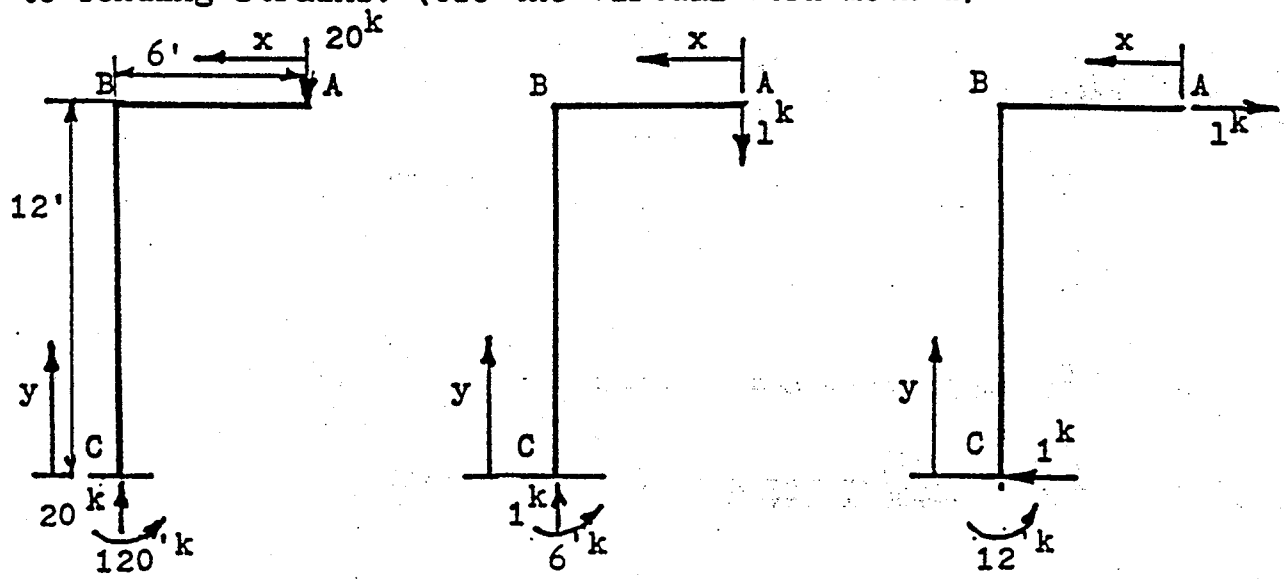
M_Q = Moment in beam due to a unit load Q.

F_P = Forces in frame due to actual real loads.

M_P = Moment in beam due to actual real loads.

EXAMPLE

Find the vertical and horizontal deflection of point A due to bending strains. (Use the virtual work method)



P Forces $\qquad$ Q_V Forces $\qquad$ Q_H Forces

Member AB	x increases A to B $0 < x < 6$	M_P 20x	M_{Q_V} x	M_{Q_H} 0
BC	y increases C to B $0 < y < 12$	-120	-6	-12+y

$$Q\Delta_{AV} = \int_0^L M_Q M_P \frac{ds}{EI}$$

$$EI(1^k)\Delta_{AV} = \int_0^6 (x)(20x)dx + \int_0^{12}(-6)(-120)dy$$

$$EI\Delta_{AV} = \frac{20x^3}{3}\Big|_0^6 + 720y\Big|_0^{12}$$

$$EI\Delta_{AV} = 1440 + 8640$$

$$\Delta_{AV} = \frac{1440 + 8640}{30000 \times 240} \times 1728 = 2.4'' \text{ Ans.}$$

Example (Cont.)

$$Q\Delta_{AH} = \int_0^L M_Q M_P \frac{ds}{EI}$$

$$EI(1^k)\Delta_{AH} = \cancel{\int_0^6 (20x)(0)dx}^{\,0} + \int_0^{12} (-12+y)(-120)dy$$

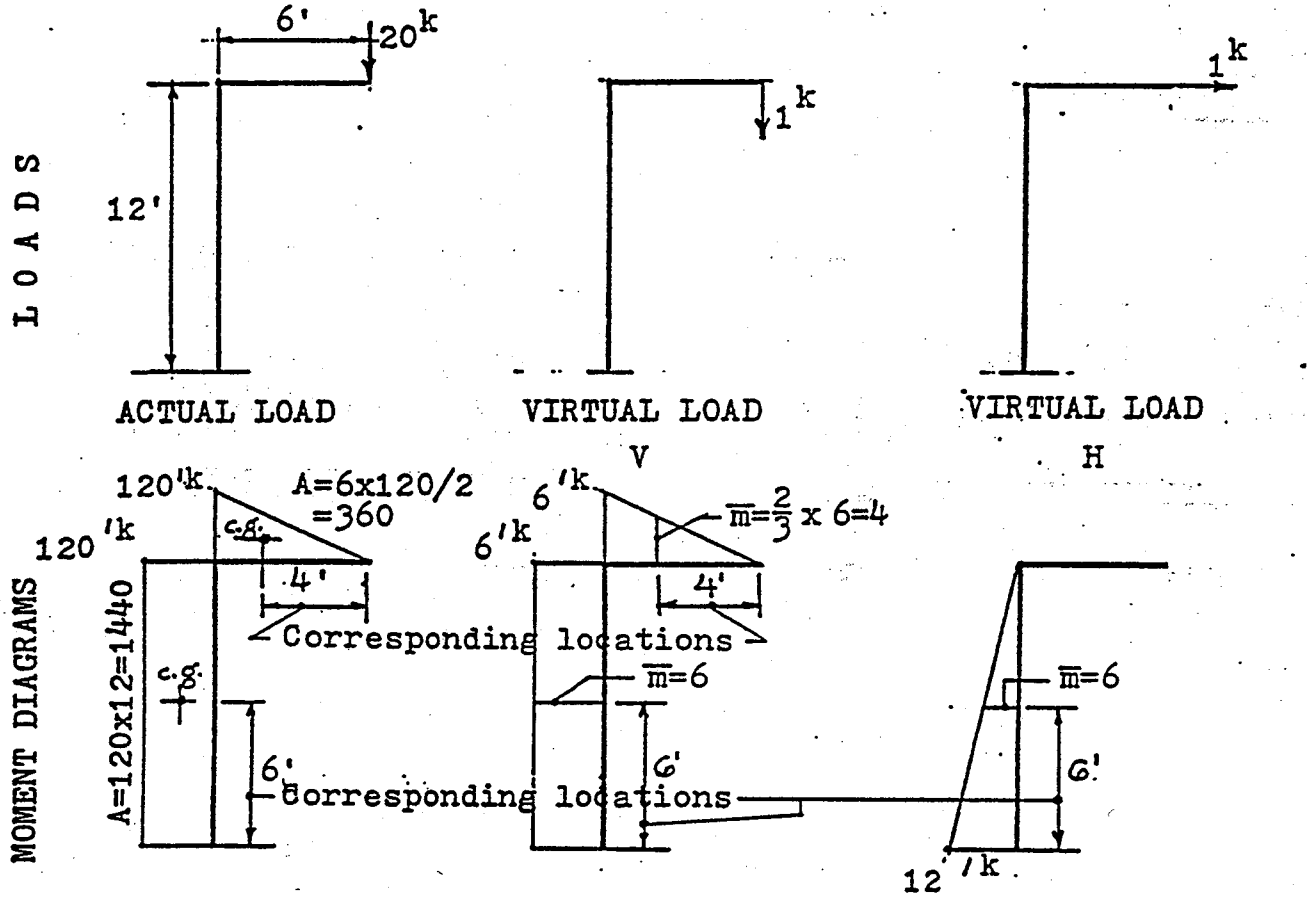
$$EI\,\Delta_{AH} = 1440y - 60y^2 \Big|_0^{12}$$

$$EI\,\Delta_{AH} = 17280 - 8640 = 8640$$

$$\Delta_{AH} = \frac{8640}{30000 \times 240} \times 1728 = 2.07'' \text{ Ans.}$$

SEMI-GRAPHICAL METHOD OF INTEGRATION

A variation of the virtual work approach to this type of problem is a semi-graphical integration method. It works well for moment diagrams composed of easily defined geometrical shapes. Such is the case for this particular problem.



A_{RLM} = Areas of real load moment diagrams.

$\overline{m}$ = Ordinates taken on virtual load moment diagrams at location corresponding to centers of gravity of real load moment diagrams.

$$1 \times \Delta = \frac{A_{RLM} \times \overline{m}}{EI}$$

$\overline{m}$ for triangular moment diagram

$\overline{m}$ for rectangular mom. diag.

$$\Delta_V = \frac{360 \times 4 + 1440 \times 6}{1 \times 30 \times 10^3 \times 240} \times 1728 = 2.42''$$

$\overline{m}$ for top leg mom. diag.

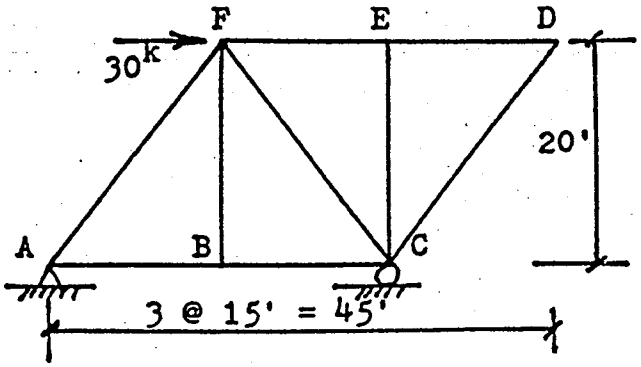
$\overline{m}$ for vert. leg mom. diag.

$$\Delta_H = \frac{360 \times 0 + 1440 \times 6}{1 \times 30 \times 10^3 \times 240} \times 1728 = 2.07''$$

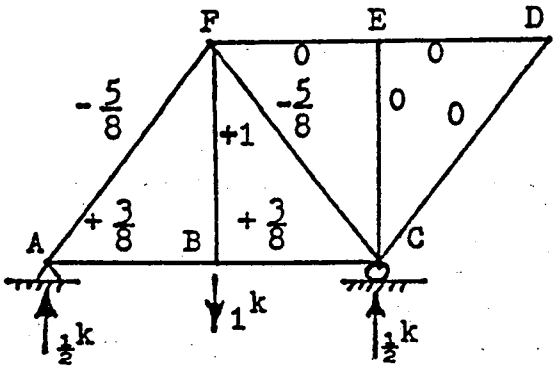
It can be seen that the graphical solution for this frame is much faster that the analytical one.

EXAMPLE

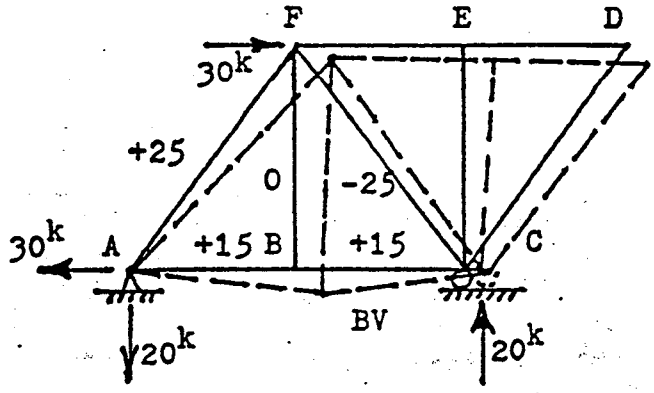
Compute the vertical deflection of joint B by virtual work.



Area of all bars = 5sq in.
$E = 30 \times 10^3$ ksi
Support C settles $\frac{1}{4}$".
Bar FB fabricated 0.2" too long.



Q Forces



P Forces

$$Q\Delta_{BV} + W_R = \sum(F_Q \Delta L) = \frac{F_Q F_P L}{AE} + F_Q \Delta L$$

$W_e = W_i$

Note: The deltas on the right-hand side of the equation denote changes in length, not deflections.

	(ft)	(in^2)	($\frac{ft}{sq\ in}$)	(k)	(k)	($\frac{k^2 ft}{sq\ in}$)	(in)	(in k)
	L	A	$\frac{L}{A}$	F_Q	F_P	$F_Q F_P (\frac{L}{A})$	L	$F_Q \Delta L$
AB	15	5	3	$+\frac{3}{8}$	+15	+16.875		
BC	15	5	3	$+\frac{3}{8}$	+15	+16.875		
AF	25	5	5	$-\frac{5}{8}$	+25	-78.125		
FC	25	5	5	$-\frac{5}{8}$	-25	+78.125		
FB	20	5	4	1	0	0	0.20	0.20
					$\sum =$	+33.750		0.20

Example (cont.)

$$(1)\ \Delta_{BV} + (0.25)(-\tfrac{1}{2}) = \frac{+33.75}{30000} \times 12 + 0.20$$

Note: The settlement at C is 0.25 inches and is positive since it is in the same direction as the assumed deflection.

The force at C due to the Q force is $\frac{1}{2}$ and it is negative because it is opposite in direction to the actual settlement.

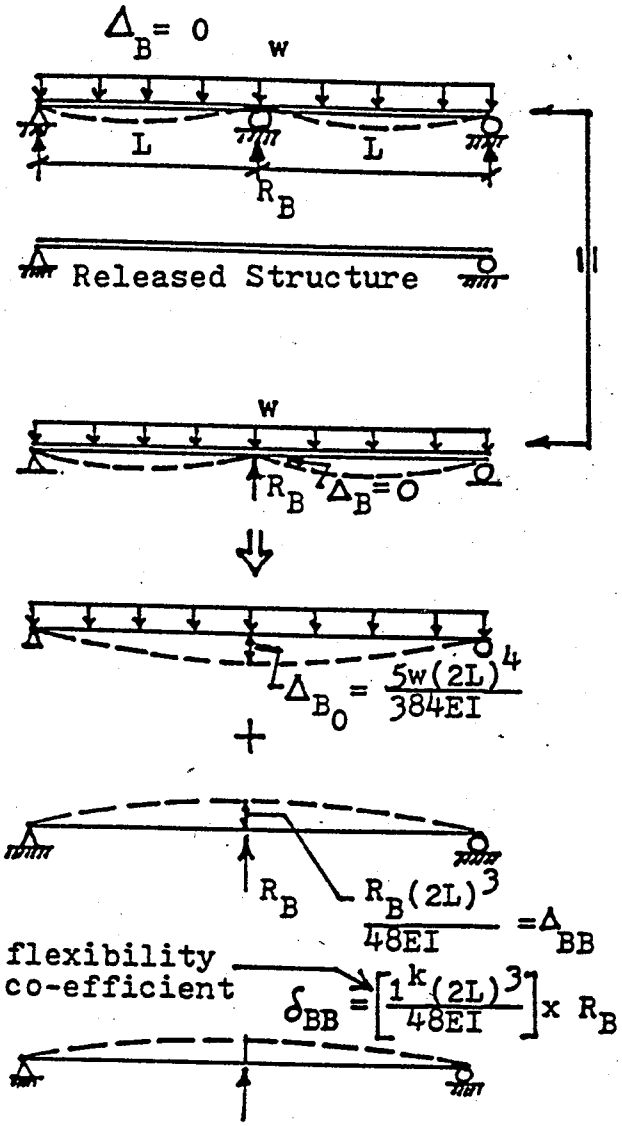
$$\Delta_{BV} - 0.125 = 0.0135 + 0.20 = 0.2135"$$

$$\therefore\ \Delta_{BV} = 0.2135 + 0.125 = 0.339" \text{ Ans.}$$

INDETERMINATE ANALYSIS

FLEXIBILITY METHOD (Also called consistent deformation method)

Outline of analysis



1. Establish the degree of indeterminancy.
   4 react. - 3 eq. stat.=1°
2. Establish a RELEASED struct. (determinate and stable)
3. Equivalent to original structure in all respects.
4. Superimpose the displ's produced by the original load and that by the redundant R_B.

The deflection due to the redundant R_B may also be expressed by using flexibility coefficients.

SIGN CONVENTION: Forces and displacements in the direction of the redundant are positive.

Outline of analysis (cont.)

Compatibility or geometry requires .....

$$\Delta_B = 0$$

$$\Delta_B = \Delta_{BO} + \Delta_{BB}$$

$$0 = \frac{-5w(2L)^4}{384EI} + \frac{R_B(2L)^3}{48EI}$$

$$R_B = 1.25\ wL \text{ Ans.}$$

$$\Delta_B = \Delta_{BO} + \delta_{BB} \cdot R_B$$

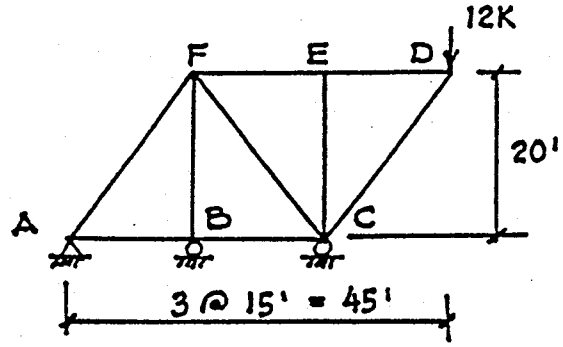
$$0 = \frac{-5w(2L)^4}{384\ EI} + \frac{R_B(2L)^3}{48\ EI}$$

$$R_B = 1.25\ wL \text{ Ans.}$$

Analysis of an indeterminate truss by Flexibility Method.

Determine the reactions for the given truss.

Area of all members = 5 sq in. E = 30,000 ksi

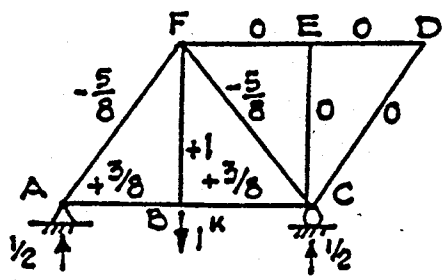


Use the reaction at B as the redundant.

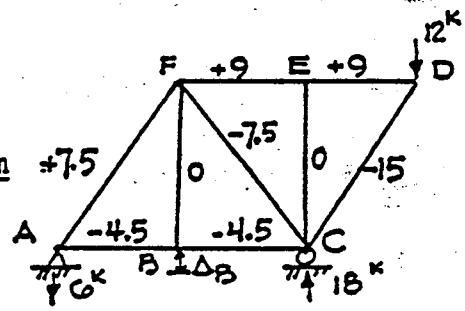
$$\Delta_B = 0 = \Delta_{BO} + \delta_{BB} R_B$$

Compute Δ_{BO} (The deflection of B in the released structure)

Q System

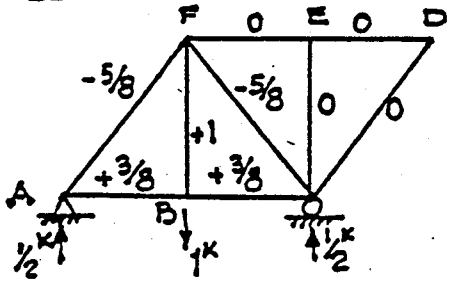


P System

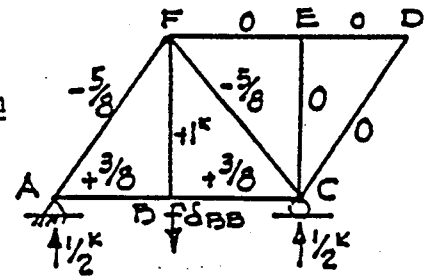


Compute δ_{BB} (The vertical defl. at B due to a unit load at B)

Q System



P System



Δ_{BO} $\quad 1^k \Delta_{BO} = \sum F_Q \frac{F_P L}{AE}$

δ_{BB} $\quad 1^k \delta_{BB} = \sum F_Q \frac{F_Q L}{AE}$

Bar	L	F_Q	$F_Q F_Q L$	F_P	$F_Q F_P L$
AB	15	$+\frac{3}{8}$	+ 2.11	- 4.5	- 25.31
BC	15	$+\frac{3}{8}$	+ 2.11	- 4.5	- 25.31
CD	25	0	0	-15	0
DE	15	0	0	+ 9	0
EF	15	0	0	+ 9	0
FA	25	$-\frac{5}{8}$	+ 9.77	+ 7.5	-117.19
FB	20	+ 1	+20	0	0
FC	25	$-\frac{5}{8}$	+ 9.77	- 7.5	+117.19
EC	20	0	0	0	0
			$\sum F_Q F_Q L =$ +43.76		$\sum F_Q F_P L =$ - 50.63

$$\Delta_B = 0 = \Delta_{BO} + \delta_{BB} R_B$$

$$0 = -\frac{50.63}{AE} + \frac{43.76}{AE} R_B$$

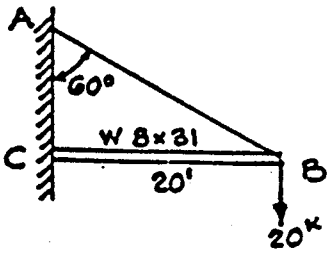
$$AE \neq 0; \therefore -50.63 + 43.76 R_B = 0$$

$$R_B = \frac{50.63}{43.76} = 1.16^k$$

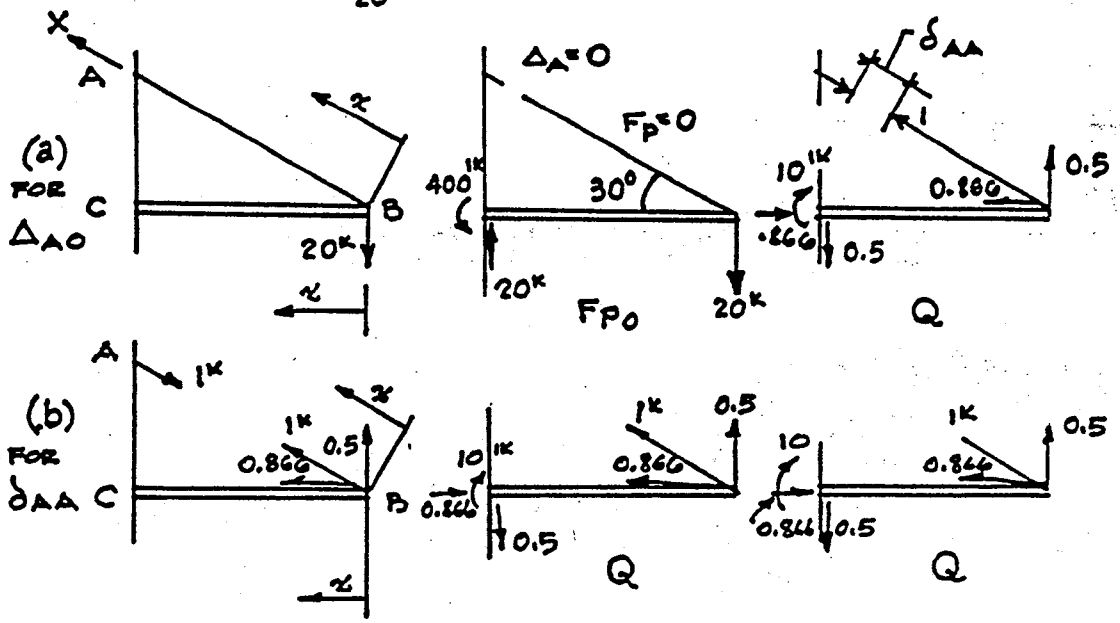
R_B acts in the direction of the 1^k load, i.e. $\downarrow$

EXAMPLE

Determine the maximum stress in the beam and in the cable.
(Neglect the weight of the beam and the cable)



Area of the cable = 0.5 sq in
E of the cable = 22×10^3 ksi
W8x31 (AISC Manual 8th Ed.)
A = 9.12 sq in; I = 110 in^4;
S = 27.4 in^3; E = 30×10^3 ksi



Compatability Equation: $\Delta_A = 0 = \Delta_{AO} + X(\delta_{AA})$

By virtual work; for Δ_{AO} consider (a).

$$1^k \Delta_{AO} = \sum F_Q \frac{F_P L}{AE} + \sum \int_0^L M_Q M_P \frac{ds}{EI}$$

From B to C	L = 20'	$0 < x < 20$
	$F_P = 0$	$M_P = 20x$
	$F_Q = 0.866$	$M_Q = -(0.5)x$
From B to A	L = 23.09'	$0 < x < 23.09$
	$F_P = 0$	$M_P = 0$
	$F_Q = 1$	$M_Q = 0$

Example (cont.)

$$1^k\Delta_{AO} = 0 + \int_0^{20} -(0.5x)(20x)dx \frac{1728}{EI}$$

$$= -10 \left.\frac{x^3}{3}\right]_0^{20} \left(\frac{1728}{30 \times 1000 \times 110}\right)$$

$$\Delta_{AO} = -13.96"$$

For δ_{AA} consider (b).

$$1^k\delta_{AA} = \sum F_Q{}^2 \frac{L}{AE} + \sum \int_0^L M_Q{}^2 \frac{ds}{EI}$$

From B to C $\quad L = 20' \quad 0 < x \leq 20$

$F_P = -0.866 \quad M_P = -(0.5x)$

$F_Q = -0.866 \quad M_Q = -(0.5x)$

From B to A $\quad L = 23.09' \quad 0 < x \leq 23.09$

$F_P = 1 \quad M_P = 0$

$F_Q = 1 \quad M_Q = 0$

$$1^k\delta_{AA} = \frac{(-0.866)(-0.866)(20)(12)}{(9.12)(30 \times 10^3)} + \frac{(1)(1)(23.09)(12)}{(0.5)(22 \times 10^3)}$$

$$+ \int_0^{20} \frac{(0.5x)^2 dx}{(30 \times 10^3)(110)} \times (1728)$$

$$= .0007 + .0252 + \left(\left.\frac{.25\,x^3}{3}\right]_0^{20} \frac{1728}{(30 \times 1000 \times 110)}\right.$$

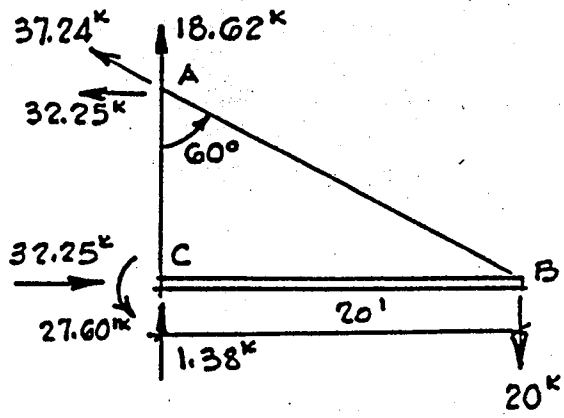
$$\delta_{AA} = .0007 + .0252 + .3491 = 0.375"$$

$$\therefore \Delta_A = \Delta_{AO} + X\delta_{AA} = 0$$

$$-13.96 + 0.375\,X = 0$$

$$X = \frac{13.96}{0.375} = 37.24^k \text{ Ans.}$$

Example (cont.)



Components of the cable tension:

$$A_V = 37.24 \sin 30^\circ = 37.24 \times 0.5 = 18.62^k$$

$$A_H = 37.24 \cos 30^\circ = 37.24 \times 0.866 = 32.25^k$$

Then for the whole structure;

$$+\uparrow F_y = 18.62 - 20 + C_V = 0$$

$$C_V = 1.38^k \uparrow$$

$$\overset{+}{\rightarrow} F_x = -32.25 + C_H = 0$$

$$C_H = 32.25^k \rightarrow$$

$$\circlearrowleft + M_C = (20 - 18.62)(20) = 27.60'^k \circlearrowleft$$

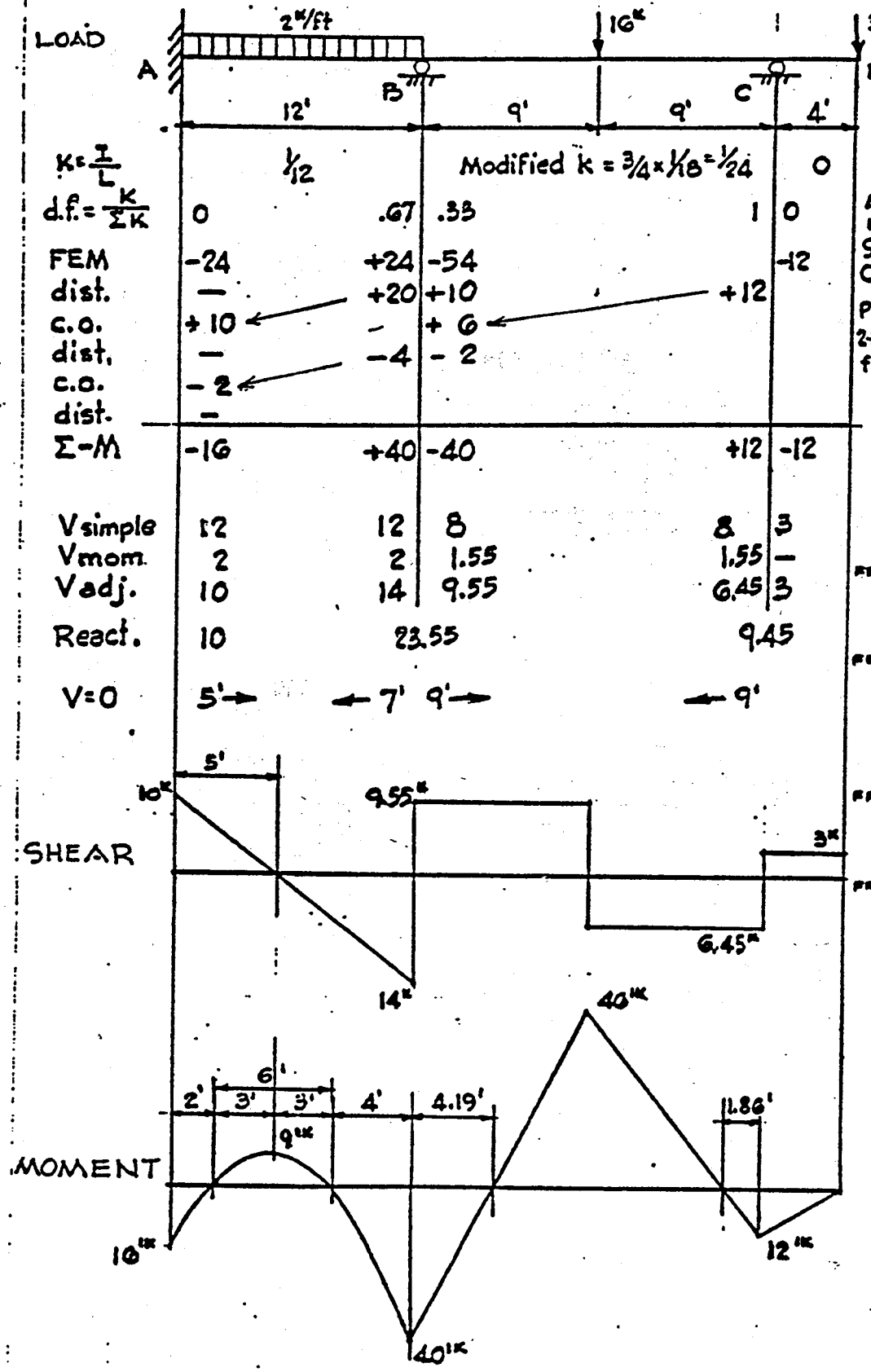
The stress in the beam due to bending $= \frac{M}{S} = \frac{(27.60)(12)}{27.4} = \pm 12.09$ ksi

The direct stress in the beam $= \frac{P}{A} = \frac{-32.25}{9.12} = -3.54$ ksi

∴ The Max. stress in the beam is:

$$\frac{P}{A} \pm \frac{M}{S} = -3.54 \pm 12.09 = \begin{cases} 15.63 \text{ ksi compr.} & \text{Ans.} \\ 8.55 \text{ ksi tension} & \text{Ans.} \end{cases}$$

Draw the Shear and Moment curves for the beam below. Analyse by Moment Distribution.



$K = \frac{I}{L}$		1/12	Modified $K = \frac{3}{4} \times \frac{1}{18} = \frac{1}{24}$		0
d.f. $= \frac{K}{\Sigma K}$	0	.67	.33	1	0
FEM	-24	+24	-54		-12
dist.	—	+20	+10	+12	
c.o.	+10	—	+6		
dist.	—	-4	-2		
c.o.	-2				
dist.	—				
Σ-M	-16	+40	-40	+12	-12
Vsimple	12	12	8	8	3
Vmom.	2	2	1.55	1.55	—
Vadj.	10	14	9.55	6.45	3
React.	10	23.55		9.45	
V=0	5'→	←7'	9'→	←9'	

AISC P 2-14 ff

I = constant = 1

Sign convention

+ ( ) +

$FEM_{AB} = -FEM_{BA}$

$-\frac{wL^2}{12} = -\frac{2 \times 12^2}{12} = -24'$

$FEM_{BC} = -\frac{3PL}{16}$

$= -\frac{3}{16} \times 16 \times 18 = -54^{k}$

$FEM_{CB} = 0$

$FEM_{CD} = -PL$

$= -3 \times 4 = -12^{k}$

+M (SPAN AB) = FROM END A

$\frac{1}{2}(10)(5) - 16 = 9^{k}$

CHECK FROM END B

$\frac{1}{2}(14)(7) - 40 = 9^{k}$ ✓

+M (SPAN BC) = FROM END B

$9.55 \times 9 - 40 = 46^{k}$

CHECK FROM END C

$6.45 \times 9 - 12 = 46^{k}$

PTS. OF INFLECTIO

SPAN AB

LENGTH OF +M SPAN

$L = \sqrt{\frac{8M}{w}}$

$L = \sqrt{\frac{8 \times 9}{2}} = 6'$

∴ PTS. OF INFL. ARE

$5 - 3 = 2'$; $7 - 3 = 4'$

SPAN BC

$PI_{LEFT} = \frac{40}{86} \times 9 = 4.19'$

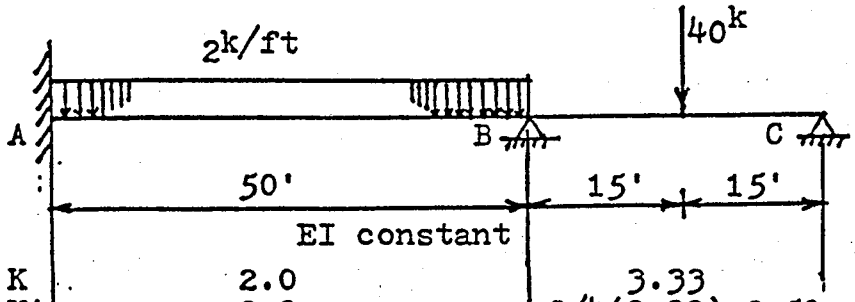
$PI_{RIGHT} = \frac{12}{58} \times 9 = 1.86'$

MODIFIED STIFFNESS FACTORS FOR VARIOUS END CONDITIONS

1. One end simply supported: $K' = 3/4(K)$
2. One end symmetrical to other end: $K' = 1/2(K)$
3. One end antisymmetrical to other end: $K' = 3/2(K)$

e.g.

Case 1.



$$-FEM_{AB} = +FEM_{BA} = \frac{2 \times 50^2}{12}$$

$$= 416.7'k$$

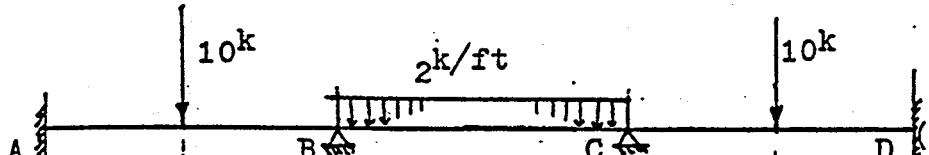
$$FEM_{BC} = -3/16(40 \times 30)$$

$$= -225.0'k$$

(Diag. 13 p.2-118, AISC Manual 8th ed.)

K	2.0		3.33
K'	2.0		3/4(3.33)=2.50
$D_f = \frac{K'}{K'}$	2.0/4.5 =	0.44	0.56= 2.5/4.5
FEM	-416.7	+416.7	-225.0
d1		- 84.3	-107.4
c.o.	- 42.2		
-M	-458.9	+332.4	-332.4

Case 2.



$$FEM_{AB} = -\frac{10 \times 16}{8} = -20'k$$

$$FEM_{BA} = -FEM_{AB} = +20'k$$

(Diag. 16 p.2-119)

$$FEM_{BC} = -\frac{2 \times 16^2}{12} = -42.6'k$$

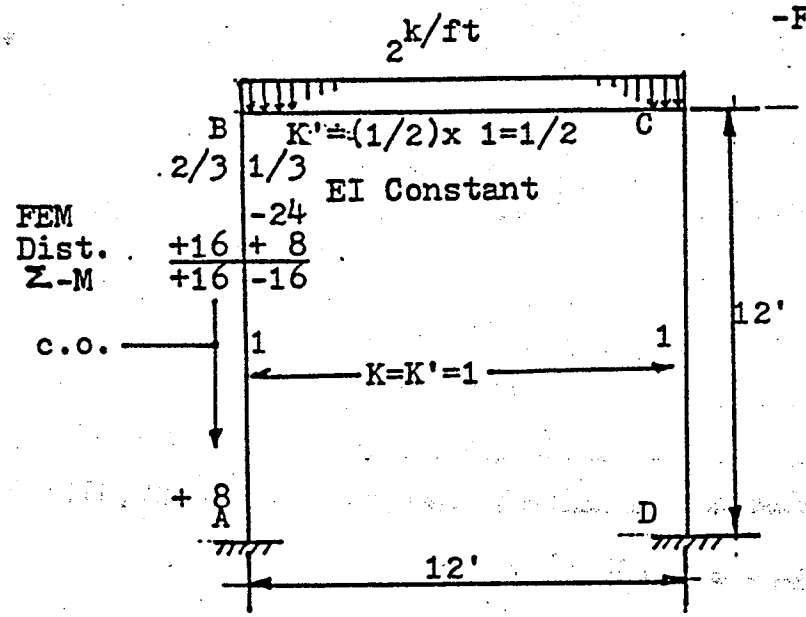
(Diag.15 p.2-119)

K	1		1		1	
K'	1		1 x (1/2)=1/2		1	
K'/K'	0	2/3	1/3			
FEM	-20.0	+20.0	-42.6			
Dist.		+15.1	+ 7.5			
c.o.	+ 7.5					
-M	-12.5	+35.1	-35.1	+35.1	-35.1	+12.5

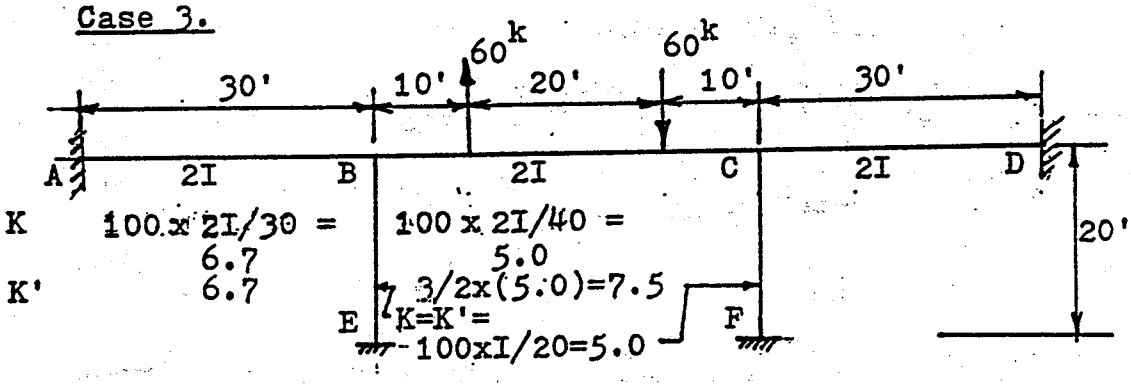
Case 2.

$-FEM_{BC} = +FEM_{CB} = \frac{2 \times 12^2}{12} = 24'k$

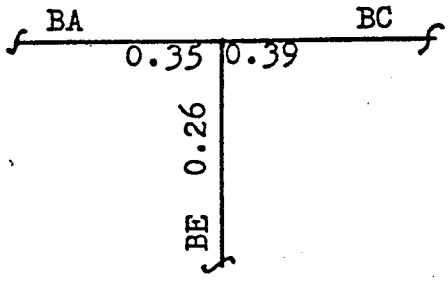
(Diag. 15 p.2-119)

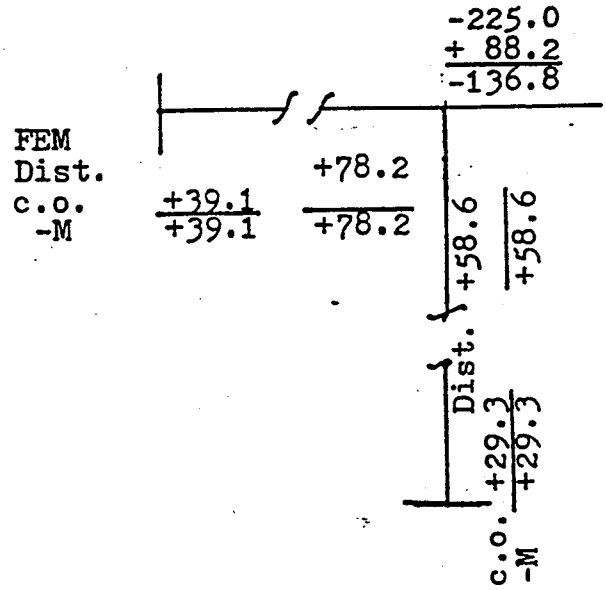


Case 3.

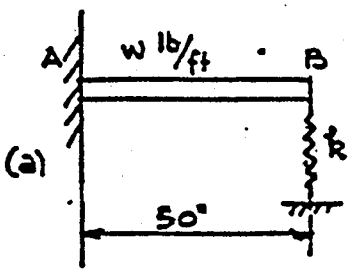


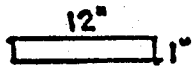
Dist. Factors





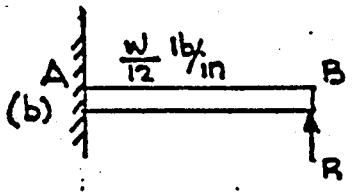
A spring placed under a cantilever beam as shown limits the deflection to one-half the value obtained if the beam could deflect freely. The beam is made of steel and is 12 inches wide by 1" deep. Find the spring constant.
Steel, $E = 30{,}000{,}000$ psi





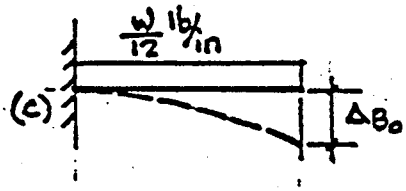
$$I = \frac{bd^3}{12} = \frac{12 \times 1^3}{12} = 1 \text{ in}^4$$

Consider the spring support as the redundant.
∴ The primary structure is as shown in (b).
The compatibility equation, based on the conditions shown in (c) and (d) and the initial deflection condition stated in the problem, is:-



$$\Delta_{Bo} - \delta_{bb} \times R = \tfrac{1}{2}\Delta_{Bo}$$

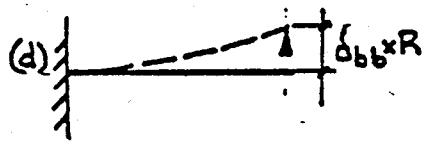
$$\therefore \delta_{bb} \times R = \tfrac{1}{2}\Delta_{Bo}$$



① $R = F = (\text{force in spring}) = \dfrac{\Delta_{Bo}}{2\delta_{bb}}$

$$\Delta_{Bo} = \frac{wL^4}{8EI}$$ AISC Tables p 2-120 No. 19

δ_{bb} = deflection at B due to a unit load acting upward at B.



$$\therefore \delta_{bb} = -\frac{1}{3}\frac{L^3}{EI}$$ AISC Tables p.2-121 No. 22

Then, from eq. ① above

$$F = \frac{\frac{1}{2}\frac{w}{12}\frac{L^4}{8EI}}{\frac{(1)\,L^3}{3EI}} = \frac{wL}{64}$$

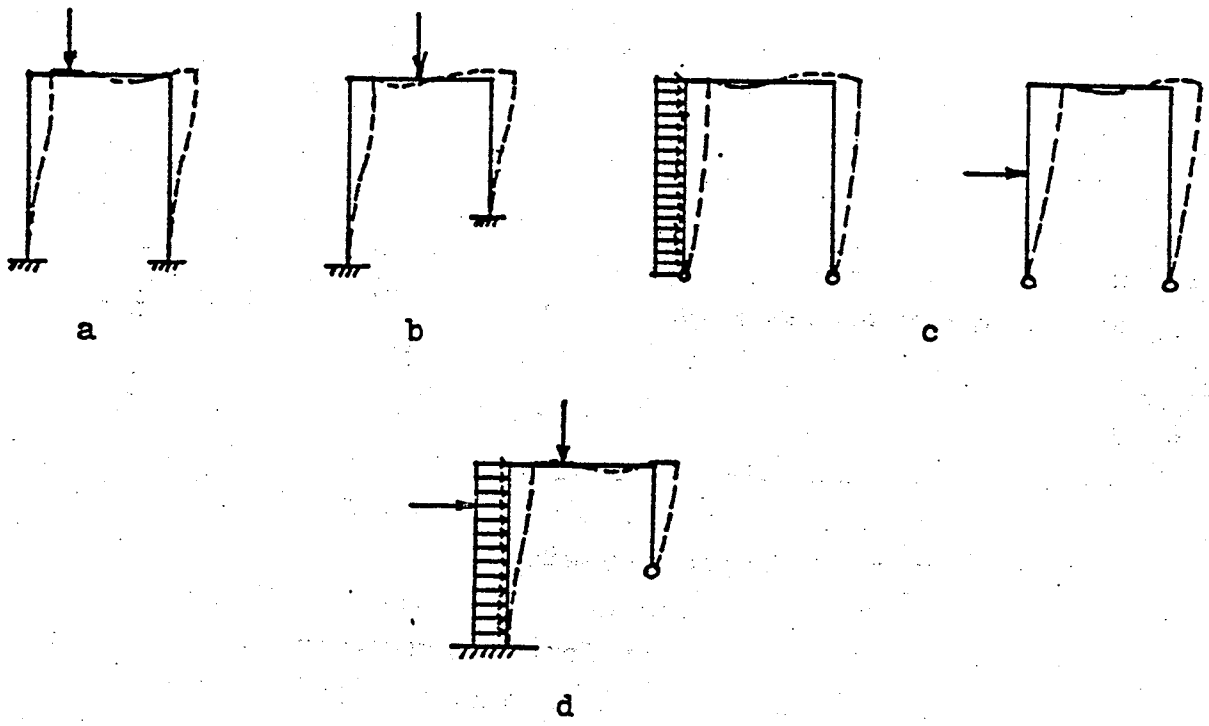
The force in the spring $= k\delta = k\frac{1}{2}\Delta_{Bo}$

$$\therefore \quad \frac{wL}{64} = k\,\frac{1}{2}\,\frac{w}{12}\,\frac{L^4}{8EI}$$

$$k = \frac{3EI}{L^3} = \frac{3 \times 30{,}000{,}000 \times 1}{50 \times 50 \times 50} = 720 \text{ lb/in}$$

FRAME SIDESWAY (By moment-distribution method)

Structural frames are subjected to sidesway because of unsymmetrical position of loads (Fig. a), unsymmetrical frame geometry (Fig. b), lateral loadings (Fig. c), or possibly a combination of these (Fig. d).



An analysis of these frames by an accepted method, say moment-distribution, will result in inconsistencies. When the values of the horizontal components at the supports are computed, the sum of these forces and any horizontal forces acting on the structure will not be equal to zero. This inequality, of course, means that the structure is not in equilibrium.

By using the principle of superposition it is possible to use an indirect approach to the solution of problems of this type. The method of moment-distribution lends itself readily to such a procedure.

FRAME SIDESWAY (cont.)

The general procedure for solution of problems of this type using the moment-distribution method is as follows:

1. Assume an imaginary support that prevents the structure from swaying.

2. Distribute the fixed-end moments due to the applied external loadings.

3. Compute and sum the shears at the bases of the columns. The unbalanced shear is the imaginary force required to prevent sidesway. If this force is removed the frame will undergo sidesway and the ends of the columns will rotate and induce moments at the joints.

4. Assume values for these moments. These values MUST BE such that the moments are proportional to the $\frac{I}{L^2}$ of the columns. This is especially important if the columns are of different lengths.
   Any moment may be assumed but the value should be consistent with the proportional requirements.
   It is generally easier if the magnitude is assumed to be of the same order as the moments due to the external loadings as figured in Step 2.

5. Analyse these moments and compute the shears at the bases of the columns, again you will have an unbalanced shear. Establish the ratio of

$$\frac{\text{Unbalanced shear in Step 3}}{\text{Unbalanced shear in Step 5}}$$

6. Multiply the moments in Step 5 by this ratio and add the result to the moments of Step 2. These are the correct moments

7. Figure the column base shears resulting from the final moments and isolate the frame members as free bodies. Apply all loadings, moments and shears. All members should be in equilibrium. This constitutes a final check.

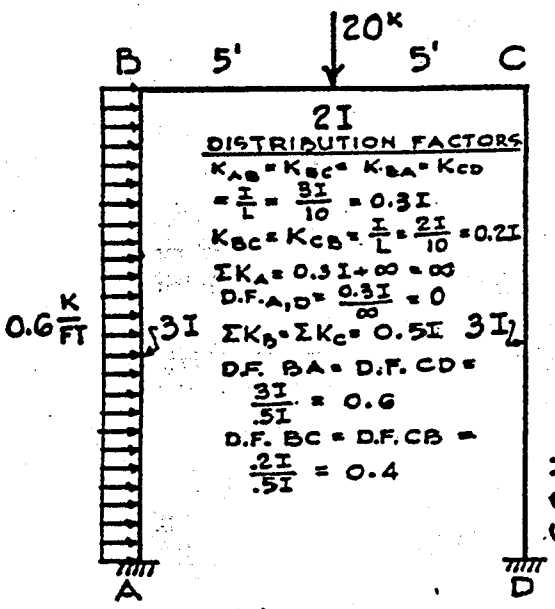
EXAMPLE

Determine the end moments M_A and M_B for the given frame. (Fig. a)
Use the moment-distribution method.

Sign convention: Fixed-end Moments (+).

$$FEM_{AB} = -FEM_{BA} = \frac{WL}{12} = \frac{(0.6)(10)(10)}{12} = -5'^{k}$$

$$FEM_{BC} = -FEM_{CB} = \frac{PL}{8} = \frac{20 \times 10}{8} = -25'^{k}$$



DISTRIBUTION FACTORS

$K_{AB} = K_{BC} = K_{BA} = K_{CD}$

$= \frac{I}{L} = \frac{3I}{10} = 0.3I$

$K_{BC} = K_{CB} = \frac{I}{L} = \frac{2I}{10} = 0.2I$

$\Sigma K_A = 0.3I + \infty = \infty$

$D.F._{A,D} = \frac{0.3I}{\infty} = 0$

$\Sigma K_B = \Sigma K_C = 0.5I$

D.F. BA = D.F. CD = $\frac{3I}{.5I} = 0.6$

D.F. BC = D.F. CB = $\frac{.2I}{.5I} = 0.4$

Fig. a

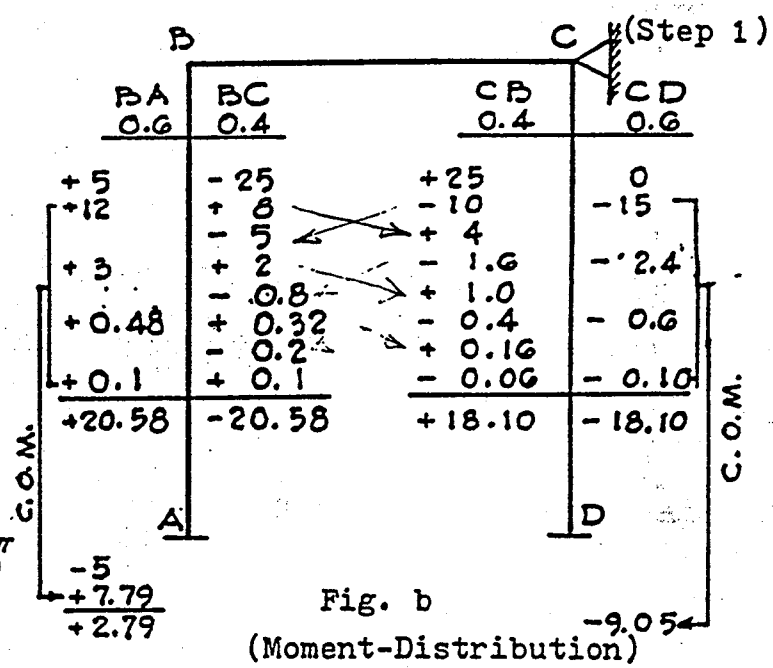


Fig. b
(Moment-Distribution)
(Step 2)

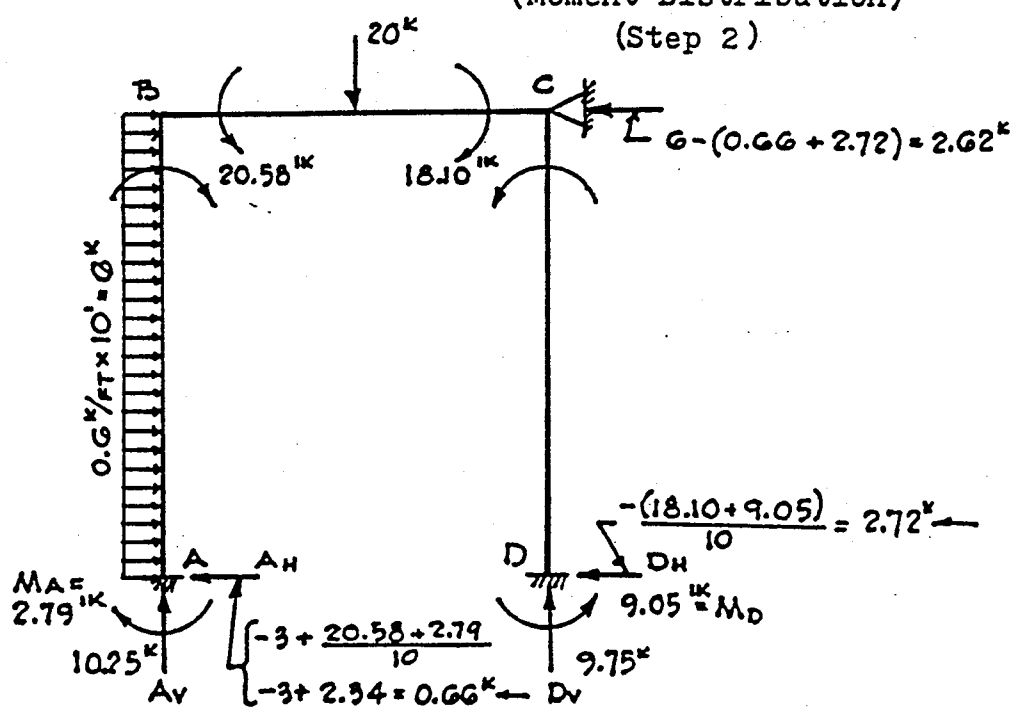


Fig. c
(Shears, moments,
and reactions)
(Step 3)

EXAMPLE (cont.)

Applying a force at A equal and opposite to the restraining force of 2.62 kips will cause the frame to deflect as shown. This will induce negative moments into the columns. Since the column inertias and lengths are the same the moments will be the same.

Assume a moment in each column equal to -100 ft-k.

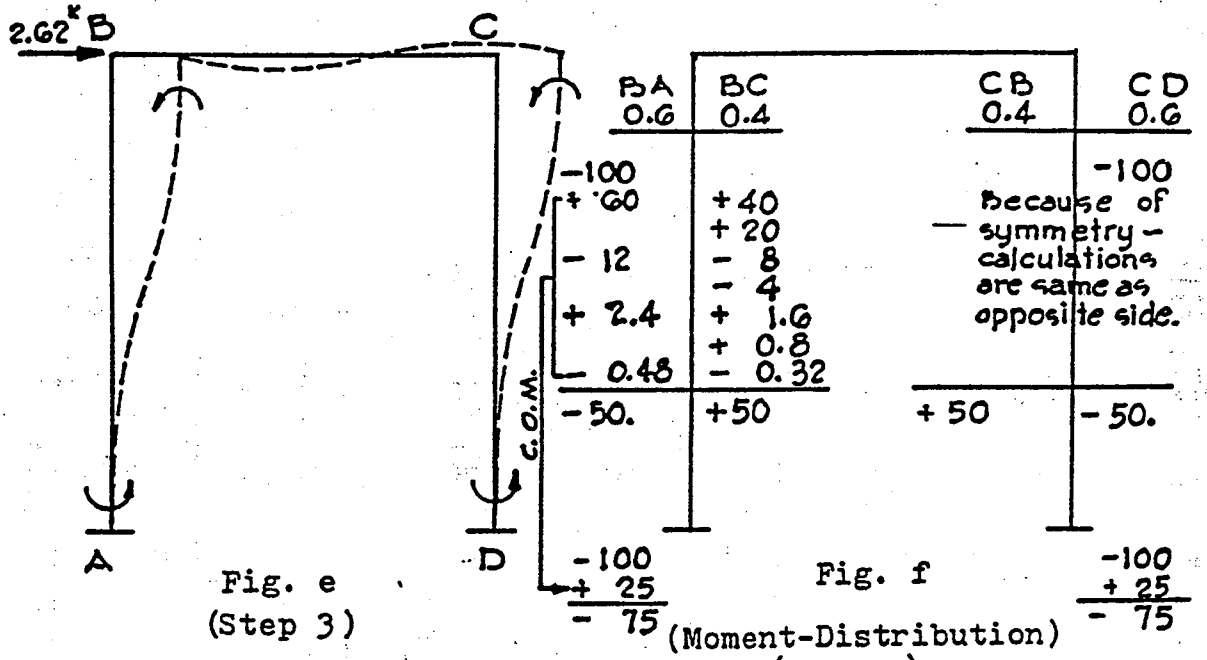


Fig. e
(Step 3)

Fig. f
(Moment-Distribution)
(Step 4)

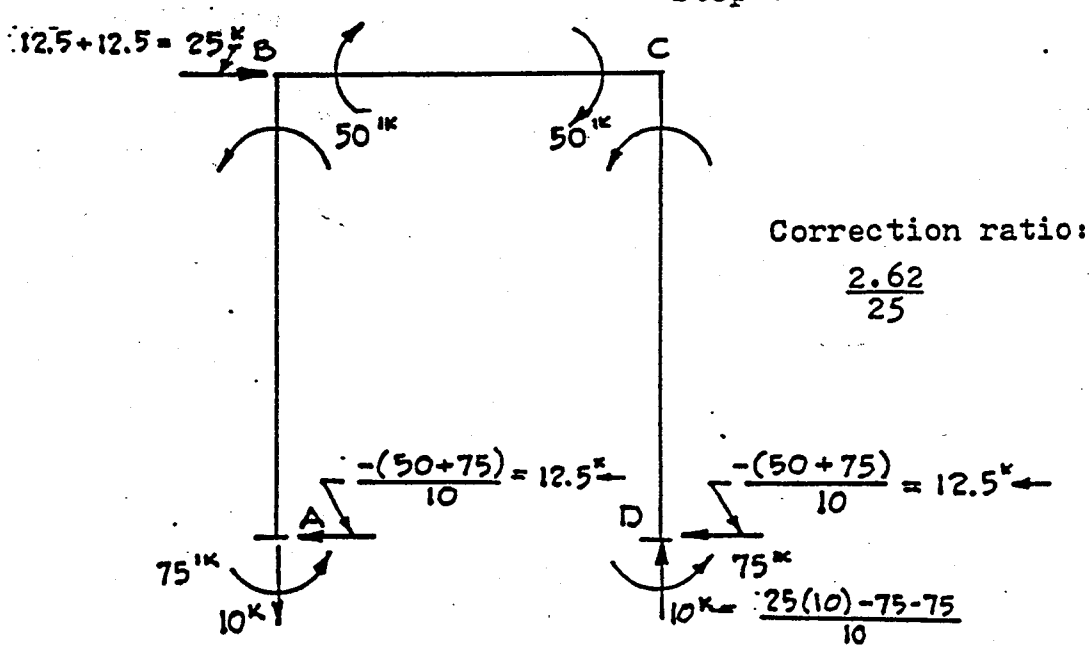


Fig. g
(Shears, moments and reactions)
(Step 5)

EXAMPLE (cont.)

Applying the correction factor to the assumed moments and shears will give the correct moments and shears for the restraining force.

$$25 \times \frac{2.62}{25} = 2.62^k$$

(a) $-50 \times \frac{2.62}{25} = -5.24'^k$ at the top of each column.

$-75 \times \frac{2.62}{25} = -7.86'^k$ at the bottom of each column.

Adding these to the moments and shears from the external loading (Step 2) gives:

(b)

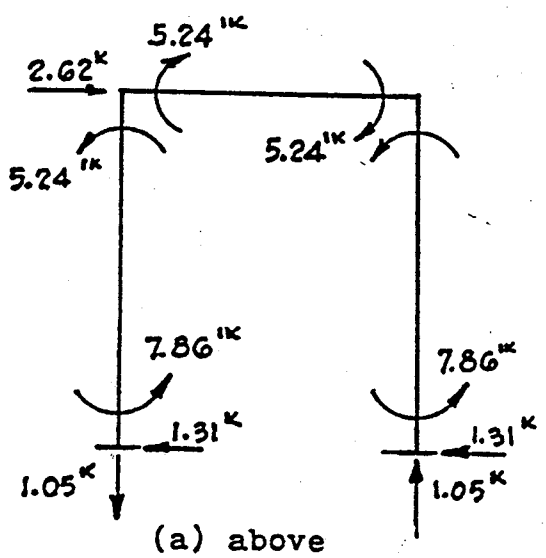
$+20.58 - 5.24 = +15.34'^k$ at the top of column AB.

$+ 2.79 - 7.86 = - 5.07'^k$ at the bottom of column AB.

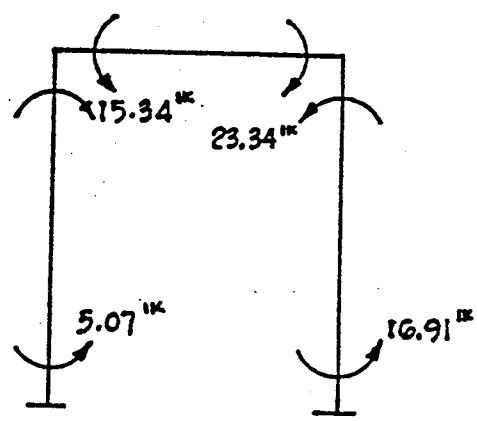
$-18.10 - 5.24 = -23.34'^k$ at the top of column DC.

$- 9.05 - 7.86 = -16.91'^k$ at the bottom of column DC.

$+ 2.62 - 2.62 = 0$ thus eliminating the restraining force.



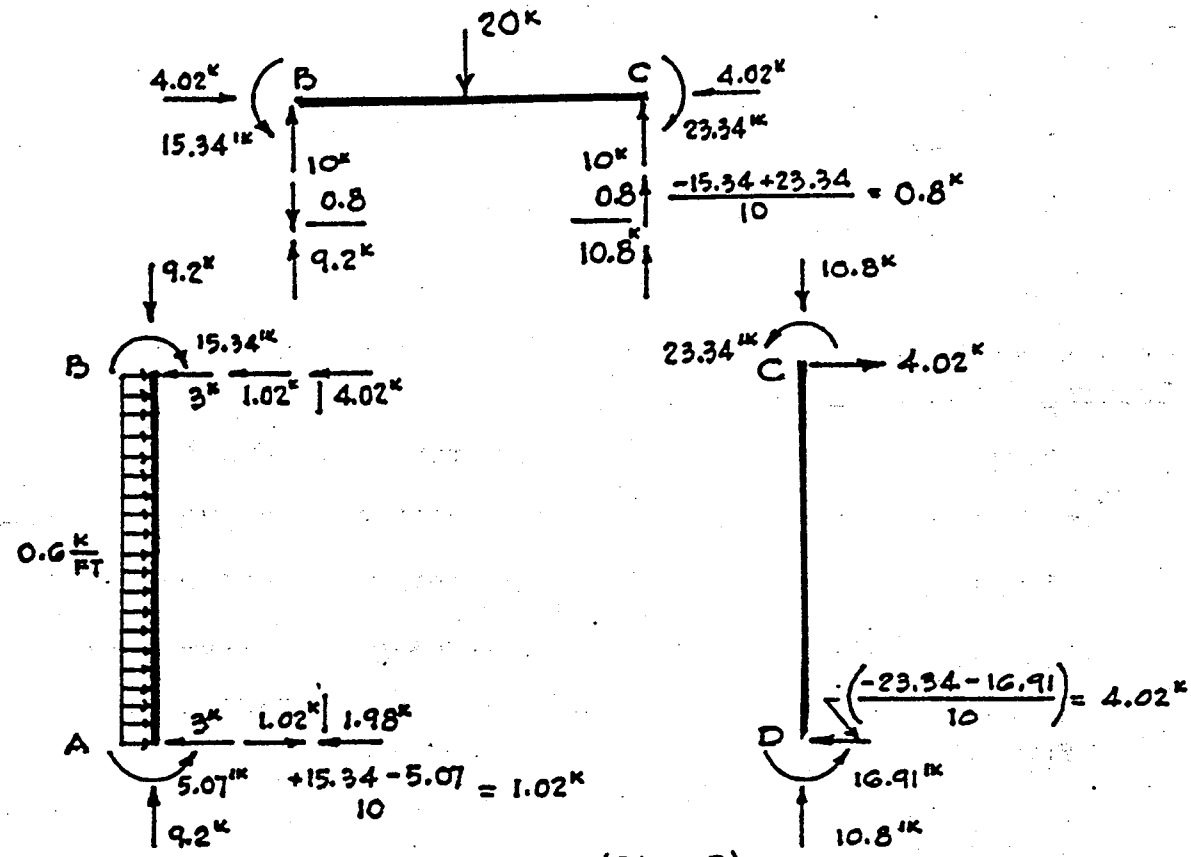
(a) above



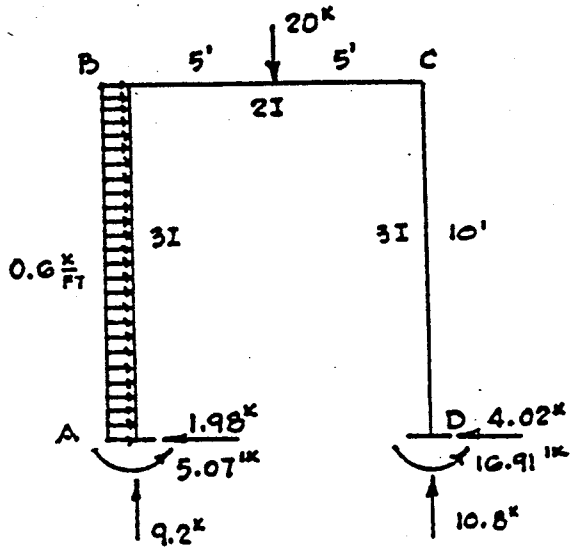
(b) above

(S t e p 6)

EXAMPLE (cont.)



Free-body Diagrams (Step 7)



Answer

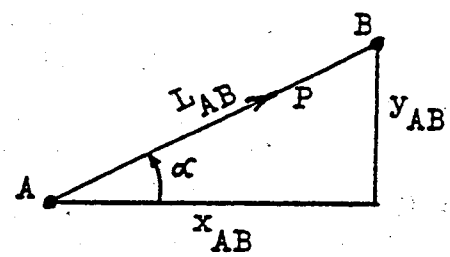
TENSION CO-EFFICIENTS

For over a half a century British engineers have used a variation of the method of joints which is easier to apply and which, for hand calculations, has no equal in solving space trusses. The procedure is called the method of tension co-efficients.

A tension co-efficient of a member is defined to be the force in the member divided by the length of the member. In using this approach the forces in the members are all considered to be tension and are therefore positive in the calculations. If the force is compression then the tension co-efficient will have a negative value.

Consider member AB:

$$t_{AB} = \frac{P_{AB}}{L_{AB}}$$



The horizontal and vertical components of the tension force P_{AB} are, $P_{AB}\cos\alpha$ and $P_{AB}\sin\alpha$ respectively. The horizontal and vertical projections of the length of the member are x_{AB} and y_{AB}.

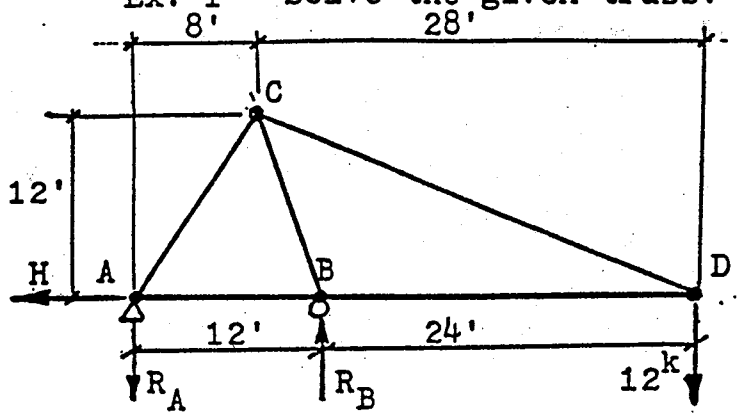
Writing the equations for horizontal and vertical equilibrium at pin A,

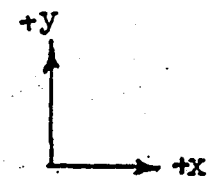
$$P_{AB}\cos\alpha = P\frac{x_{AB}}{L_{AB}} = t_{AB}x_{AB}$$

$$P_{AB}\sin\alpha = P\frac{y_{AB}}{L_{AB}} = t_{AB}y_{AB}$$

From these we can see that the equilibrium equations for ΣF_H and ΣF_V can be written in terms of tension co-efficients using the projected lengths of the members, which lengths we know, rather than sines and cosines of the angle α.

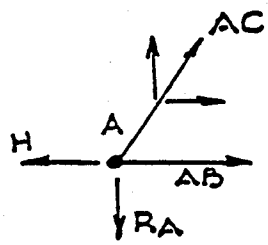
Ex. 1 Solve the given truss.





Solution:
Just as in the method of joints, each joint is isolated and equations for ΣF_x and ΣF_y are written.

Joint A $\qquad \Sigma F_x = 0$



Assuming all unknowns are tension, and summing forces in the x direction, for force AB_x we write its equivalent,

$$t_{AB}x_{AB} = t_{AB}12$$

for AC_x we write,

$$t_{AC}x_{AC} = t_{AC}8$$

and, since $\Sigma F_x = 0$;

$$12t_{AB} + 8t_{AC} - H = 0$$

The values for $12t_{AB}$ and $8t_{AC}$ are positive since they are directed in the positive x direction. The H is negative because its assumed direction is in the negative x direction. The remainder of the equations for the x and y directions at each joint are found in the same way and are recorded in Table I.

T A B L E I

(1)	ΣA_x;	$12t_{AB}$	$+ 8t_{AC}$	$- H$	$= 0$
(2)	ΣA_y;		$12t_{AC}$	$- R_A$	$= 0$
(3)	ΣB_x;	$-12t_{AB}$	$- 4t_{BC}$	$+ 24t_{BD}$	$= 0$
(4)	ΣB_y;		$12t_{BC}$	$+ R_B$	$= 0$
(5)	ΣC_x;	$- 8t_{AC}$	$+ 4t_{BC}$	$+ 28t_{CD}$	$= 0$
(6)	ΣC_y;	$-12t_{AC}$	$-12t_{BC}$	$- 12t_{CD}$	$= 0$
(7)	ΣD_x;	$-28t_{DC}$	$-24t_{BD}$		$= 0$
(8)	ΣD_y;	$12t_{DC}$	-12		$= 0$

These equations are now solved for the respective values of the tension co-efficients of the various members and the values are recorded in Table II.

Example 1 (cOnt.)

The length L of each member is found from

$$L = (x^2 + y^2)^{\frac{1}{2}}$$

The length of each member is then multiplied by the respective tension co-efficient for that member and the product is the force in the member. The values of the reactions R_A, R_B, and H are found in the process of solving the equations in Table I.

TABLE II

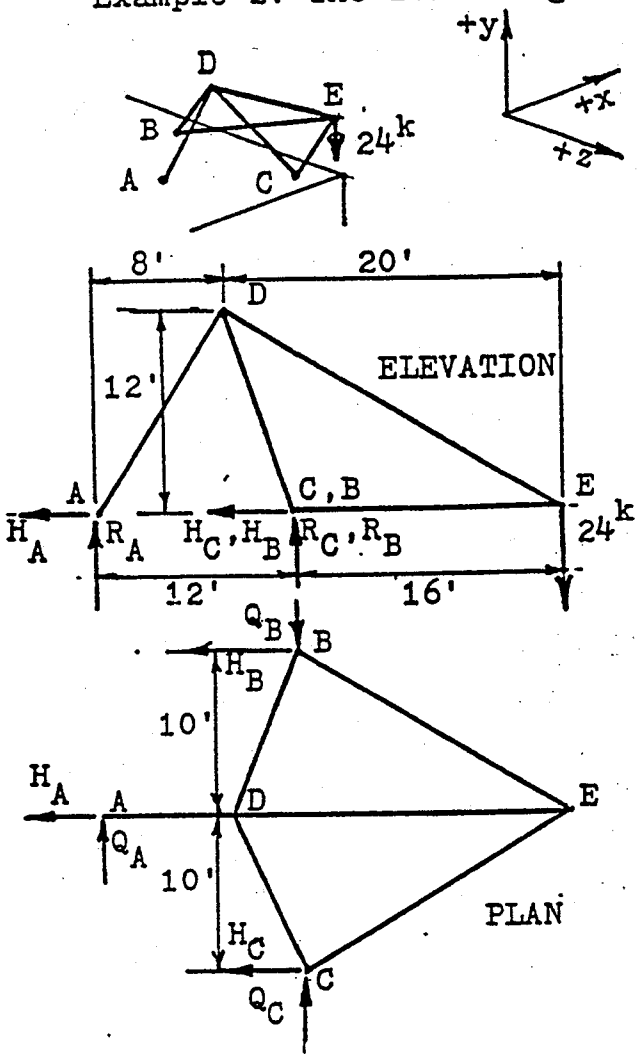
Member	Tens. cO-eff.	L	P
AB	-1.33	12.00	16.00 C
AC	2.00	14.42	28.84 T
BC	-3.00	12.65	37.95 C
BD	-1.17	24.00	28.00 C
CD	1.00	30.46	30.46 T
R_A			24 ↓
R_B			36 ↑
H			0

In solving the equations in Table I the following order was used:

(8), (7), (5) & (6) simultaneously, (4), (3), (2), and (1).

Tension co-e... .ents continued;

Example 2. The following tripod problem is solved in the same way.



Order of solving equations in Table I;
E_y;E_z;E_x;D_z;D_x&D_y simultaneously
∴ A_x; A_y etc. in any order.

T A B L E I

A_x;	$8t_{AD}-H_A = 0$
A_y;	$12t_{AD}+R_A = 0$
A_z;	$-Q_A = 0$
B_x;	$-4t_{BD}+16t_{BE}-H_B = 0$
B_y;	$12t_{BD}+R_B = 0$
B_z;	$10t_{BD}+10t_{BE}+Q_B = 0$
C_x;	$-4t_{CD}+16t_{CE}-H_C = 0$
C_y;	$12t_{CD}+R_C = 0$
C_z;	$-10t_{CD}-10t_{CE}-Q_C = 0$
D_x;	$-8t_{DA}+20t_{DE}+4t_{DB}+4t_{DC} = 0$
D_y;	$-12t_{DA}-12t_{DC}-12t_{DB}-12t_{DE} = 0$
D_z;	$-10t_{DB}+10t_{DC} = 0$
E_x;	$-16t_{EB}-16t_{EC}-20t_{ED} = 0$
E_y;	$12t_{ED}-24 = 0$
E_z;	$-10t_{EB}+10t_{EC} = 0$

The length of the frame members is obtained from;

$$L = (x^2+y^2+z^2)^{\frac{1}{2}}$$

T A B L E II

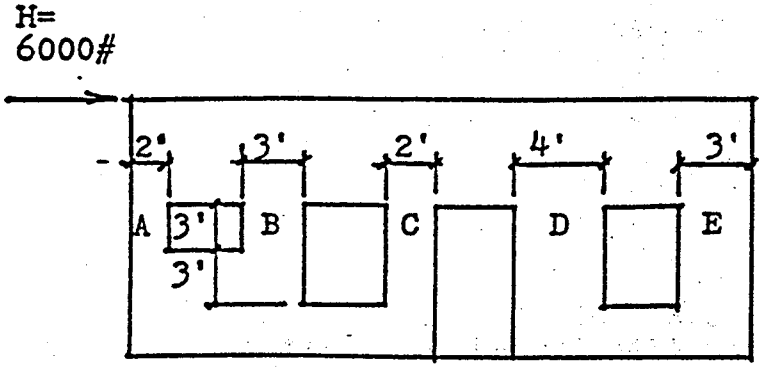
Member	Tens. co-eff.	Length	Force in member
AD	2.67	14.42	38.50 T
BD	-2.33	16.12	37.56 C
BE	-1.25	18.87	23.59 C
CD	-2.33	16.12	37.56 C
CE	-1.25	18.87	23.59 C
DE	2.00	23.32	46.64 T

TABLE III

Reactions		
H_A	21.33	←
R_A	32.00	↓
Q_A	0	
H_B	10.67	→
R_B	28.00	↑
Q_B	35.83	↓
H_C	10.67	→
R_C	28.00	↑
Q_C	35.83	↑

RIGIDITY OF MASONRY WALLS

Distribution of lateral forces to individual piers



t = wall thickness
(generally constant)
Fig. 1

H = horizontal load

P_n = force to each pier
(n = (A through E))
= design shear for each pier

M_n = design moment for each pier, where
$M_n = \frac{P_n h}{2}$ for top and bottom of fixed piers.

h = height of pier

d = width of pier

Rigidity of wall = $1/\Delta_F$ or $1/\Delta_c$

where $\Delta_F = 0.001\left[(h/d)^3 + 3(h/d)\right]$ for fixed ended piers.

$\Delta_c = 0.001\left[4(h/d)^3 + 3(h/d)\right]$ for fixed at one end only.

For example given in figure 1:

Pier	h (in.)	d (in.)	h/d	R	% of H to each pier	P (lbs)	M (in-lbs)
A	36	24	1.5	127	21.1	1266	22,788
B	36	36	1.0	250	41.4	2484	44,712
C	72	24	3.0	28	4.6	276	9,936
D	72	48	1.5	127	21.1	1266	45,576
E	72	36	2.0	71	11.8	708	25,488

$\Sigma R = 603$

DEFLECTION FORMULAS

The maximum deflection for a simply supported beam having a uniformly distributed load may be found from:

$$\Delta_{max} = \frac{180.00\ M\ L^2}{E\ I}$$

M=moment in ft-kips
L=span in feet
E=mod. of elast. in ksi
I=moment of inertia in in^4
Δ=deflection in inches

This formula is valid for beams of any material, except concrete, providing the beams have a constant cross section.

With the proper use of equivalent tabular load co-efficients, this formula provides a method of rapid calculation of maximum deflections for certain non-uniformly distributed load patterns.

To simplify the deflection formula, we may write:

$$\Delta_{max.} = M\ L^2 / k\ I$$

where k = E / 180.00.

Then k for steel and various E's for wood is as follows:

E	K
29000	161.111
1200	6.667
1300	7.222
1400	7.778
1500	8.333
1600	8.889
1700	9.444
1800	10.000
1900	10.556
2000	11.111

Note: Use one-half the values for k for dead load deflections for wood beams, since, for creep effect, the E_{creep} values are taken as E/2.

The maximum deflection for the wood design problem on page SD-30 may be found by using the formula above, with a modification to accommodate the concentrated load. The method is as follows:

The M in the formula above is the TOTAL MOMENT for a uniform load. The moment in the problem consists, by superposition, of two parts, the uniform load moment, and the concentrated load moment. By changing the concentrated load to an equivalent uniform load and adding its uniform load moment to the moment caused by the actual uniform load we would then have the total uniform load moment required for use in the deflection formula.

The concentrated load may be changed to an EUL (equivalent uniform load) by multplying it by an EUL co-efficient of 2.0 (See "f" co-eff. for n=2, on p.2-295 of AISC Manual of Steel Construction, 9th ed.), ASD; p.3-129 LRFD.

Using the total uniform load of $W_{udl} = w \times L$, and a total EUL for the concentrated load of 2.0 x P, the moments are, for the UDL, $M=W_{udl} \times L/8$, and for the concentrated load, $M_{EUL}=W_{EUL} \times L/8$.

For the dead load:

UDL =0.060 k/ft	Conc. ld. = 0.600^k
$W_{udl}=0.060 \times 18=1.080^k$	EUL = $2.0 \times 0.600=1.2^k$
$M_{udl}=1.080 \times 18/8=2.43^{ft-k}$	$M_{EUL} = 1.2 \times 18/8=2.70^{ft-k}$

Now, we must modify the EUL moment for the concentrated load by multiplying it by a co-efficent for deflection since the real load is a concentrated load.

The deflection co-efficient "g", from the reference above, is, 0.800.

$$\Delta_{DL(max)} = \frac{(2.43+(.800(2.70))(18)^2}{(4.444)(889.89)} = 0.38"$$

For the live load:

UDL =0.200 k/ft	Conc. ld. = 0.900^k
$W_{udl}=0.200 \times 18=3.600^k$	EUL = $2.0 \times 0.900=1.8^k$
$M_{udl}=3.600 \times 18/8=8.10^{ft-k}$	$M_{EUL} = 1.8 \times 18/(8)=4.05^{ft-k}$

$$\Delta_{LL(max)} = \frac{(8.10+(.800)(4.05))(18)^2}{(8.889)(889.89)} = 0.46"$$

$$\Delta_{total} = 0.38 + 0.46 = 0.84"$$

To validate the procedure, the total uniform load moment of our modified beam loading is, by superposition:

$$M_{UDL}+M_{EUL} = (2.43+8.10)+((2.70+4.05)/1.15)$$

$$= 10.53 + 5.87 = 16.40^{ft-k}$$

which is exactly the maximum moment for the "with snow" condition on p. SD-30.

STRUCTURAL

DESIGN

BEAM DESIGN by AISC Specs.

Reference: Manual of Steel Construction, American Institute of Steel Construction, Ninth Edition, ASD. Chapter F.

1. Basic Equation:

$$S = \frac{M}{F_b}$$

S = Section Modulus = I/c

M = Design Moment

F_b = Allowable bending stress

2. F_b (allowable bending stress)

The allowable value of F_b is a function of bending strength, which may be limited by plate buckling or lateral-torsional buckling (LTB).

2.1 Values of F_b are discussed below:

a. Compact Section

$$F_b = .66\ F_y, \text{ but } F_y \leq 65 \text{ ksi}$$

The cross-section can undergo plastification without local plate buckling or without LTB, (See Section B5.1., ASD Manual p.5-35). Generally, hot-rolled shapes of F_y = 36 ksi steel are compact except for W6x15.

b. Semi-compact Section

$$.66\ F_y > F_b > .6\ F_y$$

$$F_b = F_y\left(.79 - .002\frac{b_f}{2t_f}\sqrt{F_y}\right)$$

See Section F1.2 & Formula F1-3, p.5-46 ASD Manual.

All properties and bracing requirements are identical to those of a compact section except for the compression flange width to thickness ratio which is:

$$\frac{65}{\sqrt{F_y}} < \frac{b_f}{2t_f} < \frac{95.0}{\sqrt{F_y}}$$

See Sect. B5.1, p. 5-35 & Table B5-1, p.5-36, ASDM

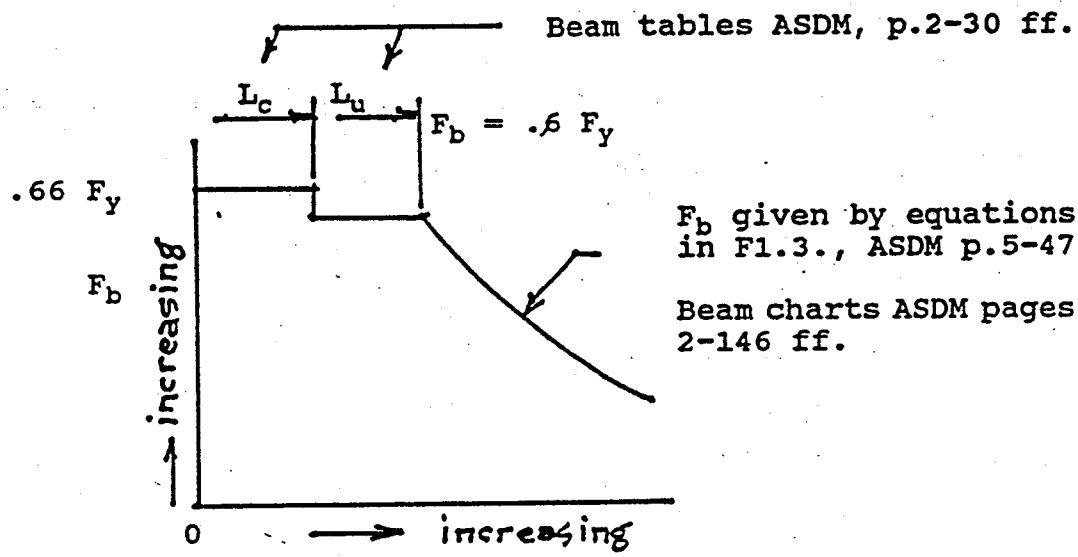
c. $F_b < .6F_y$ <u>LTB controls</u>

When the unbraced lenth of the compression flange is large, equations, F1-6,7,and 8, in Sect. F1.3., p.-47, ASDM, are used to determine the allowable bending stress, F_b. A simplified method is presented after this discussion.

d. If the compression flange width to thickness ratio is large, plate buckling may control F_b. In this case:

$F_b = .6F_yQ_s$ Equations for computing Q_s for slender compression members are given in Appendix B, page 5-98, ASDM. Normally, bending members in which Q_s must be calculated would be inefficient for use in structures.

The variation of allowable bending stress as a function of the unbraced length of the compression flange is shown below for a W16 x 36 of A-36 steel.



Unbraced length of Compression Flange

VARIATION OF F_b WITH UNBRACED LENGTH OF COMPRESSION FLANGE

The AISC Manual provides information that greatly simplifies the application of these seemingly complex lateral support equations. Of particular use are the L_c and L_u values which are provided in the beam and column sections of the manual. In subparagraph 5 of Section 1.5.1.4 the AISC states that for flanged beams the distances between points of lateral bracing should not exceed $76b_f/\sqrt{F_y}$ nor $20{,}000/[(d/A_f)F_y]$ if the members are to be assumed to have adequate lateral bracing. The least of these two values is referred to as L_c.

Should the distance between points of lateral bracing be greater than L_c the AISC says that the allowable bending stress must be reduced from $0.66F_y$, with the appropriate formula but in no case may it exceed $0.60F_y$. When these formulas are used, however, there is a range in which they give a value above $0.60F_y$. For each beam there is an unbraced length for which the controlling formula yields an allowable stress exactly equal to $0.60F_y$. This length is called L_u throughout the manual. Based on this information it is possible to make the following simplifying statements.

1. If the unbraced length $\leqslant L_c$, F_b equals $0.66F_y$ assuming the other requirements of AISC Section 1.5.1.4 are met.
2. If the unbraced length is $> L_c$ but $\leqslant L_u$, F_b equals $0.60F_y$.
3. If the unbraced length is $> L_c$ and $> L_u$, F_b will be less than $0.60F_y$ and can be determined by the appropriate formulas from Section 1.5.1.4.5 of the AISC Specification. The designer will often find that the curves given in the beam part of the AISC Manual entitled "Allowable Moments in Beams with Unbraced Length Greater Than L_u" and discussed later in this section will be helpful in this regard.

Example 6-6 illustrates the calculations necessary to determine the allowable stresses in a W section that has different unbraced lengths.

Example 6-6

Determine the allowable bending stresses in a W33×130 (see Fig. 6-6) for simple spans of 10, 13, 20, and 30 ft without lateral support. Use A36 steel and the AISC Specification.

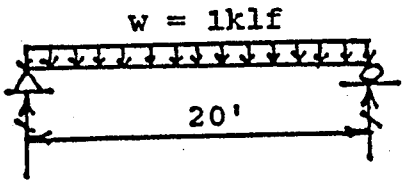
SOLUTION

Computing properties of the section

$$A_f + \frac{1}{6}A_w = (11.510)(0.855) + \left(\frac{1}{3}\right)\left(\frac{31.38}{2}\right)(0.580) = 12.87 \text{ in.}^2$$

$$r_T = \sqrt{\frac{\frac{1}{2}\times 218}{12.87}} = 2.91$$

<u>EXAMPLE</u> Design of a laterally supported steel beam.



Select a W section to support a uniformly distributed load of 1^k per linear foot.
Steel is A36, full lateral support for top flange.

Solution:

Full lateral support: $\therefore F_b = 24$ ksi.

Assume beam weight = 50 plf.

Total w = 1.00 + 0.05 = 1.05 klf

$$M = wL^2/8 = ((1.05)(20)(20))/8 = 52.50'^k$$

Method 1

Compute S required.

$$S = M/F_b = (52.50 \times 12)/24 = 26.25 \text{ in}^3$$

From table of S values, ASDM p. 2-12, try a W14x22, $S_x = 29.0$ in^3. We could also use the Moment of Resistance values, M_r, which are given in this table. For a W14x22, which is the lightest weight section of its group (bold type indication at top of group), the value of M_r is $58'^k$ which is greater than the applied moment of $52.50'^k$. Therefore, the W14x22 is satisfactory for bending.

Method 2

From the Beam Tables, Part 2, Beam and Girder Design, of the ASDM, on page 2-66 the allowable total uniform load for a W14x22 having a span of 22 feet is 23^k, since this is greater than the applied total load W = 20 x 1.05 = 21^k, the section is satisfactory for bending.

TABLE B5.1
Limiting Width-Thickness Ratios for Compression Elements

Description of Element	Width-Thickness Ratio	Limiting Width-Thickness Ratios: Compact	Limiting Width-Thickness Ratios: Noncompact[c]
Flanges of I-shaped rolled beams and channels in flexure[a]	b/t	$65/\sqrt{F_y}$	$95/\sqrt{F_y}$
Flanges of I-shaped welded beams in flexure	b/t	$65/\sqrt{F_y}$	$95/\sqrt{F_{yf}/k_c}$[e]
Outstanding legs of pairs of angles in continuous contact; angles or plates projecting from rolled beams or columns; stiffeners on plate girders	b/t	NA	$95/\sqrt{F_y}$
Angles or plates projecting from girders, built-up columns or other compression members; compression flanges of plate girders	b/t	NA	$95/\sqrt{F_y/k_c}$
Stems of tees	d/t	NA	$127/\sqrt{F_y}$
Unstiffened elements simply supported along one edge, such as legs of single-angle struts, legs of double-angle struts with separators and cross or star-shaped cross sections	b/t	NA	$76/\sqrt{F_y}$
Flanges of square and rectangular box and hollow structural sections of uniform thickness subject to bending or compression[d]; flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$190/\sqrt{F_y}$	$238/\sqrt{F_y}$
Unsupported width of cover plates perforated with a succession of access holes[b]	b/t	NA	$317/\sqrt{F_y}$
All other uniformly compressed stiffened elements, i.e., supported along two edges	b/t h/t_w	NA	$253/\sqrt{F_y}$
Webs in flexural compression[a]	d/t	$640/\sqrt{F_y}$	—
	h/t_w	—	$760/\sqrt{F_b}$
Webs in combined flexural and axial compression	d/t_w	for $f_a/F_y \le 0.16$: $\frac{640}{\sqrt{F_y}}\left(1 - 3.74\frac{f_a}{F_y}\right)$	—
		for $f_a/F_y > 0.16$: $257/\sqrt{F_y}$	
	h/t_w	—	$760/\sqrt{F_b}$
Circular hollow sections In axial compression In flexure	D/t	 $3{,}300/F_y$ $3{,}300/F_y$	 — —

[a]For hybrid beams, use the yield strength of the flange F_{yf} instead of F_y.
[b]Assumes net area of plate at widest hole.
[c]For design of slender sections that exceed the noncompact limits see Appendix B5.
[d]See also Sect. F3.1.
[e]$k_c = \frac{4.05}{(h/t)^{0.46}}$ if $h/t > 70$, otherwise $k_c = 1.0$.

B6. ROTATIONAL RESTRAINT AT POINTS OF SUPPORT

At points of support, beams, girders and trusses shall be restrained agains tation about their longitudinal axis.

B7. LIMITING SLENDERNESS RATIOS

For members whose design is based on compressive force, the slenderness r Kl/r preferably should not exceed 200. If this limit is exceeded, the allow stress shall not exceed the value obtained from Equation (E2-2).

For members whose design is based on tensile force, the slenderness ratio preferably should not exceed 300. The above limitation does not apply to in tension. Members which have been designed to perform as tension meml in a structural system, but experience some compression loading, need not isfy the compression slenderness limit.

B8. SIMPLE SPANS

Beams, girders and trusses designed on the basis of simple spans shall hav effective length equal to the distance between centers of gravity of the m bers to which they deliver their end reactions.

B9. END RESTRAINT

When designed on the assumption of full or partial end restraint due to con uous, semi-continuous or cantilever action, the beams, girders and trusses well as the sections of the members to which they connect, shall be designe carry the shears and moments so introduced, as well as all other forces, with exceeding at any point the unit stresses prescribed in Chapters D throug except that some non-elastic but self-limiting deformation of a part of the nection is permitted when this is essential to avoid overstressing of fasten

B10. PROPORTIONS OF BEAMS AND GIRDERS

Rolled or welded shapes, plate girders and cover-plated beams shall, in eral, be proportioned by the moment of inertia of the gross section. No dec tion shall be made for shop or field bolt or rivet holes in either flange provi that

$$0.5F_u A_{fn} \ge 0.6F_y A_{fg} \qquad \text{(B1}$$

where A_{fg} is the gross flange area and A_{fn} is the net flange area, calculate accordance with the provisions of Sects. B1 and B2.

If

$$0.5F_u A_{fn} < 0.6F_y A_{fg} \qquad \text{(B1}$$

the member flexural properties shall be based on an effective tension fla area A_{fe}

$$A_{fe} = \frac{5}{6}\frac{F_u}{F_y} A_{fn} \qquad \text{(B1}$$

SIMPLIFIED APPROACH TO AISC BENDING FORMULAS
Frank W. Stockwell Jr., AISC Eng. Journal, 3rd Quarter, 1974.

A new term, Q, is defined to be:

$$Q = \frac{F_y(\frac{\ell}{r_T})^2}{510 \times 10^3 C_b}$$

If $Q \leq 0.2$ $\quad F_b = 0.6\ F_y$
If $0.2 \leq Q \leq 1.0$ $\quad$ Use equation F1-6, ASDM p.5-47
If $Q > 1.0$ $\quad$ Use equation F1-7, ASDM p.5-47
Formula F1-6 is:

$$F_b = (\frac{2}{3} - \frac{F_y(\frac{\ell}{r_T})^2}{1530 \times 10^3 C_b})$$

The second term in the parentheses is Q/3, so F1-6 becomes:

$$F_b = (\frac{2 - Q}{3}) F_y$$

Formula F1-7 is:

$$F_b = \frac{170 \times 10^3 C_b}{(\frac{\ell}{r_T})^2}$$

Solving for $(\ell/r_T)^2$ in the expression given above for Q, results in:

$$(\frac{\ell}{r_T})^2 = \frac{510 \times 10^3 C_b Q}{F_y}$$

and, substituting this into F1-7:

$$F_b = \frac{170 \times 10^3\ {}_b}{\frac{510 \times 10^3 C_b Q}{F_y}} = \frac{F_y}{3Q}$$

Formula F1-8, ASDM p. 5-47, is:

$$F_b = \frac{12 \times 10^3 C_b}{\frac{\ell d}{A_f}}$$

This equation should be checked when the compression flange is solid, approximately rectangular in shape, and its area is not less than the area of the tension flange.

SIMPLIFIED APPROACH TO AISC BENDING FORMULAS (cont.)

Summary:

The four steps for finding F_b when the length of the compression flange is greater than L_u are:

1. Note: F_y is in ksi.

$$Q = \frac{F_y\left(\frac{\ell}{r_T}\right)^2}{510\times10^3 C_b}$$

2. (a) If $Q \le 0.2$ $\quad F_b = .6\ F_y$

   (b) If $0.2 \le Q \le 1.0$ $\quad F_b = ((2-Q)/3)F_y$

   (c) If $Q \ge 1.0$ $\quad F_b = F_y/(3Q)$

3. If applicable, Check Formula F1-8.

$$F_b = \frac{12\times10^3 C_b}{\frac{\ell d}{A_f}}$$

4. F_b = the larger of the values from Steps 2 and 3 but not over $0.6\ F_y$.

If F_y = 36 ksi:

For Step 2(a) above;

$F_b = 0.6 \times 36 = 21.6$ ksi

For Step 2(b) above:

$F_b = ((2 - Q)/3)(36) = 12(2 - Q)$

For Step 2(c) above:

$F_b = 36/(3Q) = 12/Q$

TABLE 4
VALUES OF C_c

For Use with Equations (E2-1) and (E2-2) and in Table 3

F_y (ksi)	C_c	F_y (ksi)	C_c
33	131.7	46	111.6
35	127.9	50	107.0
36	126.1	55	102.0
39	121.2	60	97.7
40	119.6	65	93.8
42	116.7	90	79.8
45	112.8	100	75.7

TABLE 5
Slenderness Ratios of Elements as a Function of F_y

Specification Section and Ratios	F_y (ksi)					
	36	42	46	50	60	65
Table B5.1						
$65/\sqrt{F_y}$	10.8	10.0	9.6	9.2	8.4	8.1
$190/\sqrt{F_y}$	31.7	29.3	28.0	26.9	24.5	23.6
$640/\sqrt{F_y}$	106.7	98.8	94.4	90.5	82.6	79.4
$257/\sqrt{F_y}$	42.8	39.7	37.9	36.3	33.2	31.9
Sect. F1.2						
$\sqrt{\frac{102 \times 10^3 C_b}{F_y}}$	$53\sqrt{C_b}$	$49\sqrt{C_b}$	$47\sqrt{C_b}$	$45\sqrt{C_b}$	$41\sqrt{C_b}$	$40\sqrt{C_b}$
$\sqrt{\frac{510 \times 10^3 C_b}{F_y}}$	$119\sqrt{C_b}$	$110\sqrt{C_b}$	$105\sqrt{C_b}$	$101\sqrt{C_b}$	$92\sqrt{C_b}$	$89\sqrt{C_b}$
Table B5.1						
$76/\sqrt{F_y}$	12.7	11.7	11.2	10.7	9.8	9.4
$95/\sqrt{F_y}$	15.8	14.7	14.0	13.4	12.3	11.8
$127/\sqrt{F_y}$	21.2	19.6	18.7	18.0	16.4	15.8
Table B5.1						
$238/\sqrt{F_y}$	39.7	36.7	35.1	33.7	30.7	29.5
$317/\sqrt{F_y}$	52.8	48.9	46.7	44.8	40.9	39.3
$253/\sqrt{F_y}$	42.2	39.0	37.3	35.8	32.7	31.4
Table B5.1—Appendix B5.2b						
$3300/F_y$	91.7	78.6	71.7	66.0	55.0	50.8
$13000/F_y$	361	310	283	260	217	200
Sect. G1						
$\frac{14000}{\sqrt{F_y(F_y + 16.5)}}$	322	282	261	243	207	192
$2000/\sqrt{F_y}$	333	309	295	283	258	248

TABLE 6
Values of C_b

For Use in Equations (F1-6), (F1-7) and (F1-8)

$\frac{M_1}{M_2}$	C_b	$\frac{M_1}{M_2}$	C_b	$\frac{M_1}{M_2}$	C_b
−1.00	1.00	−0.45	1.34	0.10	1.86
−0.95	1.02	−0.40	1.38	0.15	1.91
−0.90	1.05	−0.35	1.42	0.20	1.97
−0.85	1.07	−0.30	1.46	0.25	2.03
−0.80	1.10	−0.25	1.51	0.30	2.09
−0.75	1.13	−0.20	1.55	0.35	2.15
−0.70	1.16	−0.15	1.60	0.40	2.22
−0.65	1.19	−0.10	1.65	0.45	2.28
−0.60	1.23	−0.05	1.70	≥0.47	2.30
−0.55	1.26	0	1.75		
−0.50	1.30	0.05	1.80		

Note 1: $C_b = 1.75 + 1.05(M_1/M_2) + 0.3\,(M_1/M_2)^2 \leq 2.3$
Note 2: M_1/M_2 positive for reverse curvature and negative for single curvature.

TABLE 7
Values of C_m

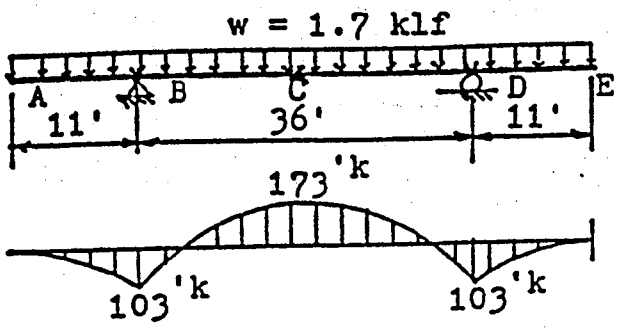
For Use in Equation (H1-1)

$\frac{M_1}{M_2}$	C_m	$\frac{M_1}{M_2}$	C_m	$\frac{M_1}{M_2}$	C_m
−1.00	1.00	−0.45	0.78	0.10	0.56
−0.95	0.98	−0.40	0.76	0.15	0.54
−0.90	0.96	−0.35	0.74	0.20	0.52
−0.85	0.94	−0.30	0.72	0.25	0.50
−0.80	0.92	−0.25	0.70	0.30	0.48
−0.75	0.90	−0.20	0.68	0.35	0.46
−0.70	0.88	−0.15	0.66	0.40	0.44
−0.65	0.86	−0.10	0.64	0.45	0.42
−0.60	0.84	−0.05	0.62	0.50	0.40
				0.60	0.36
−0.55	0.82	0	0.60	0.80	0.28
−0.50	0.80	0.05	0.58	1.00	0.20

Note 1: $C_m = 0.6 - 0.4(M_1/M_2)$
Note 2: M_1/M_2 is positive for reverse curvature and negative for single curvature.

EXAMPLE

Select a W section to carry the uniform load. The beam is laterally braced at supports B and D, and at mid-span of BD. $F_y = 36$ ksi



With lateral support at C, $L_u = \frac{36}{2} = 18'$

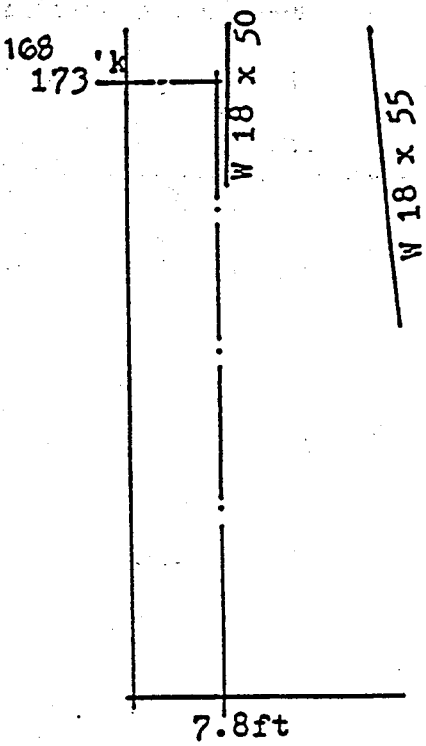
C_b for a cantilever is 1.0 ASDM p. 5-47

C_b for sections BC and CD =

$$C_b = 1.75 + 1.05\frac{M_1}{M_2} + 0.3\left(\frac{M_1}{M_2}\right)^2 \leq 2.3$$ ASDM p. 5-47

$$C_b = 1.75 + 1.05\left(\frac{103}{173}\right) + 0.3\left(\frac{103}{173}\right)^2 = 2.43 \quad \text{Use } 2.3$$

We will use the beam diagram on page 2-168. Since the graph is drawn for $C_b = 1.0$, divide the actual l_u by the actual C_b and enter the graph with this revised length and the moment. The nearest heavy line to the right of the intersection of length and moment will give a trial section.



$$L_{rev.} = \frac{18}{2.3} = 7.8'$$

The nearest trial size is a W18 x 50 which has an $S_x = 88.9$ in^3. p. 1-22

$$\therefore f_b = \frac{173 \times 12}{88.9} = 23.4 \text{ ksi}$$

The L_u for $C_b = 1.0$ for this section is 11 feet. The revised L_u for $C_b = 2.3$ is $11 \times 2.3 = 25.4' > 18'$, so $F_b = 21.6$ ksi.

However, 21.6 ksi < 23.4 ksi, so the section is unsatisfactory.

EXAMPLE (cont.)

The next heavy line to the right is a W 18 x 55 the properties of which are:

$L_u = 12.1$ ft

$S = 98.3$ in^3

$r_T = 1.95$ in

$\frac{d}{A_f} = 3.82$

$$f_b = \frac{173 \times 12}{98.3} = 21.12 \text{ ksi}$$

$$\frac{\ell}{r_T} = \frac{18 \times 12}{1.95} = 111$$

Using the simplified method for the allowable bending stress:

$$Q = \frac{F_y \left(\frac{\ell}{r_T}\right)^2}{510 \times 10^3 \, C_b} = \frac{36 \, (111)^2}{510{,}000 \times 2.3} = 0.38$$

$$F_b = 12 \, (2 - 0.38) = 19.46 \text{ ksi}$$

Check Formula F1-8 p. 5-47

$$F_b = \frac{12000 \times 2.3}{(18 \times 12)(3.82)} = 33.45 \text{ ksi} \quad \text{Use } F_b = 21.6 \text{ksi}$$

$$f_b = 21.12 < 21.6 \text{ksi} = F_b$$

The W 18 x 55 is satisfactory.

This could also have been determined by considering a revised unsupported length because of C_b.

$$\ell_u = 12.1 \times 2.3 = 27.8' > 18'$$

$$\therefore F_b = 21.6 \text{ ksi}$$

and, proceeding as above, the W 18 x 55 is found to be OK.

TABLE 2-7 from AISC manual

ALLOWABLE STRESS DESIGN SELECTION TABLE

For shapes used as beams

S_x

F_y = 50 ksi			S_x	Shape	Depth d	F'_y	F_y = 36 ksi		
L_c	L_u	M_R					L_c	L_u	M_R
Ft	Ft	Kip-ft	In.^{3}		In.	Ksi	Ft	Ft	Kip-ft
10.3	11.1	1230	448	W 33×141	33¼	—	12.2	15.4	887
8.8	11.0	1210	439	W 36×135	35½	—	12.3	13.0	869
9.4	13.4	1200	436	W 30×148	30⅝	—	11.1	18.7	863
10.3	35.5	1150	419	W 18×211	20⅝	—	12.2	49.3	830
11.2	27.1	1150	417	W 21×182	22¾	—	13.2	37.6	826
11.6	21.1	1140	414	W 24×162	25	—	13.7	29.3	820
12.5	16.6	1130	411	W 27×146	27⅜	—	14.7	23.0	814
9.9	10.8	1120	406	W 33×130	33⅛	—	12.1	13.8	804
9.4	11.8	1050	380	W 30×132	30¼	—	11.1	16.1	752
11.1	25.1	1050	380	W 21×166	22½	—	13.1	34.8	752
10.3	32.7	1050	380	W 18×192	20⅜	—	12.1	45.4	752
11.6	18.9	1020	371	W 24×146	24¾	—	13.6	26.3	735
8.8	10.7	987	359	W 33×118	32⅞	—	12.0	12.6	711
9.4	10.8	976	355	W 30×124	30⅛	—	11.1	15.0	703
9.0	13.3	949	345	W 27×129	27⅝	—	10.6	18.4	683
10.2	30.0	946	344	W 18×175	20	—	12.0	41.7	681
9.4	9.9	905	329	W 30×116	30	—	11.1	13.8	651
11.5	16.8	905	329	W 24×131	24½	—	13.6	23.4	651
11.2	21.8	905	329	W 21×147	22	—	13.2	30.3	651
10.1	27.5	853	310	W 18×158	19¾	—	11.9	38.3	614
8.9	9.8	822	299	W 30×108	29⅞	—	11.1	12.3	592
9.0	11.5	822	299	W 27×114	27¼	—	10.6	15.9	592
11.1	19.6	811	295	W 21×132	21⅞	—	13.1	27.2	584
11.5	14.9	800	291	W 24×117	24¼	—	13.5	20.8	576
10.0	25.3	776	282	W 18×143	19½	—	11.8	35.1	558
11.1	18.3	751	273	W 21×122	21⅝	—	13.1	25.4	541
7.9	9.7	740	269	W 30× 99	29⅝	—	10.9	11.4	533
9.0	10.2	734	267	W 27×102	27⅛	—	10.6	14.2	529
11.4	13.2	710	258	W 24×104	24	58.5	13.5	18.4	511
10.0	23.1	704	256	W 18×130	19¼	—	11.8	32.2	507
11.1	16.6	685	249	W 21×111	21½	—	13.0	23.3	493
7.2	9.6	674	245	W 30× 90	29½	58.1	10.0	11.4	485
8.1	12.0	674	245	W 24×103	24½	—	9.5	16.7	485
8.9	9.5	668	243	W 27× 94	26⅞	—	10.5	12.8	481
10.1	21.0	635	231	W 18×119	19	—	11.9	29.1	457
11.0	15.4	624	227	W 21×101	21⅜	—	13.0	21.3	449
8.1	10.9	610	222	W 24× 94	24¼	—	9.6	15.1	440
8.0	9.4	586	213	W 27× 84	26¾	—	10.5	11.0	422
10.0	18.7	561	204	W 18×106	18¾	—	11.8	26.0	404
8.1	9.6	539	196	W 24× 84	24⅛	—	9.5	13.3	388
7.5	12.1	528	192	W 21× 93	21⅝	—	8.9	16.8	380
13.1	31.7	523	190	W 14×120	14½	—	15.5	44.1	376
10.0	17.4	517	188	W 18× 97	18⅝	—	11.8	24.1	372

ALLOWABLE STRESS DESIGN SELECTION TABLE

For shapes used as beams

S_x

F_y = 50 ksi			S_x	Shape	Depth d	F'_y	F_y = 36 ksi		
L_c	L_u	M_R					L_c	L_u	M_R
Ft	Ft	Kip-ft	In.^{3}		In.	Ksi	Ft	Ft	Kip-ft
8.1	8.6	484	176	W 24× 76	23⅞	—	9.5	11.8	348
9.3	20.2	481	175	W 16×100	17	—	11.0	28.1	347
13.1	29.2	476	173	W 14×109	14⅜	58.6	15.4	40.6	343
7.5	10.9	470	171	W 21× 83	21⅜	—	8.8	15.1	339
9.9	15.5	457	166	W 18× 86	18⅜	—	11.7	21.5	329
13.0	26.7	432	157	W 14× 99	14⅛	48.5	15.4	37.0	311
9.3	18.0	426	155	W 16× 89	16¾	—	10.9	25.0	307
7.4	8.5	424	154	W 24× 68	23¾	—	9.5	10.2	305
7.4	9.6	415	151	W 21× 73	21¼	—	8.8	13.4	299
9.9	13.7	402	146	W 18× 76	18¼	64.2	11.6	19.1	289
13.0	24.5	385	143	W 14× 90	14	40.4	15.3	34.0	283
7.4	8.9	385	140	W 21× 68	21⅛	—	8.7	12.4	277
9.2	15.8	369	134	W 16× 77	16½	—	10.9	21.9	265
5.8	6.4	360	131	W 24× 62	23¾	—	7.4	8.1	259
7.4	8.1	349	127	W 21× 62	21	—	8.7	11.2	251
6.8	11.1	349	127	W 18× 71	18½	—	8.1	15.5	251
9.1	20.2	338	123	W 14× 82	14¼	—	10.7	28.1	244
10.9	25.0	325	118	W 12× 87	12½	—	12.8	36.2	234
6.8	10.4	322	117	W 18× 65	18⅜	—	8.0	14.4	232
9.2	13.9	322	117	W 16× 67	16⅜	—	10.8	19.3	232
5.0	6.3	314	114	W 24× 55	23⅝	—	7.0	7.5	226
9.0	18.6	308	112	W 14× 74	14⅛	—	10.6	25.9	222
5.9	6.7	305	111	W 21× 57	21	—	6.9	9.4	220
6.8	9.6	297	108	W 18× 60	18¼	—	8.0	13.3	214
10.8	24.0	294	107	W 12× 79	12⅜	62.6	12.8	33.3	212
9.0	17.2	283	103	W 14× 68	14	—	10.6	23.9	204
6.7	8.7	270	98.3	W 18× 55	18⅛	—	7.9	12.1	195
10.8	21.9	268	97.4	W 12× 72	12¼	52.3	12.7	30.5	193
5.6	6.0	260	94.5	W 21× 50	20⅞	—	6.9	7.8	187
6.4	10.3	254	92.2	W 16× 57	16⅜	—	7.5	14.3	183
9.0	15.5	254	92.2	W 14× 61	13⅞	—	10.6	21.5	183
6.7	7.9	244	88.9	W 18× 50	18	—	7.9	11.0	176
10.7	20.0	238	87.9	W 12× 65	12⅛	43.0	12.7	27.7	174
4.7	5.3	224	81.6	W 21× 44	20⅝	—	6.6	7.0	162
6.3	9.1	223	81.0	W 16× 50	16¼	—	7.5	12.7	160
5.4	6.8	217	78.8	W 18× 46	18	—	6.4	9.4	156
9.0	17.5	215	78.0	W 12× 58	12¼	—	10.6	24.4	154
7.2	12.7	214	77.8	W 14× 53	13⅞	—	8.5	17.7	154
6.3	8.2	200	72.7	W 16× 45	16⅛	—	7.4	11.4	144
9.0	15.9	194	70.6	W 12× 53	12	55.9	10.6	22.0	140
7.2	11.5	193	70.3	W 14× 48	13¾	—	8.5	16.0	139

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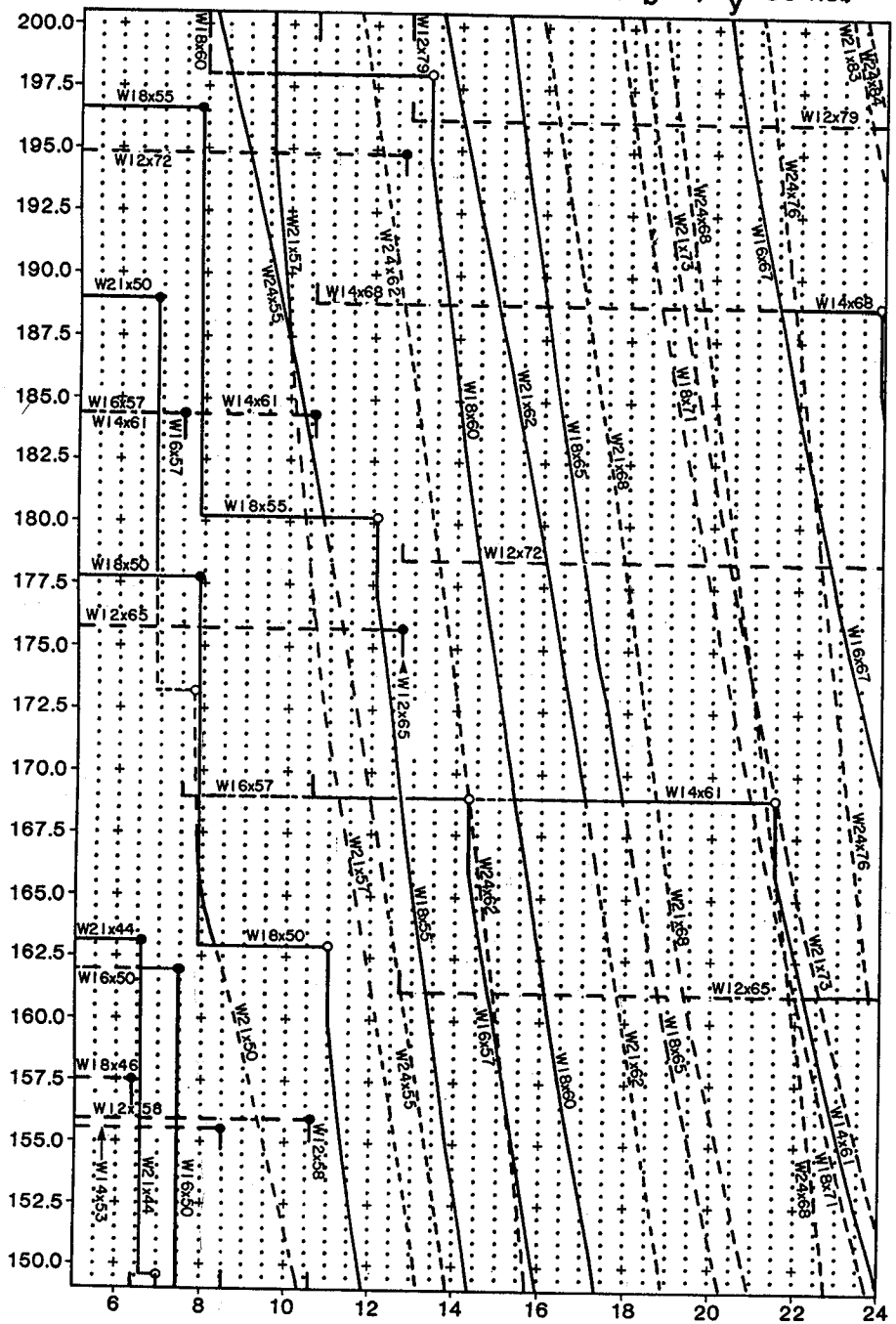
ALLOWABLE MOMENTS IN BEAMS (C_b =1, F_y=36 ksi)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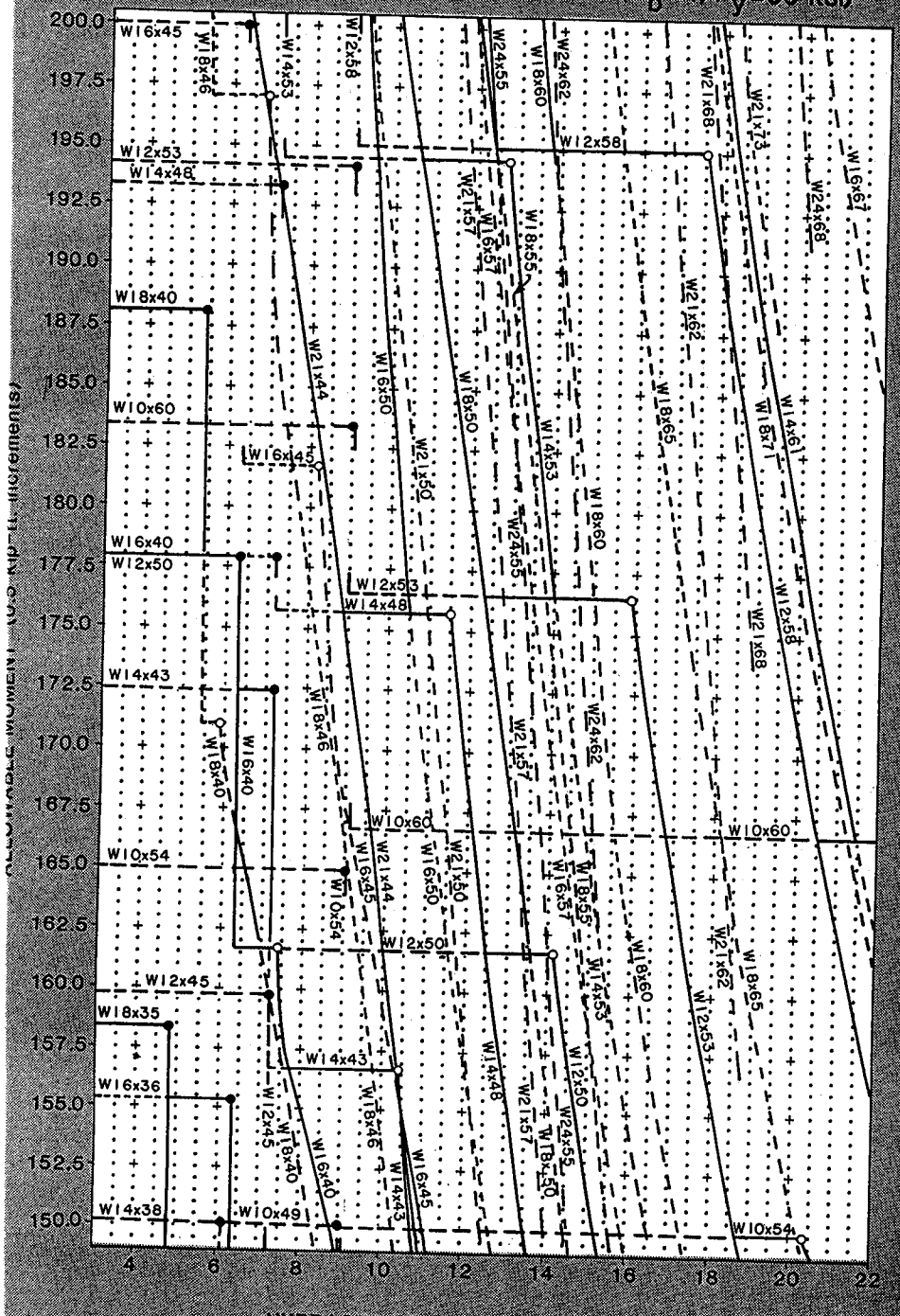
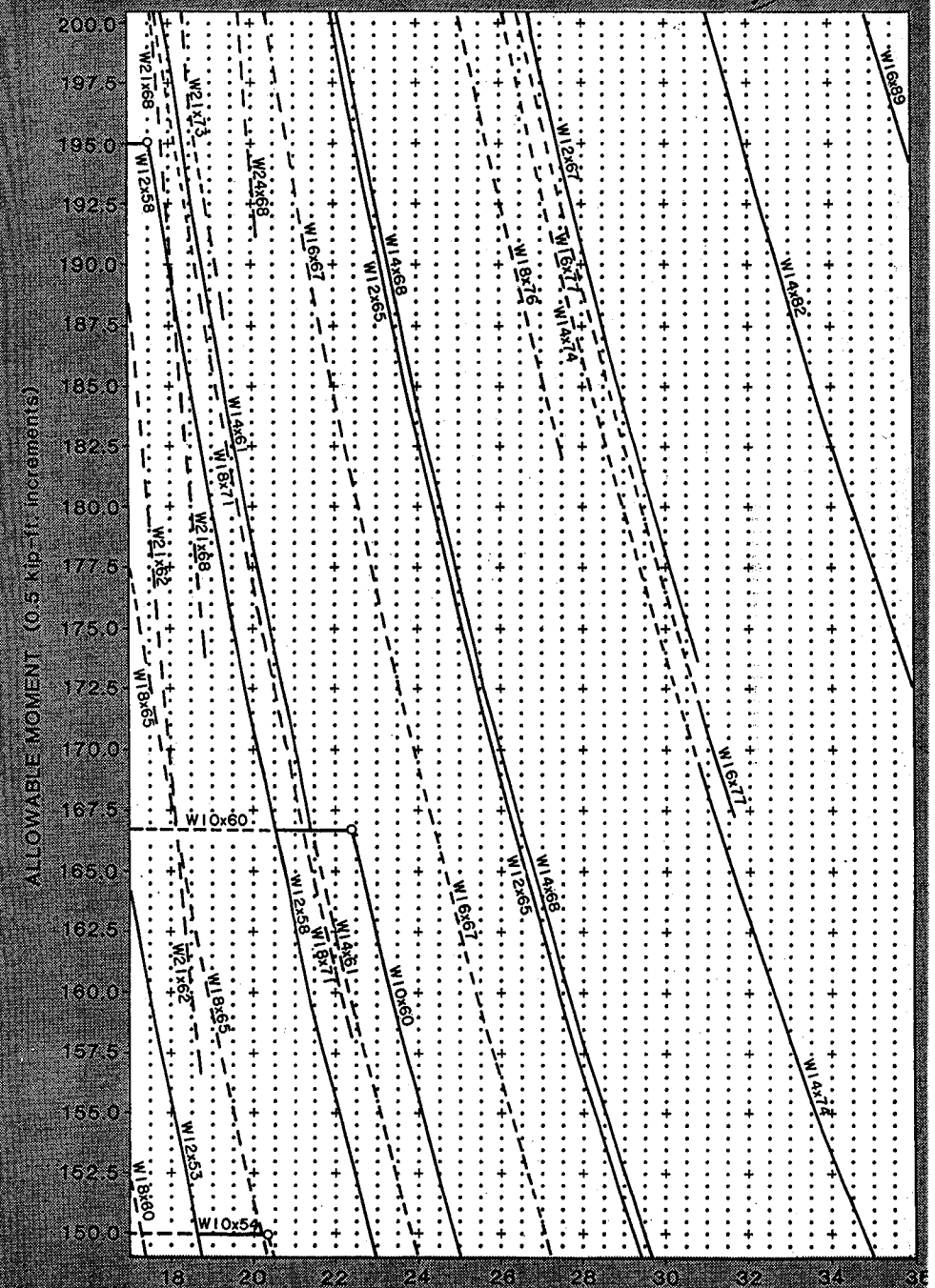
W21x8758980
W24x8687300
W16x8758980
W18x8687300
W14x8758980
W21x8800855
W24x8729000
W16x8800855
W18x8729000
W14x8800855
W21x8842830
W24x8770800
W16x

ALLOWABLE MOMENTS IN BEAMS (C_b=1, F_y=50 ksi)



ALLOWABLE MOMENTS IN BEAMS (C_b=1, F_y=50 ksi)



I_x MOMENT OF INERTIA SELECTION TABLE
For W and M shapes

Shape	I_x In.^{4}	Shape	I_x In.^{4}	Shape	I_x In.^{4}	Shape	I_x In.^{4}
W 36×848	**67400**	**W 44×248**	**21400**	**W 40×167**	**11600**	**W 33×118**	**5900**
		W 30×391	20700	W 33×201	11500	W 30×132	5770
W 36×798	**62600**	W 27×448	20400	W 36×182	11300	W 24×176	5680
		W 36×300	20300	W 27×258	10800	W 27×146	5630
W 40×655	**56500**	W 40×249	19500	W 21×364	10800	W 18×258	5510
W 36×720	55300	W 33×318	19500	W 14×605	10800	W 14×370	5440
				W 24×306	10700	W 30×124	5360
W 40×593	**50400**			W 36×170	10500	W 21×201	5310
W 36×650	48900	**W 44×224**	**19200**	W 30×211	10300	W 24×162	5170
		W 40×244	19200				
W 40×531	**44300**	W 24×492	19100				
W 36×588	43500	W 36×280	18900				
W 33×619	41800	W 30×357	18600	**W 40×149**	**9780**	**W 30×116**	**4930**
		W 27×407	18100	W 36×160	9750	W 18×234	4900
W 40×480	**39500**	W 33×291	17700	W 27×235	9660	W 14×342	4900
W 36×527	38300	W 36×260	17300	W 21×333	9610	W 27×129	4760
W 33×567	37700	W 24×450	17100	W 24×279	9600	W 21×182	4730
		W 36×256	16800	W 14×550	9430	W 24×146	4580
W 40×436	**35400**	W 30×326	16800	W 33×169	9290		
W 36×485	34700			W 30×191	9170		
W 33×515	33700			W 36×150	9040		
W 30×581	33000	**W 44×198**	**16700**	W 27×217	8870	**W 30×108**	**4470**
		W 40×215	16700	W 24×250	8490	W 18×211	4330
W 40×397	**32000**	W 40×221	16600	W 21×300	8480	W 14×311	4330
W 36×439	31000	W 36×245	16100	W 14×500	8210	W 21×166	4280
W 33×468	30100	W 27×368	16100	W 30×173	8200	W 27×114	4090
W 30×526	29300	W 33×263	15800	W 33×152	8160	W 12×336	4060
		W 24×408	15100	W 27×194	7820	W 24×131	4020
W 40×362	**28900**	W 36×230	15000				
W 36×393	27500	W 36×232	15000				
W 33×424	26900	W 40×199	14900				
		W 30×292	14900			**W 30×99**	**3990**
		W 27×336	14500	**W 36×135**	**7800**	W 18×192	3870
W 40×328	**26800**	W 14×730	14300	W 24×229	7650	W 14×283	3840
W 30×477	26100	W 33×241	14200	W 21×275	7620	W 21×147	3630
				W 33×141	7450		
W 40×324	**25600**			W 14×455	7190		
W 27×539	25500	**W 40×192**	**13500**	W 27×178	6990		
W 36×359	24800	W 24×370	13400	W 18×311	6960	**W 30×90**	**3620**
				W 24×207	6820	W 27×102	3620
W 44×285	**24600**			W 21×248	6760	W 12×305	3550
W 33×387	24300	**W 40×183**	**13300**			W 24×117	3540
W 40×298	24200	W 36×210	13200			W 18×175	3450
W 40×297	23200	W 30×261	13100			W 14×257	3400
W 30×433	23200	W 27×307	13100	**W 33×130**	**6710**	W 27×94	3270
W 27×494	22900	W 33×221	12800	W 30×148	6680	W 21×132	3220
W 36×328	22500	W 14×665	12400	W 14×426	6600	W 12×279	3110
		W 21×402	12200	W 27×161	6280	W 24×104	3100
W 40×277	**21900**	W 36×194	12100	W 24×192	6260	W 18×158	3060
W 33×354	21900	W 27×281	11900	W 18×283	6160	W 14×233	3010
		W 24×335	11900	W 14×398	6000	W 24×103	3000
W 40×268	**21500**	W 30×235	11700	W 21×223	5950	W 21×122	2960

MENT OF INERTIA SELECTION TABLE
For W and M shapes I_x

Shape	I_x In.^{4}	Shape	I_x In.^{4}	Shape	I_x In.^{4}	Shape	I_x In.^{4}
W 27×84	**2850**	**W 21×50**	**984**	**W 16×26**	**301**	**M 12×10**	**61.6**
W 18×143	2750	W 18×60	984	W 14×30	291	W 10×12	53.8
W 12×252	2720	W 16×67	954	W 12×35	285	W 6×25	53.4
W 24×94	2700	W 12×106	933	W 10×49	272	W 8×15	48.0
W 21×111	2670	W 18×55	890	W 8×67	272	W 6×20	41.4
W 14×211	2660	W 14×82	882	W 10×45	248	W 8×13	39.6
W 18×130	2460					M 6×20	29.0
W 21×101	2420						
W 12×230	2420	**W 21×44**	**843**	**W 14×26**	**245**	**M 10×9**	**38.8**
W 14×193	2400	W 12×96	833	W 12×30	238		
		W 18×50	800	W 8×58	228	**M 10×8**	**34.5**
W 24×84	**2370**	W 14×74	796	W 10×39	209		
W 18×119	2190	W 16×57	758			**M 10×7.5**	**32.8**
W 14×176	2140	W 12×87	740			W 6×16	32.1
W 12×210	2140	W 14×68	723	**W 12×26**	**204**	W 8×10	30.8
		W 10×112	716			W 6×15	29.1
W 24×76	**2100**	W 18×46	712			W 5×19	26.2
W 21×93	2070	W 12×79	662	**W 14×22**	**199**	M 5×18.9	24.1
W 18×106	1910	W 16×50	659	W 8×48	184	W 6×12	22.1
W 14×159	1900	W 14×61	640	W 10×30	170	W 5×16	21.3
W 12×190	1890	W 10×100	623	W 10×33	170		
						M 8×6.5	**18.5**
W 24×68	**1830**	**W 18×40**	**612**	**W 12×22**	**156**	W 6×9	16.4
W 21×83	1830	W 12×72	597			W 4×13	11.3
W 18×97	1750	W 16×45	586				
W 14×145	1710	W 14×53	541	**M 14×18**	**148**	**M 6×4.4**	**7.20**
W 12×170	1650	W 10×88	534	W 8×40	146		
W 21×73	1600	W 12×65	533	W 10×26	144		
				W 12×19	130		
W 24×62	**1550**			W 8×35	127		
W 18×86	1530	**W 16×40**	**518**	W 10×22	118		
W 14×132	1530			W 8×31	110		
W 16×100	1490	**W 18×35**	**510**				
W 21×68	1480	W 14×48	485	**W 12×16**	**103**		
W 12×152	1430	W 12×58	475	W 8×28	98.0		
W 14×120	1380	W 10×77	455	W 10×19	96.3		
		W 16×36	448				
W 24×55	**1350**	W 14×43	428				
W 21×62	1330	W 12×53	425	**W 12×14**	**88.6**		
W 18×76	1330	W 12×50	394	W 8×24	82.8		
W 16×89	1300	W 10×68	394	W 10×17	81.9		
W 14×109	1240	W 14×38	385	W 8×21	75.3		
W 12×136	1240						
W 21×57	1170						
W 18×71	1170	**W 16×31**	**375**	**M 12×11.8**	**71.9**		
W 16×77	1110	W 12×45	350	W 10×15	68.9		
W 14×99	1110	W 10×60	341				
W 18×65	1070	W 14×34	340				
W 12×120	1070	W 12×40	310	**M 12×10.8**	**65.0**		
W 14×90	999	W 10×54	303	W 8×18	61.9		

Lacing, including flat bars, angles, channels or other shapes employed as lacing, shall be so spaced that the ratio l/r of the flange included between their connections shall not exceed ¾ times the governing ratio for the member as a whole. Lacing shall be proportioned to resist a shearing stress normal to the axis of the member equal to 2% of the total compressive stress in the member. The ratio l/r for lacing bars arranged in single systems shall not exceed 140. For double lacing this ratio shall not exceed 200. Double lacing bars shall be joined at their intersections. For lacing bars in compression the unsupported length of the lacing bar shall be taken as the distance between fasteners or welds connecting it to the components of the built-up member for single lacing, and 70% of that distance for double lacing. The inclination of lacing bars to the axis of the member shall preferably be not less than 60° for single lacing and 45° for double lacing. When the distance between the lines of fasteners or welds in the flanges is more than 15 in., the lacing preferably shall be double or be made of angles.

The function of tie plates and lacing may be performed by continuous cover plates perforated with access holes. The unsupported width of such plates at access holes, as defined in Sect. B5, is assumed available to resist axial stress, provided that: the width-to-thickness ratio conforms to the limitations of Sect. B5; the ratio of length (in direction of stress) to width of holes shall not exceed 2; the clear distance between holes in the direction of stress shall be not less than the transverse distance between nearest lines of connecting fasteners or welds; and the periphery of the holes at all points shall have a minimum radius of 1½ in.

E5. PIN-CONNECTED COMPRESSION MEMBERS

Pin-connections of pin-connected compression members shall conform to the requirements of Sect. D3.

E6. COLUMN WEB SHEAR

Column connections must be investigated for concentrated force introduction in accordance with Sect. K1.

CHAPTER F

BEAMS AND OTHER FLEXURAL MEMBERS

Beams shall be distinguished from plate girders on the basis of the web slenderness ratio h/t_w. When this value is greater than $760/\sqrt{F_b}$ the allowable bending stress is given in Chapter G. The allowable shear stresses and stiffener requirements are given in Chapter F unless tension field action is used, then the allowable shear stresses are given in Chapter G.

This chapter applies to singly or doubly symmetric beams including hybrid beams and girders loaded in the plane of symmetry. It also applies to channels loaded in a plane passing through the shear center parallel to the web or restrained against twisting at load points and points of support. For members subject to combined flexural and axial force, see Sect. H1.

F1. ALLOWABLE STRESS: STRONG AXIS BENDING OF I-SHAPED MEMBERS AND CHANNELS

1. Members with Compact Sections

For members with compact sections as defined in Sect. B5.1 (excluding hybrid beams and members with yield points greater than 65 ksi) symmetrical about, and loaded in, the plane of their minor axis the allowable stress is

$$F_b = 0.66\,F_y \qquad \text{(F1-1)}$$

provided the flanges are connected continuously to the web or webs and the laterally unsupported length of the compression flange L_b does not exceed the value of L_c, as given by the smaller of:

$$\frac{76b_f}{\sqrt{F_y}} \text{ or } \frac{20{,}000}{(d/A_f)\,F_y} \qquad \text{(F1-2)}$$

Members (including composite members and excluding hybrid members and members with yield points greater than 65 ksi) which meet the requirements for compact sections and are continuous over supports or rigidly framed to columns, may be proportioned for 9/10 of the negative moments produced by gravity loading when such moments are maximum at points of support, provided that, for such members, the maximum positive moment is increased by 1/10 of the average negative moments. This reduction shall not apply to moments produced by loading on cantilevers. If the negative moment is resisted by a column rigidly framed to the beam or girder, the 1/10 reduction is permitted in proportioning the column for the combined axial and bending loading, provided that the stress f_a due to any concurrent axial load on the member, does not exceed $0.15F_a$.

2. Members with Noncompact Sections

For members meeting the requirements of Sect. F1.1 except that their flanges are noncompact (excluding built-up members and members with yield points greater than 65 ksi), the allowable stress is

$$F_b = F_y \left[0.79 - 0.002 \frac{b_f}{2t_f} \sqrt{F_y}\right] \qquad \text{(F1-3)}$$

For built-up members meeting the requirements of Sect. F1.1 except that their flanges are noncompact and their webs are compact or noncompact, (excluding hybrid girders and members with yield points greater than 65 ksi) the allowable stress is

$$F_b = F_y \left[0.79 - 0.002 \frac{b_f}{2t_f} \sqrt{\frac{F_y}{k_c}}\right] \qquad \text{(F1-4)}$$

where

$$k_c = \frac{4.05}{(h/t_w)^{0.46}} \text{ if } h/t_w > 70, \text{ otherwise } k_c = 1.0.$$

For members with a noncompact section (Sect. B5), but not included above, and loaded through the shear center and braced laterally in the region of compression stress at intervals not exceeding $76b_f/\sqrt{F_y}$, the allowable stress is

$$F_b = 0.60\ F_y \qquad \text{(F1-5)}$$

3. Members with Compact or Noncompact Sections with Unbraced Length Greater than L_c

For flexural members with compact or noncompact sections as defined in Sect. B5.1, and with unbraced lengths greater than L_c as defined in Sect. F1.1, the allowable bending stress in tension is determined from Equation (F1-5).

For such members with an axis of symmetry in, and loaded in the plane of their web, the allowable bending stress in compression is determined as the larger value from Equations (F1-6) or (F1-7) and (F1-8), except that Equation (F1-8) is applicable only to sections with a compression flange that is solid and approximately rectangular in cross section and that has an area not less than the tension flange. Higher values of the allowable compressive stress are permitted if justified by a more precise analysis. Stresses shall not exceed those permitted by Chapter G, if applicable.

For channels bent about their major axis, the allowable compressive stress is determined from Equation (F1-8).

When

$$\sqrt{\frac{102 \times 10^3 C_b}{F_y}} \le \frac{l}{r_T} \le \sqrt{\frac{510 \times 10^3 C_b}{F_y}}:$$

$$F_b = \left[\frac{2}{3} - \frac{F_y\,(l/r_T)^2}{1530 \times 10^3 C_b}\right] F_y \le 0.60\ F_y \qquad \text{(F1-6)}$$

When

$$\frac{l}{r_T} \ge \sqrt{\frac{510 \times 10^3 C_b}{F_y}}:$$

SEE Pg 5-120

$$F_b = \frac{170 \times 10^3 C_b}{(l/r_T)^2} \le 0.60\ F_y \qquad \text{(F1-7)}$$

For any value of l/r_T:

$$F_b = \frac{12 \times 10^3 C_b}{ld/A_f} \le 0.60\ F_y \qquad \text{(F1-8)}$$

where

l = distance between cross sections braced against twist or lateral displacement of the compression flange, in. For cantilevers braced against twist only at the support, l may conservatively be taken as the actual length.

r_T = radius of gyration of a section comprising the compression flange plus ⅓ of the compression web area, taken about an axis in the plane of the web, in.

A_f = area of the compression flange, in.^{2}

C_b = $1.75 + 1.05\,(M_1/M_2) + 0.3\,(M_1/M_2)^2$, but not more than 2.3;* where M_1 is the smaller and M_2 the larger bending moment at the ends of the unbraced length, taken about the strong axis of the member, and where M_1/M_2, the ratio of end moments, is positive when M_1 and M_2 have the same sign (reverse curvature bending) and negative when they are of opposite signs (single curvature bending). When the bending moment at any point within an unbraced length is larger than that at both ends of this length, the value of C_b shall be taken as unity. When computing F_{bx} to be used in Equation (H1-1), C_b may be computed by the equation given above for frames subject to joint translation, and it shall be taken as unity for frames braced against joint translation. C_b may conservatively be taken as unity for cantilever beams.**

*It is conservative to take C_b as unity. For values smaller than 2.3, see Table 6 in the Numerical Values Section.

**For the use of larger C_b values, see Galambos (1988).

For hybrid plate girders, F_y for Equations (F1-6) and (F1-7) is the yield stress of the compression flange. Equation (F1-8) shall not apply to hybrid girders.

Sect. F1.3 does not apply to tee sections if the stem is in compression anywhere along the unbraced length.

F2. ALLOWABLE STRESS: WEAK AXIS BENDING OF I-SHAPED MEMBERS, SOLID BARS AND RECTANGULAR PLATES

Lateral bracing is not required for members loaded through the shear center about their weak axis nor for members of equal strength about both axes.

1. Members With Compact Sections

For doubly symmetrical I- and H-shape members with compact flanges (Sect. B5) continuously connected to the web and bent about their weak axes (except members with yield points greater than 65 ksi); solid round and square bars; and solid rectangular sections bent about their weaker axes, the allowable stress is

$$F_b = 0.75\,F_y \tag{F2-1}$$

2. Members With Noncompact Sections

For members not meeting the requirements for compact sections of Sect. B5 and not covered in Sect. F3, bent about their minor axis, the allowable stress is

$$F_b = 0.60\,F_y \tag{F2-2}$$

Doubly symmetrical I- and H-shape members bent about their weak axes (except members with yield points greater than 65 ksi) with noncompact flanges (Sect. B5) continuously connected to the web may be designed on the basis of an allowable stress of

$$F_b = F_y\left[1.075 - 0.005\left(\frac{b_f}{2t_f}\right)\sqrt{F_y}\right] \tag{F2-3}$$

F3. ALLOWABLE STRESS: BENDING OF BOX MEMBERS, RECTANGULAR TUBES AND CIRCULAR TUBES

1. Members With Compact Sections

For members bent about their strong or weak axes, members with compact sections as defined in Sect. B5 and flanges continuously connected to the webs, the allowable stress is

$$F_b = 0.66\,F_y \tag{F3-1}$$

To be classified as a compact section, a box-shaped member shall have, in addition to the requirements in Sect. B5, a depth not greater than 6 times the width, a flange thickness not greater than 2 times the web thickness and a laterally unsupported length L_b less than or equal to

$$L_c = \left(1{,}950 + 1{,}200\,\frac{M_1}{M_2}\right)\frac{b}{F_y} \tag{F3-2}$$

except that it need not be less than 1,200 (b/F_y), where M_1 is the smaller and M_2 the larger bending moment at the ends of the unbraced length, taken about the strong axis of the member, and where M_1/M_2, the ratio of end moments, is positive when M_1 and M_2 have the same sign (reverse curvature bending) and negative when they are of opposite signs (single curvature bending).

2. Members With Noncompact Sections

For box-type and tubular flexural members that meet the noncompact section requirements of Sect. B5, the allowable stress is

$$F_b = 0.60\,F_y \tag{F3-3}$$

Lateral bracing is not required for a box section whose depth is less than 6 times its width. Lateral-support requirements for box sections of larger depth-to-width ratios must be determined by special analysis.

F4. ALLOWABLE SHEAR STRESS

For $h/t_w \le 380/\sqrt{F_y}$, on the overall depth times the web thickness, the allowable shear stress is

$$F_v = 0.40\,F_y \tag{F4-1}$$

For $h/t_w > 380/\sqrt{F_y}$, the allowable shear stress is on the clear distance between flanges times the web thickness is

$$F_v = \frac{F_y}{2.89}(C_v) \le 0.40F_y \tag{F4-2}$$

where

$$C_v = \frac{45{,}000k_v}{F_y(h/t_w)^2} \text{ when } C_v \text{ is less than } 0.8$$

$$= \frac{190}{h/t_w}\sqrt{\frac{k_v}{F_y}} \text{ when } C_v \text{ is more than } 0.8$$

$$k_v = 4.00 + \frac{5.34}{(a/h)^2} \text{ when } a/h \text{ is less than } 1.0$$

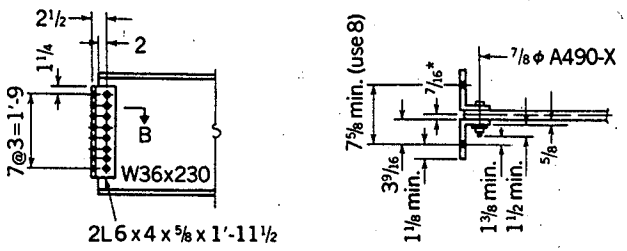
$$= 5.34 + \frac{4.00}{(a/h)^2} \text{ when } a/h \text{ is more than } 1.0$$

t_w = thickness of web, in.
a = clear distance between transverse stiffeners, in.
h = clear distance between flanges at the section under investigation, in.

For shear rupture on coped beam end connections see Sect. J4.

Maximum h/t_w limits are given in Chapter G.

An alternative design method for plate girders utilizing tension field action is given in Chapter G.



SECTION AT B

angles at length L = 23½ in. From Table II-C, the net shear capacity with ⅞-in. dia. bolts and n = 8 on angles ⅝-in. thick and 23½-in. long is 348 kips > 340 kips **o.k.**

Bearing capacity of the beam may be checked from Table I-E, where the value 60.9 kips/in. thickness is read for F_u = 58 ksi and ⅞-in. dia. bolts at 3-in. spacing. Allowable bearing R = 60.9 × 0.760 × 8 = 370 kips > 340 kips **o.k.**

Bearing on the angle material is checked with Table I-F. The 1¼-in. vertical edge distance on the top bolt is less than 1½d (1 5/16 in.), so the bearing value for the top bolt is 36.3 kips/in. The value for the remaining seven bolts is 60.9 kips/in. The capacity of the connection angles is ⅝ [36.3 + (7 × 60.9)] = 289 kips > 340/2 = 170 kips **o.k.**

Since this beam is not coped, beam web tear-out does not require checking.

The connection as sketched is adequate. Attention must also be given to:

a. Insertion and driving clearance; see Table of Assembling Clearances (Manual, Part 4).

b. Adequacy of this connection with respect to the supporting member.

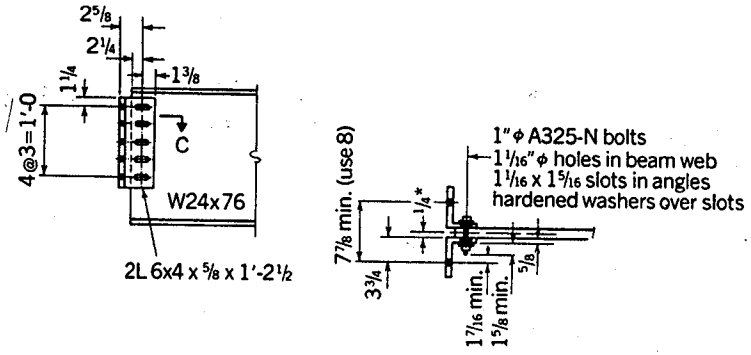
c. If a smaller gage is required on the outstanding legs of the connection angles, the bolts must be staggered with those in the beam web. An even stagger will permit the gage to be reduced to 7 in. and still maintain driving clearances while maintaining the 3-in. pitch. See Table of Stagger for Tightening (Manual, Part 4).

EXAMPLE 3

Given:

Beam: W24 × 76, t_w = 0.440 in.
A572 Gr. 50 (F_y = 50 ksi and F_u = 65 ksi)
Connection angles: A36 (F_y = 36 ksi and F_u = 58 ksi)
Beam reaction = 156 kips
Bolts: 1-in. dia. A325-N
Holes: 1 1/16 × 1 5/16 (short slots), long axis perpendicular to transmitted force

* This dimension (see sketch, Section at B) is one-half the decimal web thickness rounded to the next higher 1/16 in., as in Ex. 1.



SECTION AT C

Solution:

From Table II-A, select a connection with 5 rows (n = 5) of 1-in. dia. A325-N bol and shear value of 165 kips using ⅝-in. thick angles. This exceeds the 156-kip rea tion.

Bearing capacity of the beam may be checked from Table I-E, where the value 78 kips/in. thickness is read for F_u = 65 ksi and 1-in. dia. bolts at 3-in. spacing. Allow ble bearing R = 78.0 × 0.440 × 5 = 172 kips > 156 kips **o.k.**

Check capacity of the A36 connection angles using Table I-F with F_u = 58 ksi ar l_v = 1¼-in. edge distance. The top bolt has a material bearing value of 36.3 kips/i Allowable bearing on the connection angles then is ⅝ × [36.3 + (4 × 69.6)] = 1 kips > 156/2 = 78 kips. **o.k.**

Since this beam is not coped, beam web tear-out (block shear) does not requi checking.

The net shear capacity of the angles need not be checked, using Table II-C, for tl reason given in Ex. 1.

Note that short slots are permitted in any and all plies of slip-critical or bearing-typ connections using high-strength bolts. To simplify the fabrication, slots are put in th angles, rather than the beam web. Hardened washers are required over these slo because they are in an outer ply.

The connection as sketched is adequate. Attention must also be given to the follov ing:

a. Insertion and driving clearances; see Table of Assembling Clearances (Manua Part 4).

b. Adequacy of this connection with respect to the supporting member.

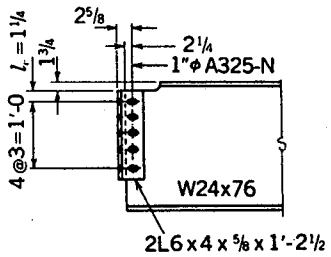
c. An even stagger of web bolts with those in the outstanding legs will permit us of a 7-in. gage and still maintain driving clearances while maintaining the 3-ir pitch. See the Table of Stagger for Tightening (Manual, Part 4).

* This dimension (see sketch, Section at C) is one-half the decimal web thickness rounded to the nex higher 1/16 in., as in Ex. 1.

EXAMPLE 4

Given:

The same conditions as Ex. 3, except the beam is coped 1¾ in. deep at the top flange. Find the allowable reaction on the modified beam.



Solution:

The allowable reaction of the coped beam will be the least value determined by the bolt shear, bolt bearing, beam web tear-out (block shear) or shear capacity of the connection angles.

1. Bolt shear: From Table II-A, R = 165 kips.

2. Bolt bearing of beam web:
   From Table I-F, for l_v = 1¼ in. and F_u = 65 ksi:
   Allowable R = 0.440 in. × [(1 bolt × 40.6) + (4 bolts × 78.0)] = 155 kips

3. Bolt bearing on connection angles:
   From Table I-F, for l_v = 1¼ in. and F_u = 58 ksi:
   Allowable R = ⅝ in. × 2 angles × [(1 bolt × 36.3) + (4 bolts × 69.6)] = 393 kips

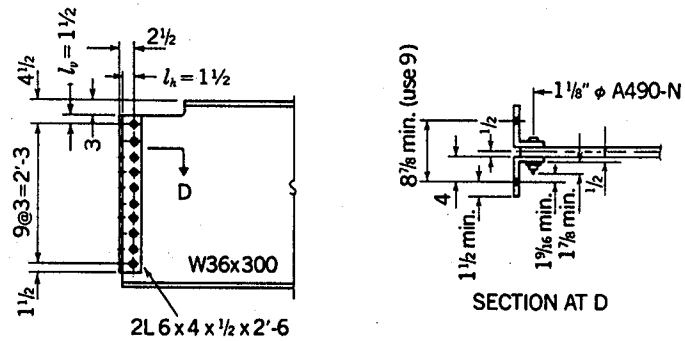
4. Web tear-out:
   From Table I-G, for l_v = 1¼ in., l_h = 2¼ in., t = 0.440 in. and F_u = 65 ksi:
   Coefficient C_1 = 1.50
   Coefficient C_2 = 1.90
   Total 3.40
   $R_{BS} = (C_1 + C_2)F_u t$
   $= 3.40 \times 65 \times 0.440 = 97.2$ kips

5. Shear capacity of angles:
   From Table II-C, for 1-in. bolts, angle thickness = ⅝ in., L = 14½ in. (n = 5):
   Shear capacity = 200 kips

The allowable reaction is limited to 97.2 kips and the conditions assumed would require a redesign of this connection.

EXAMPLE 5

Given: Beam: W 36 × 300, t_w = 0.945 in.
A36 (F_y = 36 ksi and F_u = 58 ksi)
Top flange coped 3 in. deep
Beam reaction = 300 kips
Bolts: 1⅛-in. dia. A490-N in 1³⁄₁₆-in. dia. holes



Solution:

Since values are not tabulated in this Manual for 1⅛-in. dia. fasteners, they must be derived from the AISC ASD Specification. The following is offered as a guide and outline. References are to sections of the AISC ASD Specification and Commentary.

	Specification and Commentary Reference
1. Bolt shear:	
Area of 1⅛-in. dia. bolt = 0.994 in.^{2}	
F_v = 28.0 ksi	
Single shear capacity = 0.994 × 28 = 27.8 kips/bolt	Table J3.2
No. bolts req'd = 300/(27.8 × 2) = 5.4.	
Since this beam is coped, judgement and experience suggest trying 10 bolts because web tear-out is likely to be critical.	
2. Bolt spacing:	**J3.8**
Minimum spacing = 2⅔d = 3 in.	
Preferred spacing = 3d = 3⅜ in.	
To satisfy bearing on web:	
Spacing = $2P/F_u t + d/2$	
= (2 × 300/10)/(58 × 0.945) + (1⅛)/2 = 1.66 in.	
Use 3-in. spacing, which is standard but less than the preferred spacing.	
3. Edge distance:	**J3.9**
Minimum distance in the direction of reactive force:	
$l_v = 2P/F_u t$ = (2 × 300/10)/(58 × 0.945) = 1.09 in.	
Use 1½ in. (minimum for gas cut edge)	Table J3.5

4. Since this beam is coped, failure by shear and tension through the fastener holes must be checked.

ASD Specification Reference

Allowable block shear $R_{BS} = 0.30A_vF_u + 0.50A_tF_u$ — J4

This equation expands to the general expression shown at Table I-G

$$R_{BS} = \{(0.3\ l_v + 0.5\ l_h) + 0.3\ [(n-1)(s-d_h) - d_h/2] - d_h/4\}\ F_u t$$

$$= \{[(0.3 \times 1\tfrac{1}{2}) + (0.5 \times 1\tfrac{1}{2})] + 0.3[(10-1)(3-1\tfrac{3}{16}) - \tfrac{19}{32}] - \tfrac{19}{64}\}\ 58 \times 0.945$$

$$= 308 \text{ kips}$$

5. Connection angles:

Try 2 L 6 × 4 × ½ × 2′- 6, A36.

Allowable bearing:

From AISC ASD Specification Table J3.5 for 1⅛-in. dia. bolts, $l_v = 1\tfrac{1}{2}$ in. < $1\tfrac{1}{2}\ d = 1.69$,

Top Bolt: $F_u \times t/2 \times$ edge distance
$58 \times 0.5/2 \times 1\tfrac{1}{2} = 21.8$ kips

Remaining Bolts: $1.2 \times F_u \times d \times t$
$1.2 \times 58 \times 1\tfrac{1}{8} \times \tfrac{1}{2} = 39.2$ kips

R = (1 bolt × 21.8) + (9 bolts × 39.2) = 374 kips **o.k.**

Shear on plane through fasteners:

Allowable shear is $0.4F_y = 14.5$ ksi on the gross area or $0.3F_u = 17.4$ ksi on the net area. Shear on the *net* area governs when $(d + \tfrac{1}{16}$ in.$) > L/6n$. Then 30/(6 × 10) = 0.5 in. and *net* area governs.

Net area $= 2t\,[L - n(d + \tfrac{1}{16})]$
$= (2 \times \tfrac{1}{2})\,[30 - (10 \times 1\tfrac{3}{16})] = 18.1$ in.^{2}

$f_v = 300/18.1 = 16.6$ ksi < 17.4 ksi

6. The connection as sketched is adequate. Attention must also be given to the following:

a. Insertion and driving clearances; see Table of Assembling Clearances (Manual, Part 4).

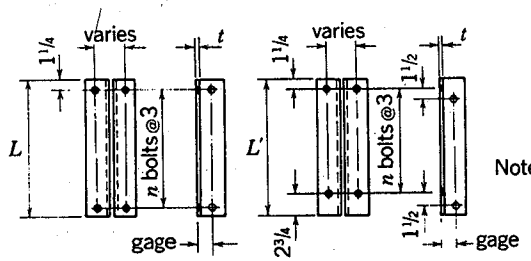
b. Need for reinforcement at the coped section in bending.

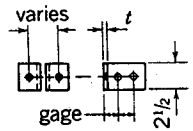
c. Adequacy of this connection with respect to the supporting member.

d. If a smaller gage is required on the outstanding legs, a new design is required, since the maximum number of fasteners has been used in the beam web. Consideration can be given to larger diameter bolts and to the use of a second gage line in the web. The effect of web tear-out (block shear) on a connection with two gage lines has not been determined at this time.

Bolted

TABLE II Allowable loads in kips





Note: For $L = 2\tfrac{1}{2}$ use one half the tabular load value shown for $L = 5\tfrac{1}{2}$, for the same bolt type, diameter, and thickness.

TABLE II-A Bolt Shear[a]

For A307 bolts in standard or slotted holes and for A325 and A490 bolts in **slip-critical** connections with standard holes and Class A, clean mill scale surface condition.

Bolt Type			A307			A325-SC			A490-SC		
F_v, Ksi			10.0			17.0			21.0		
Bolt Dia., d in.			¾	⅞	1	¾	⅞	1	¾	⅞	1
Angle Thickness t, in.			¼	¼	¼	¼	5/16	½	5/16	½	⅝
L in.	L' in.	n									
29½	31	10	88.4	120	157	150	204	267	186	253	330
26½	28	9	79.5	108	141	135	184	240	167	227	297
23½	25	8	70.7	96.2	126	120	164	214	148	202	264
20½	22	7	61.9	84.2	110	105	143	187	130	177	231
17½	19	6	53.0	72.2	94.2	90.1	123	160	111	152	198
14½	16	5	44.2	60.1	78.5	75.1	102	134	92.8	126	165
11½	13	4	35.3	48.1	62.8	60.1	81.8	107	74.2	101	132
8½	10	3	26.5	36.1	47.1[b]	45.1	61.3	80.1	55.7	75.8	99.0
5½	7	2	17.7	24.1	31.4[b]	30.0	40.9	53.4	37.1	50.5	66.0

Note: For slip-critical connections with oversize or slotted holes, see Table II-B.

Notes:

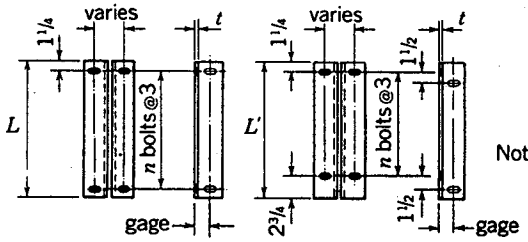
[a]Tabulated load values are based on double shear of bolts unless noted. See RCSC Specification for other surface conditions.

[b]Capacity shown is based on double shear of the bolts; however, for length L, net shear on the angle thickness specified is critical. See Table II-C.

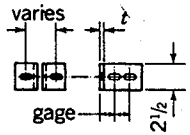
slip critical connection.

Bolted

TABLE II Allowable loads in kips



STAGGERED BOLT ALTERNATE



Note: For $L=2\frac{1}{2}$ use one half the tabular load value shown for $L=5\frac{1}{2}$, for the same bolt type, diameter, and thickness.

TABLE II-A Bolt Shear[a]

For bolts in **bearing-type** connections with standard or slotted holes.

Bolt Type			A325-N			A490-N			A325-X			A490-X		
F_v, Ksi			21.0			28.0			30.0			40.0		
Bolt Dia., d In.			¾	⅞	1	¾	⅞	1	¾	⅞	1	¾	⅞	1
Angle Thickness t, In.			5/16	⅜	⅝	⅜	½	⅝	⅜	⅝	⅝	½	⅝	⅝
L In.	L' In.	n												
29½	31	10	186	253	330	247	337	440[b]	265	361	[d]	353	453[c]	[d]
26½	28	9	167	227	297	223	303	396[b]	239	325	[d]	318	408[c]	[d]
23½	25	8	148	202	264	198	269	352[b]	212	289	[d]	283	363[c]	[d]
20½	22	7	130	177	231	173	236	308[b]	186	253	[d]	247	317[c]	[d]
17½	19	6	111	152	198	148	202	264[b]	159	216	272[c]	212	272[c]	272[c]
14½	16	5	92.8	126	165	124	168	220[b]	133	180	227[c]	177	227[c]	227[c]
11½	13	4	74.2	101	132	99.0	135	176[b]	106	144	181[c]	141	181[c]	181[c]
8½	10	3	55.7	75.8[b]	99.0	74.2	101[b]	132[b]	79.5[b]	108	136[c]	106[b]	136[c]	136[c]
5½	7	2	37.1	50.5[b]	66.0	49.5	67.3[b]	88.0[b]	53.0[b]	72.2	90.6[c]	70.7[b]	90.6[c]	90.6[c]

[a]Tabulated load values are based on double shear of bolts unless noted. See RCSC Specification for other surface conditions.

[b]Capacity shown is based on double shear of the bolts; however, for length L, net shear on the angle thickness specified is critical. See Table II-C.

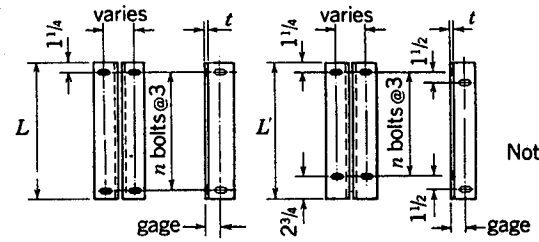
[c]Capacity shown is based on bearing capacity of 1¼ in. end distance [Specification Equation (J3-6)] on A36 angles of thickness specified; however, for length L, net shear on this angle is critical. See Table II-C.

[d]Capacity is governed by net shear on angles for lengths L and L'. See Table II-C.

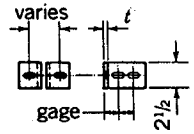
Bearing type connection

Bolted

TABLE II Allowable loads in kips



STAGGERED BOLT ALTERNATE



Note: For $L=2\frac{1}{2}$ use one half the tabular load value shown for $L=5\frac{1}{2}$, for the same bolt type, diameter, and thickness.

TABLE II-B Bolt Shear[a]

For bolts in **slip-critical** connections with oversize, short-slotted, or long-slotted holes and Class A, clean mill scale surface condition.

Hole Type			Long-slotted Holes						Oversize and Short-slotted Holes					
Bolt Type			A325-SC			A490-SC			A325-SC			A490-SC		
F_v, Ksi			12.0			15.0			15.0			18.0		
Bolt Dia., d, In.			¾	⅞	1	¾	⅞	1	¾	⅞	1	¾	⅞	1
Angle Thickness, t, In.			¼	¼	5/16	¼	5/16	½	¼	5/16	½	5/16	⅜	⅝
L In.	L' In.	n												
29½	31	10	106	144	188	133	180	236	133	180	236	159	216	283
26½	28	9	95.4	130	170	119	162	212	119	162	212	143	195	254
23½	25	8	84.8	115	151	106	144	188	106	144	188	127	173	226
20½	22	7	74.2	101	132	92.8	126	165	92.8	126	165	111	152	198
17½	19	6	63.6	86.6	113	79.5	108	141	79.5	108	141	95.4	130	170
14½	16	5	53.0	72.2	94.2	66.3	90.2	118	66.3	90.2	118	79.5	108	141
11½	13	4	42.4	57.7	75.4	53.0	72.2	94.2	53.0	72.2	94.2	63.6	86.6	113
8½	10	3	31.8	43.3	56.5	39.8	54.1	70.7	39.8	54.1	70.7	47.7	64.9	84.8
5½	7	2	21.2	28.9	37.7[b]	26.5	36.1	47.1	26.5	36.1	47.1	31.8	43.3[c]	56.5

[a]Tabulated load values are based on double shear of bolts unless noted. See RCSC Specification, for other surface conditions. Slotted holes are parallel to beam flange. When slotted holes are not parallel to beam flanges, the above connection angle values and details will differ. See AISC ASD Specification, Sects. J3.8 and J3.9.

[b]Capacity shown is based on double shear of the bolts; however, for length L, net shear on the angle thickness specified is critical. See Table II-C.

[c]Capacity shown is based on double shear of bolts; however, for L = 5½ in., with oversize holes, net shear in the angle thickness specified is critical and tabulated load reduces to 44.0 kips.

Bearing type connection

TABLE II Allowable loads in kips

TABLE II-C
Allowable Shear in Connection Angles for A36 Material

Bolt Dia., d in.		¾				⅞					1				
Angle Thickness, t in.		¼	5/16	⅜	½	¼	5/16	⅜	½	⅝	¼	5/16	⅜	½	⅝
L in.	n														
9½	10	186	232	279	372	175	219	263	350	438	164	205	246	328	411
6½	9	167	209	250	334	157	196	236	314	393	147	184	221	295	368
3½	8	148	185	222	296	139	174	209	278	348	131	163	196	261	326
0½	7	129	161	193	258	121	152	182	243	303	114	142	170	227	284
7½	6	110	137	165	220	103	129	155	207	258	96.8	121	145	194	242
4½	5	90.8	114	136	182	85.4	107	128	171	213	79.9	99.9	120	160	200
1½	4	71.8	89.7	108	144	67.4	84.3	101	135	169	63.1	78.8	94.6	126	158
8½	3	52.7	65.9	79.1	105	49.5	61.9	74.2	99.0	124	46.2	57.8	69.3	92.4	116
5½	2	33.7	42.1	50.6	67.4	31.5	39.4	47.3	63.1	78.8	29.4	36.7	44.0	58.7	73.4
L' in.	n														
1	10	199	249	299	398	188	235	282	376	470	177	222	266	355	443
8	9	180	225	270	360	170	213	255	340	425	160	201	241	321	401
5	8	161	201	241	322	152	190	228	305	381	144	179	215	287	359
2	7	142	177	213	284	134	168	201	269	336	127	158	190	253	317
9	6	123	154	184	246	116	145	175	233	291	110	137	165	220	275
6	5	104	130	156	208	98.4	123	148	197	246	93.0	116	139	186	232
3	4	84.8	106	127	170	80.5	101	121	161	201	76.1	95.2	114	152	190
0	3	65.8	82.2	98.7	132	62.5	78.2	93.8	125	156	59.3	74.1	88.9	119	148
7	2	46.8	58.5	70.1	93.5	44.6	55.7	66.9	89.2	111	42.4	53.0	63.6	84.8	106

NOTES: Table based on an allowable shear of $0.3F_u$ (17.4 ksi for A36 angles) of the net section of two angles.

Net section based on diameter of fastener + 1/16 in.

Welded–E70XX electrodes for combination with Table II connections

TABLE III

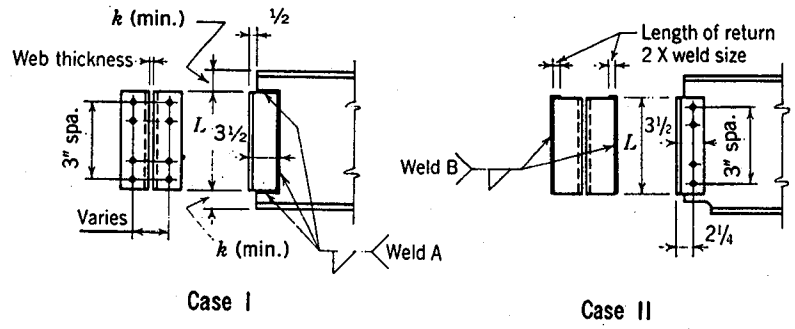


Table III is arranged to permit substitution of welds for bolts in the connections shown in Table II which fall within the weld capacities. Welds A replace fasteners in the beam web legs (Case I). Welds B replace fasteners in the outstanding legs (Case II).

To accommodate usual gages, angle leg widths will generally be 4 × 3½, with the 4-in. leg outstanding. Width of web legs in Case I may be reduced optionally from 3½ to 3 in. Width of outstanding legs in Case II may be reduced optionally from 4 to 3 in. for values of L = 5½ in. through 1′-5½.

Angle thickness is equal to weld size plus 1/16 in., or thickness of angle from applicable Table II, whichever is greater.

Angle length L must be as tabulated in Table III.

Holes for erection bolts may be placed as required in legs to be field-welded (optional).

When bolts are used, investigate bearing capacity of supporting member.

Although it is permissible to use Welds A and B in combination to obtain all-welded connections, it is recommended such connections be chosen from Table IV. This table will usually provide greater economy and allow increased flexibility in selection of angle lengths and connection capacities.

Allowable capacity for Weld A utilizes instantaneous center solutions based on the same criteria developed for Tables XIX to XXVI, Eccentric Loads on Weld Groups. However, capacity for Weld B is computed using traditional vector analysis techniques.

CASE I

EXAMPLE 6

Given:

Beam: W 36 × 150 (not coped); t_w = 0.625 in.
F_y = 36 ksi; F_u = 14.5 ksi
Reaction: 200 kips
Bolts: ⅞ in. dia., ASTM A325-N
Welds: E70XX

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S AND THREADED PARTS
Bearing
Allowable loads in kips

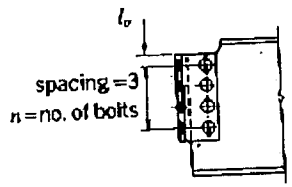
TABLE I-E. BEARING
Slip-critical and Bearing-type Connections

Material Thickness	F_u = 58 ksi Bolt dia.			F_u = 65 ksi Bolt dia.			F_u = 70 ksi Bolt dia.			F_u = 100 ksi Bolt dia.		
	¾	⅞	1	¾	⅞	1	¾	⅞	1	¾	⅞	1
⅛	6.5	7.6	8.7	7.3	8.5	9.8	7.9	9.2	10.5	11.3	13.1	15.0
3/16	9.8	11.4	13.1	11.0	12.8	14.6	11.8	13.8	15.8	16.9	19.7	22.5
¼	13.1	15.2	17.4	14.6	17.1	19.5	15.8	18.4	21.0	22.5	26.3	30.0
5/16	16.3	19.0	21.8	18.3	21.3	24.4	19.7	23.0	26.3	28.1	32.8	37.5
⅜	19.6	22.8	26.1	21.9	25.6	29.3	23.6	27.6	31.5	33.8	39.4	45.0
7/16	22.8	26.6	30.5	25.6	29.9	34.1	27.6	32.2	36.8		45.9	52.5
½	26.1	30.5	34.8	29.3	34.1	39.0	31.5	36.8	42.0			60.0
9/16	29.4	34.3	39.2	32.9	38.4	43.9		41.3	47.3			
⅝	32.6	38.1	43.5		42.7	48.8		45.9	52.5			
11/16		41.9	47.9		46.9	53.6			57.8			
¾		45.7	52.2			58.5						
13/16			56.6									
⅞			60.9									
15/16												
1	52.2	60.9	69.6	58.5	68.3	78.0	63.0	73.5	84.0	90.0	105.0	120.0

Notes:
This table is applicable to all mechanical fasteners in both slip-critical and bearing-type connections utilizing standard holes. Standard holes shall have a diameter nominally 1/16-in. larger than the nominal bolt diameter (d + 1/16 in.).
Tabulated bearing values are based on $F_p = 1.2\ F_u$.
F_u = specified minimum tensile strength of the connected part.
In connections transmitting axial force whose length between extreme fasteners measured parallel to the line of force exceeds 50 in., tabulated values shall be reduced 20%.
Connections using high-strength bolts in slotted holes with the load applied in a direction other than approximately normal (between 80 and 100 degrees) to the axis of the hole and connections with bolts in oversize holes shall be designed for resistance against slip at working load in accordance with AISC ASD Specification Sect. J3.8.
Tabulated values apply when the distance l parallel to the line of force from the center of the bolt to the edge of the connected part is not less than 1½ d and the distance from the center of a bolt to the center of an adjacent bolt is not less than 3d. See AISC ASD Commentary J3.8.
Under certain conditions, values greater than the tabulated values may be justified under Specification Sect. J3.7.
Values are limited to the double-shear bearing capacity of A490-X bolts.
Values for decimal thicknesses may be obtained by multiplying the decimal value of the unlisted thickness by the value given for a 1-in. thickness.

BOLTS AND RIVETS
Bearing
Allowable loads in kips

TABLE I-F. EDGE DISTANCE



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Edge Distance[b] l_v in.	Allowable Loads, Kips[a] (for one fastener, 1-in. thick material) F_u = 58	F_u = 65	F_u = 70	F_u = 100
1	29.0	32.5	35.0	50.0
1⅛	32.6	36.6	39.4	56.3
1¼	36.3	40.6	43.8	62.5
<1½	43.5	48.8	52.5	75.0

Bolt Dia.	1½ d in.	Values when edge distance is 1½ d or greater[c]			
1	1½	69.6	78.0	84.0	120
⅞	1 5/16	60.9	68.3	73.5	105
¾	1⅛	52.2	58.5	63.0	90.0

[a] Total allowable load = $\Sigma\left[(\text{tabular value}) \times n\right] t$
where
t = thickness of critical connected part, in.
n = number of fasteners.

[b] $l_v \geq 2P/F_u t$ (AISC ASD Spec. J3.9) distance center of hole to free edge of connected part in direction of force, in.

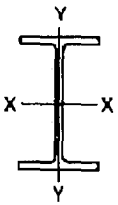
where
F_u = specified minimum tensile strength of material, ksi
P = force transmitted by one fastener to the critical connected part, kips

[c] $P = 1.2\ F_u\ d$ (AISC ASD Spec. Sect. J3.7).

F_y = 36 ksi

F_y = 50 ksi

COLUMNS
W shapes
Allowable axial loads in kips



Designation		W10											
Wt./ft		60		54		49		45		39		33	
F_y		36	50	36	50	36	50	36	50	36	50	36	50
Effective length in ft KL with respect to least radius of gyration r_y	0	380	528	341	474	311	432	287	399	248	345	210	291
	6	353	482	317	433	289	394	260	351	224	303	189	255
	7	348	472	312	423	284	385	253	340	218	293	184	248
	8	341	461	306	414	279	376	247	328	213	283	179	237
	9	335	450	300	403	273	367	240	316	206	272	173	228
	10	328	437	294	392	268	357	232	303	200	260	167	217
	11	321	425	288	381	262	348	224	289	193	248	161	207
	12	313	412	281	369	258	335	216	274	186	235	155	196
	13	306	398	274	356	249	324	208	259	178	221	149	184
	14	297	383	267	343	242	312	199	243	170	207	142	171
	15	289	368	259	330	235	299	190	227	162	193	135	159
	16	280	353	251	316	228	286	180	209	154	177	127	145
	17	271	337	243	301	221	273	170	191	145	161	120	131
	18	262	320	235	286	213	259	160	172	136	144	112	117
	19	253	303	226	271	205	245	149	154	126	130	103	105
	20	243	285	217	255	197	230	138	139	116	117	95	95
	22	222	248	199	221	180	198	115	115	97	97	79	79
	24	201	209	179	186	161	167	97	97	81	81	66	66
	26	177	178	158	159	142	143	82	82	69	69	56	56
	28	154	154	137	137	123	123	71	71	60	60	48	48
	30	134	134	119	119	107	107	62	62	52	52	42	42
	32	118	118	105	105	94	94	54	54	46	46	37	37
	33	111	111	99	99	88	88	51	51	43	43		
	34	104	104	93	93	83	83						
	36	93	93	83	83	74	74						
Properties													
U		2.55	2.55	2.56	2.56	2.57	2.57	3.25	3.25	3.28	3.28	3.35	3.35
P_{wo} (kips)		99	148	83	116	73	101	79	109	64	88	55	77
P_{wi} (kips/in.)		15	21	13	19	12	17	13	18	11	16	10	15
P_{wb} (kips)		239	282	163	193	127	149	138	163	101	119	79	93
P_{fb} (kips)		104	145	85	118	71	98	86	120	63	88	43	59
L_c (ft)		10.6	9.0	10.6	9.0	10.6	9.0	8.5	7.2	8.4	7.2	8.4	7.1
L_u (ft)		31.1	22.4	28.2	20.3	26.0	18.7	22.8	16.4	19.8	14.2	16.5	11.9
A (in.^{2})		17.6		15.8		14.4		13.3		11.5		9.71	
I_x (in.^{4})		341		303		272		248		209		170	
I_y (in.^{4})		116		103		93.4		53.4		45.0		36.6	
r_y (in.)		2.57		2.56		2.54		2.01		1.98		1.94	
Ratio r_x/r_y		1.71		1.71		1.71		2.15		2.16		2.16	
B_x } Bending		0.264		0.263		0.264		0.271		0.273		0.277	
B_y } factors		0.765		0.767		0.770		1.000		1.018		1.055	
$a_x/10^6$		50.5		45.0		40.6		37.2		31.2		25.4	
$a_y/10^6$		17.3		15.4		13.8		8.0		6.7		5.4	
F'_{ex} $(K_xL_x)^2/10^2$ (kips)		200		198		196		194		189		182	
F'_{ey} $(K_yL_y)^2/10^2$ (kips)		68.5		68.0		66.9		41.9		40.7		39.0	

Note: Heavy line indicates Kl/r of 200.

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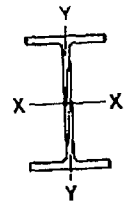
F_y = 36 ksi

F_y = 50 ksi

COLUMNS
W shapes
Allowable axial loads in kips



Designation		W8											
Wt./ft		67		58		48		40		35		31	
F_y		36	50	36	50	36	50	36	50	36	50	36	50
Effective length in ft KL with respect to least radius of gyration r_y	0	426	591	369	513	305	423	253	351	222	309	197	274
	6	387	525	336	455	276	378	229	310	201	272	178	241
	7	379	510	328	442	270	363	223	300	197	264	174	234
	8	370	494	320	428	263	352	218	290	191	255	170	226
	9	360	477	312	413	256	339	212	279	186	246	165	217
	10	350	459	303	397	249	326	205	268	180	236	160	208
	11	339	440	293	380	241	312	199	256	174	225	154	199
	12	328	420	283	363	233	297	192	244	168	214	149	189
	13	316	399	273	344	224	282	184	231	162	202	143	179
	14	304	378	263	325	215	266	177	217	155	190	137	168
	15	292	355	251	305	206	249	169	203	146	177	131	156
	16	279	331	240	284	196	232	160	188	141	164	124	145
	17	265	307	228	263	186	214	152	172	133	150	117	132
	18	251	281	216	240	176	195	143	158	125	136	110	119
	19	236	254	203	217	165	175	134	140	117	122	103	107
	20	221	230	190	196	154	156	124	126	109	110	95	97
	22	190	190	162	162	131	131	104	104	91	91	80	80
	24	159	159	136	136	110	110	88	88	76	76	67	67
	26	136	136	116	116	94	94	75	75	65	65	57	57
	28	117	117	100	100	81	81	64	64	56	56	49	49
	30	102	102	87	87	70	70	56	56	49	49	43	43
	32	90	90	76	76	62	62	49	49	43	43	38	38
	33	84	84	72	72	58	58	46	46	40	40	35	35
	34	79	79	68	68	55	55	44	44				
	35	75	75	64	64								
Properties													
U		2.48	2.48	2.50	2.50	2.54	2.54	2.56	2.56	2.59	2.59	2.61	2.61
P_{wo} (kips)		147	205	120	167	86	119	69	96	56	78	48	67
P_{wi} (kips/in.)		21	29	18	26	14	20	13	18	11	16	10	14
P_{wb} (kips)		744	873	533	628	257	303	187	221	120	141	93	110
P_{fb} (kips)		197	273	148	205	106	147	71	98	55	77	43	59
L_c (ft)		8.7	7.4	8.7	7.4	8.6	7.3	8.5	7.2	8.5	7.2	8.4	7.2
L_u (ft)		39.9	28.7	35.3	25.4	30.3	21.8	25.3	18.2	22.6	16.3	20.1	14.5
A (in.^{2})		19.7		17.1		14.1		11.7		10.3		9.13	
I_x (in.^{4})		272		228		184		146		127		110	
I_y (in.^{4})		88.6		75.1		60.9		49.1		42.6		37.1	
r_y (in.)		2.12		2.10		2.08		2.04		2.03		2.02	
Ratio r_x/r_y		1.75		1.74		1.74		1.73		1.73		1.72	
B_x } Bending		0.326		0.329		0.326		0.330		0.330		0.332	
B_y } factors		0.921		0.934		0.940		0.959		0.972		0.985	
$a_x/10^6$		40.6		33.9		27.4		21.7		18.9		16.4	
$a_y/10^6$		13.2		11.2		9.1		7.3		6.3		5.6	
F'_{ex} $(K_xL_x)^2/10^2$ (kips)		144		138		135		129		128		125	
F'_{ey} $(K_yL_y)^2/10^2$ (kips)		46.6		45.7		44.9		43.2		42.7		42.3	

Note: Heavy line indicates Kl/r of 200.

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F_y = 36 ksi

BEAMS
W Shapes
Allowable uniform loads in kips for beams laterally supported

For beams laterally unsupported, see page 2-146

W 30

Designation	W 30						Deflection in.
Wt./ft	292	261	235	211	191	173	
Flange Width	15¼	15⅛	15	15⅛	15	15	
L_c	16.1	16.0	15.9	15.9	15.9	15.8	
L_u	41.0	36.7	33.3	29.7	26.9	24.2	
Span in Feet (14)						574	.16
15	940	847	748	691	627	569	.18
16	919	819	739	656	592	534	.21
17	865	771	695	618	557	502	.24
18	817	728	656	583	526	474	.27
19	774	689	622	553	499	449	.30
20	735	655	591	525	474	427	.33
21	700	624	563	500	451	407	.36
22	668	595	537	477	431	388	.40
23	639	570	514	457	412	371	.43
24	612	546	492	438	395	356	.47
25	588	524	473	420	379	342	.51
26	565	504	454	404	364	328	.55
27	544	485	438	389	351	316	.60
28	525	468	422	375	338	305	.64
29	507	452	407	362	327	294	.69
30	490	437	394	350	316	285	.74
31	474	423	381	339	306	275	.79
32	459	409	369	328	296	267	.84
34	432	385	348	309	279	251	.95
36	408	364	328	292	263	237	1.06
38	387	345	311	276	249	225	1.18
40	367	327	295	263	237	213	1.31
42	350	312	281	250	226	203	1.45
44	334	298	269	239	215	194	1.59
46	320	285	257	228	206	186	1.73
48	306	273	246	219	197	178	1.89
50	294	262	236	210	189	171	2.05
52	283	252	227	202	182	164	2.22
54	272	243	219	194	175	158	2.39
56	262	234	211	188	169	152	2.57
58	253	226	204	181	163	147	2.76
60	245	218	197	175	158	142	2.95
62	237	211	191	169	153	138	3.15
64	230	205	185	164	148	133	3.36
66	223	198	179	159	144	129	3.57
68	216	193	174	154	139	126	3.79
70	210	187	169	150	135	122	4.01
72	204	182	164	146	132	119	4.25

Properties and Reaction Values

S_x in.³	928	827	746	663	598	539	For explanation of deflection, see page 2-32
V kips	470	423	374	345	314	287	
R_1 kips	159	135	111	97.8	81.7	73.0	
R_2 kips/in.	24.2	22.1	19.7	18.4	16.9	15.6	
R_3 kips	286	235	189	160	133	112	
R_4 kips/in.	11.0	10.1	7.45	7.00	6.02	5.30	
R kips	244	212	180	162	141	128	

Load above heavy line is limited by maximum allowable web shear.

W 30

F_y = 36 ks

BEAMS
W Shapes
Allowable uniform loads in kips for beams laterally supported

For beams laterally unsupported, see page 2-146

Designation	W 30							Deflection in.
Wt./ft	148	132	124	116	108	99	90	
Flange Width	10½	10½	10½	10½	10½	10½	10⅜	
L_c	11.1	11.1	11.1	11.1	11.1	10.9	10.0	
L_u	18.7	16.1	15.0	13.8	12.3	11.4	11.4	
Span in Feet (9)						444	400	.07
10				488	468	426	388	.08
11		537	508	474	431	387	353	.10
12	574	502	469	434	395	355	323	.12
13	531	463	433	401	364	328	299	.14
14	493	430	402	372	338	304	277	.16
15	460	401	375	347	316	284	259	.18
16	432	376	351	326	296	266	243	.21
17	406	354	331	307	279	251	228	.24
18	384	334	312	290	263	237	216	.27
19	363	317	296	274	249	224	204	.30
20	345	301	281	261	237	213	194	.33
21	329	287	268	248	226	203	185	.36
22	314	274	256	237	215	194	176	.40
23	300	262	244	227	206	185	169	.43
24	288	251	234	217	197	178	162	.47
25	276	241	225	208	189	170	155	.51
26	266	232	216	200	182	164	149	.55
27	256	223	208	193	175	158	144	.60
28	247	215	201	186	169	152	139	.64
29	238	208	194	180	163	147	134	.69
30	230	201	187	174	158	142	129	.74
31	223	194	181	168	153	137	125	.79
32	216	188	176	163	148	133	121	.84
34	203	177	165	153	139	125	114	.95
36	192	167	156	145	132	118	108	1.06
38	182	158	148	137	125	112	102	1.18
40	173	150	141	130	118	107	97	1.31
42	164	143	134	124	113	101	92	1.45
44	157	137	128	118	108	97	88	1.59
46	150	131	122	113	103	93	84	1.73
48	144	125	117	109	99	89	81	1.89
50	138	120	112	104	95	85	78	2.05
52	133	116	108	100	91	82	75	2.22
54	128	111	104	97	88	79	72	2.39
56	123	107	100	93	85	76	69	2.57
58	119	104	97	90	82	73	67	2.76
60	115	100	94	87	79	71	65	2.95
62	111	97	91	84	76	69	63	3.15
64	108	94	88	81	74	67	61	3.36
66	105	91	85	79	72	65	59	3.57
68	102	89	83	77	70	63	57	3.79
70	99	86	80	74	68	61	55	4.01
72	96	84	78	72	66	59	54	4.25

Properties and Reaction Values

S_x in.³	436	390	355	329	299	269	245	For explanation of deflection see page 2-3
V kips	287	268	254	244	234	222	200	
R_1 kips	77.2	63.9	58.6	54.5	50.6	44.4	36.6	
R_2 kips/in.	15.4	14.6	13.9	13.4	12.9	12.4	11.2	
R_3 kips	116	98.4	88.0	79.9	71.6	62.6	51.3	
R_4 kips/in.	4.64	4.70	4.37	4.33	4.37	4.33	3.53	
R kips	131	115	103	95	87	78	64	

Load above heavy line is limited by maximum allowable web shear.

Solution:

Enter Table III under Weld A and note that a value that satisfies the reaction is 222 kips. This requires 5/16-in. welds and 23½-in. long angles. Use ⅜-in. thick angles to meet the weld requirement stipulated in AISC ASD Specification Sect. J2.2b. The 0.625-in. web thickness is less than the minimum required 0.64 in., so the reduction in capacity is (0.625/0.64) × 222 kips = 217 kips.

Note, in Table II-A, the angle length provides for 8 rows of ⅞-in. dia. ASTM A325-N bolts with a capacity of 202 kips. The ⅜-in. required angle thickness is the same as the ⅜-in. angle thickness required due to Weld A.

Detail data: Two L 4 × 3½ × ⅜ × 1′−11½
F_y = 36 ksi
Sixteen ⅞-in. dia. ASTM A325-N bolts (threads included in shear plane)
5/16-in fillet weld, E70XX

EXAMPLE 7

Given:

Beam:	W 16 × 26 (not coped); t_w = 0.25 in.
	F_y = 36 ksi; F_v = 14.5 ksi
Reaction:	46 kips
Bolts:	⅞-in. dia. ASTM A307
Welds:	E70XX

Solution:

See Table II-A and note 4 rows of bolts with 11½-in. long angles are compatible with a 16-in. deep section. Capacity of the ⅞-in. dia. ASTM A307 bolts with ¼-in. thick angles is 48.1 kips.

Note in Table III that 72.7 kips capacity is designated for 3/16-in. Weld A and 11½-in. long angles. The 0.25-in. web thickness is less than the minimum 0.38 in. listed. The reduced capacity is (0.25/0.38) × 72.7 kips = 47.8 kips. The ¼-in. angle thickness required for bolts is satisfactory for the 3/16-in. weld.

Detail data: Two L 4 × 3½ × ¼ × 0′−11½
F_y = 36 ksi
Eight ⅞-in. dia. ASTM A307 bolts
3/16-in fillet weld, E70XX

CASE II

EXAMPLE 8

Given:

Beam:	W 36 × 150 (coped); t_w = 0.625 in.
	F_y = 36 ksi; F_v = 14.5 ksi
Reaction:	155 kips
Bolts:	⅞-in. dia., ASTM A490-N
Welds:	E70XX

Solution:

Enter Table III under Weld B and note the value most nearly satisfying the rea is 156 kips. This requires 5/16-in. Weld B and 20½-in. long, ⅜-in thick angles. T II-A shows a bolt capacity for a 7 fastener connection of 236 kips, which is more the 155 kips required. Therefore, a 20½-in. long angle is selected from Table

Check bearing on beam web = 155/7 = 22.1 kips per bolt in double shear bea From Table I-E, the allowable bearing 60.9 kips for 1-in. thickness and 3-in. spa 60.9 × 0.625 in. = 38.1 kips per bolt allowable. Assuming the beam is coped, using 1¼-in. edge distance for the top bolt, the single shear bearing capacity o ⅜-in. connection angle (Table I-F) is,

⅜ [(1 bolt × 36.3) + (6 bolts × 60.9 kips)] = 151 kips **o.k.**

Check block shear:

$$(C_1 + C_2)\, F_u\, t = (1.25 + 3.34)\, 58 \times 0.625 = 166.4 \text{ kips} \quad \textbf{o.k.}$$

Detail data: Two L 4 × 3½ × ⅜ × 1′−8½
F_y = 36 ksi
Seven ⅞-in. dia. ASTM A490-N bolts
5/16-in. fillet weld, E70XX

EXAMPLE 9

Given:

Beam:	W 16 × 31 (not coped); t_w = 0.275 in.
	F_y = 50 ksi; F_v = 20 ksi
Reaction:	39 kips
Bolts:	¾-in. dia., ASTM A325-N
Welds:	E70XX

Solution:

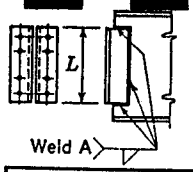
Enter Table III under Weld B and note the value most nearly satisfying the reac is 40.3 kips. This requires 5/16-in. Weld B and 8½-in. long, ⅜-in. thick angles.

Enter Table II-A for 3 rows of fasteners and note that the angle length is compat with beam size. Capacity of three ¾-in. dia. ASTM A325-N bolts is 55.7 kips.

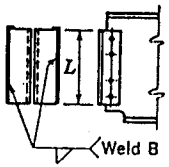
Check bearing on beam web = 39/3 = 13 kips per bolt in double shear. From T I-E, the allowable bearing is 58.5 kips for 1-in. thickness at 3-in. spacing; 58. 0.275 = 16.1 kips per bolt allowable. Assuming 1¼-in. edge distance for the bolt, the single shear bearing capacity of ⅜-in. connecting angle (Table I-F) is,

3/8(3 × 52.2) = 58.7 kips **o.k.**

Detail data: Two L 4 × 3½ × ⅜ × 0′−8½
F_y = 36 ksi
Three ¾-in. dia. ASTM A325-N bolts
5/16-in. fillet weld, E70XX



FRAMED BEAM CONNECTIONS
Welded—E70XX electrodes for combination with Table II connections
TABLE III Allowable loads in kips



Weld A		Weld B		Angle Length L In.	[a]Minimum Web Thickness for Welds A		Maximum Number of Fasteners in One Vertical Row (Table II)
Capacity, Kips	[b]Size, In.	[c]Capacity, Kips	Size, In.		F_y = 36 ksi F_v = 14.5 ksi	F_y = 50 ksi F_v = 20 ksi	
266	5/16	296	3/8	29½	.64	.46	10
213	¼	247	5/16	29½	.51	.37	
160	3/16	197	¼	29½	.38	.28	
245	5/16	261	3/8	26½	.64	.46	9
196	¼	217	5/16	26½	.51	.37	
147	3/16	173	¼	26½	.38	.28	
222	5/16	223	3/8	23½	.64	.46	8
178	¼	186	5/16	23½	.51	.37	
133	3/16	149	¼	23½	.38	.28	
198	5/16	187	3/8	20½	.64	.46	7
158	¼	156	5/16	20½	.51	.37	
119	3/16	125	¼	20½	.38	.28	
174	5/16	152	3/8	17½	.64	.46	6
139	¼	126	5/16	17½	.51	.37	
104	3/16	101	¼	17½	.38	.28	
148	5/16	115	3/8	14½	.64	.46	5
118	¼	95.7	5/16	14½	.51	.37	
88.7	3/16	76.6	¼	14½	.38	.28	
121	5/16	80.1	3/8	11½	.64	.46	4
97.0	¼	66.9	5/16	11½	.51	.37	
72.7	3/16	53.4	¼	11½	.38	.28	
92.1	5/16	48.2	3/8	8½	.64	.46	3
73.7	¼	40.3	5/16	8½	.51	.37	
55.3	3/16	32.2	¼	8½	.38	.28	
61.8	5/16	21.9	3/8	5½	.64	.46	2
49.5	¼	18.3	5/16	5½	.51	.37	
37.1	3/16	14.6	¼	5½	.38	.28	

[a]When the beam web thickness is less than the minimum, multiply the connection capacity furnished by Weld A by the ratio of the actual web thickness to the tabulated minimum thickness. Thus, if 5/16 in. Weld A, with a connection capacity of 148 kips and a 14½-in. long angle, is considered for a beam of web thickness of 0.375 in. with F_y = 36 ksi, the connection capacity must be multiplied by 0.375/0.64, giving 86.7 kips.

[b]Should the thickness of material to which connection angles are welded exceed the limits set by AISC ASD Specification Sects. J2.1b or J2.2b for weld sizes specified, increase the weld size as required, but not to exceed the angle thickness.

[c]When welds are used on outstanding legs, connection capacity may be limited by the shear capacity of the supporting member as stipulated by AISC ASD Specification Sect. F4. See Ex. 13 and 14 for Table IV.

Note 1: Connection angles: Two L 4 × 3½ × thickness × L; F_y = 36 ksi. See discussion preceding examples for Table III for limiting values of thickness and optional width of legs.

Note 2: Capacities shown in this table apply only when the material welded is F_y = 36 ksi or F_y = 50 ksi steel.

FRAMED BEAM CONNECTIONS
Welded–E70XX electrodes
TABLE IV

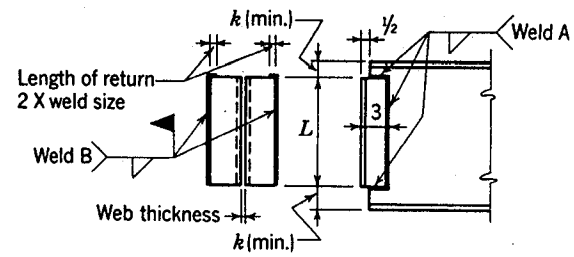


Table IV lists capacities and details for angle connections welded to both the be web and the supporting member.

Holes for erection bolts may be placed as required in legs that are to be fi welded (optional).

Allowable capacity for Weld A utilizes instantaneous center solutions based the same criteria developed for Tables XIX to XXVI, Eccentric Loads on W Groups. However, Weld B capacity is computed using traditional vector analy technique.

EXAMPLE 10

Given:

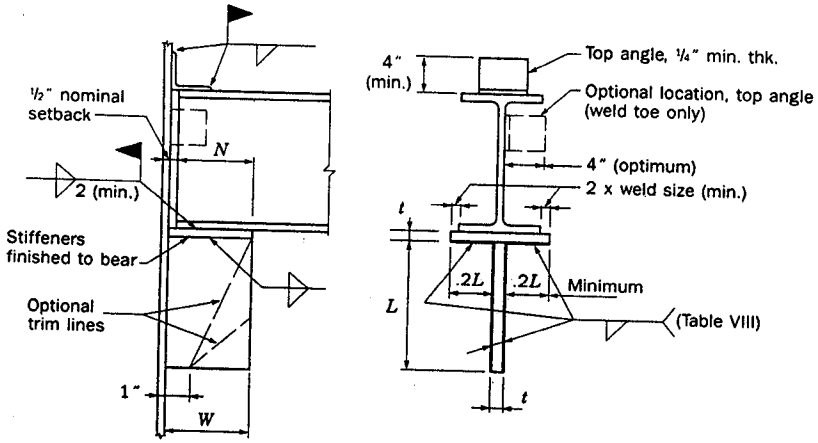
Beam: W 36 × 150; t_w = 0.625 in.; T = 32⅛ in.
F_y = 36 ksi; F_v = 14.5 ksi
Weld: E70XX
Reaction: 170 kips

Solution:

Enter Table IV and select a Weld A capacity of 174 kips (weld size = ¼ in.). W B has a capacity of 191 kips and is satisfactory. The angle length (24 in.) is less th T for the W 36 × 150 and is satisfactory. The beam web thickness (0.625 in.) excee the minimum web thickness (0.51 in.), so no reduction in Weld A capacity is quired.

Detail data: Two L 4 × 3 × ⅜ × 2'-0 ; F_y = 36 ksi
Weld A = ¼ in., E70XX
Weld B = 5/16 in., E70XX

STIFFENED SEATED BEAM CONNECTIONS
Welded–E70XX Electrodes
TABLE VIII



Seated connections should be used only when the beam is supported by a top angle placed as shown above, or in the optional location as indicated.

Allowable loads in Table VIII are based on the use of E70XX electrodes. The table may be used for other electrodes, provided the tabular values are adjusted for the electrodes used (e.g., for E60XX electrodes, multiply tabular values by 18/21, or 0.86, etc.) and the welds and base metal meet the provisions of AISC ASD Specification Sect. J2.4.

Allowable weld capacities in Table VIII are computed using traditional vector analysis.

Based on F_y = 36 ksi bracket material, minimum stiffener plate thickness, t, for supported beams with unstiffened webs should not be less than the supported beam web thickness for F_y = 36 ksi beams, and not less than 1.4 times the beam web thickness for beams with F_y = 50 ksi. Based on bracket material of F_y = 50 ksi or greater, the minimum stiffener plate thickness, t, for supported beams with unstiffened webs should be the beam web thickness multiplied by the ratio of F_y of the beam to F_y of the bracket (e.g., F_y (beam) = 65 ksi; F_y (bracket) = 50 ksi; $t = t_w$ (beam) × 65/50, minimum). The minimum stiffener plate thickness, t, should be at least two times the required E70XX weld size when F_y of the bracket is 36 ksi, and should be at least 1.5 times the required E70XX weld size when F_y of the bracket is 50 ksi.

Thickness t of the horizontal seat plate, or tee flange, should not be less than ⅜ in.

If seat and stiffener are separate plates, finish stiffener to bear against seat. Welds connecting the two plates should have a strength equal to, or greater than, the horizontal welds to the support under the seat plate.

Welds attaching beam to seat may be replaced by bolts.

ASTM A307 bolts may be used in seated connections, if the stipulations of AISC ASD Specification Sect. J1.12 are observed.

For stiffener seats in line on opposite sides of a column web of F_y = 36 ksi material, select E70XX weld size no greater than 0.50 of column web thickness. For column web of F_y = 50 ksi, select E70XX weld size no greater than 0.67 of column web thickness. To investigate the resistance of the column web for stiffened seats on one side see Hopper, Bruce E., Gordon B. Batson and Heino Ainso "Bracket Loaded Webs with Low Slenderness Ratios," AISC *Engineering Journal*, First Quarter, 1985, pg. 11-18 and Hoptay, Joseph M. and Heino Ainso, "An Experimental look at Bracket–loaded Webs," AISC *Engineering Journal*, Vol. 18, No. 1, First Quarter, 1981, pg. 1-7.

Should combinations of material thickness and weld size selected from Table VIII exceed the limits set by AISC ASD Specification Sects. J2.2 and J2.4, increase the weld size or material thickness as required.

In addition to the welds shown, temporary erection bolts may be used to attach beams to seats (optional).

To permit selection of the most economical connection, the reaction values should be given on the contract drawings. If the reaction values are not given, the connections must be selected to support the beam end reaction calculated from the Allowable Uniform Load Tables for the given shape, span, and steel specification of the beam in question. The effect of concentrated loads near an end connection *must* also be considered.

EXAMPLE 19

Given:

Beam: W 30 × 116 (flange = 10.495 in. × 0.85 in.; web = 0.565 in.)
ASTM A36 steel (F_y = 36 ksi)
Welds: E70XX
Reaction: 100 kips

Design a two-plate welded stiffener seat using ASTM A36 steel.

Solution:

From the F_y = 36 ksi, Allowable Uniform Load Table for W30 × 116, note that R_1, = 54.5 kips, R_2 = 13.4 kips/in., R_3 = 79.9 kips, R_4 = 4.33 kips/in.

For yielding N, req'd = $(R - R_1)/R_2$
= (100 − 54.5)/13.4 = 3.40 in.

For buckling N, req'd = $(R - R_3)/R_4$
= (100 − 79.9)/4.33 = 4.64 in.

Stiffener width = 4.64 + 0.5 (setback) = 5.14 in.

Use W = 6 in.

Enter Table VIII with W = 6 in. and a reaction of 100 kips; select a 5/16-in. weld with L = 15 in., which has a capacity of 103 kips. From this, the minimum length of weld between seat plate and support is 2 × 0.2L = 6 in. This also establishes the minimum weld between the seat plate and the stiffener as 6 in. total, or 3 in. on each side of stiffener.

Stiffener plate thickness t to develop welds is 2 × 5/16 = ⅝ in., or 0.625 in. This is greater than the beam web thickness of 0.565 in.; thus, the stiffener plate thickness need not be increased.

Use: ⅝-in. plate for the stiffener and ⅜-in. plate for seat.

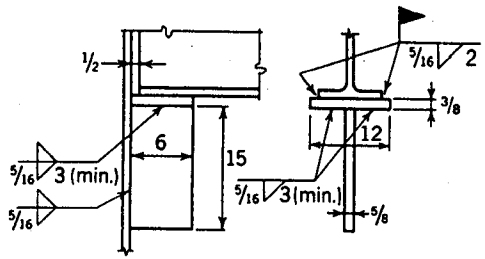
Welds attaching the beam flange to the seat must be 5/16 in. to conform to AISC ASD Specification Sect. J2.2, due to the 0.85-in. flange thickness of the W 30 × 116 beam.

Seat plate width to permit field welding of beam to seat = flange width + (4 × weld size) = 10.5 + (4 × 5/16) = 11.75 in.

Use: 3/8 × 6 × 1′-0 plate

This is adequate for the required minimum weld length.

Detail data:



Use: L 4 × 4 × 3/8 × 0′-4 top angle (F_y = 36 ksi) with 5/16-in. welds along toes of angle only.

EXAMPLE 20

Given:

Beam: W 21 × 68 (flange = 8.27 in. × 0.685 in.; web = 0.43 in.)
ASTM A572, Grade 50 steel (F_y = 50 ksi)
Welds: E70XX electrodes
Reaction: 83 kips

Design a two-plate welded stiffener seat using ASTM A36 steel.

Solution:

From the F_y = 50 ksi, Allowable Uniform Load Table for W21 × 68, note R_1 = 51.0 kips, R_2 = 14.2 kips/in., R_3 = 56.1 kips, R_4 = 3.96 kips/in.

For yielding N, req'd = $(R - R_1)/R_2$

= (83 − 51.0)/14.2 = 2.25 in.

For buckling N, req'd = $(R - R_3)/R_4$

= (83 − 56.1)/3.96 = 6.79 in.

Stiffener width = 6.79 + 0.5 (setback) = 7.29 in.

Use W = 8 in.

Enter Table VIII with W = 8 in. and a reaction of 83 kips; satisfying these requirements are a 5/16-in. weld, L = 15 in. (84.4 kips), or a 3/8-in. weld, L = 14 in. (89.8 kips), or an even larger weld size. Generally, the 5/16-in. weld is the better selection,

as this can be made in one pass using manual welding. Select 5/16-in. weld. From thi the minimum length of 5/16-in. weld between seat plate and support is 2 × 0.2 = 6 in.

Use: 6 in. weld length. This also establishes the minimum weld between the se plate and the stiffener as 6 in. total, or 3 in. on each side.

Stiffener plate thickness t to develop welds is 2 × 5/16 = 5/8 in., or 0.625 in. Tl minimum thickness t for a bracket of F_y = 36 ksi with a beam of F_y = 50 ksi 1.4 times the beam web thickness = 1.4 × 0.43 = 0.602 in.

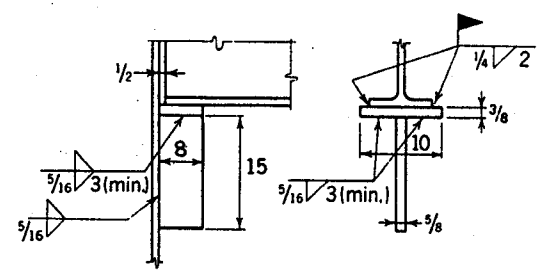
Use: 5/8-in. plate for the stiffener and 3/8-in. plate for the seat.

Welds attaching the beam flange to the seat can be 1/4 in. for a flange of 0.685 in., p AISC ASD Specification Sect. J2.2.

Seat plate width, to permit field welding of beam to seat = flange width + (4 × we size) = 8.27 + (4 × 1/4) = 9.27 in.

Use: 3/8 × 8 × 0′-10 plate

This is adequate for the required minimum weld length.



Detail data:

Use: L 4 × 4 × 3/8 × 0′-4 top angle (F_y = 36 ksi) with 5/16-in. welds along toes of angl only.

the remaining beam depth. A similar discussion can be made where holes are cut in beam webs for duct work or other items.

4. Theoretically, very heavily loaded short beams can have excessive shears but practically this does not occur too often unless it is like case 1.

5. Shear may very well be a problem even for ordinary loadings when very thin webs are used as in plate girders or in light gage cold formed steel members.

Should calculated shearing stresses exceed allowable values ($F_v = 0.40F_y$ in the AISC) shear plates can be connected to the webs in the zones of excessive stress.

7-2. WEB CRIPPLING

Beams that support heavy concentrated loads sometimes fail by web crippling or crushing unless the web is stiffened near the loads. Web crippling occurs due to the stress concentrations at the junction of the flange and the web, where the beam is trying to transfer compression in the relatively wide flange to the narrow web. Failure will occur when the metal begins to fail at the toe of the fillet in bearing and the flange and web have a tendency to fold over each other.

Most specifications assume the reaction or load spreads out from its place of application along a 45° plane (see Fig. 7-3). The toe of the fillet is the most dangerous location for failure because the resisting area has its smallest value there. The AISC does not permit the compression at this point in beams without web stiffeners to exceed $0.75F_y$. Should the allowable value be exceeded it is necessary to use stiffeners, or the bearing of the load or reaction must be spread over a greater length. The expressions for the length of bearing required are shown in Fig. 7-3. The top flanges of a beam must have lateral support at the reaction points or the web crippling strength is greatly reduced.

The minimum bearing length at the end of a certain beam is calcu-

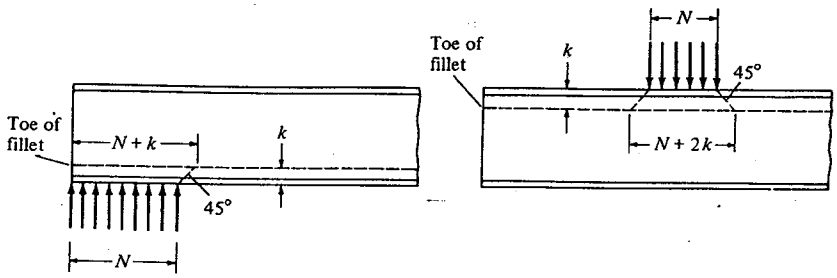


Figure 7-3 Left: Reaction (AISC Formula 1.10-9). N=length of bearing, which cannot be less than k; k=distance from toe of fillet to outside of flange; t=thickness of web; R=reaction; $R/t(N+k)$ may not exceed $0.75F_y$. Right: Concentrated load (AISC Formula 1.10-8). R=concentrated load; $R/t(N+2k)$ may not exceed $0.75F_y$.

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failures have occurred, p
bearing. Another fact to k
large enough to keep the b
of the supporting materia
this chapter.

Example 7-1

A W33×130 consisting of
span of Fig. 7-4. (a) Dete
the reactions. (b) The 50-
over a width of 6 in. Is thi

SOLUTION

Properties of a W33×

W

(a) Minimum bearing

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(b) Web-crippling str

t(N

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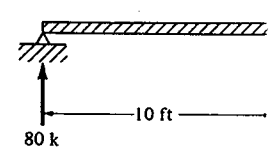

Pg-9-96

STIFFENED SEATED BEAM CONNECTIONS

Welded—E70XX electrodes

TABLE VIII Allowable loads in kips

L In.	Width of Seat W, In.														
	4					5						6			
	Weld Size, In.					Weld Size, In.						Weld Size, In.			
	3/16	1/4	5/16	3/8	7/16	3/16	1/4	5/16	3/8	7/16	1/2	5/16	3/8	7/16	1/2
6	17.0	22.7	28.4	34.0	39.7	14.1	18.8	23.5	28.1	32.8	37.5	19.9	23.9	27.9	31.9
7	22.4	29.9	37.4	44.9	52.4	18.7	25.0	31.2	37.5	43.7	50.0	26.7	32.0	37.3	42.7
8	28.3	37.8	47.2	56.7	66.1	23.9	31.9	39.8	47.8	55.8	63.7	34.3	41.1	48.0	54.8
9	34.6	46.1	57.6	69.2	80.7	29.5	39.3	49.1	59.0	68.8	78.6	42.5	51.1	59.6	68.1
10	41.1	54.9	68.6	82.3	96.0	35.4	47.2	59.0	70.8	82.6	94.4	51.4	61.7	72.0	82.3
11	47.9	63.9	79.8	95.8	112	41.6	55.5	69.4	83.3	97.1	111	60.9	73.1	85.2	97.4
12	54.8	73.1	91.4	110	128	48.1	64.1	80.2	96.2	112	128	70.8	85.0	99.2	113
13	61.9	82.5	103	124	144	54.8	73.0	91.3	110	128	146	81.1	97.4	114	130
14	69.0	92.0	115	138	161	61.6	82.1	103	123	144	164	91.9	110	129	147
15	76.2	101	127	152	178	68.5	91.4	114	137	160	183	103	123	144	165
16	83.5	111	139	167	195	75.6	100	126	151	176	202	115	138	160	183
17	90.7	121	151	181	212	82.7	110	138	165	193	221	126	151	176	201
18	98.0	131	163	196	229	89.9	119	150	180	210	240	137	164	192	219
19	105	140	175	211	246	97.1	129	162	194	227	259	149	179	208	238
20	112	150	188	225	263	104	139	174	209	243	278	161	193	225	257
21	119	160	200	240	280	111	148	189	223	260	298	173	207	242	276
22	127	169	212	254	296	118	158	198	238	277	317	185	221	258	295
23	134	179	224	269	313	126	168	210	252	294	336	197	236	275	315
24	141	189	236	283	330	133	177	222	267	311	356	209	250	292	334
25	148	198	248	297	347	140	187	234	281	328	375	221	265	309	353
26	155	208	260	312	364	148	197	247	296	345	394	233	279	326	373
27	163	217	272	326	380	155	206	259	310	362	414	245	294	343	392

Note: Loads shown are for E70XX electrodes. For E60XX electrodes, multiply tabular loads by 0.86, or enter table with 1.17 times the given reaction. For E80XX electrodes, multiply tabular loads by 1.14 or enter table with 0.875 times the given reaction.

STIFFENED SEATED BEAM CONNECTIONS

Welded—E70XX electrodes

TABLE VIII Allowable loads in kips

L In.	Width of Seat W, In.											
	7				8				9			
	Weld Size, In.				Weld Size, In.				Weld Size, In.			
	5/16	3/8	7/16	1/2	5/16	3/8	1/2	5/8	5/16	3/8	1/2	5/8
11	54.0	64.8	75.6	86.3	48.4	58.0	77.3	96.6	43.7	52.4	69.9	87.
12	63.1	75.7	88.4	101	56.7	68.1	90.7	113	51.4	61.7	82.2	103
13	72.7	87.2	102	117	65.5	78.7	105	131	59.6	71.5	95.3	119
14	82.6	99.1	116	132	74.8	89.8	120	149	68.2	81.8	109	136
15	92.9	112	130	149	84.4	101	135	169	77.2	92.6	123	154
16	104	124	145	166	94.4	113	151	189	86.5	104	138	173
17	114	137	160	183	105	126	167	209	96.2	115	154	192
18	126	151	176	201	115	138	184	230	106	127	170	212
19	137	164	192	219	126	151	202	252	117	140	186	233
20	148	178	208	237	137	165	219	274	127	152	203	254
21	160	192	224	256	148	178	237	296	138	165	220	276
22	172	206	240	274	159	192	255	319	149	178	238	297
23	183	220	257	293	171	205	274	342	160	192	256	320
24	195	234	274	312	183	219	292	365	171	205	274	342
25	207	249	290	331	195	233	311	389	182	219	292	365
26	219	263	307	351	206	248	330	412	194	233	310	388
27	231	278	324	370	218	262	349	436	206	247	329	411
28	243	292	341	389	230	276	368	460	217	261	348	435
29	256	307	358	409	242	291	387	484	229	275	367	458
30	268	321	375	428	254	305	406	508	241	289	386	482
31	280	336	392	447	266	319	426	532	253	303	405	506
32	292	350	409	467	278	334	445	556	265	318	424	530

Note: Loads shown are for E70XX electrodes. For E60XX electrodes, multiply tabular loads by 0.86, or enter table with 1.17 times the given reaction. For E80XX electrodes, multiply tabular loads by 1.14 or enter table with 0.875 times the given reaction.

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b. Limitations

The minimum effective throat thickness of a partial-penetration groove weld shall be as shown in Table J2.3. Minimum effective throat thickness is determined by the thicker of the two parts joined, except that the weld size need not exceed the thickness of the thinnest part joined. For this exception, particular care shall be taken to provide sufficient preheat for soundness of the weld.

TABLE J2.1
Effective Throat Thickness of Partial-penetration Groove Welds

Welding Process	Welding Position	Included Angle at Root of Groove	Effective Throat Thickness
Shielded metal arc Submerged arc Gas metal arc Flux-cored arc	All	J or U joint	Depth of chamfer
		Bevel or V joint ≥ 60°	
		Bevel or V joint < 60° but ≥ 45°	Depth of chamfer minus ⅛-in.

TABLE J2.2
Effective Throat Thickness of Flare Groove Welds

Type of Weld	Radius (R) of Bar or Bend	Effective Throat Thickness
Flare bevel groove	All	$\frac{5}{16}R$
Flare V-groove	All	$\frac{1}{2}R$[a]

[a]Use $\frac{5}{8}R$ for Gas Metal Arc Welding (except short circuiting transfer process) when $R \geq \frac{1}{2}$-in.

TABLE J2.3
Minimum Effective Throat Thickness of Partial-penetration Groove Welds

Material Thickness of Thicker Part Joined (in.)	Minimum Effective Throat Thickness[a] (in.)
To ¼ inclusive	⅛
Over ¼ to ½	3/16
Over ½ to ¾	¼
Over ¾ to 1½	5/16
Over 1½ to 2¼	⅜
Over 2¼ to 6	½
Over 6	⅝

[a]See Sect. J2.

[redacted]

a. Effective Area

The effective area of fillet welds shall be taken as the effective length times the effective throat thickness.

The effective length of fillet welds, except fillet welds in holes and slots, shall be the overall length of full-size fillets, including returns.

The effective throat thickness of a fillet weld shall be the shortest distance from the root of the joint to the face of the diagrammatic weld, except that for fillet welds made by the submerged arc process, the effective throat thickness shall be taken equal to the leg size for ⅜-in. and smaller fillet welds, and equal to the theoretical throat plus 0.11-in. for fillet welds larger than ⅜-in.

For fillet welds in holes and slots, the effective length shall be the length of the centerline of the weld along the center of the plane through the throat. In the case of overlapping fillets, the effective area shall not exceed the nominal cross-sectional area of the hole or slot in the plane of the faying surface.

b. Limitations

The *minimum size of fillet welds* shall be as shown in Table J2.4. Minimum weld size is dependent upon the thicker of the two parts joined, except that the weld size need not exceed the thickness of the thinner part. For this exception, particular care shall be taken to provide sufficient preheat for soundness of the weld. Weld sizes larger than the thinner part joined are permitted if required by calculated strength. In the as-welded condition, the distance between the edge of the base metal and the toe of the weld may be less than 1/16-in. provided the weld size is clearly verifiable.

The *maximum size of fillet welds* that is permitted along edges of connected parts shall be:

- Material less than ¼-in. thick, not greater than the thickness of the material.
- Material ¼-in. or more in thickness, not greater than the thickness of the material minus 1/16-in., unless the weld is especially designated on the drawings to be built out to obtain full-throat thickness.

TABLE J2.4
Minimum Size of Fillet Welds

Material Thickness of Thicker Part Joined (in.)	Minimum Size of Fillet Weld[a] (in.)
To ¼ inclusive	⅛
Over ¼ to ½	3/16
Over ½ to ¾	¼
Over ¾	5/16

[a]Leg dimension of fillet welds. Single-pass welds must be used.

CHAPTER H

COMBINED STRESSES

The strength of members subjected to combined stresses shall be determined according to the provisions of this chapter.

This chapter pertains to doubly and singly symmetrical members only. See Chapter E for determination of F_a and Chapter F for determination of F_{bx} and F_{by}.

H1. AXIAL COMPRESSION AND BENDING

Members subjected to both axial compression and bending stresses shall be proportioned to satisfy the following requirements:

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \leq 1.0 \qquad \text{(H1-1)}$$

$$\frac{f_a}{0.60F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \qquad \text{(H1-2)}$$

When $f_a/F_a \leq 0.15$, Equation (H1-3) is permitted in lieu of Equations (H1-1) and (H1-2):

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \qquad \text{(H1-3)}$$

In Equations (H1-1), (H1-2) and (H1-3), the subscripts x and y, combined with subscripts b, m and e, indicate the axis of bending about which a particular stress or design property applies, and

F_a = axial compressive stress that would be permitted if axial force alone existed, ksi

F_b = compressive bending stress that would be permitted if bending moment alone existed, ksi

$$F'_e = \frac{12\,\pi^2 E}{23(Kl_b/r_b)^2}$$

= Euler stress divided by a factor of safety, ksi (In the expression for F'_e, l_b is the actual unbraced length *in the plane of bending* and r_b is the corresponding radius of gyration. K is the effective length factor *in the plane of bending*.) As in the case of F_a, F_b and $0.60F_y$, F'_e may be increased ⅓ in accordance with Sect. A5.2.

f_a = computed axial stress, ksi

f_b = computed compressive bending stress at the point under consideration, ksi

C_m = Coefficient whose value shall be taken as follows:

a. For compression members in frames subject to joint translation (sidesway), $C_m = 0.85$.

b. For rotationally restrained compression members in frames braced against joint translation and not subject to transverse loading between their supports in the plane of bending,

$$C_m = 0.6 - 0.4\,(M_1/M_2)$$

where M_1/M_2 is the ratio of the smaller to larger moments at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature, negative when bent in single curvature.

c. For compression members in frames braced against joint translation in the plane of loading and subjected to transverse loading between their supports, the value of C_m may be determined by an analysis. However, in lieu of such analysis, the following values are permitted:

i. For members whose ends are restrained against rotation in the plane of bending ..... $C_m = 0.85$

ii. For members whose ends are unrestrained against rotation in the plane of bending ..... $C_m = 1.0$

H2. AXIAL TENSION AND BENDING

Members subject to both axial tension and bending stresses shall be proportioned at all points along their length to satisfy the following equation:

$$\frac{f_a}{F_t} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \qquad \text{(H2-1)}$$

where f_b is the computed bending tensile stress, f_a is the computed axial tensile stress, F_b is the allowable bending stress and F_t is the governing allowable tensile stress defined in Sect. D1.

However the computed bending compressive stress arising from an independent load source relative to the axial tension, taken above, shall not exceed the applicable value required in Chapter F.

Fa (SEE PG 3-16)

TABLE 8
Values of F'_e

For Use in Equation (H1-1), for Steel of Any Yield Stress

All grades of steel

$\frac{Kl_b}{r_b}$	F'_e (ksi)	$\frac{Kl_b}{r_b}$	F'_e (ksi)	$\frac{Kl_b}{r_b}$	F'_e (ksi)	$\frac{Kl_b}{r_b}$	F'_e (ksi)	$\frac{Kl_b}{r_b}$	F'_e (ksi)	$\frac{Kl_b}{r_b}$	F'_e (ksi)
21	338.62	51	57.41	81	22.76	111	12.12	141	7.51	171	5.11
22	308.54	52	55.23	82	22.21	112	11.90	142	7.41	172	5.05
23	282.29	53	53.16	83	21.68	113	11.69	143	7.30	173	4.99
24	259.26	54	51.21	84	21.16	114	11.49	144	7.20	174	4.93
25	238.93	55	49.37	85	20.67	115	11.29	145	7.10	175	4.88
26	220.90	56	47.62	86	20.19	116	11.10	146	7.01	176	4.82
27	204.84	57	45.96	87	19.73	117	10.91	147	6.91	177	4.77
28	190.47	58	44.39	88	19.28	118	10.72	148	6.82	178	4.71
29	177.56	59	42.90	89	18.85	119	10.55	149	6.73	179	4.66
30	165.92	60	41.48	90	18.44	120	10.37	150	6.64	180	4.61
31	155.39	61	40.13	91	18.03	121	10.20	151	6.55	181	4.56
32	145.83	62	38.85	92	17.64	122	10.03	152	6.46	182	4.51
33	137.13	63	37.62	93	17.27	123	9.87	153	6.38	183	4.46
34	129.18	64	36.46	94	16.90	124	9.71	154	6.30	184	4.41
35	121.90	65	35.34	95	16.55	125	9.56	155	6.22	185	4.36
36	115.22	66	34.28	96	16.20	126	9.41	156	6.14	186	4.32
37	109.08	67	33.27	97	15.87	127	9.26	157	6.06	187	4.27
38	103.42	68	32.29	98	15.55	128	9.11	158	5.98	188	4.23
39	98.18	69	31.37	99	15.24	129	8.97	159	5.91	189	4.18
40	93.33	70	30.48	100	14.93	130	8.84	160	5.83	190	4.14
41	88.83	71	29.62	101	14.64	131	8.70	161	5.76	191	4.09
42	84.65	72	28.81	102	14.35	132	8.57	162	5.69	192	4.05
43	80.76	73	28.02	103	14.08	133	8.44	163	5.62	193	4.01
44	77.13	74	27.27	104	13.81	134	8.32	164	5.55	194	3.97
45	73.74	75	26.55	105	13.54	135	8.19	165	5.49	195	3.93
46	70.57	76	25.85	106	13.29	136	8.07	166	5.42	196	3.89
47	67.60	77	25.19	107	13.04	137	7.96	167	5.35	197	3.85
48	64.81	78	24.54	108	12.80	138	7.84	168	5.29	198	3.81
49	62.20	79	23.93	109	12.57	139	7.73	169	5.23	199	3.77
50	59.73	80	23.33	110	12.34	140	7.62	170	5.17	200	3.73

Note: $F'_e = \frac{12\pi^2 E}{23(Kl_b/r_b)^2}$

Commentary

ON THE SPECIFICATION FOR STRUCTURAL STEEL BUILDINGS
ALLOWABLE STRESS DESIGN AND PLASTIC DESIGN
(June 1, 19

INTRODUCTION

This Commentary provides information on the basis and limitations of various pr sions of the Specification, so that designers, fabricators and erectors (users) make more efficient use of the Specification. The Commentary and Specificati termed as documents, do not attempt to anticipate and/or set forth all the questi or possible problems that may be encountered, or situations in which special con eration and engineering judgment should be exercised in using and applying the c uments. Such a recitation could not be made complete and would make the dc ments unduly lengthy and cumbersome.

Warning is given that AISC assumes the users of its documents are competen their fields of endeavor and are informed on current developments and findings lated to their fields.

COMBINED STRESS
SEE PG 5-55

Table C-36
Allowable Stress
For Compression Members of 36-ksi Specified Yield Stress Steel[a]

$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)
1	21.56	41	19.11	81	15.24	121	10.14	161	5.76
2	21.52	42	19.03	82	15.13	122	9.99	162	5.69
3	21.48	43	18.95	83	15.02	123	9.85	163	5.62
4	21.44	44	18.86	84	14.90	124	9.70	164	5.55
5	21.39	45	18.78	85	14.79	125	9.55	165	5.49
6	21.35	46	18.70	86	14.67	126	9.41	166	5.42
7	21.30	47	18.61	87	14.56	127	9.26	167	5.35
8	21.25	48	18.53	88	14.44	128	9.11	168	5.29
9	21.21	49	18.44	89	14.32	129	8.97	169	5.23
10	21.16	50	18.35	90	14.20	130	8.84	170	5.17
11	21.10	51	18.26	91	14.09	131	8.70	171	5.11
12	21.05	52	18.17	92	13.97	132	8.57	172	5.05
13	21.00	53	18.08	93	13.84	133	8.44	173	4.99
14	20.95	54	17.99	94	13.72	134	8.32	174	4.93
15	20.89	55	17.90	95	13.60	135	8.19	175	4.88
16	20.83	56	17.81	96	13.48	136	8.07	176	4.82
17	20.78	57	17.71	97	13.35	137	7.96	177	4.77
18	20.72	58	17.62	98	13.23	138	7.84	178	4.71
19	20.66	59	17.53	99	13.10	139	7.73	179	4.66
20	20.60	60	17.43	100	12.98	140	7.62	180	4.61
21	20.54	61	17.33	101	12.85	141	7.51	181	4.56
22	20.48	62	17.24	102	12.72	142	7.41	182	4.51
23	20.41	63	17.14	103	12.59	143	7.30	183	4.46
24	20.35	64	17.04	104	12.47	144	7.20	184	4.41
25	20.28	65	16.94	105	12.33	145	7.10	185	4.36
26	20.22	66	16.84	106	12.20	146	7.01	186	4.32
27	20.15	67	16.74	107	12.07	147	6.91	187	4.27
28	20.08	68	16.64	108	11.94	148	6.82	188	4.23
29	20.01	69	16.53	109	11.81	149	6.73	189	4.18
30	19.94	70	16.43	110	11.67	150	6.64	190	4.14
31	19.87	71	16.33	111	11.54	151	6.55	191	4.09
32	19.80	72	16.22	112	11.40	152	6.46	192	4.05
33	19.73	73	16.12	113	11.26	153	6.38	193	4.01
34	19.65	74	16.01	114	11.13	154	6.30	194	3.97
35	19.58	75	15.90	115	10.99	155	6.22	195	3.93
36	19.50	76	15.79	116	10.85	156	6.14	196	3.89
37	19.42	77	15.69	117	10.71	157	6.06	197	3.85
38	19.35	78	15.58	118	10.57	158	5.98	198	3.81
39	19.27	79	15.47	119	10.43	159	5.91	199	3.77
40	19.19	80	15.36	120	10.28	160	5.83	200	3.73

[a]When element width-to-thickness ratio exceeds noncompact section limits of Sect. B5.1, see Appendix B5.
Note: C_c = 126.1

Table C-50
Allowable Stress
For Compression Members of 50-ksi Specified Yield Stress Steel[a,b]

$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)
1	29.94	41	25.69	81	18.81	121	10.20	161	5.76
2	29.87	42	25.55	82	18.61	122	10.03	162	5.69
3	29.80	43	25.40	83	18.41	123	9.87	163	5.62
4	29.73	44	25.26	84	18.20	124	9.71	164	5.55
5	29.66	45	25.11	85	17.99	125	9.56	165	5.49
6	29.58	46	24.96	86	17.79	126	9.41	166	5.42
7	29.50	47	24.81	87	17.58	127	9.26	167	5.35
8	29.42	48	24.66	88	17.37	128	9.11	168	5.29
9	29.34	49	24.51	89	17.15	129	8.97	169	5.23
10	29.26	50	24.35	90	16.94	130	8.84	170	5.17
11	29.17	51	24.19	91	16.72	131	8.70	171	5.11
12	29.08	52	24.04	92	16.50	132	8.57	172	5.05
13	28.99	53	23.88	93	16.29	133	8.44	173	4.99
14	28.90	54	23.72	94	16.06	134	8.32	174	4.93
15	28.80	55	23.55	95	15.84	135	8.19	175	4.88
16	28.71	56	23.39	96	15.62	136	8.07	176	4.82
17	28.61	57	23.22	97	15.39	137	7.96	177	4.77
18	28.51	58	23.06	98	15.17	138	7.84	178	4.71
19	28.40	59	22.89	99	14.94	139	7.73	179	4.66
20	28.30	60	22.72	100	14.71	140	7.62	180	4.61
21	28.19	61	22.55	101	14.47	141	7.51	181	4.56
22	28.08	62	22.37	102	14.24	142	7.41	182	4.51
23	27.97	63	22.20	103	14.00	143	7.30	183	4.46
24	27.86	64	22.02	104	13.77	144	7.20	184	4.41
25	27.75	65	21.85	105	13.53	145	7.10	185	4.36
26	27.63	66	21.67	106	13.29	146	7.01	186	4.32
27	27.52	67	21.49	107	13.04	147	6.91	187	4.27
28	27.40	68	21.31	108	12.80	148	6.82	188	4.23
29	27.28	69	21.12	109	12.57	149	6.73	189	4.18
30	27.15	70	20.94	110	12.34	150	6.64	190	4.14
31	27.03	71	20.75	111	12.12	151	6.55	191	4.09
32	26.90	72	20.56	112	11.90	152	6.46	192	4.05
33	26.77	73	20.38	113	11.69	153	6.38	193	4.01
34	26.64	74	20.10	114	11.49	154	6.30	194	3.97
35	26.51	75	19.99	115	11.29	155	6.22	195	3.93
36	26.38	76	19.80	116	11.10	156	6.14	196	3.89
37	26.25	77	19.61	117	10.91	157	6.06	197	3.85
38	26.11	78	19.41	118	10.72	158	5.98	198	3.81
39	25.97	79	19.21	119	10.55	159	5.91	199	3.77
40	25.83	80	19.01	120	10.37	160	5.83	200	3.73

[a]When element width-to-thickness ratio exceeds noncompact section limits of Sect. B5.1, see Appendix B5.
[b]Values also applicable for steel of any yield stress ≥ 39 ksi.
Note: C_c = 107.0

F_y = 50 ksi

COMBINED STRESSES
CHAPTERS E AND H

H1-1 $$\frac{f_a}{F_a} + \frac{C_m f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_m f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} < 1.0$$

H1-2 $$\frac{f_a}{0.6 F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} < 1.0$$

$f_a / F_a < 0.15$

H1-3 $$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} < 1.0$$

F_a depends on Kl/r_x or Kl/r_y P. 3-16 & 3-17

F'_e depends on Kl_b/r_b axis of applied moment P. 5-122

$C_m = 0.85$ SIDESWAY, NO TRANSV. LD.

$C_m = 0.6 - 0.4 \frac{M_1}{M_2} > 0.4$ NO SIDESWAY

CHAPTER E

COLUMNS AND OTHER COMPRESSION MEMBERS

This section applies to prismatic members with compact and noncompact sections subject to axial compression through the centroidal axis. For members with slender elements, see Appendix B5.2. For members subject to combined axial compression and flexure, see Chap. H. For tapered members, see Appendix F7.

E1. EFFECTIVE LENGTH AND SLENDERNESS RATIO

The effective-length factor K shall be determined in accordance with Sect. C2.

In determining the slenderness ratio of an axially loaded compression member, the length shall be taken as its effective length Kl and r as the corresponding radius of gyration. For limiting slenderness ratios, see Sect. B7.

E2. ALLOWABLE STRESS

On the gross section of axially loaded compression members whose cross sections meet the provisions of Table B5.1, when Kl/r, the largest effective slenderness ratio of any unbraced segment is less than C_c, the allowable stress is:

$$F_a = \frac{\left[1 - \frac{(Kl/r)^2}{2C_c^2}\right] F_y}{\frac{5}{3} + \frac{3(Kl/r)}{8C_c} - \frac{(Kl/r)^3}{8C_c^3}} \qquad \text{(E2-1)}$$

where

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

On the gross section of axially loaded compression members, when Kl/r exceeds C_c, the allowable stress is:

$$F_a = \frac{12\pi^2 E}{23(Kl/r)^2} \qquad \text{(E2-2)}$$

E3. FLEXURAL-TORSIONAL BUCKLING

Singly symmetric and unsymmetric columns, such as angles or tee-shaped columns, and doubly symmetric columns such as cruciform or built-up columns with very thin walls, may require consideration of flexural-torsional and torsional buckling.

E4. BUILT-UP MEMBERS

All parts of built-up compression members and the transverse spacing of the lines of fasteners shall meet the requirements of Sect. B7.

For spacing and edge distance requirements for weathering steel members, se Sect. J3.10.

At the ends of built-up compression members bearing on base plates or mille surfaces, all components in contact with one another shall be connected by ri ets or bolts spaced longitudinally not more than 4 diameters apart for a di tance equal to 1½ times the maximum width of the member, or by continuo welds having a length not less than the maximum width of the member.

The longitudinal spacing for intermediate bolts, rivets or intermittent welds built-up members shall be adequate to provide for the transfer of calculate stress. The maximum longitudinal spacing of bolts, rivets or intermittent wel connecting two rolled shapes in contact shall not exceed 24 in. In addition, f painted members and unpainted members not subject to corrosion where t outside component consists of a plate, the maximum longitudinal spacing sh not exceed:

$127/\sqrt{F_y}$ times the thickness of the outside plate nor 12 in. when fastene are not staggered along adjacent gage lines.

$190/\sqrt{F_y}$ times the thickness of the outside plate nor 18 in. when fastene are staggered along adjacent gage lines.

Compression members composed of two or more rolled shapes separated b intermittent fillers shall be connected at these fillers at intervals such that t slenderness ratio Kl/r of either shape, between the fasteners, does not excee ¾ times the governing slenderness ratio of the built-up member. The least r dius of gyration r shall be used in computing the slenderness ratio of each co ponent part. At least two intermediate connectors shall be used along t length of the built-up member.

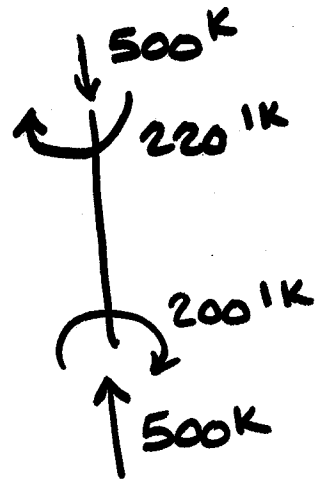
All connections, including those at the ends, shall be welded or shall utili high-strength bolts tightened to the requirements of Table J3.7.

Open sides of compression members built up from plates or shapes shall b provided with lacing having tie plates at each end and at intermediate points the lacing is interrupted. Tie plates shall be as near the ends as practicable. main members carrying calculated stress, the end tie plates shall have a leng of not less than the distance between the lines of fasteners or welds connectir them to the components of the member. Intermediate tie plates shall have length not less than ½ of this distance. The thickness of tie plates shall not b less than 1/50 of the distance between the lines of fasteners or welds connectir them to the components of the member. In bolted and riveted constructio the spacing in the direction of stress in tie plates shall not be more than 6 dia eters and the tie plates shall be connected to each component by at least 3 fas eners. In welded construction, the welding on each line connecting a tie pla shall aggregate not less than ⅓ the length of the plate.

COMBINED STRESSES

EXAMPLE

COLUMN



TRY W14x159

$A = 46.7$

$S_x = 254 \quad r_x = 6.38$

$S_y = 96.2 \quad r_y = 4.00$

$L_c = 16.4$

$L = 14'$

$K_x = 1.92$

$K_y = 0.80$

SUBJECT TO SIDESWAY
BUT NO TRANSVERSE LOADING

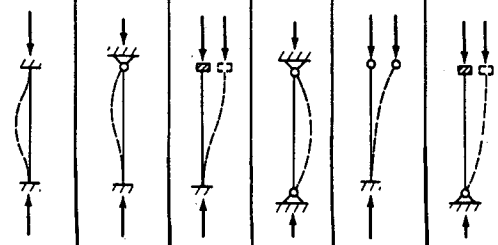
$$f_a = \frac{500}{46.7} = 10.71 \text{ ksi}$$

$$f_{b_x} = \frac{220 * 12}{254} = 10.39 \text{ ksi} < 24 \text{ ksi} \qquad L_c = 16.4 > 14$$

$$\frac{KL}{r_x} = \frac{1.92 * 14 * 12}{6.38} = 50.6 \longrightarrow F_a = 18.3 \text{ ksi}, \; F_e = 58.42$$

$$\frac{KL}{r_y} = \frac{0.8 * 14 * 12}{4.0} = 33.6$$

Table 5-1 Effective Lengths for Main Members Only

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code		Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free				

SOURCE: *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings November 1, 1978.* (*New York: AISC*, 1978), *p.* 5–124 *in the AISC Manual.*

"Commentary on AISC Specification" in the Steel Handbook gives recommended effective lengths when ideal conditions are approximated. This table is reproduced here as Table 5-1 with the permission of the AISC. K is the theoretical value to be multiplied times the column length to give its theoretical effective length; however, more conservative values are recommended in the table for actual design practices. The use of these latter values will be illustrated in several design problems in the pages to follow and Kl will be referred to as the effective length of the column.

The selection of the K value to be used for a particular column has given practicing engineers as much trouble as any other part of the AISC Specification. On many occasions K values have been selected which are too large, resulting in overdesigned columns with resulting economy losses. The difficulty seems to lie in distinguishing symmetrical buckling from sidesway buckling. When sidesway buckling occurs a smaller load can be supported.

When translation of the tops of the columns is clearly prevented as by diagonal bracing, shear walls, attachment to adjacent buildings, etc. (read Section 1.8.2 of the AISC Specification), symmetrical buckling will occur and the structure is referred to as a *braced frame*. For such cases as these the column effective lengths can be no greater than their actual lengths. Values of K equal to 1.0 can be conservatively assumed or lesser values

estimated from parts (a), (b), or (d) of Table 5-1 (AISC Table C.1.8.1) by charts prepared for this purpose such as the "sidesway prevented" ch given by the Structural Stability Research Council in their publicati *Guide to Stability Design Criteria for Metal Structures*. A large percenta of the columns designed by the usual structural designer fall into this cla

It should be realized that most of the columns which the structu engineer has to design serve as members of frames and have effecti lengths that are controlled by the amounts of restraint applied to their er by the other members of the frame and by the walls of the structure its As no column ends are completely fixed or perfectly pinned, the desig may wish to interpolate between the values given in the table, the inter lation to be based on his judgment of the actual restraint conditio Another reasonable method for estimating effective lengths involves careful sketching of the anticipated deflected shape of a particular colu and the measuring of the distance between the inflection points on sketch.

For most buildings, sidesway is substantially eliminated by maso walls, but for buildings built with light curtain walls and large colu spacings or for tall buildings built without a positive system of lat bracing, sidesway is appreciable. Such frames are referred to as *unbra frames*. For such cases the bending stiffness of the structural fra provides most of the lateral support.[5] The effective lengths of columns such laterally unsupported continuous frames must always be greater t 1.0 because of sidesway. This topic is more applicable to beam colu and is thus continued in Chapter 8 which deals with those types members.

5-8. MAXIMUM SLENDERNESS RATIOS

The AISC Specification (1.8.4) states that the slenderness ratio Kl/r of compression members may not exceed 200. The AASHTO Specificati (1.7.5) states that the ratio may not exceed 120 for main members. secondary members whose purpose is to brace the structure against lat or longitudinal forces or to reduce the unbraced length of other mem the ratio may not exceed 140.

5-9. DESIGN OF COLUMNS WITH AISC FORMULAS

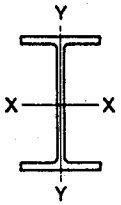
Examples 5-6 through 5-10 illustrate the design of different columns us the AISC expressions. In part (a) of Example 5-6 the solution is made substituting into the formulas but in the remainder of this example an the other examples the AISC Manual tables are used to simplify calculations. It should be remembered that the yield points of sev

[5] *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings No ber 1, 1978* (New York: AISC, 1978), pp. s-123 through s-127 AISC Manual.

F_y = 36 ksi

F_y = 50 ksi

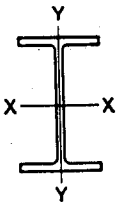
COLUMNS
W shapes
Allowable axial loads in kips



Designation		W14							
Wt./ft		193		176		159		145	
F_y		36	50	36	50	36	50	36	50
Effective length in ft KL with respect to least radius of gyration r_y	0	1227	1704	1119	1554	1009	1401	922	1281
	6	1178	1820	1074	1477	968	1331	884	1217
	7	1167	1603	1064	1461	959	1317	877	1203
	8	1157	1584	1054	1444	950	1301	869	1189
	9	1146	1565	1044	1426	941	1285	860	1174
	10	1134	1545	1034	1407	931	1268	851	1159
	11	1122	1524	1022	1388	921	1250	842	1142
	12	1110	1502	1011	1368	911	1232	832	1125
	13	1097	1479	999	1347	900	1213	822	1108
	14	1083	1455	987	1325	889	1193	812	1090
	15	1069	1431	974	1302	877	1173	801	1071
	16	1055	1406	961	1279	865	1152	790	1051
	17	1040	1380	947	1255	853	1130	779	1031
	18	1025	1353	933	1231	840	1107	767	1011
	19	1010	1326	919	1205	827	1085	755	990
	20	994	1298	904	1179	814	1061	743	968
	22	961	1239	874	1125	786	1012	718	923
	24	927	1178	842	1069	758	960	691	875
	26	891	1113	809	1009	727	906	663	825
	28	853	1046	775	947	696	850	634	773
	30	814	976	739	882	663	791	604	719
	32	774	902	701	815	629	729	573	662
	34	732	826	662	744	594	665	540	603
	36	688	745	622	670	558	598	507	541
	38	643	669	580	601	520	537	472	486
	40	596	604	537	543	480	484	435	438
Properties									
U		2.29	2.29	2.31	2.31	2.32	2.32	2.34	2.34
P_{wo} (kips)		340	473	299	415	251	349	214	298
P_{wl} (kips/in.)		32	45	30	42	27	37	24	34
P_{wb} (kips)		1542	1817	1250	1474	904	1066	688	810
P_{fb} (kips)		467	648	386	536	319	443	267	371
L_c (ft)		16.6	14.1	16.5	14.0	16.4	13.9	16.4	13.9
L_u (ft)		68.1	49.0	62.6	45.0	57.2	41.2	52.6	37.9
A (in.^{2})		56.8		51.8		46.7		42.7	
I_x (in.^{4})		2400		2140		1900		1710	
I_y (in.^{4})		931		838		748		677	
r_y (in.)		4.05		4.02		4.00		3.98	
Ratio r_x/r_y		1.60		1.60		1.60		1.59	
B_x } Bending		0.183		0.184		0.184		0.184	
B_y } factors		0.477		0.484		0.485		0.489	
$a_x/10^6$		358		319		283		255	
$a_y/10^6$		139		125		111		101	
F'_{ex} $(K_xL_x)^2/10^2$ (kips)		438		429		422		416	
F'_{ey} $(K_yL_y)^2/10^2$ (kips)		170		168		166		164	

F_y = 36 ksi

F_y = 50 ksi



COLUMNS
W shapes
Allowable axial loads in kips

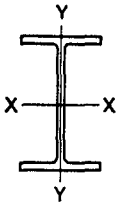
Designation		W14									
Wt./ft		132		120		109		99		90	
F_y		36	50	36	50	36	50	36	50	36	50
Effective length in ft KL with respect to least radius of gyration r_y	0	838	1164	762	1059	691	960	629	873	572	795
	6	801	1101	729	1002	661	908	600	825	547	751
	7	794	1088	722	990	654	897	595	815	541	742
	8	786	1074	714	977	647	885	589	805	536	732
	9	777	1060	707	963	640	873	582	793	530	722
	10	768	1044	699	949	633	860	575	782	524	711
	11	759	1028	690	935	626	847	568	769	517	700
	12	750	1011	682	919	618	833	561	757	511	689
	13	740	994	673	903	609	818	554	743	504	676
	14	730	976	663	887	601	803	546	730	497	664
	15	719	958	654	870	592	788	538	715	489	651
	16	708	938	644	852	583	772	529	701	482	637
	17	697	919	633	834	574	755	521	685	474	624
	18	686	898	623	815	564	739	512	670	466	609
	19	674	877	612	796	554	721	503	654	458	595
	20	662	856	601	776	544	703	494	637	449	580
	22	637	811	578	735	523	665	475	603	432	548
	24	610	764	554	692	501	626	454	567	413	515
	26	583	714	528	647	478	585	433	529	394	481
	28	554	663	502	599	454	541	411	489	374	444
	30	524	608	475	549	429	496	388	448	353	406
	32	493	551	446	497	403	449	365	404	331	366
	34	461	492	416	443	376	399	340	359	308	325
	36	427	439	385	395	348	356	314	320	285	290
	38	392	394	353	355	319	320	287	288	260	261
Properties											
U		2.47	2.47	2.48	2.48	2.49	2.49	2.50	2.28	2.52	2.29
P_{wo} (kips)		196	272	173	240	148	205	125	174	109	151
P_{wl} (kips/in.)		23	32	21	30	19	26	17	24	16	22
P_{wb} (kips)		587	692	449	529	316	373	249	294	186	220
P_{fb} (kips)		239	332	199	276	166	231	137	190	113	158
L_c (ft)		15.5	13.2	15.5	13.1	15.4	13.1	15.4	13.0	15.3	13.0
L_u (ft)		47.7	34.4	44.1	31.7	40.6	29.2	37.0	26.7	34.0	24.5
A (in.^{2})		38.8		35.3		32.0		29.1		26.5	
I_x (in.^{4})		1530		1380		1240		1110		999	
I_y (in.^{4})		548		495		447		402		362	
r_y (in.)		3.76		3.74		3.73		3.71		3.70	
Ratio r_x/r_y		1.67		1.67		1.67		1.66		1.66	
B_x } Bending		0.186		0.186		0.185		0.185		0.185	
B_y } factors		0.521		0.523		0.523		0.527		0.531	
$a_x/10^6$		228.0		204.8		184.5		165.1		148.9	
$a_y/10^6$		81.7		73.6		66.3		59.7		54.1	
F'_{ex} $(K_xL_x)^2/10^2$ (kips)		409		404		401		395		391	
F'_{ey} $(K_yL_y)^2/10^2$ (kips)		147		145		144		143		142	

†Flange is noncompact; see discussion preceding column load tables.

F_y = 36 ksi

F_y = 50 ksi

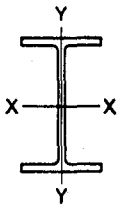
COLUMNS
W shapes
Allowable axial loads in kips



Designation		W12											
Wt./ft		210		190		170		152		136		120	
F_y		36	50	36	50	36	50	36	50	36	50	36	50
Effective length in ft KL with respect to least radius of gyration r_y	0	1335	1854	1205	1674	1080	1500	966	1341	862	1197	762	1059
	6	1266	1736	1142	1566	1023	1402	914	1253	815	1117	721	987
	7	1251	1711	1129	1543	1011	1381	903	1233	805	1100	712	972
	8	1236	1684	1115	1518	998	1359	891	1213	795	1082	702	956
	9	1219	1655	1100	1492	984	1335	879	1192	784	1062	692	938
	10	1202	1625	1084	1465	970	1310	866	1169	772	1042	682	920
	11	1185	1594	1068	1437	956	1285	853	1146	760	1021	671	901
	12	1166	1562	1051	1407	940	1258	839	1122	747	999	660	881
	13	1147	1528	1034	1376	924	1230	825	1096	734	976	648	860
	14	1127	1493	1016	1344	908	1200	810	1070	721	952	636	839
	15	1107	1457	997	1311	891	1170	794	1042	707	927	624	817
	16	1086	1419	978	1276	873	1139	778	1014	693	901	611	794
	17	1064	1381	958	1241	855	1107	762	985	678	875	597	770
	18	1042	1341	937	1204	837	1074	745	955	662	848	584	746
	19	1019	1300	916	1167	817	1039	728	924	647	819	569	720
	20	995	1257	894	1128	798	1004	710	892	630	790	555	694
	22	946	1169	849	1047	757	931	673	825	597	730	525	640
	24	894	1078	802	962	714	853	633	754	561	666	493	583
	26	840	977	752	872	668	771	592	680	524	598	460	522
	28	783	874	700	776	621	684	549	601	485	527	425	457
	30	723	766	645	679	571	597	504	524	444	459	388	398
	32	661	673	588	597	519	525	457	461	402	403	349	350
	34	596	596	529	529	465	465	408	408	357	357	310	310
	36	532	532	472	472	415	415	364	364	319	319	277	277
	38	477	477	423	423	372	372	327	327	286	286	248	248
	40	431	431	382	382	336	336	295	295	258	258	224	224
Properties													
U		2.47	2.47	2.49	2.49	2.51	2.51	2.53	2.53	2.55	2.55	2.56	2.56
P_{wo} (kips)		558	774	465	646	389	540	333	462	276	383	232	322
P_{wl} (kips/in.)		42	59	38	53	35	48	31	44	28	40	26	36
P_{wb} (kips)		4255	5014	3084	3635	2291	2700	1705	2010	1277	1505	927	1092
P_{fb} (kips)		812	1128	677	941	548	761	441	613	352	488	275	382
L_c (ft)		13.5	11.5	13.4	11.3	13.3	11.3	13.2	11.2	13.1	11.1	13.0	11.0
L_u (ft)		75.9	54.6	71.2	51.3	64.3	46.3	58.6	42.2	53.2	38.3	48.2	34.7
A (in.^{2})		61.8		55.8		50.0		44.7		39.9		35.3	
I_x (in.^{4})		2140		1890		1650		1430		1240		1070	
I_y (in.^{4})		664		589		517		454		398		345	
r_y (in.)		3.28		3.25		3.22		3.19		3.16		3.13	
Ratio r_x/r_y		1.80		1.79		1.78		1.77		1.77		1.76	
B_x } Bending		0.212		0.212		0.213		0.214		0.215		0.217	
B_y } factors		0.594		0.600		0.608		0.614		0.621		0.630	
$a_x/10^6$		319.5		281.6		245.5		213.4		185.1		159.7	
$a_y/10^6$		99.1		87.8		77.2		67.8		59.4		51.5	
F'_{ex} $(K_xL_x)^2/10^2$ (kips)		360		351		342		332		323		315	
F'_{ey} $(K_yL_y)^2/10^2$ (kips)		112		110		108		106		104		102	

F_y = 36 ksi

F_y = 50 ksi



COLUMNS
W shapes
Allowable axial loads in kips

Designation		W12											
Wt./ft		106		96		87		79		72		65	
F_y		36	50	36	50	36	50	36	50	36	50	36	50†
Effective length in ft KL with respect to least radius of gyration r_y	0	674	936	609	846	553	768	501	696	456	633	413	573
	6	637	872	575	788	522	715	473	647	430	589	389	533
	7	629	858	568	775	515	703	467	637	424	579	384	524
	8	620	844	560	762	508	691	460	626	418	569	378	514
	9	611	828	552	748	501	678	453	614	412	558	373	504
	10	602	812	544	733	493	665	446	601	406	547	367	494
	11	593	795	535	718	485	650	439	588	399	535	361	483
	12	583	777	526	701	477	636	431	575	392	522	354	472
	13	572	759	516	685	468	620	423	561	385	509	348	460
	14	561	740	506	667	459	604	415	546	377	496	341	448
	15	550	720	496	649	450	588	407	531	369	482	334	435
	16	539	699	486	630	440	570	398	515	361	468	326	422
	17	527	678	475	611	430	553	389	499	353	453	319	408
	18	514	656	464	591	420	534	379	482	344	438	311	394
	19	502	634	452	570	409	515	370	465	336	422	303	380
	20	489	611	440	549	398	496	360	447	326	406	294	365
	22	462	562	416	505	376	455	339	410	308	372	277	334
	24	433	511	390	458	352	412	317	371	288	336	259	301
	26	404	457	362	408	327	367	294	329	267	297	240	266
	28	372	399	334	356	301	319	270	285	245	258	220	230
	30	340	348	304	310	273	278	245	249	222	225	199	201
	32	305	306	272	273	244	244	219	219	197	197	176	176
	34	271	271	242	242	216	216	194	194	175	175	156	156
	36	241	241	215	215	193	193	173	173	156	156	139	139
	38	217	217	193	193	173	173	155	155	140	140	125	125
	40	196	196	175	175	156	156	140	140	126	126	113	113
Properties													
U		2.59	2.59	2.60	2.60	2.62	2.62	2.63	2.63	2.65	2.65	2.66	2.42
P_{wo} (kips)		185	257	161	223	139	193	122	169	106	148	92	128
P_{wl} (kips/in.)		22	31	20	28	19	26	17	24	15	22	14	20
P_{wb} (kips)		588	693	431	508	354	417	269	317	206	243	154	181
P_{fb} (kips)		221	306	182	253	148	205	122	169	101	140	82	114
L_c (ft)		12.9	10.9	12.8	10.9	12.8	10.9	12.8	10.8	12.7	10.8	12.7	10.7
L_u (ft)		43.3	31.2	39.9	28.7	36.2	26.0	33.3	24.0	30.5	21.9	27.7	20.0
A (in.^{2})		31.2		28.2		25.6		23.2		21.1		19.1	
I_x (in.^{4})		933		833		740		662		597		533	
I_y (in.^{4})		301		270		241		216		195		174	
r_y (in.)		3.11		3.09		3.07		3.05		3.04		3.02	
Ratio r_x/r_y		1.76		1.76		1.75		1.75		1.75		1.75	
B_x } Bending		0.215		0.215		0.217		0.217		0.217		0.217	
B_y } factors		0.633		0.635		0.645		0.648		0.651		0.656	
$a_x/10^6$		139.1		124.3		110.4		98.6		88.6		79.3	
$a_y/10^6$		45.0		40.1		36.0		32.2		29.1		26.0	
F'_{ex} $(K_xL_x)^2/10^2$ (kips)		310		307		300		296		292		289	
F'_{ey} $(K_yL_y)^2/10^2$ (kips)		100		99.0		97.7		96.5		95.8		94.6	

†Flange is noncompact; see discussion preceding column load tables.

COMBINED STRESSES

$$\frac{10.71}{18.3} + \frac{0.85(10.39)}{\left(1 - \frac{10.71}{58.42}\right)24} = 1.036 > 1 \quad \text{N.G.}$$

$0.59 + 0.446$

USE W14X176

$A = 51.8$

$S_x = 281 \qquad r_x = 6.43$

$S_y = 107 \qquad r_y = 4.02$

$L_c = 16.5$

$$\left.\begin{array}{l} f_a = 9.66 \\ F_a = 18.33 \end{array}\right\} 0.53 \qquad \Delta = 0.06 \quad \underline{\text{OK}}$$

USE W14X176

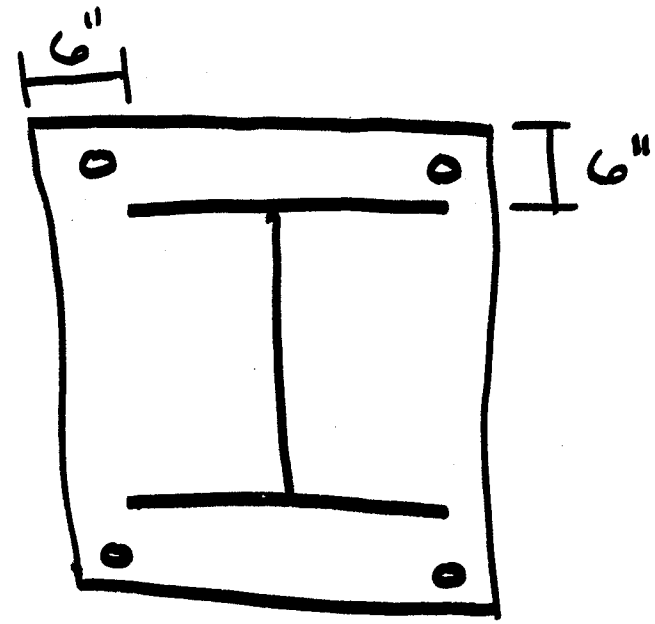
DESIGN BASE PL

W14x176

$b_f = 15.65$ $d = 15.22$

MOMENT BASE PLATE

PL = 16 + 12 = 28" SQ.



$$P = \frac{P}{A} \pm \frac{M}{S}$$

$$= \frac{500}{28^2} \pm \frac{200 * 12 * 6}{28^3}$$

$$0.64 \pm 0.66 = 1.3$$

CONCRETE = 1.3/0.35 = 3.7 KSI USE 4 KSI

OR 1.3/.7 = 1.86 KSI COULD USE 2 KSI

if $F_p = 0.35 f'_c \sqrt{\frac{A_2}{A_1}} < 0.7 f'_c$

BASE PLATE

$$T = C = M/d = \frac{200 * 12}{16+6} = 109.1^{K}/2 = 54.55^{K}/\text{bolt}$$

$1\frac{1}{4}''\phi$ A325 $T = 54^{K}$ 1% >

$1\frac{3}{8}''\phi$ A325 $T = 65.3^{K}$

$$C = \frac{500}{4} = 125^{K}/\text{bolt}$$

$C = 125 + 54.6 = 179.6^{KSI}$

$T = -125 + 54.6 = -70.4^{KSI}$ compression

check compression of bolt

$P = 180^{K}$ $\quad l = 2 * 1\frac{1}{4}''\phi = 2\frac{1}{2}''$

$$P_c = \frac{\pi^2 E}{K l/r}$$

$E = 29\,000$ KSI

$K = 2$

$r = \frac{1.25}{4} = 0.32$

$$P_c = \frac{286219}{15.63} = 18312 \quad FS = 100$$

BASE PLATE

$M = 2 * 180 * 3$

$M = 1080''^{K}$

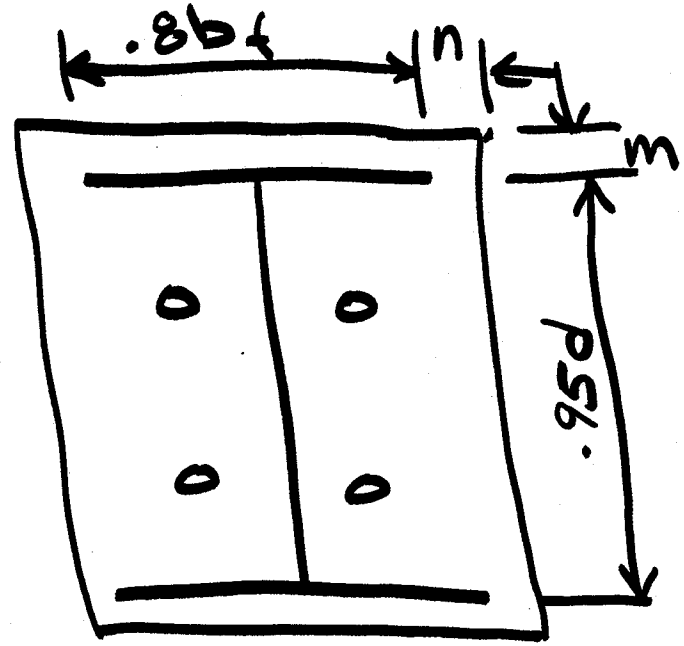
$M = 1080/28 = 38.58''^{K}/''$

$$t = \sqrt{\frac{M * 6}{F_b}}$$

$t = (8.58)^{\frac{1}{2}} = 2.93''$ <u>USE 3"</u>

PL 28" x 3" x 2'-4"

BASE PLATE (HINGED)



$P = 500^k$

Try = 15.65 + 3 = 18.65

18" x 20"

~~$m = \frac{1}{2}(18 - 15.22) = 1.39$~~

$m = \frac{1}{2}(18 - 0.95 * 15.22) = 1.77$

$n = \frac{1}{2}(20 - 0.8 * 15.65) = 3.74$

$$t_p = 2 * 3.74 \sqrt{\frac{500/18*20}{36}}$$

$t_p = 1.47"$ 1½" PL 18x20

18x18

$$t_p = 2 * 2.74 \sqrt{\frac{500/18^2}{36}}$$

$t_p = 1.13"$ 1¼ PL 18x18 ← USE

Design Procedure

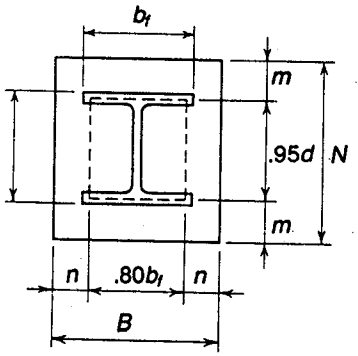


Figure 1

P = Total column load, kips
$A_1 = B \times N$ = Area of plate, in.^{2}
A_2 = Full cross-sectional area of concrete support, in.^{2}
F_b = Allowable bending stress in base plate, ksi
F_p = Allowable bearing pressure on support, ksi
f_p = Actual bearing pressure, ksi
f'_c = Compressive strength of concrete, ksi
t_p = Thickness of plate, in.

el base plates are generally used under columns for distribution of the column d over a sufficient area of the concrete pier or foundation.

Unless the m and n dimensions are small, the base plate is designed as a cantile- beam, fixed at the edges of a rectangle whose sides are $0.80b_f$ and $0.95d$. The col- n load P is assumed to be distributed uniformily over the base plate within the tangle. Letting F_b equal $0.75F_y$, the required thickness is found from the formulas

$$t_p = 2m\sqrt{\frac{f_p}{F_y}} \quad \text{and} \quad t_p = 2n\sqrt{\frac{f_p}{F_y}}$$

mensions of the base plate are optimized if $m = n$. This condition is approached en $N \approx \sqrt{A_1} + \Delta$, where $\Delta = 0.5\,(0.95d - 0.80b_f)$ and $B = A_1/N$.

When the values of m and n are small (the base plate is just large enough in area accommodate the column profile), a different model may be required. With this type base plate, the column load is assumed to be distributed to the concrete area, as shown cross-hatching in Fig. 2:

$$A = 2(d + b - 2L)L$$

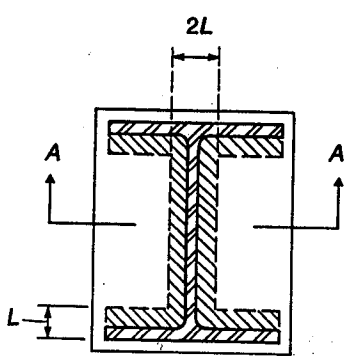


Figure 2

whe

d = depth of column section, in.
b = flange width of column section, in.

The dimension L can be found from the following equation, where P equals the column load:

$$F_p = \frac{P}{2(d + b - 2L)L}$$

This procedure assumes the plate is fixed at the column web; a free-body diagram for a 1-in. strip of plate is shown in Fig. 3.

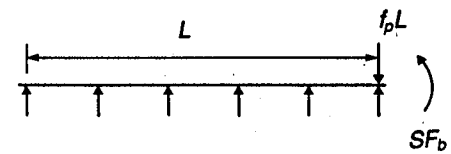


Figure 3

Designing for $f_p = F_p$, the required plate thickness may be calculated as follows:

$$f_p\,(L)\,L/2 = SF_b$$

$$S_{req'd} = \frac{1}{6}t^2 = \frac{f_pL^2}{2F_b}$$

$$t = L\sqrt{3f_p / F_b}$$

The allowable bearing strength F_p of the concrete depends on f'_c and the percent of support area occupied by the base plate. From AISC ASD Specification Sect. J9,

$$F_p = 0.35\,f'_c$$

when the entire area of a concrete support is covered, and

$$F_p = 0.35\,f'_c\sqrt{A_2/A_1} \le 0.7\,f'_c$$

when less than the full area is covered. By substituting $P/A_1 \le F_p$, this formula may be rewritten as:

$$\left(\frac{P}{0.35\,f'_c}\right)^2 \le A_1A_2 \le 4A_1^2$$

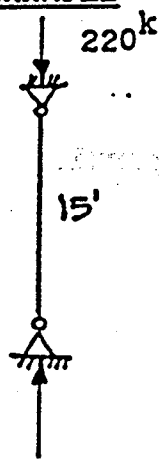
The first two terms give the general equation:

$$A_1 \ge \frac{1}{A_2}\left(\frac{P}{0.35\,f'_c}\right)^2$$

The first and third terms give the equation for the minimum base plate area for the upper concrete bearing limit:

$$A_1 \ge \frac{P}{0.7\,f'_c}$$

EXAMPLE



Determine the lightest weight W10 column, F_a, and P allowable for the column shown. F_y = 36 ksi.

The column has pinned ends, so the effective length factor, K, is equal to 1.0. Table C-C2.1 ASDM p. 5-135
From Column Tables ASDM p 3-30, with KL = 1 x 15 = 15';
a W10x49 has a P allowable of 235^k.
From ASDM p. 1-31, a W10x49 has the following properties:

r_y=2.54 in, r_x/r_y=1.71, A=14.4 sq in.

K_xL_x=1.0 x 15 = 15'

The equivalent minor axis effective length for the major axis is $K_xL_x/(r_x/r_y)$=15/1.71=8.77'< K_yL_y=15'

Then the minor axis length controls.

$$\frac{K_yL_y}{r_y} = 1.0 \times 15 \times 12/2.54 = 70.9$$

$$C_c = \left(\frac{2\pi^2 E}{F_y}\right)^{\frac{1}{2}} = \left(\frac{2\pi^2 29{,}000}{36}\right)^{\frac{1}{2}} = 126.1$$

or, ASDM p.5-120, C_c for F_y=36, = 126.1

$$\frac{K_yL_y}{r_y} < C_c$$

By Formula E2-1 ASDM p. 5-42

$$F_a = \frac{\left[1 - \frac{(Kl/r)^2}{2C_c^2}\right] F_y}{\frac{5}{3} + \frac{3Kl/r}{8C_c} - \frac{(Kl/r)^3}{8C_c^3}}$$

$$F_a = \frac{[1-(70.9)^2/2(126.1)^2]\,36}{1.67 + .375(70.9/126.1)-(70.9)^3/8(126.1)^3}$$

F_a = 16.34 ksi

Then P allowable = F_axA=16.34 x 14.4 = $235^k > 220^k$

Or, from Table C-36, ASDM p. 3-16
for Kl/r = 70.9, by interpolation,

F_a = 16.34 ksi, and P allow. is again, 235^k

COMPOSITE BEAM

$$n = \frac{E_s}{E_c}$$

$$E_s = 29{,}000 \text{ KSI}$$

$$E_c = 57\sqrt{f'_c}$$

$$f_s = \frac{M_D}{S_s} \pm \frac{M_{SDL} + M_L}{S_{TR}}$$

$$f_c = \frac{M_{SDL} + M_L}{n\, S_{TR}}$$

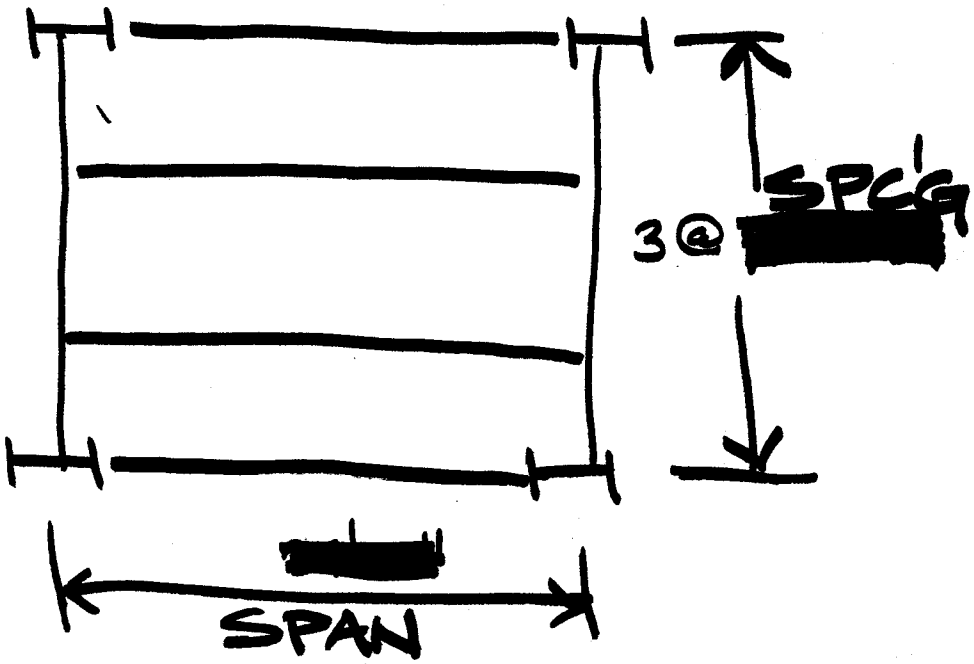
NO SHORING

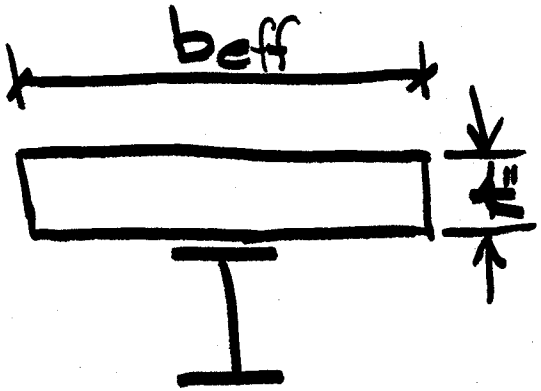
$F_{b_s} = 1.35 * F_b$ OR $0.81\, F_Y$ NON COMPACT

$0.89\, F_Y$ COMPACT

COMPOSITE BEAM

FRAMING



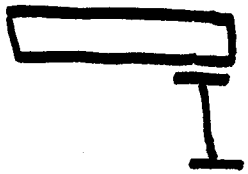


$b_{eff} = \frac{1}{4}$ SPAN

$b_{eff} =$ [redacted] SPACING

$b_{eff} = b_f + 16t$

IF END SPAN



$b_{eff} = b_f + \frac{1}{12}$ SPAN

$b_{eff} = b_f + \frac{1}{2}$ SPC'G

$b_{eff} = b_f + 6t$

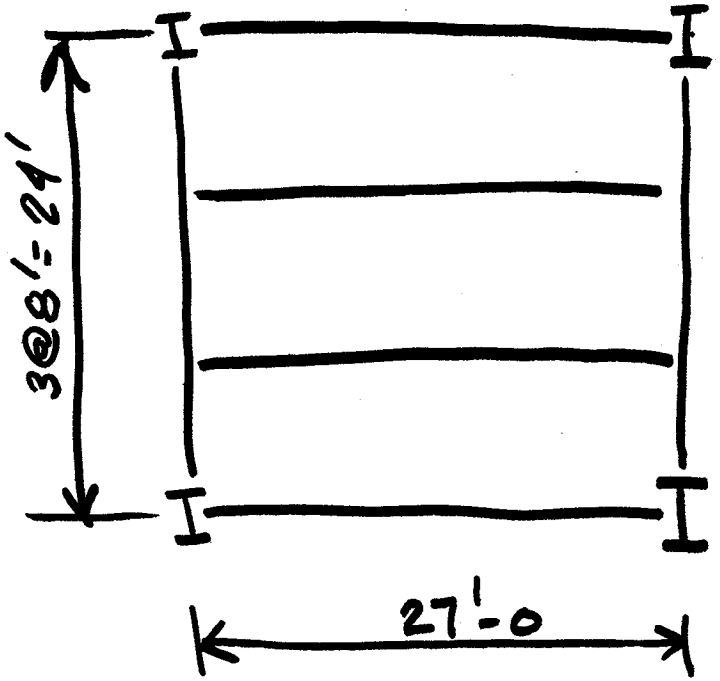
EXAMPLE

4" CONC SLAB

W16X31 $A = 9.12 \text{ in}^2$

$d = 15.88$ $b_f = 5.525''$

$I = 375 \text{ in}^4$ $S_x = 47.2 \text{ in}^3$



LOADS

LL = 100 PSF

PARTITIONS = 18 PSF

4" SLAB = 50 PSF

$f'_c = 3$ KSI $f_c = 1350$ psi

$n = 9$ $\frac{29000}{57\sqrt{3000}} = 9.3$

A36 STEEL

$f_c = 0.45 f'_c$

COMPOSITE BEAM

MOMENTS

4" SLAB = 8 * 50 = 400 plf

beam 31

431 plf

$$M_D = \frac{0.431 * 27^2}{8} = 39.3^{'K}$$

Partition = 8 * 18 = 144 plf

LL = 8 * 100 = 800

944 plf

$$M_L = \frac{0.944 * 27^2}{8} = 86.0^{'K}$$

$$M_T = 39.3 + 86 = 125.3^{'K}$$

COMPOSITE BEAM

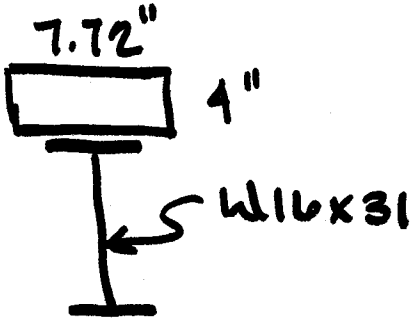
$$b = \frac{1}{4}\,27 \times 12 = 81''$$

$$b = 8.0 * 12 = 96''$$

$$b = 5.5^{25} + 64 = 69.525'' \leftarrow \text{CONTROLS}$$

TRANSFORMED AREA

$$\frac{69.52}{9} = 7.72 \text{ in}$$



COMPOSITE SECTION PROP.

$$A = 9.12 + 4 * 7.72 = 40 \text{ in}^2$$

$$\bar{y} = \frac{9.12 * 7.94 + 30.88 * 17.88}{40}$$

$$\bar{y} = 15.61 \text{ in}$$

$$I = 375 + 9.12(7.67)^2 + \frac{1}{12}(7.72) * 4^3 + 30.88 * 2.27^2$$

$$I = 1112 \text{ in}^4$$

COMPOSITE BEAM

PRIOR TO SET

$$f_s = \frac{39.3 * 12}{47.2} = 10^{ksi} < 24^{ksi} \quad \underline{OK}$$

AFTER SET

$$f_c = \frac{86 * 12 * (19.88 - 15.61)}{9 * 1112} = 0.44^{ksi} < 1.35^{ksi}$$

$$f_s = 10 + \frac{12 * 86 * 15.61}{1112} = 24.48 \text{ ksi} > 24 \text{ ksi} \quad \text{N.G.}$$

BY AISC CHAPTER I

STEEL STRESS CAN BE INCREASED IF TOTAL SERVICE LOAD MOMENT USING S_{TR} PROVIDED IT DOES NOT EXCEED MAX. PERMISSIBLE VALUE

$$S_{TR} = \frac{1112}{15.61} = 71.24 \text{ in}^3$$

$$S_{TR} = \left(1.35 + 0.35 \frac{86}{39.3}\right) 47.2 = 99.9 > 71.2 \text{ in}^3 \quad \underline{OK}$$

$$f_c = \frac{125.3 * 12 * 4.27}{9 \cancel{(\ldots)}\ 1112} = 0.64 < 1.35 \text{ OK} \qquad f_s = \frac{125.3 \times 12}{71.24} = 21.1 < 24 \quad \underline{OK}$$

Table 15-1 Allowable Horizontal Shear Load for One Connector (q), Kips[a]

Connector[b]	Specified compressive strength of concrete (f'_c), ksi		
	3.0	3.5	>4.0
$\frac{1}{2}$ in. diam.×2 in. hooked or headed stud	5.1	5.5	5.9
$\frac{5}{8}$ in. diam.×$2\frac{1}{2}$ in. hooked or headed stud	8.0	8.6	9.2
$\frac{3}{4}$ in. diam.×3 in. hooked or headed stud	11.5	12.5	13.3
$\frac{7}{8}$ in. diam.×$3\frac{1}{2}$ in. hooked or headed stud	15.6	16.8	18.0
Channel C3×4.1	$4.3w^c$	$4.7w^c$	$5.0w^c$
Channel C4×5.4	$4.6w^c$	$5.0w^c$	$5.3w^c$
Channel C5×6.7	$4.9w^c$	$5.3w^c$	$5.6w^c$

SOURCE: *Specification for the Design, Fabrication and Erection of Structural Steel Buildings*, November 1, 1978, Table 1.11.4. AISC, Inc.
[a]Applicable only to concrete made with ASTM C33 aggregates.
[b]The allowable horizontal loads tabulated may also be used for studs longer than shown.
[c]w-length of channel, in.

over the web their diameters shall not be greater than 2.5 times the flange thickness to which they are welded. Tests have shown if this rule is not met they will tend to tear out before their full shear-resisting capacity is reached.

Shear connectors must be capable of resisting both horizontal and vertical movement because there is a tendency for the slab and beam to separate vertically as well as to slip horizontally. Most designers feel that because of the tendency to slip vertically the longitudinal spacing of connectors should not exceed approximately 2 ft. The upset heads of the common studs help to prevent vertical separation.

Careful attention should be given to the AISC method of determining the horizontal shear to be taken by the connectors. Rather than basing their designs on the shear computed by the VQ/I formula, shear is estimated at ultimate load conditions. When a composite beam is being tested, failure will probably occur with a crushing of the concrete. At that time it seems reasonable to assume that the concrete and steel have both reached a plastic stress condition.

For this discussion reference is made to Fig. 15-7. Should the neutral axis fall in the slab the maximum horizontal shear (or horizontal force on

Table 15-2 Coefficients for Use with Concrete Made with C330 Aggregates

Specified compressive strength of concrete (f'_c)	Air dry unit weight of concrete, pcf						
	90	95	100	105	110	115	120
<4.0 ksi	0.73	0.76	0.78	0.81	0.83	0.86	0.88
>5.0 ksi	0.82	0.85	0.87	0.91	0.93	0.96	0.99s

SOURCE: *Specification for the Design, Fabrication, and Erection of Structural Steel Buildings*, November 1, 1978, Table 1.11.4A. AISC, Inc.

COMPOSITE BEAM

SHEAR CONNECTORS:

$$V_h = \frac{0.85\,(3)\;69.52 * 4}{2} = 354.6^K$$

$$V_h = \frac{9.12 * 36}{2} = 164.2^K \leftarrow \text{CONTROLS}$$

TRY 3/4" ϕ x 3" HSA

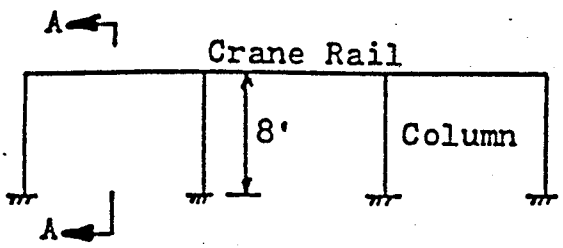
$V_R = 11.5^K$/STUD

$$N = \frac{164.2}{11.5} = 14.3 \text{ EA SIDE } ℄$$

USE 15 HSA EA SIDE

30 HSA TOTAL

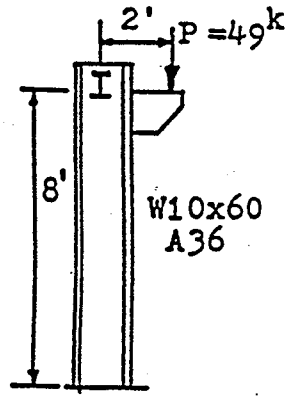
EXAMPLE Beam-column Design



Using AISC Specs., determine if the column has adequate strength to support the loads as shown.

Sidesway is not prevented in a direction perpendicular to the crane rail.

The column top is braced against sidesway in the direction along the crane rail.



SECTION A-A

Properties of W10x60: ASDM p. 1-30

$A=17.6$ sq in, $d=10.22$ in, $S_x=66.7$ in^3,

$r_x=4.39$ in, $r_y=2.57$ in, $r_T=2.77$ in,

$d/A_f=1.49$, $b_f/2t_f=7.4$

See Section H.1 ASDM p. 5-54

Determine which interaction equation controls.

Check f_a/F_a

$$f_a=P/A=49/17.6 = 2.78 \text{ ksi}$$

F_a depends on Kl/r, see p.5-135 for values of K, in this case we use condition (e) of Table C-C2.1, i.e. K=2.1.

$$\frac{K_x l_x}{r_x} = \frac{2.1 \times 8 \times 12}{4.39} = 45.92 \quad \underline{\text{Governs}}$$

$$\frac{K_y l_y}{r_y} = \frac{0.8 \times 8 \times 12}{2.57} = 29.9$$

Condition (b) of Table C-C2.1 gives K_y.

Interpolation in Table C-36, ASDM p.3-16, gives

$$F_a = 18.71 \text{ ksi}$$

$$f_a/F_a = 2.78/18.71 = 0.149 < 0.15$$

Then by Section H.1 we need only to check Formula H1-3

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} = \leq 1.0 \qquad \text{H1-3, ASDM p. 5-54}$$

EXAMPLE (cont)

Since we have no bending about the y-axis, the third term on the left of Formula H1-3 goes to zero.

$$f_b = \frac{M}{S} = \frac{49^k \times 24''}{66.7} = 17.63 \text{ ksi}$$

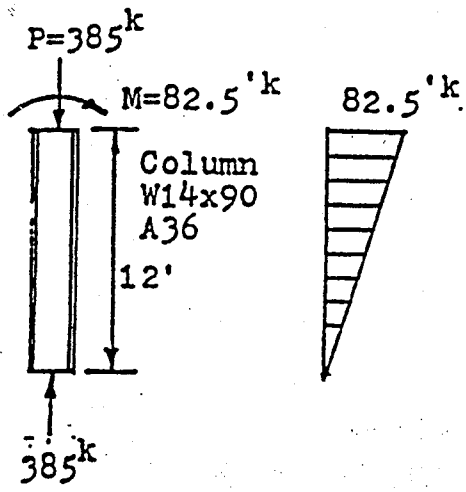
From p.3-30 ASDM, L_c = 10.6' which is greater than the actual length of 8'; therefore,

$$F_b = 24 \text{ ksi}$$

Substituting in Formula H1-3:

$$0.149 + \frac{17.63}{24} = 0.149 + 0.735 = 0.884 < 1.0 \quad \therefore \text{ OK}$$

EXAMPLE (Beam-column)



Can the W14x90 carry the load safely? Assume the column is hinged at both ends. See p.1-27 for properties of the W14x90.

$A=26.5\ in^2$

$d=14.02\ in$

$S_x=143\ in^3$

$r_x=6.14\ in$

$r_y=3.70\ in$

$r_T=3.99\ in$

$d/A_f=1.36$

From the Column Tables, p.3-23, $L_c=15.3' > 12'$ $\therefore F_b = 24ksi$

Since column is hinged at both ends, $K_x=K_y=1.0$

$$\frac{K_x l_x}{r_x} = \frac{1 \times 12 \times 12}{6.14} = 23.4 \qquad F'_e = 273 \text{ ksi}$$ (by interpolation from p.5-122)

$$\frac{K_y l_y}{r_y} = \frac{1 \times 12 \times 12}{3.70} = 39.0 \qquad F_a = 19.27 \text{ ksi}$$ (from p.3-16)

$$f_a = \frac{P}{A} = \frac{385}{26.5} = 14.5 \text{ ksi} \qquad \frac{f_a}{F_a} = \frac{14.5}{19.27} = .75 > .15$$

$\therefore$ must check HI-1 and HI-2 p.5-54

HI-1

$$\frac{f_a}{F_a} + \frac{C_m f_{bx}}{(1 - \frac{f_a}{F'_{ex}})F_{bx}} \leq 1.0$$

HI-2

$$\frac{f_a}{0.6F_y} + \frac{f_{bx}}{F_b} \leq 1.0$$

p.5-55; $C_m = 0.6 - \cancel{0.4\frac{M_1}{M_2}}^{\,0} = 0.6$

$$f_{bx} = \frac{M}{S_x} = \frac{82.5 \times 12}{143} = 6.9 \text{ ksi}$$

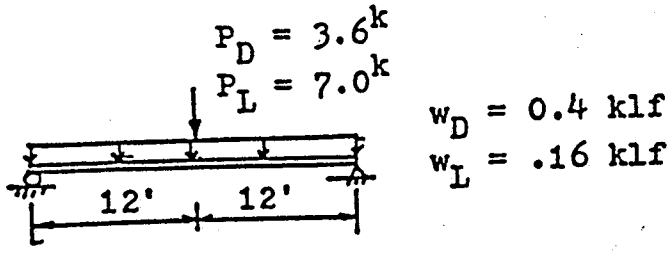
Check HI-1 $\quad 0.75 + \frac{.6 \times 6.9}{(1-\frac{14.5}{273})24} = 0.75 + 0.18 = 0.93 < 1.0$ OK

Check HI-2 $\quad (14.5/21.6)+(6.9/24) = 0.66 + 0.29 = 0.95 < 1.0$ OK

EXAMPLE

Proportion the beam below for moment requirements, the use condition is exterior exposure, and the loads shown are service (working) loads.

$f'_c = 3{,}000$ psi $\qquad f_y = 50{,}000$ psi

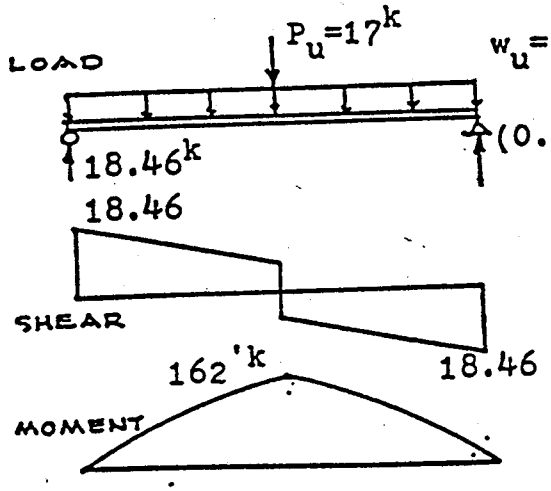


1. Establish the ultimate loads by multiplying service loads by the appropriate load factors found in Section 9.2 of the ACI Code, p. 92.

$$P_u = 3.6 \times 1.4 + 7.0 \times 1.7 = 16.94^k \text{ call } 17^k$$

$$w_u = 0.4 \times 1.4 + 0.16 \times 1.7 = 0.83 \text{ klf}$$

2. Shear and Moment Curves



$(0.83 \times 24)/2 = 9.96$
$17/2 = 8.50$
18.46

$M_{max} = [(18.46 + 8.50)/2](12)$
$= 161.76'^k$ call $162'^k$

3. Establish the minimum depth.

By Table 9.5(a) ACI Code p.96 see note at bottom of table "For f_y other than 60,000 etc."

$$h_{min} = \frac{\ell}{16}(0.4 + f_y/100{,}000)$$

$$h_{min} = \frac{24 \times 12}{16}\left(0.4 + \frac{50{,}000}{100{,}000}\right) = 16.2''$$

4. Establish b and d, at this point the designer selects a value for the percentage of steel.

$\rho \leq .75\ \rho_b$ Sections 10.3.2; 10.3.3 ACI p.109

$$\rho_b = \frac{0.85\beta_1 f_c'}{f_y}\left[\frac{87,000}{87,000+f_y}\right]$$

$$\rho_b = \frac{(.85)(.85)(3)}{50}\left[\frac{87}{87+50}\right] = 0.0275$$

$$0.75\rho_b = 0.75(0.0275) = 0.0204$$

Try $\rho = 0.018$, if this amount of steel is difficult to fit into the beam, a smaller ρ can be selected.

Using a capacity reduction factor $\phi = 0.90$, Sect. 9.3.2, p.94

$$M_u = \phi\rho f_y bd^2\left[1 - \frac{\rho f_y}{f_c'}\right] =$$

$$(162)(12) = (0.90)(0.018)(50\ bd^2)\left[1 - \frac{(0.018)(50)}{(1.7)(3)}\right]$$

$$2920 = bd^2$$

NOTE: f_c' and f_y are in KIPS PER SQ. IN.

If b = 12; d = 15.6 ← Try
If b = 10; d = 17.0

Points to consider in selecting proportions:
a) deeper beam is more efficient, deflects less.
b) shallower beam provides more headroom

5. Area of steel required:

$$A_s = \rho bd = (0.018)(12)(15.6) = 3.38 \text{ sq in.}$$

Use 2 - #10 and 1 - #9 rebars, A_s supplied = 3.53 sq in.

6. Check Z if $f_y > 40,000$ psi. This paramater limits the size of cracks. Section 10.6, ACI Code p.112

$$Z = f_s \sqrt[3]{d_c A}$$

Where, $f_s = 0.6f_y = 0.6(50) = 30$ ksi
d_c is defined in Sect. 10.0, ACI Code, p.105
A is defined in the same section.

EXAMPLE (cont.)

6. (cont.)

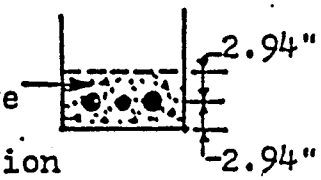
d_c = 2.0" Cover (ACI 7.7.1)

.375" Stirrup dia.

.5625" = $\frac{9}{8} \times \frac{1}{2}$

2.94"

A is the effective area of the tension concrete



$A = \frac{A_t}{\text{No. bars}^*}$

*When the main reinforcement consists of several bar sizes, the number of bars shall be computed as the total steel area divided by the area of the largest bar.
(Def. of A, Sect.10.0
ACI Code p. 105

A =(2.94 x 2 x 12)/(3.58/1.27)

A = 25.03

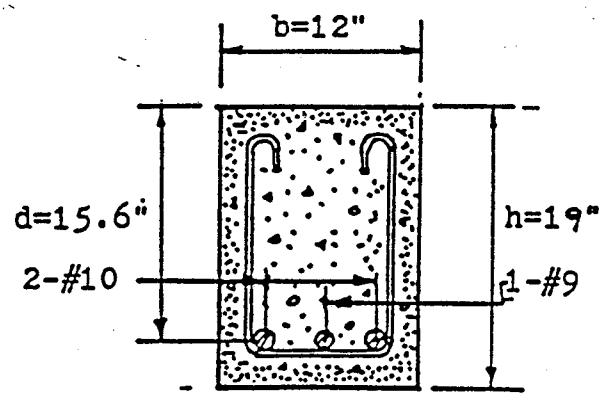
$Z = 30 \sqrt[3]{2.94 \times 25.03}$

Z =126 <145 = Z_{max} for exterior exposure. (10.6.4 p.113)

∴ The design is OK

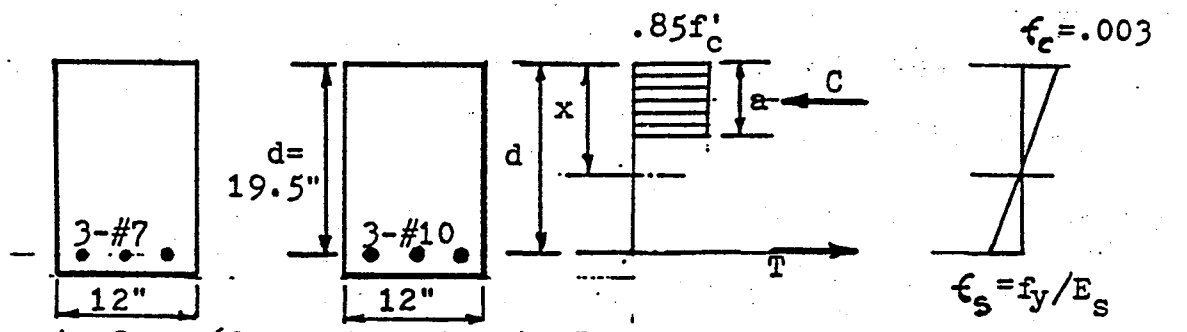
7. Sketch of the cross-section of the beam is:

h= 15.6
+2.94
18.54"
SAY 19"



EXAMPLE

For each of the beams, using f'_c=4,000 psi, f_y=60,000 psi, determine (a) the theoretical capacity M_n, (b) if loading is 60% LL and the basic provision for dead load plus live load controls, what is the service moment capacity?



A_s=3 x .60 =1.80 sq in A_s = 3 x 1.27 = 3.81 sq in

Balanced condition:

$$x_b = \frac{.003}{\frac{60}{29,000} + .003}(19.5) = 11.52"$$

$$a_b = .85(11.52) = 9.80"$$

$$C_b = .85f'_c ba_b = (.85)(4)(12)(9.80) = 400^k$$

$$A_{sb} = \frac{C_b}{f_y} = \frac{400}{60} = 6.67 \text{ sq in}$$

$$A_{s_{max}} = 0.75\ A_{sb} = 0.75(6.67) = 5 \text{ sq in}$$

5 sq in > 1.80 sq in or 3.81 sq in ∴OK

(a) $C = 0.85f'_c ab = 0.85(4)(12)(a) = 40.8a$

$$T = A_s f_y = 1.80 \times 60 = 108^k$$

$$a = 108/40.8 = 2.65"; \quad \frac{a}{2} = 1.33"$$

$$x = \frac{a}{.85} = 2.65/.85 = 3.12"$$

$$M_n = T(d - \frac{a}{2}) = 108(19.5 - 1.33)/12 = 163.5'^k$$

call $164'^k$

EXAMPLE (cont.)

(b) $C = 40.8a$

$T = (3.81)(60) = 228.6^k$

$a = 228.6/40.8 = 5.60"; \quad \frac{a}{2} = 2.80"$

$x = 5.60/.85 = 6.59"$

$M_n = 228.6(19.5 - 2.8)/12 = 318'^k$

Safe service load moment:

$$M_n = \frac{1.4(0.4M_w) + 1.7(0.6M_w)}{0.90} = \frac{0.56M_w + 1.02M_w}{0.90}$$

$$= 1.76M_w$$

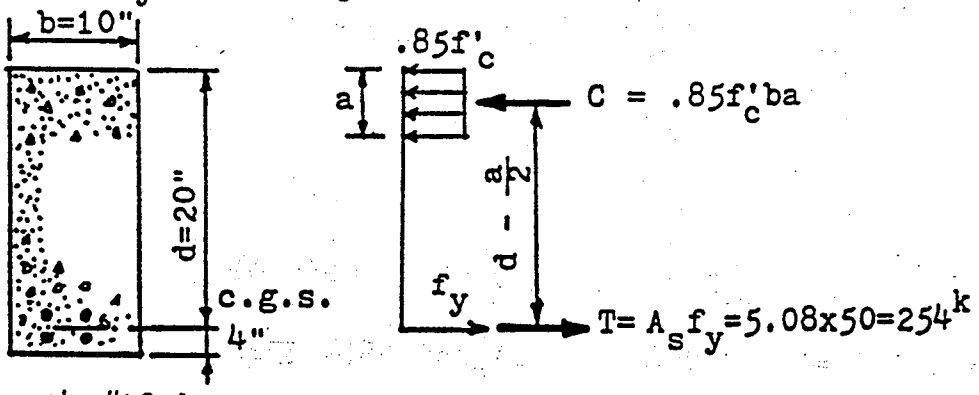
(a) $M_w = 164/1.76 = 93'^k$ for beam with 3 - #7

(b) $M_w = 318/1.76 = 181'^k$ for beam with 3 -#10

EXAMPLE

Using strength design, determine the maximum distance a simply supported beam can span if it has the cross-section shown below. Consider bending only. The beam must carry a uniform dead load of 0.8 klf and a live load of 0.9 klf.

f_y=50ksi; f'_c=3.75 ksi. Use ACI 318-89



4-#10 bars

A_s=1.27 x 4 = 5.08 sq in.

Equate C =T to get "a".

$$.85f'_c ba = A_s f_y \quad \text{which gives,} \quad a = \frac{A_s f_y}{.85 f'_c b} = \frac{254}{.85 \times 3.75 \times 10} = 7.97"$$

$$M_u = \left[A_s f_y \left(d - \frac{a}{2}\right)\right]\phi \quad \text{where } \phi\text{, the reduction factor, } = 0.90.$$

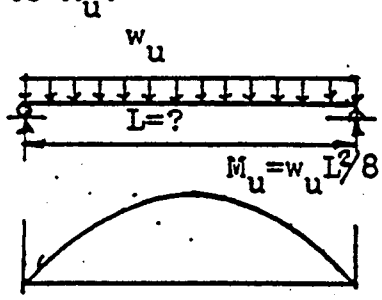
$$M_u = 0.90(254)\left(20 - \frac{7.97}{2}\right) = 3661''^k = 305'^k$$

The weight of the beam is (10x24)/144 (.150) = .25 klf.

$$w_u = 1.4w_D + 1.7w_L = 1.4(0.8+0.25) + 1.7(0.9)$$

$$w_u = 3.00 \text{ klf}$$

Equate the internal moment in terms of load and span length to M_u.



$$M_u = \frac{w_u L^2}{8}$$

$$306'^k = \frac{3.00L^2}{8}$$

$$816 = L^2$$

$$28.6' = L$$

EXAMPLE

Design a square tied concrete column to support an axial loading as follows:

DL = 300 kips; LL = 120 kips

f'_c = 4,000 psi, f_y = 60,000 psi

Section 10.3.5.2 ACI Code p.110

$$P_u = \phi P_{n(max)} = 0.80\phi \left[0.85 f'_c (A_g - A_{st}) + f_y A_{st} \right]$$

Assume 2% of steel, i.e. $A_{st} = .02A_g$

$$P_u = 1.4(300) + 1.7(120) = 420 + 204 = 624^k$$

$$624 = (0.80)(0.70)\left[0.85(4)(A_g - 0.02A_g) + 60(0.02A_g)\right]$$

$$624 = 0.56(3.40A_g - 0.068A_g + 1.20A_g) = 0.56(4.532A_g)$$

$$624 = 2.538A_g$$

$$A_g = 245.9 \text{ sq in}$$

Use a 16" x 16" column, A = 256 sq in.

Area of steel, A_{st}, required:

$$624 = 0.56\ (3.40 \times 256) - 3.40A_{st} + 60A_{st}$$

$$624 = 487 + 31.7A_{st}$$

A_{st} = 4.32 sq in. Use 8 - #7, Area = 4.80 sq in.

Sections 7.10.5.1 and 7.10.5.2 ACI Code, p. 72

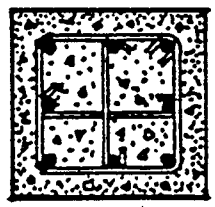
Using #3 ties, the spacing is:

48 x 3/8 = 18"

16 x 7/8 = 14" (Governs)

Least dimension of the column is 16"

Use #3 ties at 14 inches o.c.



16"x 16"column
8 - #7 vert.
#3 ties at 14"o.c.

EXAMPLE

Design a square column footing to support a 16 inch square column having a DL $=300^k$ and LL $=120^k$. The column re-bars are 8 -#7 vertical. The bottom of the footing is 5'-0" below ground. The concrete weight is 150pcf, the soil is 110, f'_c = 4,000 psi, f_y = 60,000 psi, and the soil capacity is three tons per sq ft.

Assume the footing depth = 24", then d = 24 -(3+1) = 20"

Section 15.2.2 ACI Code p.245

$$P_D=300^k \quad P_{LL}=120^k \quad P_{TOT}=300+120=420^k$$

The bearing pressure due to the footing and the soil is:

$$f_p=2.0x.150+3x.110=.300+.330=.630 \text{ ksf}$$

$$\text{Net } f_p=6^k - .630^k = 5.37 \text{ ksf}$$

$$\text{Area}_{req'd} = \frac{P_{TOT}}{\text{Net } f_p} = \frac{420}{5.37} = 78.2 \text{ sq ft}$$

Use footing 9' x 9'. Area = 81 sq ft.

Section 9.2.1 ACI Code p.91

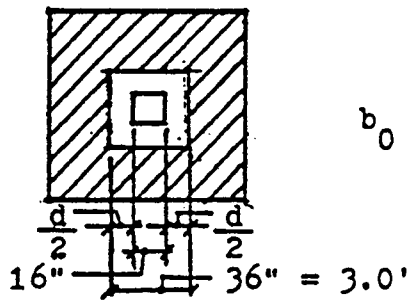
The design pressure on the base of the footing due to factored column loads is

$$f_{pu} = \frac{(1.4 \times 300) + (1.7 \times 120)}{81} = \frac{624}{81} = 7.70 \text{ ksf}$$

Depth for punching shear;

Sections 11.11.1.2; 11.12.2.1 ACI Code p.169

The shaded area is the loaded area for punching shear.



b_0 = 4 x 36=144"

EXAMPLE (cont.)

$$V_u = (81 - 3^2)(7.70) = 554.4^k$$

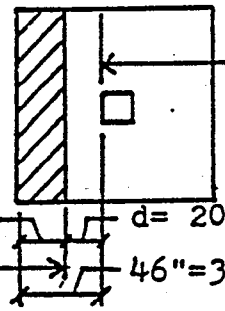
$$d = \frac{V_u}{\phi 4\sqrt{f'_c}\, b_0} = \frac{554{,}400}{(.85)(4)\sqrt{(4000)}(144)} = 17.90"$$

$17.90" < 20"$ OK

Depth of footing for beam action:

Section 11.12.1.1 ACI Code p.168

b = 108"



Section for maximum moment (Section 15.4.2a ACI Code p.246)

2'-2" = 2.17'

d= 20"=1.66'

Section for beam action shear

46"=3'-10"=3.83'

$$V_u = 9.00 \times 2.17 \times 7.7 = 150.38^k$$

$$d = \frac{V_u}{\phi 2\sqrt{f'_c}\, b} = \frac{150{,}380}{(0.85)(2)\sqrt{(4000)}(108)} = 12.95" < 20" \text{ OK}$$

Steel Area:

$$M_u = (7.70)(9.0)(3.83)(3.83)/2 = 508.28\ ^{'k}$$

$$R_u = \frac{M_u}{\phi b d^2} = \frac{508.28 \times 12000}{0.90 \times 108 \times 20 \times 20} = 156.88$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_u}{f_y}}\right) \quad \text{where } m = \frac{f_y}{.85\, f'_c} = \frac{60}{.85(4)} = 17.65$$

$$\rho = \frac{1}{17.65}\left(1 - \sqrt{1 - \frac{2 \times 17.65 \times 156.88}{60{,}000}}\right)$$

$$\rho = 0.0027; \quad \rho_{min} = \frac{200}{f_y} = \frac{200}{60{,}000} = 0.0033 \longleftarrow \text{Use}$$

$$A_{st} = A_{s(min)} = 0.0033 \times 108 \times 20 = 7.20 \text{ sq in}$$

Use 12 - #7 rebars $A_s = 12 \times 0.60 = 7.20$ sq in.

EXAMPLE (cont.)

Development length for footing rebars;
Section 12.2.2 ACI Code p.182

$$\ell_d = \frac{.04\ A_b\ f_y}{f'_c} = \frac{.04 \times .60 \times 60{,}000}{4{,}000} = 23"$$

$$\ell_{min} = .0004\ d_b f_y = .0004 \times \frac{7}{8} \times 60{,}000 = 21"$$

$$\ell_{actual} = 46 - 3 = 43" \quad \text{OK}$$

Load transfer from base of column to footing;
Sections 10.15.1 ACI Code p.133

Allowable load at the base of the column;

$P = \phi\, 0.85\ f'_c\ A_1$; where A_1 = area of column = 256 sq in.

$P = (0.70)(0.85)(4)(256) = 609^k < 624^k$

Allowable load on footing;

$P = 0.85\ f'_c\ A_1 \sqrt{\frac{A_2}{A_1}}$; where A_2 is the supporting surface, and $\sqrt{\frac{A_2}{A_1}}$ need not exceed 2.

$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{81}{1.77}} = 6.76 > 2 \quad \therefore$ Use 2

$P = (0.70)(0.85)(4)(256)(2) = 1218.6^k > 624^k$ OK

Excess to be carried by dowels;
Section 15.8.1.2 ACI Code p. 249

$624 - 609 = 15^k$

A_s of dowels $= \frac{15}{60} = 0.25$ sq in.

$A_{s(min)} = .005 \times 256 = 1.28$ sq in.

Use 4 - #6 rebars, area = 4 x .44 = 1.76 sq in.

EXAMPLE (cont.)

Development length of dowels:

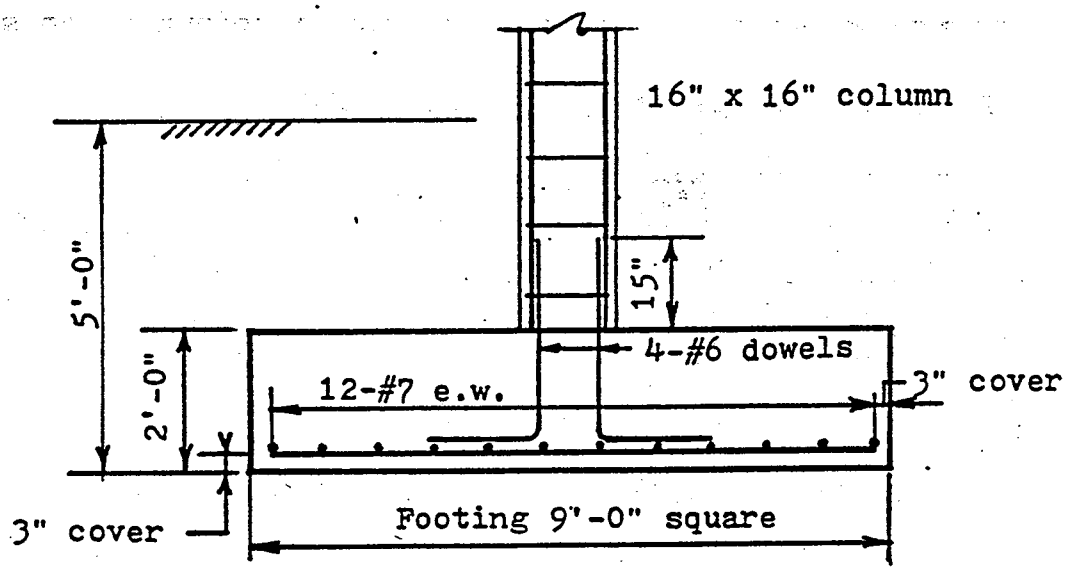
Section 12.3.2 ACI Code p. 188

$$\ell_d = \frac{0.02 f_y d_b}{\sqrt{f'_c}} = \frac{(.02)(60{,}000)(.75)}{\sqrt{4{,}000}} = 14.23'' \longleftarrow \text{Governs}$$

$$\ell_d = 0.0003\ f_y d_b = (.0003)(60{,}000)(.75) = 13.5''$$

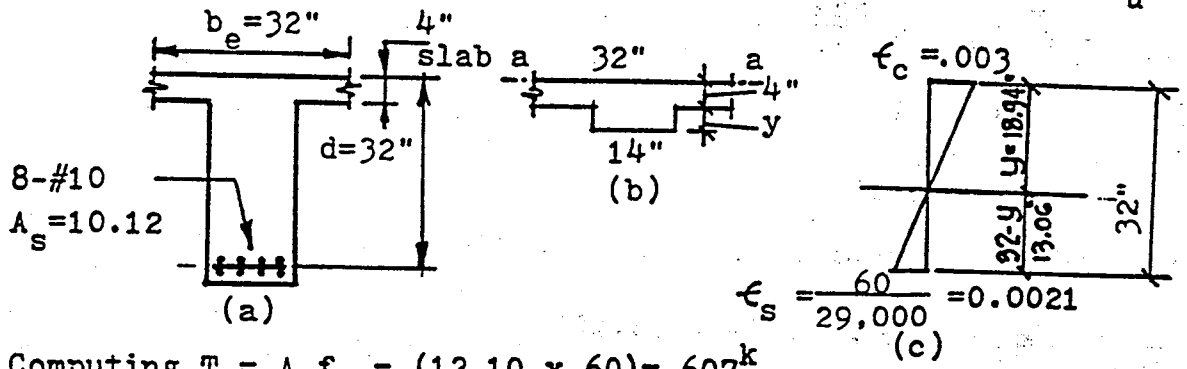
$$\ell_d = 8''$$

Use 4 - #6 dowels extending 15" up into the column and down into the footing so as to set on the rebar mat.



EXAMPLE

Given the concrete T beam shown, the span is 30 feet and $f'_c = 4{,}000$ psi, $f_y = 60{,}000$ psi. Considering bending only, find: (a) M_u = ? (b) what uniformly distributed load for M_u?



Computing $T = A_s f_y = (12.10 \times 60) = 607^k$

Since the total compression must be equal to T, and

$C = 0.85\, f'_c\, A_{(\text{concrete in compression})}$

$$A_c = \frac{T}{0.85 f'_c} = \frac{607}{.85 \times 4} = 178.5 \text{ sq in}; \; 32 \times 4 = 128 \text{ sq in.}$$

∴ The neutral axis lies below the slab.

Using (b) above:

$$14 \times y = 178.5 - 128 = 50.5$$

$$y = 3.61"$$

Compute $\bar{y}$ with respect to axis a-a.

	Area	y_{a-a}	$A(y_{a-a})$
4 x 32	128	2	256
3.61 x 14	50.5	5.81	293.15
	178.5		549.15

$$\bar{y} = \frac{549.15}{178.5} = 3.08"$$

The lever arm from T to C = 32 - 3.08 = 28.92"

$$M_n = 607^k \times \left(\frac{28.92}{12}\right) = 1463'^k$$

$$M_u = \phi M_n = 0.90 \times 1463 = 1317'^k$$

EXAMPLE (Cont.)

Check $\rho_{max} = \frac{3}{4}\rho_b$, T is a function of ρ, then check T_{max}.

Using Fig.(c), for balanced conditions:

$$\frac{y}{32} = \frac{.003}{.003 + .0021}$$

$$y = 18.94" = y_b$$

$$32 - y = 13.06"$$

The depth of the compression stress block, $a_b = .85y_b$.

$$a_b = .85 \times 18.94 = 16.10"$$

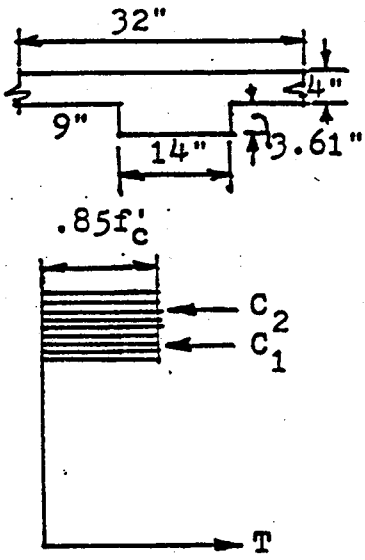
Then the area of the concrete compression block is:

$$A_c = 4(32) + (16.10 - 4)(14) = 297.4 \text{ sq in.}$$

$$C = (.85)(4)(297.4) = 1011^k$$

$$T_{max} = \frac{3}{4}C = 0.75 \times 1011 = 758^k > T_{actual} = 607^k \text{ OK}$$

Check on M_u by alternate method: see Fig.(d).



$$C_1 = (.85)(4)(14)(7.61) = 362.2^k$$

$$C_2 = (.85)(4)(2)(9)(4) = 244.8^k$$

$$M_n = \left[362.2\left(32 - \frac{7.62}{2}\right) + 244.8\left(32 - \frac{4}{2}\right)\right]/12$$

$$M_n = 1463'^k$$

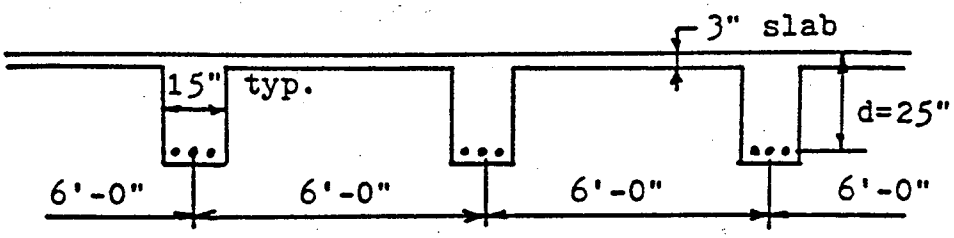
$$M_u = \phi M_n = 0.90 \times 1463 = 1317'^k \text{ Ans.}$$

$$w_u = \frac{8M}{L^2} = \frac{8 \times 1317}{30 \times 30} = 11.70 \text{ klf Ans.}$$

(d)

EXAMPLE

Given the T beam floor system as shown, and $M_{DL} = 200'^k$, $M_{LL} = 250'^k$, f'_c=4,000 psi, f_y=60,000 psi, L = 20'0".
Find the area of steel required for a T beam in this system.



Section 8.10.2 ACI Code p. 85

Effective flange width b_e:

1) $b_e = \frac{1}{4} \times 20 \times 12 = 60"$ Governs

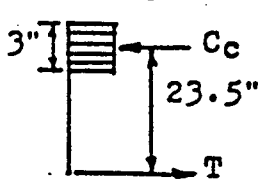
2) $b_e = (2)(8)(3) + 15 = 63"$

3) $b_e = (2)(\frac{57}{2}) + 15 = 72"$

$$M_u = 1.4(200) + 1.7(250) = 280 + 425 = 705'^k$$

$$M_n = \frac{M_u}{\phi} = \frac{705}{.90} = 783'^k$$

Assume a the same depth as the slab and compute moments about C_c.



$$T(23.5) = A_s f_y (23.5) = M_n$$

$$A_s(60)(23.5) = 783 \times 12$$

$$A_s = 6.66 \text{ sq in.}$$

Compute A_c:

$$A_c = \frac{C_c}{.85f'_c} = \frac{T}{.85f'_c} = \frac{6.66 \times 60}{(.85)(4)} = 117.6 \text{ sq in.}$$

$$a = \frac{A_c}{b_e} = \frac{117.6}{60} = 1.96" \quad a/2 = 0.98"$$

Since a<3" the neutral axis is in the flange.

EXAMPLE (Cont.)

$$M_n = T(d - \frac{a}{2}) = (A_s \times 60)(25 - 0.98)/12$$

$$783 = 120.10\ A_s$$

$$A_s = 6.52 \text{ sq in.}$$

Use 3 - #9 and 3 - #10 in two layers, A_s = 6.81 sq in.

Check a:

$$T = A_s f_y = 6.81 \times 60 = 408.6^k$$

$$A_c = \frac{408.6}{.85 \times 4} = 120.18$$

$$a = \frac{120.18}{60} = 2.00" \quad \frac{a}{2} = 1.00" \quad d - \frac{a}{2} = 24"$$

$$M_{n(allow.)} = T(d - \frac{a}{2}) = (408.6 \times 24)/12 = 817.2^{'k} > 783^{'k} \text{ OK}$$

$$A_s = \frac{M_{n(actual)}}{f_y(d - \frac{a}{2})} = \frac{783 \times 12}{60 \times 24} = 6.53 \text{ sq in} \sim 6.52 \text{ sq in OK}$$

$$M_u = \phi M_n = 0.90(817.2) = 735^{'k} > 705^{'k} \text{ OK}$$

Check $\rho_{max} = 0.75\rho_b$:

From ACI Tables, for f'_c = 4,000 psi and f_y = 60,000 psi,

$$0.75\rho_b = .0214$$

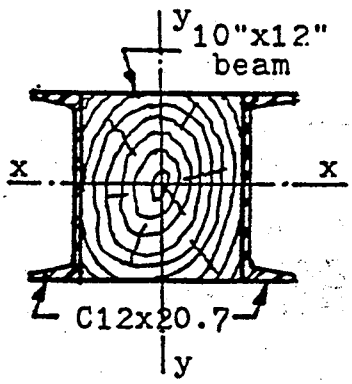
$$\rho_{actual} = \frac{A_s}{b_w d} = \frac{6.81}{15 \times 25} = .0182 < .0214 \quad \text{OK}$$

Check $\rho_{min} = 200/f_y = 200/60.000 = .0033 < .0182$ OK

EXAMPLE

A pair of C12x20.7 channels are bolted to a beam 10"x 12" as shown. Determine the safe resisting moment if bending occurs about the x-axis.

Given; F_s=18,000 psi, F_w=1200 psi, and E_s/E_w=n=20.



From ASDM p.1-40 (9th Ed.)

C12x20.7 A=6.09 sq in., d=12", b_f=2.942"

Axis x - x	Axis y - y
I = 129 in^4	I = 3.88 in^4
$\bar{y}$ = 6 in	$\bar{x}$ = 0.698 in

10x12 A = 120 sq in.

Axis x - x	Axis y - y
$I=(10x12^3)/12$	$I=(12x10^3)/12$
= 1440 in^4	= 1000 in^4
$\bar{y}$= 6 in	$\bar{x}$= 5 in

Transform the section to an equivalent wood section by using the given value of n = 20.

For x-axis bending, since the section is symmetrical with repect to this axis, we may transform the moments of inertia directly.

$$I_{tr}= nI_s+I_w = (2)(20)(129)+1440=5160+1440=6600 \text{ in}^4$$

$$S_{tr}=I_{tr}/(\tfrac{d}{2})=6600/6 = 1100 \text{ in}^3$$

If F_w = 1200 psi, $F_s=nF_w=(20)(1200)=24000>F_{s(allow.)}$=18,000 NG

If F_s =18,000 psi, $F_w=F_s/n=18,000/20=900<F_{w(allow.)}$=1200 ← Use

Then the moment capacity of the section is;

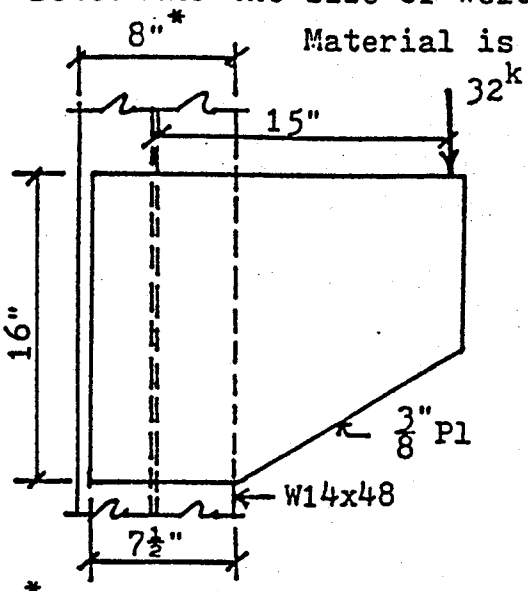
$$M_R=\frac{(F_w S_{tr})}{12,000} = 900 \frac{\text{lbs}}{\text{in}^2} \times 1100 \text{ in}^3 \times \frac{1 \text{ ft} \times 1^k}{12,000 \text{ in lbs}}$$

$$M_R= 82.50 \text{ 'k}$$

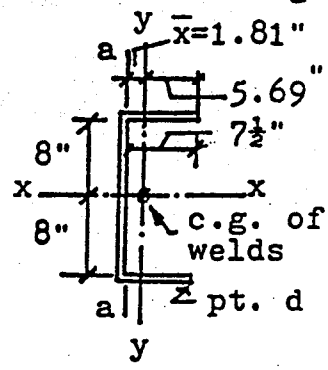
EXAMPLE

Determine the size of weld required for the bracket shown.

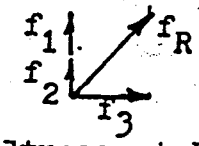
Material is A36, and welding electrodes E70.



* Flange width for detailing, from ASDM p.1-26



Assume effective thickness of weld throat, $t_e = 1"$.



Stress at Pt. d

Because of symmetry, the c.g. is half-way between the top and bottom welds. For $\bar{x}$ with respect to the y-axis: take moments about a-a.

$$\bar{x} = \frac{(2)(7.5)(1)(3.75)+(16)(1)(0)}{2(7.5)(1)+(16)(1)}$$

$$= \frac{56.25}{31} = 1.81"$$

$$I_p = I_x + I_y = \underbrace{\frac{16^3}{12}+2(7.5)(8)^2}_{I_x} + \underbrace{2\left(\frac{7.5^3}{12}\right)+2(7.5)\left(\frac{7.5}{2}-1.81\right)+16(1.81)^2}_{I_y}$$

$$I_p = 1301.33 + 179.18 = 1480$$

$$e = 15 + 3\tfrac{1}{2} - 1.81 = 16.69"$$

$$f_1 = \frac{32}{(2x7.5)+16} = 1.03 \text{ ksi} \uparrow$$

$$f_2 = \frac{Pex}{I_p} = \frac{32 \times 16.69 \times (7.50-1.81)}{1480} = 2.05 \text{ ksi} \uparrow$$

$$f_3 = \frac{Pey}{I_p} = \frac{32 \times 16.69 \times 8}{1480} = 2.89 \text{ ksi} \rightarrow$$

$$f_R = \sqrt{(f_1 + f_2)^2 + f_3^2} = \sqrt{(1.03+2.05)^2+(2.89)^2} = 4.22 \text{ ksi}$$

$$D = \frac{4.22}{.928} = 4.55 \text{ sixteenths, use } \frac{5}{16}" \text{ fillet weld.}$$

Or; a

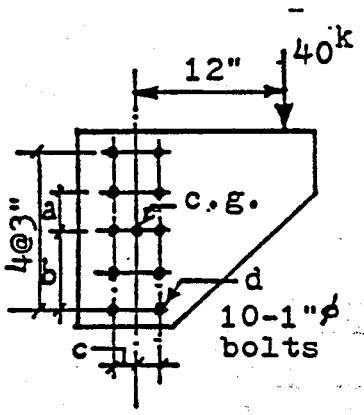
$$a = \frac{f_R\sqrt{2}}{F_{t(allow.)}} = \frac{5.97}{21} = 0.284", \text{ use } \frac{5}{16}".$$

Min. plate thickness $= \dfrac{.707 \times .3125 \times 21}{14.5} = 0.320"$, $\frac{3}{8}"$ Pl OK

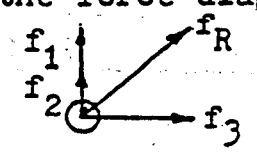
The numerator of the above fraction represents the shear force on the throat of a 5/16" fillet weld, and the denominator is the allowable shear stress for A36 steel which is in Section F4, ASDM p. 5-49 or, Table 1 p. 5-117.

EXAMPLE

Assume the plate is of sufficient size and strength to carry the applied loads. Using material having F_y=36 ksi, and 1"dia. A-325N high strength bolts, find the maximum load on the bolts.



By inspection, the center of gravity of the bolt group is as shown.
Since the resultant force in any bolt is perpendicular to the line which connects it to the center of gravity of the bolt group, the bolt having the largest load is farthest from the c.g. We will check bolt "d".
At "d", the force diagram is:



In this diagram, $f_1 = \frac{P}{n}$ Where n = total number of bolts.

$f_2 = \frac{Pex}{I_p}$; $f_3 = \frac{Pey}{I_p}$; and e =12".

$I_p = I_x + I_y$

I_x=No.bolts x a^2+No.bolts x b^2 = $4(3^2 + 6^2) = 180$

I_y=No.bolts x c^2 = 10 x 3^2 = 90

$I_p = 270$

$f_1 = \frac{40}{10} = 4^k$

$f_2 = \frac{40 \times 12 \times 3}{270} = 5.33^k$

$f_3 = \frac{40 \times 12 \times 6}{270} = 10.66^k$

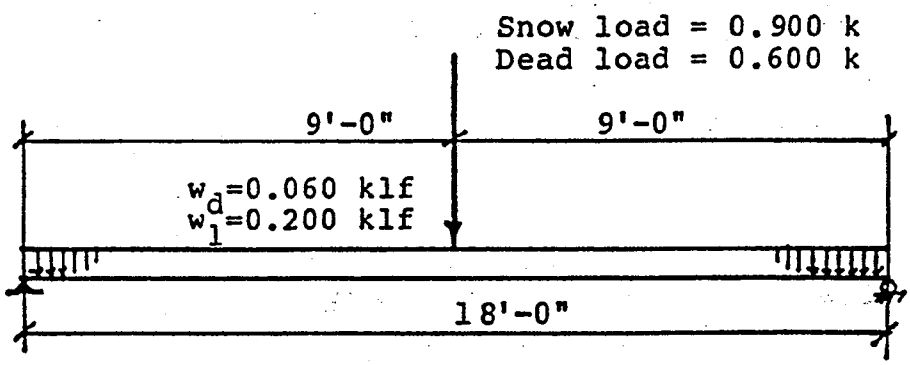
$f_R = \sqrt{(f_1 + f_2)^2 + f_3^2} = \sqrt{(4 + 5.33)^2 + (10.66)^2} = 14.17^k$

$R_{v(allow.)} = 16.5^k$, Table 1-D, ASDM p.4-5, and is the allowable load in single shear for a 1"diameter A-325N bolt. Therefore the maximum load on the given group of fasteners, 14.17^k, is satisfactory.

DESIGN OF WOOD BEAM

Design a wood beam for the given loads.

Lumber to be #2 kiln dried Southern Pine.
The beam is a floor beam, so the use condition is "dry".
The live load deflection is limited to L/360, and the dead load deflection is to be less than L/240.



Check with snow, and without snow.

With snow:

$$M_{max} = \frac{(.060+.200)(18)^2}{8} + \frac{(.900+.600)(18)}{4 \times 1.15 \text{ (snow duration factor)}}$$

$$= 10.53 + 5.87 = 16.4 \text{ ft-k}$$

W.O. snow:

$$M_{max} = 10.53 + \frac{.600 \times 18}{4} = 10.53 + 2.70 = 13.23 \text{ ft-k}$$

For single use, the bending stress F_b, for #2KD So. Pine, is 1.300 ksi.

$$S_{req.} = 16.4 \times 12/1.300 = 151.38 \text{ in}^3$$

Try multiple sticks:

Since we are using multiple sticks, the allowable bending stress F_b is increased to 1.500 ksi.

The new required section modulus is then:

$$S_{req.} = 151.38(1.3/1.5) = 131.20 \text{ in}^3$$

3 x 12's S=52.734 No. req.= 131.20/52.734 = 2.49 Use 3

2 x 12's S=31.641 No. req.= 131.20/31.648 = 4.15 Use 4

For the 2 x 12's; f_b=16.4 x 12/(4 x 31.648)=1.555 ksi.

This represent an overstress of ((1.555/1.5)-1.000)(100)= 3.667% which is acceptable.

The areas of the choices are:

$$4 - 2 \times 12\text{'s} = 4 \times 16.875 = 67.5 \text{ in}^2$$

$$3 - 3 \times 12\text{'s} = 3 \times 28.125 = 84.38 \text{ in}^2$$

Since the area of the 2 x 12 combination is less than that of the 3 x 12's, we will try the 4 -2x12's.

The maximum shear, at a distance from the support equal to the depth of the member, is:

With snow:

$$V_{max} = ((.900+.600)/2)+(.060+.200)((18/2)-(11.25/12))$$

$$= .750 + 2.096 = 2.846 \text{ k}$$

For snow duration:

$$V_{max} = 2.846/1.15 = 2.475 \text{ k}$$

W.O. snow:

$$V_{max} = (.600/2) + 2.096 = 2.396 \text{ k}$$

$$f_v = (3/2)(2.475/67.5) = 0.055 \text{ ksi} < F_v = .095 \text{ ksi} \quad \text{OK}$$

The dead load deflection is:

$$\Delta_{DL} = \frac{\dfrac{(5)(.060/12)(18\text{x}12)^4}{384} + \dfrac{.600(18\text{x}12)^3}{48}}{(1{,}600/2)(4)(177.979)}$$

(1/2)E for creep effect

$$\Delta_{DL} = 0.47''$$

The live load deflection is:

$$\Delta_{LL} = \frac{\dfrac{(5)(.200/12)(216)^4}{384} + \dfrac{.900(216)^3}{48}}{(1{,}600)(4)(177.979)}$$

$$\Delta_{LL} = 0.58''$$

$$0.58'' = L/372 < L/360 \quad \text{OK}$$

The total load deflection is:

$$\Delta_{TOT} = 0.47 + 0.58 = 1.05'' = L/206 > L/240 \quad \text{Not acceptable.}$$

Using the 3 - 3 x 12's:

The properties of this combination are:

Area = 3 x 28.125 = 84.375 in^2

Moment of inertia = 3 x 296.631 = 889.89 in^4

Section modulus = 3 x 52.734 = 158.202 in^3

The maximum shear is:

f_v = (3/2)(2.475)/84.375 = 0.044 ksi << F_v= .095 ksi OK

f_b = (16.4 x 12)/ 158.202 = 1.244 ksi < F_b= 1.500 ksi OK

Δ_{DL} = 0.47 x 711.96/889.89 = 0.38"

Δ_{LL} = 0.58 x 711.96/889.89 = 0.46"

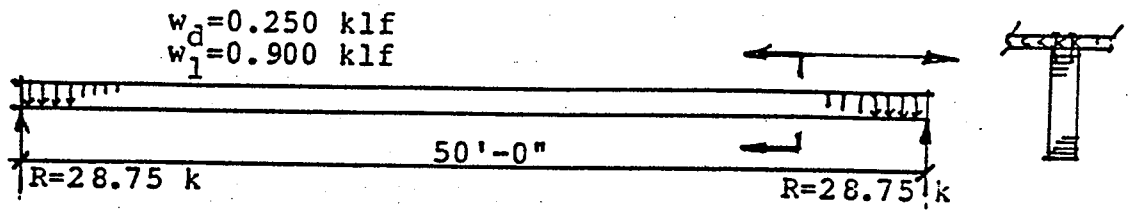
Δ_{TOT}= 0.84" = L/257 < L/240 OK

USE 3 - 3 x 12's

DESIGN OF GLULAM BEAM

Design a glulam roof beam for the given loads.

Lumber to be 22F Douglas Fir.
The use condition is "dry".
The live load deflection is limited to L/360, and the total load deflection is to be less than L/240.



By inspection, the total load condition will govern the design.

$$M_{max} = (0.250 + 0.900)(50)^2/8 = 359.4 \text{ ft-k}$$

The unsupported length of the beam is = 0, since the compression face is held by the roof planking.

Assume that the shape factor, $C_F = 0.90$,

$$F_b = 2.2 \times 0.90 \times 1.15 = 2.277 \text{ ksi}$$

where:

2.2 = F_b from allowable stress tables

0.90 = assumed shape factor

1.15 = snow load duration factor

Then, the required section modulus is:

$$S_{req.} = 359.4 \times 12 / 2.277 = 1,894 \text{ in}^3$$

Try a glulam beam 8 3/4 x 37.5 (25 - 1 1/2 lams)

Depth (in)	Area (in^2)	Moment of Inertia (in^4)	Section Modulus (in^3)
37.5	328.1	38,452.1	2050.8

$$C_F = (12/d)^{1/9} = (12/37.5)^{1/9} = 0.88$$

$$f_b = M/S = 354.4 \times 12 / 2050.8 = 2.103 \text{ ksi}$$

$$\text{Adj. } F_b = 2.2 \times .88 \times 1.15 = 2.226 > 2.103 \quad \text{OK}$$

Check shearing stress:

$$V' = 28.75 - (37.5/12)(1.150) = 25.16 \text{ k}$$

$$f_v = (3/2)(25.16)/328.1 = 0.115 \text{ ksi}$$

$$F_v = 0.165 \times 1.15 = 0.190 > 0.115 \qquad \text{OK}$$

Deflection calculations:

$$\Delta_{TL} = 180.0 \times M \times L^2 / E \times I$$

$$\Delta_{TL} = 180.0(359.4)(50)^2/(1800)(38{,}452.1)$$

$$= 2.34" = L/257 < L/240$$

$$\Delta_{LL} = 2.34\ (.900/1.150) = 1.83" = L/328 > L/360$$

Since this is about 10% over the allowable deflection value, a conservative approach would be to add an additional lamination, thus making the beam depth 39.0"

$$\text{The revised } I = 38{,}452.1 \times (39.0/37.5)^3$$

$$= 43{,}253.4 \text{ in}^4$$

$$\Delta_{LL} = 1.83 \times 38452.1/43253.4 = 1.63" = L/369 \text{ OK}$$

The dead load deflection, considering creep, and increasing the dead load deflection by 50%:

$$\Delta_{DL} = 1.5 \times 2.34 \times (.250/1.150) \times (38452.1/43253.4)$$

$$\Delta_{DL} = 0.68"$$

$$\Delta_{TL} = 0.68 + 1.63 = 2.31" = L/260 < L/240 \text{ OK}$$

The bending and shear stresses are OK by inspection.

Provide a camber equal to 1.5 times the dead load deflection,

$$\text{Camber} = 1.5 \times 0.68 = 1.02"$$

The required bearing length for the beam using $F_c = 0.385$ ksi, is:

$$\text{Adj. } F_c = 1.15 \times 0.385 =$$

$$\text{Area}_{req.} = 28.75/0.443 = 64.94 \text{ in}^2$$

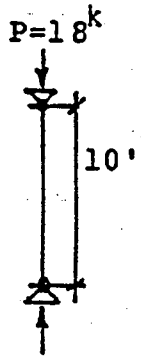
$$\text{Bearing length, } l_b = 64.94/8/75 = 7.42 \text{ say, } 7\ 1/2"$$

USE 8 3/4"x39" GL beam (26 - 1 1/2" lams)

22F Douglas Fir, Camber 1"

WOOD COLUMN DESIGN

Design a solid section wood column for the given conditions. The column supports a roof, and the total live (snow) and dead load is 18 kips.
The bracing is the same for both the x and y axes.
The lumber is No. 1 Douglas fir-larch.



Try a 4 x 6 (3 1/2 x 5 1/2) (Joist Plank category)

Area = 19.25 in^2 $(l/d_y) = 10x12/3.5 = 34.3 < 50$

F_c = 1.25 ksi $K = 0.671(E/(F_c \times LDF))^{1/2}$

E = 1800 ksi $K = 0.671(1800/(1.25 \times 1.15))^{1/2}$

$K = 23.7 < 34.3$

Since the column l/d is greater than K and less than 50 it is a long column, and the allowable compressive strength parallel to the grain is :

$$F'_c = 0.3E/(l/d)^2 = 0.3(1800)/(34.3)^2 = 0.459 \text{ ksi}$$

Then: $P_{(allow.)} = F'_c \times \text{Area} = 0.459x19.25 = 8.84 \text{ k} < 18 \text{ k}$ N.G.

Try a 6 x 6 (5 1/2 x 5 1/2) (Post & Timber category)

Area = 30.25 in^2 $(l/d_y) = 10x12/5.5 = 21.8$

F_c = 1.00 ksi $K = 0.671(1600/(1.00 \times 1.15)^{1/2}$

E = 1600 ksi $K = 25.0 > 21.8$

The column l/d is between 11 (the limit of l/d for a short column) and K, therefore, it is an intermediate column, and the allowable compressive strength is equal to:

$$F'_c = F_c(LDF)(1-1/3((l/d)/K)^4)$$

$$F'_c = (1.00)(1.15)(1-1/3(21.8/25.0)^4) = 0.928 \text{ ksi}$$

The the allowable axial load on the column is:

$$P_{(allow.)} = 0.928 \times 30.25 = 28.1 \text{ k} > 18 \text{ k} \quad \text{OK}$$

USE 6 x 6, No.1 DF-L

In composite construction, the concrete slab is attached to the steel beam by means of steel shear studs which are welded to the beam flange. The resulting bonding of the concrete and steel permits the consideration of the system as a series of T-beams.

The AISC Code, Chapter I, ASDM p.5-56, sets the parameters for designing such members.

The methods of construction which may be considered in the design process are:

1. Shored construction, in which the steel beam and the formwork are set and shored before the concrete is placed.

2. Unshored construction, in this case, the steel beams and formwork are sized to support the dead load of the beams, concrete slab, formwork, and construction loads.

In this presentation only type 2 shall be considered.

Under system two, the steel beam is stressed and deflected to accommodate the applied loads.

To determine the properties of the composite section, the concrete is transformed to an equivalent steel section by reducing its effective width by the ratio of b_{eff}/n, where the portion of the effective width of the concrete slab on each side of the beam centerline shall not exceed the smallest of:

a. One eighth of the beam span, center to center of supports,
b. One-half the distance to the centerline of the adjacent beam,
c. The distance from the beam centerline to the edge of the slab. (ASDM Section I1, p.5-58)

The web and end connections of the steel beams are designed to carry the effects of the total load.

The allowable horizontal shear load for the stud connectors is given in ASDM Table I4.1, p.5-59.

In the following problem, the shear connectors are designed for the full composite action and they must carry the total horizontal shear to be resisted from the point of maximum moment to the points of zero mement.

Section I1, ASDM p.5-56 requires that the value of this shear be the smaller of:

$$V_h = \frac{0.85 f_c' A_c}{2}$$

$$V_h = \frac{A_s F_y}{2}$$

Where,

f_c' = the compressive strength of the concrete in ksi

A_c = the actual area, in square inches, of the effective concrete flange as defined in I1, ASDM p.5-56.

PROBLEM:

The floor framing for the lobby of an office building has beams spaced 9 feet on centers and spanning 28 feet.
Design a composite section using no shoring.
The steel is A36, the concrete f'_c=3,000 psi and is normal weight concrete, the allowable horizontal shear per 3/4" x 3" stud is 11.5^k per stud.
The LL = 100 psf, partitions = 20psf, and ceiling = 10psf.
The DL = 4" concrete slab @ 50psf, steel beams and metal formwork @ 30psf, (the metal formwork is considered as not conributing to the composite action of the system), and a construction load of 20 psf.
Limit the construction and dead load deflection to 1", and the LL deflection to L/360.
Because this is a composite beam, the metal form is welded to the top flange of the steel beam, F_b may then be taken as $0.66F_y$.

Loads

Before conc. hardens		Dead load		After conc. hardens	
Constr. load	.020 ksf			LL	.100 ksf
Slab	.050 ksf	Slab	.050 ksf	Part.	.020 ksf
St. bm & dk	.030 ksf	bm & dk	.030 ksf	Ceil.	.010 ksf
				Misc.	.005 ksf
	.100 ksf		.080 ksf		.135 ksf

	Unit ld (ksf)	Bm spcg (ft)	w/ft (klf)	W (k)	V (k)	M (k-ft)
w_{LL}	.135	9	1.215	34.02	17.01	119.07
w_{DL}	.080	"	0.720	20.16	10.08	70.56
w_{CL}	.100	"	0.900	25.20	12.60	88.20
w_{TL}	.215	"	1.935	54.18	27.09	189.63

$$S_{CL}\ \text{req.} = \frac{M_{CL} \times 12}{F_b} = \frac{88.20 \times 12}{.66(36)} = 44.10\ \text{in}^3$$

The minimum required transformed section modulus is:

$$S_{tr}\ \text{req.} = \frac{M_{TL} \times 12}{F_b} = \frac{189.63 \times 12}{24} = 94.82\ \text{in}^3$$

To limit the Δ_{CL} to 1", the required moment of inertia is:

$$I_{req} = \frac{M \times L^2}{161.11 \times \Delta} = \frac{88.20 \times 28^2}{161.11 \times 1} = 429\ \text{in}^4$$

$$E_c = 57\sqrt{f'_c} = 57\sqrt{3000} = 3{,}122 \text{ ksi}; \quad E_s = 29{,}000 \text{ ksi}$$

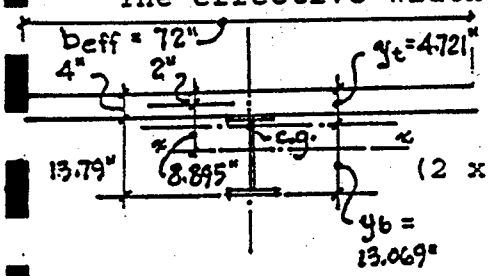
$$n = \frac{E_s}{E_c} = \frac{29{,}000}{3{,}122} = 9.2 \quad \therefore \quad n = 9$$

Select a trial section having an approximate section modulus equal to the average of the required S_{cl} and S_{tr}; in this case,

$$S_{trial} = \frac{44.10 + 94.82}{2} = 69.5 \text{ in}^3$$

Try a W14 x 48, $A=14.1 \text{ in}^2$; $S=70.3 \text{ in}^3$; $I=485 \text{ in}^4$; $b_f=8.030$ in; $d=13.79$ in; $t_w=.340$ in; $t_f=0.595"$

The effective width b_e is the smallest of:



$$28 \times \frac{12}{4} = 84"$$

$$(2 \times 4 \times 8) + 8 = 72" \text{ (governs)}$$

$$9 \times 12 = 108"$$

$$b'' = \frac{b_{eff}}{n} = \frac{72}{9} = 8"; \quad t = 4"$$

Item	A	y	Ay	Ay^2	I_o	$I_o + Ay^2$
I	8 x 4 = 32	8.895	284.64	2531.87	42.67	2574.54
II	14.1	0	0	0	485.00	485.00
	46.1		284.64		$I_{(x \text{ of bm})}$	= 3059.54
				$-Ay^2 = -(6.174 \times 284.64)$		= -1757.37
					I_o (of comp bm)	= 1302.17

$$y_{bar} = \frac{284.64}{46.1} = 6.174"$$

$$y_t = (8.895+2) - 6.174 = 4.721"$$

$$y_b = 6.895 + 6.174 = 13.069"$$

$$S_t = \frac{1302.17}{4.271} = 275.83 \text{ in}^3$$

$$S_{tr} = S_{bott} = \frac{1302.17}{13.069} = 99.64 \text{ in}^3 > \min S_{tr} = 94.82$$

$$\therefore f_{(CL)} = \frac{M_{CL}(12)}{S_s} = \frac{(88.20)(12)}{70.3} = 15.06 \text{ ksi} < F_b = 23.76 \text{ ksi} \quad \text{OK}$$

$$f_{(top)} = \frac{M_{LL}(12)}{nS_s} = \frac{(119.07)(12)}{(9)(275.83)} = 0.576 \text{ ksi} < 0.45\, f_c' \quad \text{OK}$$

$$f_{(bott)} = \frac{M_{TL}(12)}{S_{TR}} = \frac{(189.63)(12)}{99.64} = 22.84 \text{ ksi} < F_b = 23.76 \text{ ksi} \quad \text{OK}$$

$$\Delta_{(\text{const. ld})} = \frac{(M_{CL})(L^2)}{(161.11)(I_s)} = \frac{(88.20)(28^2)}{(161.11)(485)} = 0.89'' < 1'' \quad \text{OK}$$

$$\Delta_{(LL)} = \frac{(M_{LL})(L^2)}{(161.11)(I_{TR})} = \frac{(119.07)(28^2)}{(161.11)(1302.17)} = 0.45'' < \frac{L}{360} = 0.93''$$

$$\Delta_{DL} = \frac{(M_{DL})(L^2)}{(161.11)(I_s)} = \frac{(70.56)(28^2)}{(161.11)(485)} = 0.71''$$

The required shear to be carried by the studs is the smaller of:

$$V_h = \frac{0.85\, f_c'\, bt}{2} = \frac{(0.85)(3)(72)(4)}{2} = 367.2^k$$

$$V_h = \frac{A_s F_y}{2} = \frac{(14.1)(36)}{2} = 253.8^k \quad \leftarrow \text{Governs}$$

The stud diameter shall not be greater than 2.5 time the thickness of the steel beam flange, ASDM I4, p.5-60.

$(2.5)(0.595) = 1.49'' > 0.75''$, $\therefore$ 3/4" dia studs are OK.

At 11.5^k per stud, the number of studs required to carry a shear of 253.8^k is:

$$N = \frac{253.8}{11.5} = 23.07, \text{ use 24 studs each side of beam center line.}$$

Use connectors in pairs, the spacing perpendicular to beam center line, per ASDM p.5-60, is four time the stud diameter.

$$\therefore \text{spacing} = (4)(.75) = 3''$$

As per ASDM p.5-60:

Min. longitudinal spacing of studs is 6 time the stud diameter, i.e.,

$$(6)(0.75) = 4.5"$$

Max. longitudinal spacing is 8 time the slab thickness, i.e.,

$$(8)(4) = 32"$$

The actual stud sapcing, longitudinally, is:

$$\frac{(28)(12)}{\left[\frac{24}{2}\right](2)} = 14" > 4.5", \text{ and } < 32"; \quad \therefore \text{OK}$$

The total shear must be carried by the web of the steel beam, then f_v is:

$$f_v = \frac{V_{Tot}}{dt_w} = \frac{27.09}{(13.79)(0.340)} = 5.76 \text{ ksi} < F_v = 14 \text{ ksi} \quad \text{OK}$$

EXAMPLE

Using LRFD, select a beam to carry a simply supported uniformly distributed load of 1.0 K/FT LL and 0.5 K/FT DL. Use Fy = 50 ksi and consider beam as having full lateral support. The beam span = 40 FT.

Solution:

Estimated beam weight = 50 plf.

$\therefore\ w_u = 1.2(.50 + 0.05) + 1.6(1.0) = 0.66 + 1.6 = 2.26\ ^{K}/_{FT}$ LRFDM p. 6-25

$$M_u = \frac{w_u L^2}{8} = \frac{2.26(40)^2}{8} = 452^{\,'K}$$

$$Z_{x\,REQ} = \frac{M_u}{\phi_b F_y} = \frac{452 \times 12}{0.90(50)} = 121\ in^3$$

Try a W24x55 $Z_x = 134\ in^3$; $I = 1350\ in^4$

Check for a LL deflection limited to $\frac{L}{360} = \frac{40 \times 12}{360} = 1.33''$

Using service load Moment for deflection check:-

$$M_{LL} = \frac{(1.0)(40)^2}{8} = 200^{\,'K};\ \text{and}\ \Delta_{LL} = \frac{M_{LL} L^2}{161 I} = \frac{(200)(40)^2}{161(1350)} = 1.47'' > 1.33\ \underline{N.G.!!}$$

$$I\ \text{req. for}\ \Delta_{LL} = 1.33'' = \frac{200 \times 40^2}{161 \times 1.33} = 1495\ in^4$$

Try W24x62; $Z_x = 153$; $I = 1550\ in^4$

w_u revised $= 1.2(0.50 + 0.062) + 1.6(1.0) = 2.27\ ^{K}/_{FT}$

$$M_u = \frac{2.27 \times 40^2}{8} = 454^{\,'K}$$

$$M_p = Z_x F_y = (153 \times 50)/12 = 638^{\,'K}$$

$$\phi M_p = 0.90(638) = 574^{\,'K} > 454^{\,'K}\quad \text{o.k.}$$

$$\text{actual}\ \Delta_{LL} = \frac{(200)(40)^2}{161(1550)} = 1.28'' < 1.33''\ \text{o.k.}$$

Check compact section limits

From p. 1-25 LRFDM

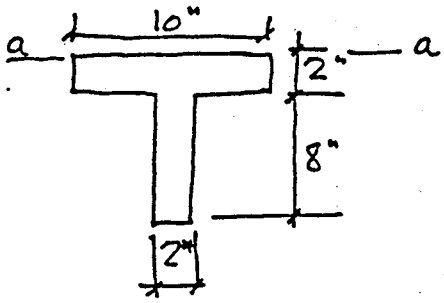
$$\frac{b}{2t_f} = 6.0;\quad \frac{h_c}{t_w} = 50.1$$

Flange: $\lambda = \frac{b}{2t_f} \le \frac{65}{\sqrt{F_y}} \Rightarrow 6.0 < \frac{65}{\sqrt{50}} = 9.19$ o.k.

Web: $\lambda = \frac{h_c}{t_w} < \frac{640}{\sqrt{F_y}} \Rightarrow 50.1 < \frac{640}{\sqrt{50}} = 90.5$ o.k.

W24x62 is satisfactory

Determine M_y, Z and M_n for a steel T beam having the following section:- If the beam span is 20^{FT} find the nominal udl w_u ~ $F_y = 36$ ksi



M_y:-

$A = (10 \times 2) + (8 \times 2) = 36^{\square''}$

$\bar{y}_{(a-a)} = \dfrac{(10 \times 2 \times 1) + (8 \times 2 \times 6)}{36} = \dfrac{20+96}{36} = 3.22'' = y_t;\ y_b = 6.78''$

$I = \frac{1}{3}(2)(1.22^3 + 6.78^3) + \frac{1}{12}(10 \times 2^3) + 20(2.22)^2$

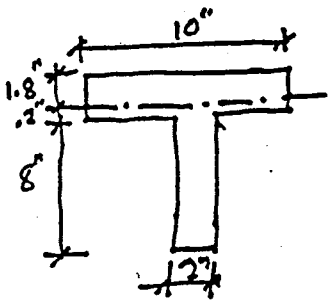
$= 208.99 + 6.67 + 98.57 = 314.22\ in^4$

Smallest $S = \dfrac{I}{y_b} = \dfrac{314.22}{6.78} = 46.35\ in^3$

$M_y = SF_y = (46.35)(36)/12 = 139.04^{'k}$

Z:- Plastic areas above and below N.A. must be =.

$20 - 16 = 4$ ∴ Top area $= 20 - 2 = 18^{\square''}$
Bot area $= 16 + 2 = 18^{\square''}$



$Z = (18 \times 0.9) + (2.0 \times 0.1) + (16 \times 4.2) = 16.2 + 0.2 + 67.2 = 83.6\ in^3$

$M_n = F_y Z = (36)(83.6)/12 = 250.8$ ft·k

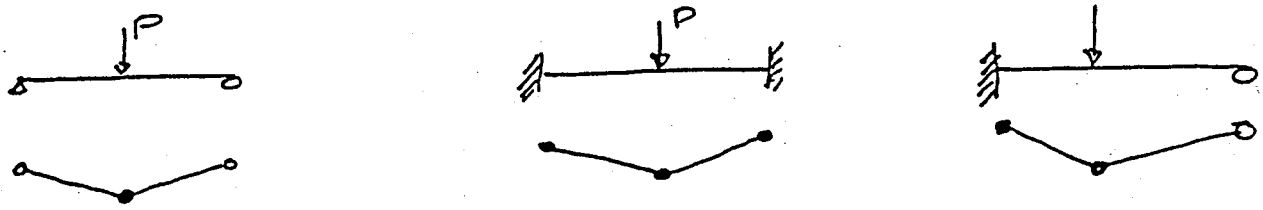
Shape Factor $= \dfrac{M_n}{M_y} = \dfrac{250.8}{139.04} = 1.80$

$w_u = \dfrac{M_n 8}{L^2} = \dfrac{250.8 \times 8}{20^2} = 5.02^{k}/FT$

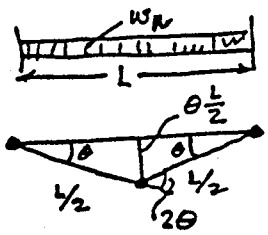
w_n

PLASTIC HINGES

○ = REAL HINGE ; ● = PLASTIC HINGE



VIRTUAL WORK METHOD :-



system is symmetrical, the end rotations, at plastic hinges, are equal. (θ)
By geometry: the middle rotation is twice that of either end. (2θ)

work = load or force × distance = tot. load × av. deflect

The total load performs work which is equal to $(w_n L)\left[\left(\frac{\theta L}{2}\right)\left(\frac{1}{2}\right)\right]$

Virtual work says $\Rightarrow$ Σext. work = Σ int. work

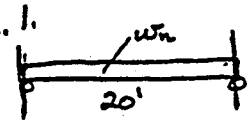
$$W_e = W_i$$

W_i is the sum of the M_n at each plastic hinge times the angle through which it works.

$$\therefore \quad (w_n L)\left[\left(\frac{\theta L}{2}\right)\left(\frac{1}{2}\right)\right] = M_n(\theta + 2\theta + \theta)$$

$$\frac{w_n L^2}{16} = M_n$$

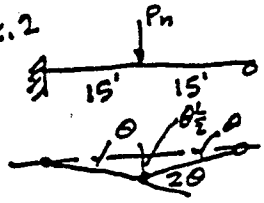
Ex. 1.



$$w_n(20)\left[\left(\frac{\theta \times 20}{2}\right)\left(\frac{1}{2}\right)\right] = M_n(4\theta)$$

$$w_n = \frac{M_n}{25}$$

Ex. 2



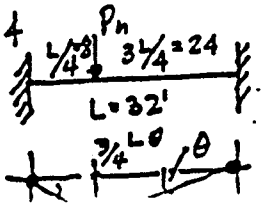
$$W_e = W_i$$

$$P_n\left(\theta\frac{L}{2}\right) = M_n(\theta + 2\theta)$$

$$\frac{P_n L}{6} = M_n$$

$$P_n = \frac{6 M_n}{30} = \frac{M_n}{5}$$

Ex. 4



$$W_e = W_i$$

$$P_n\left(3\theta \times \frac{L}{4}\right) = M_n(\theta + 4\theta + 3\theta)$$

$$P_n = \frac{M_n\, 8\theta \times 4}{L\,\theta\, 3} = \frac{10.67 M_n}{L} = \frac{10.67 M_n}{32} = .33 M_n$$

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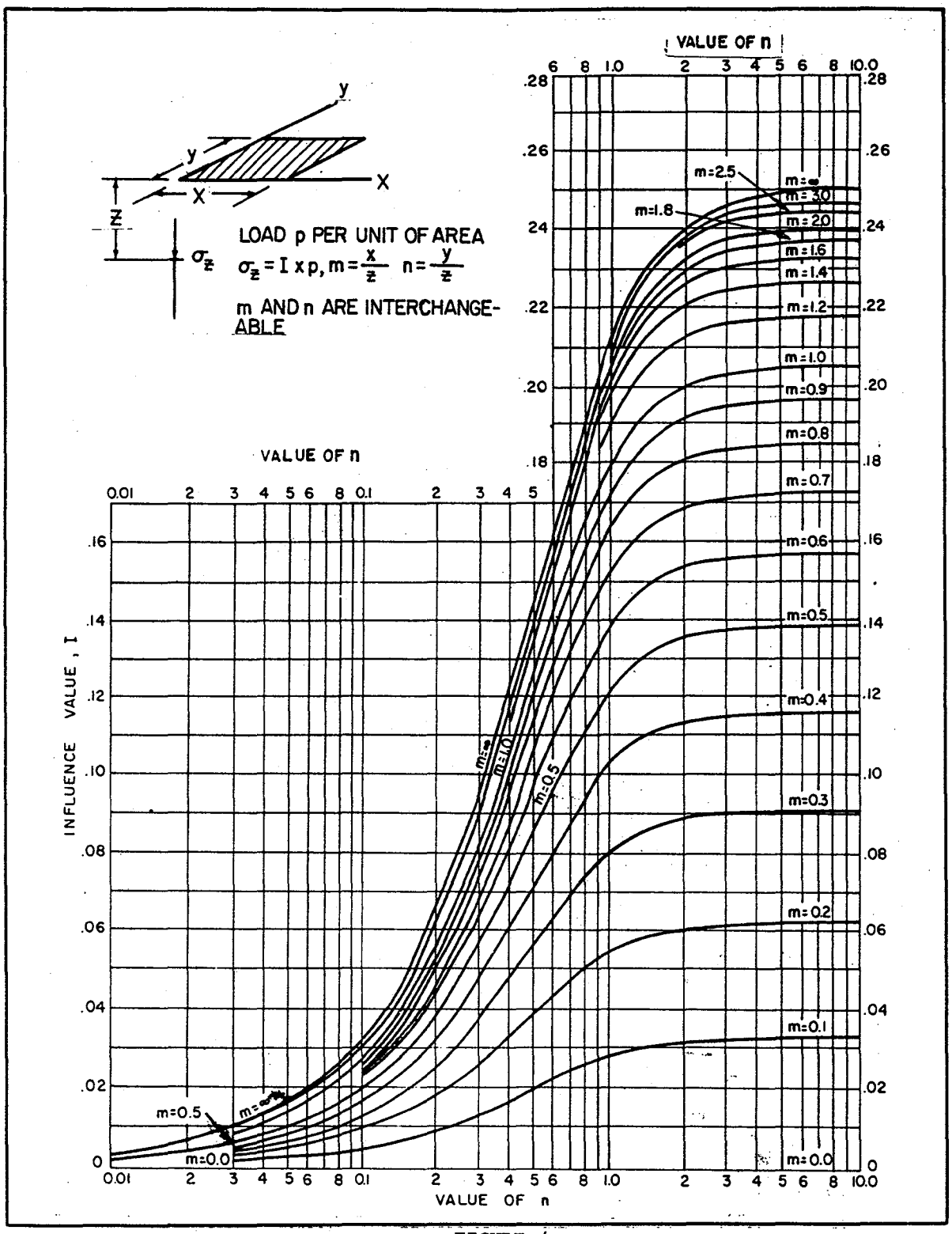
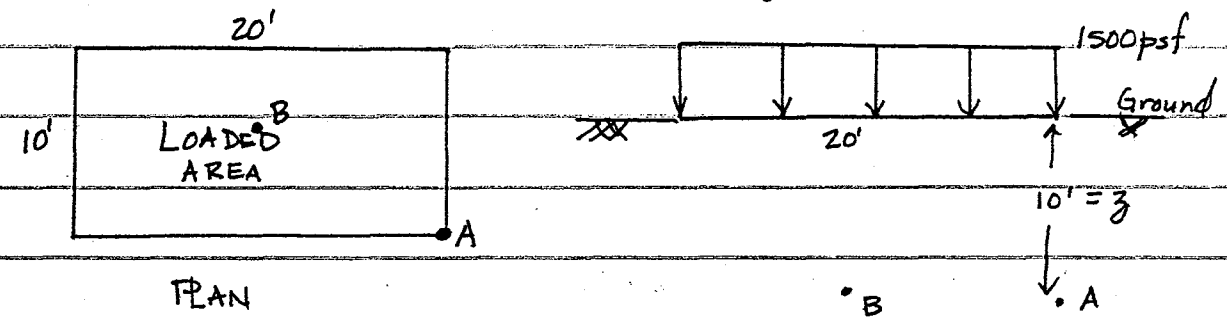


FIGURE 4
Influence Value for Vertical Stress Beneath a Corner of a Uniformly Loaded Rectangular Area (Boussinesq Case)

EXAMPLES

1.) Find the Δstress at a point 10 feet below the corner of a rectangular footing 20 feet long and 10 feet wide, carrying 1500psf. (POINT A)



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from chart, for $n = 2.0$ (bottom axis) & $m = 1.0$ (curves)

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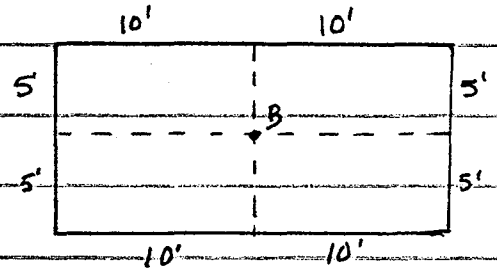

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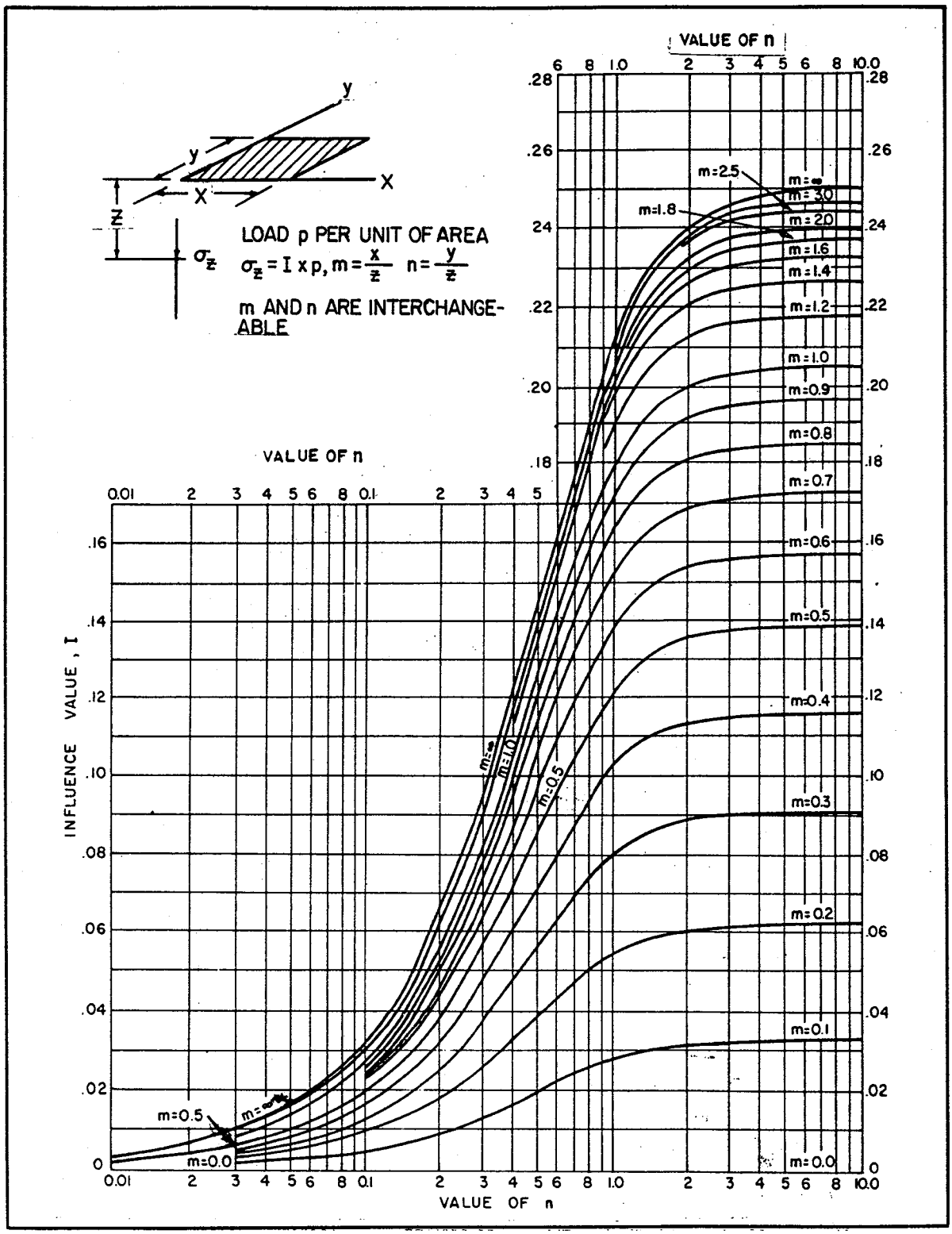
(Any shape can be modeled by breaking into rect.'s)

So $\Delta\sigma = (.125 + .125 + .125 + .125)(1500psf)$ ← one for each ▭

$= 750$ psf



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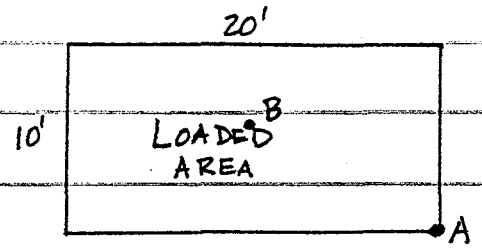
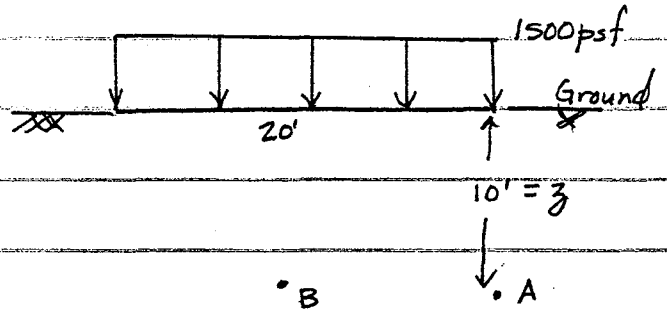


PLAN



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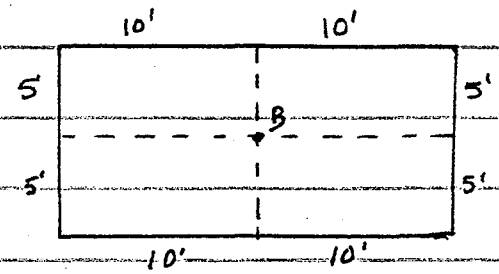

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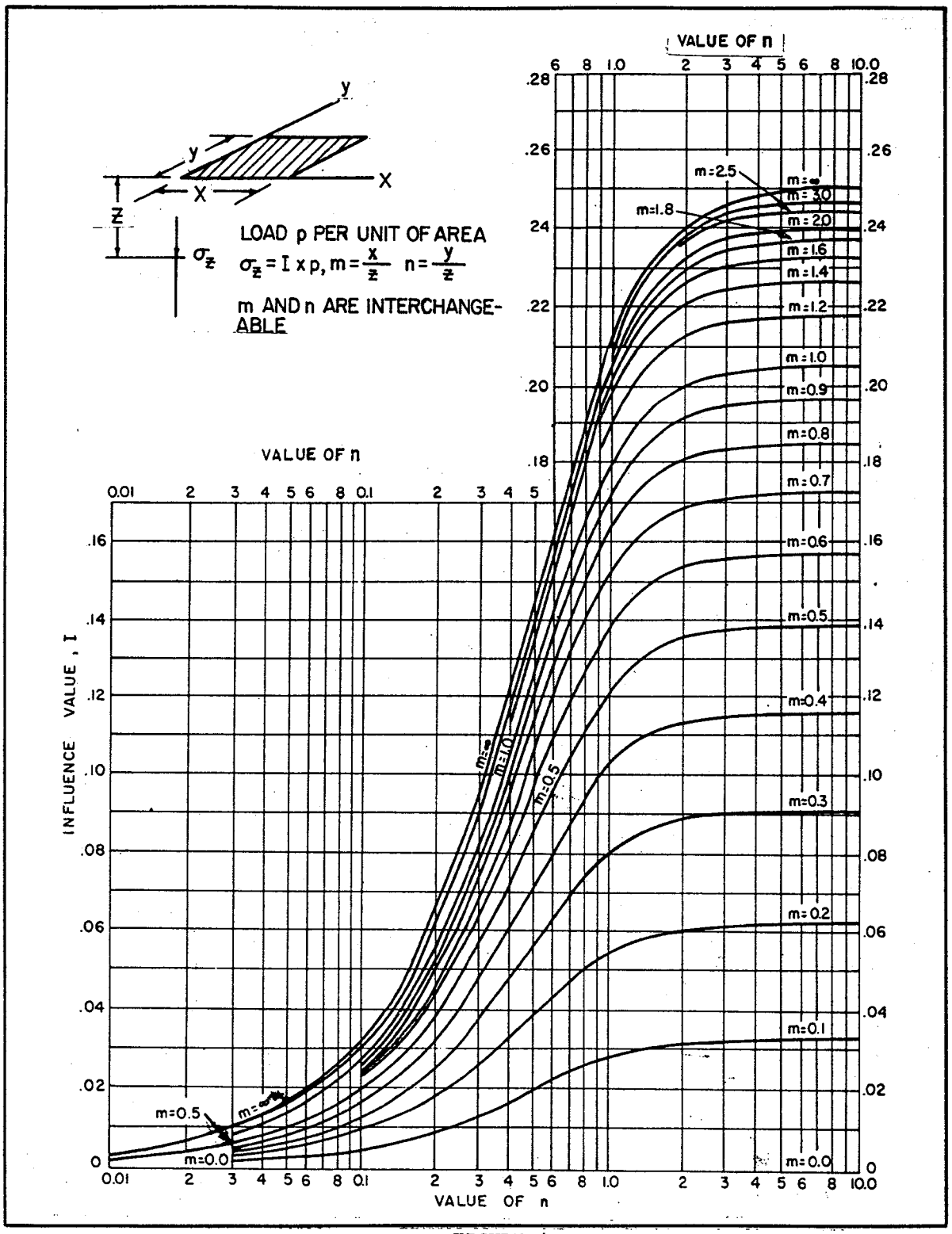

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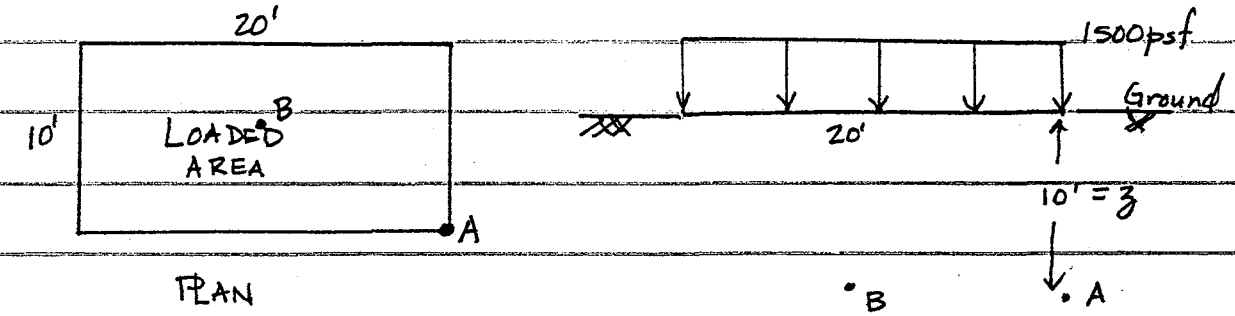

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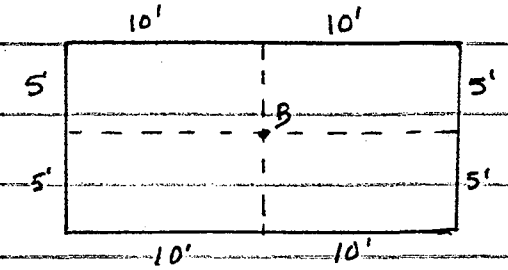

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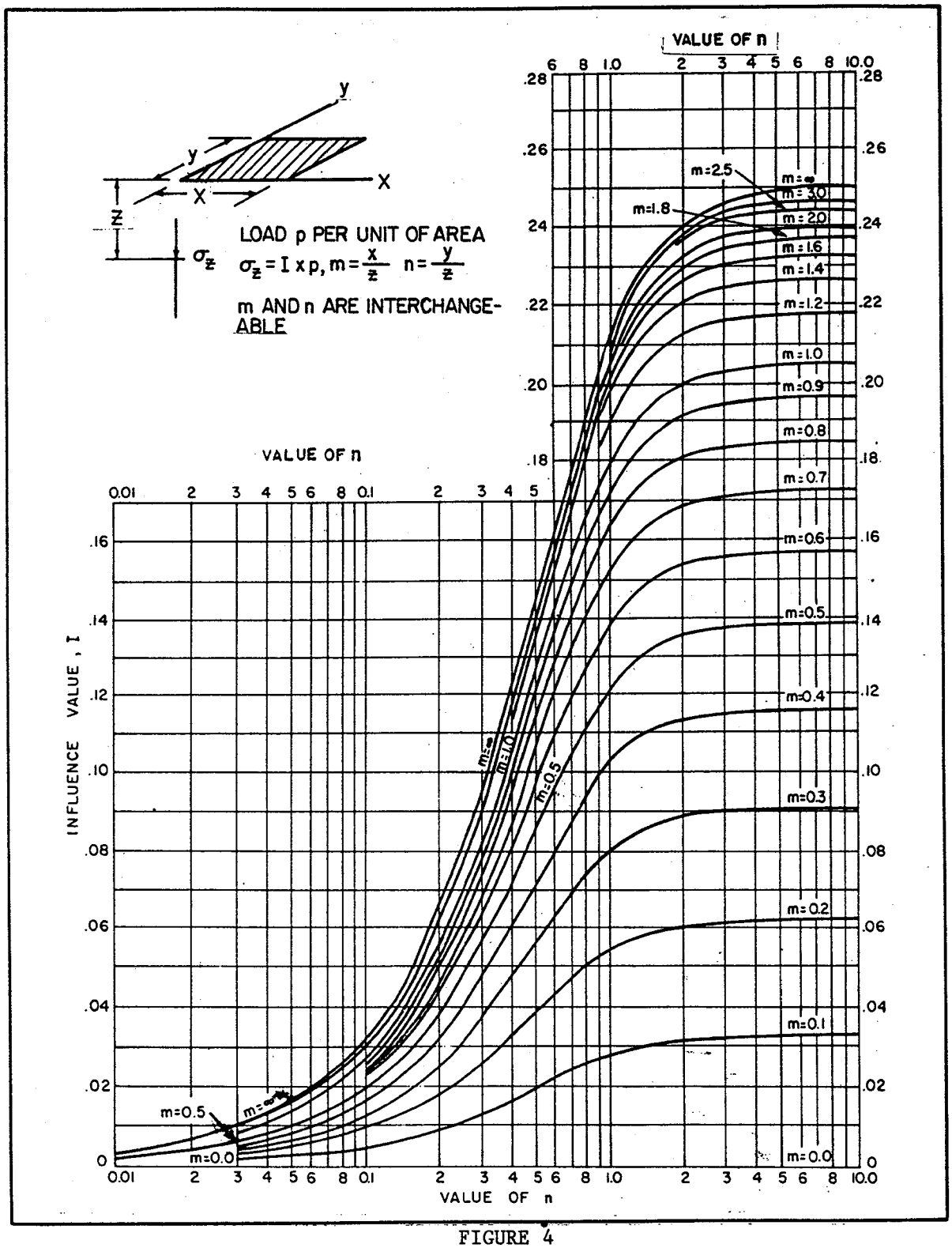

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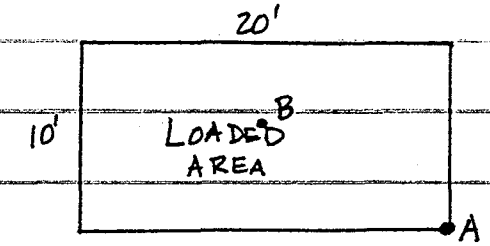
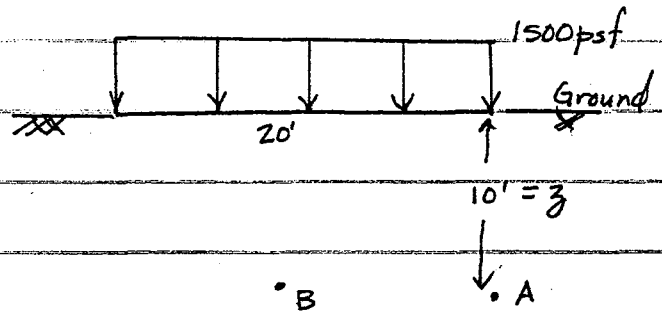


PLAN



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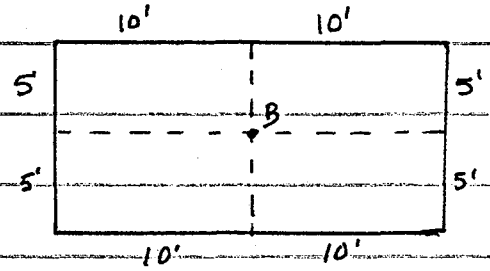

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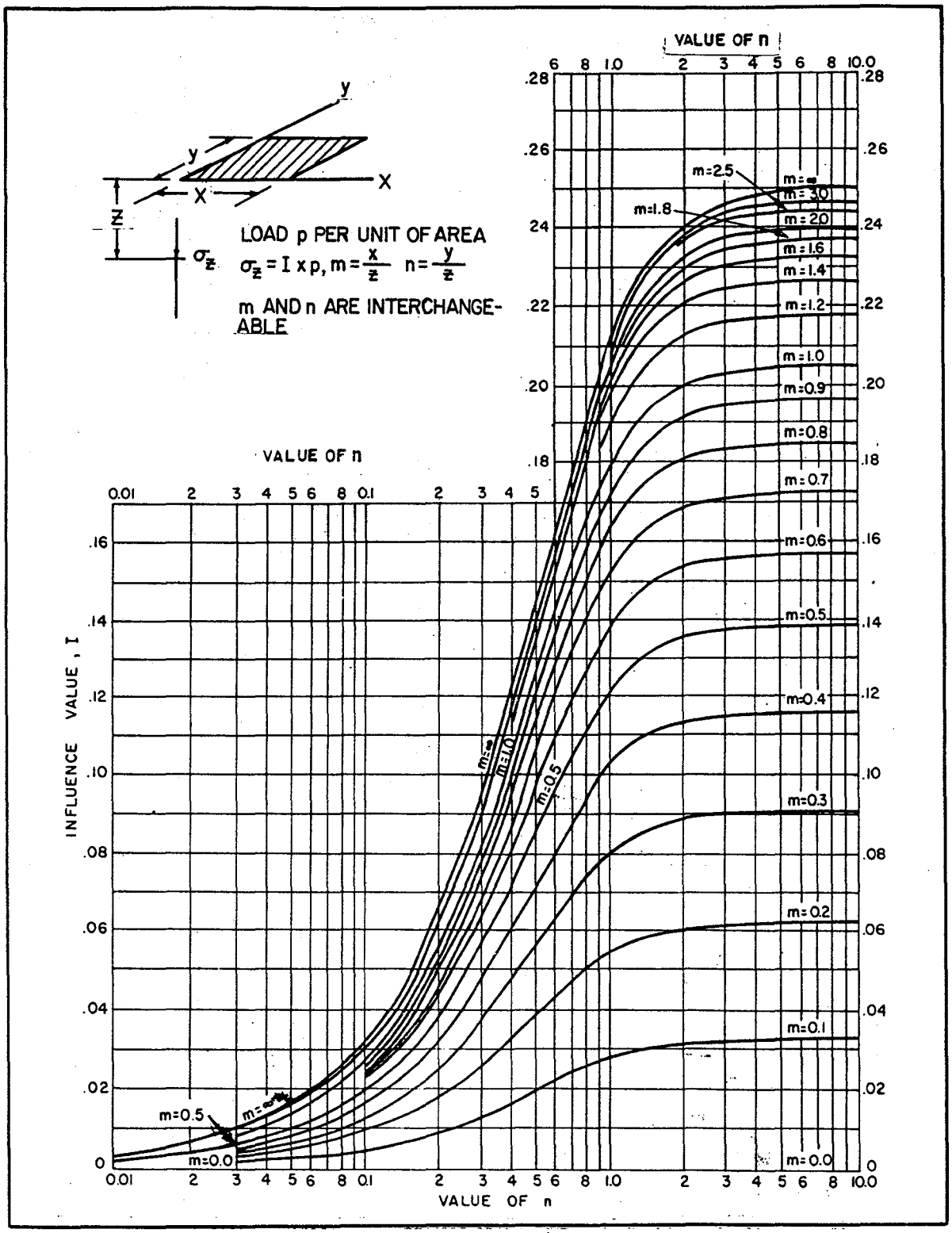

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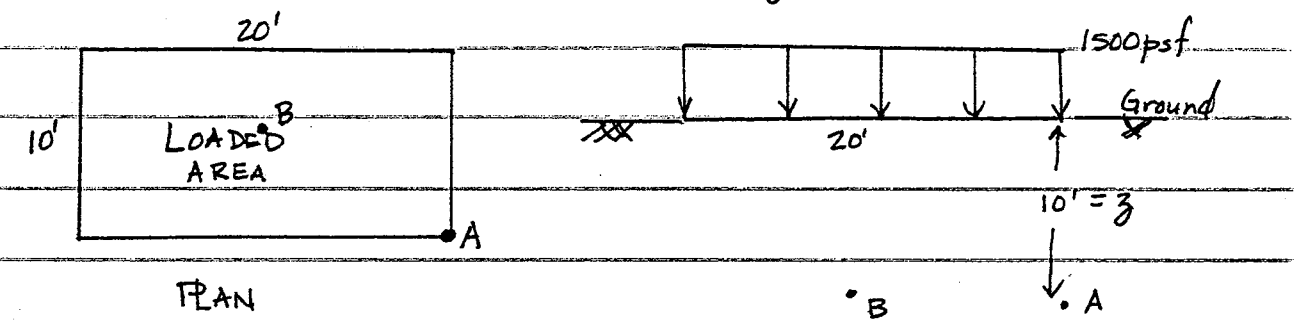

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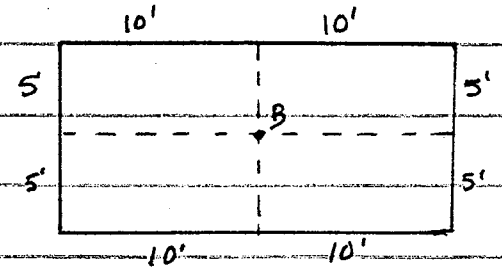

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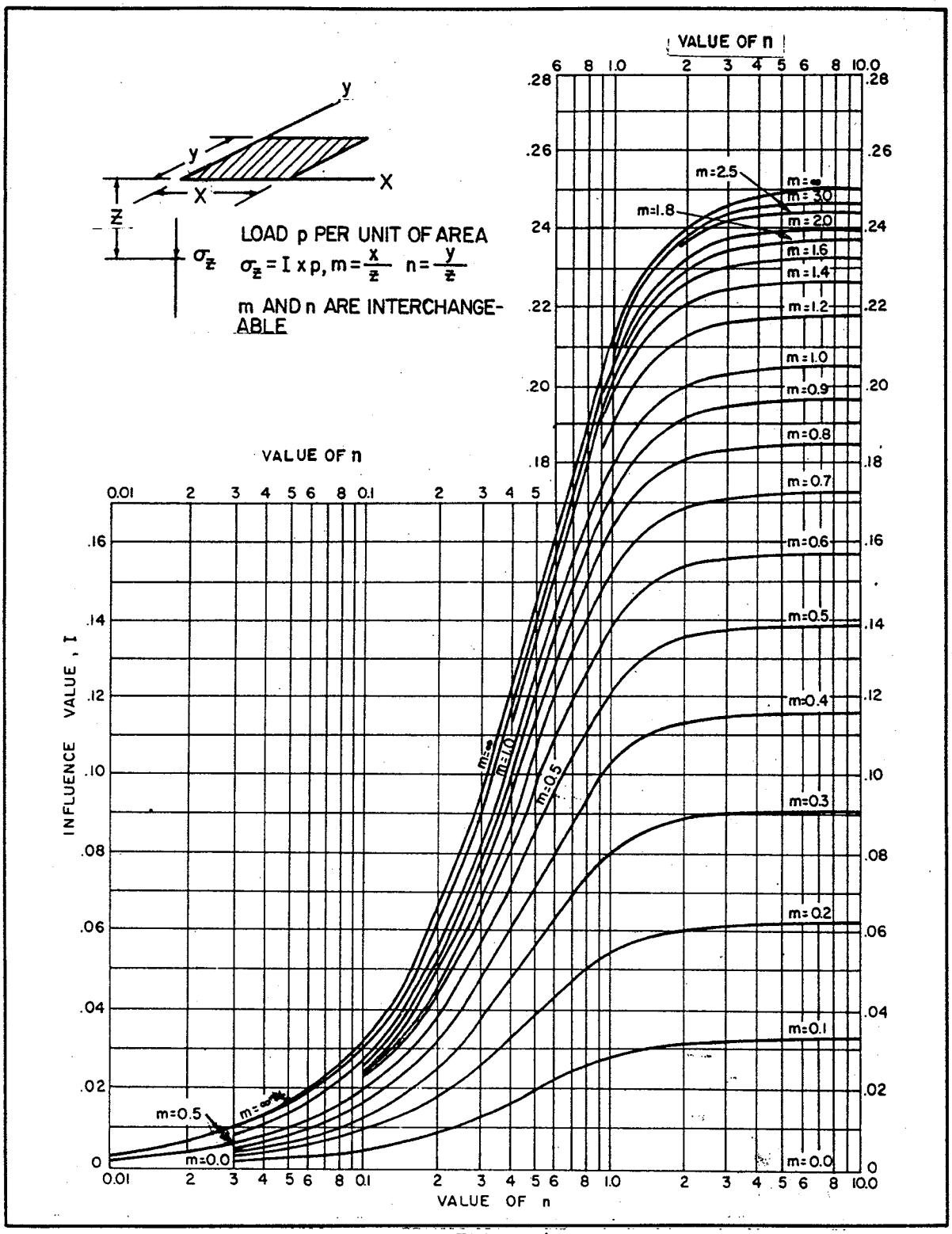

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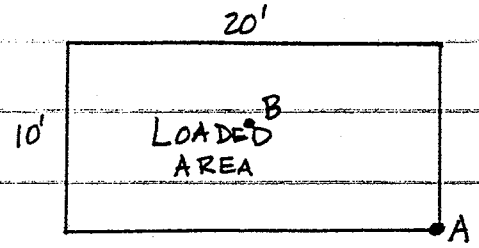
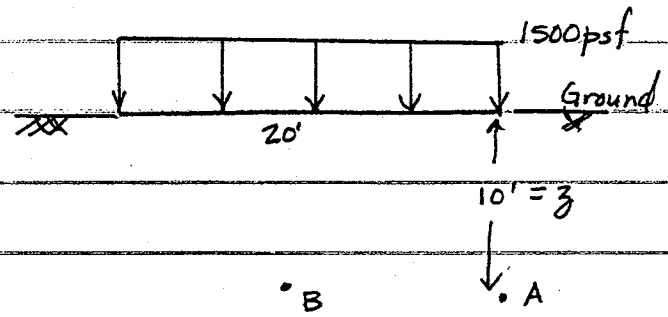


PLAN



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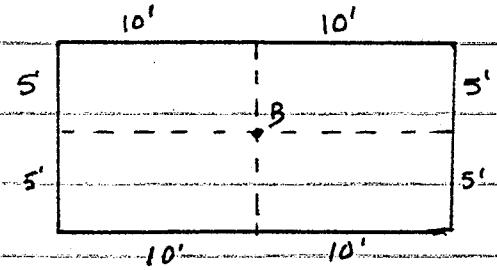

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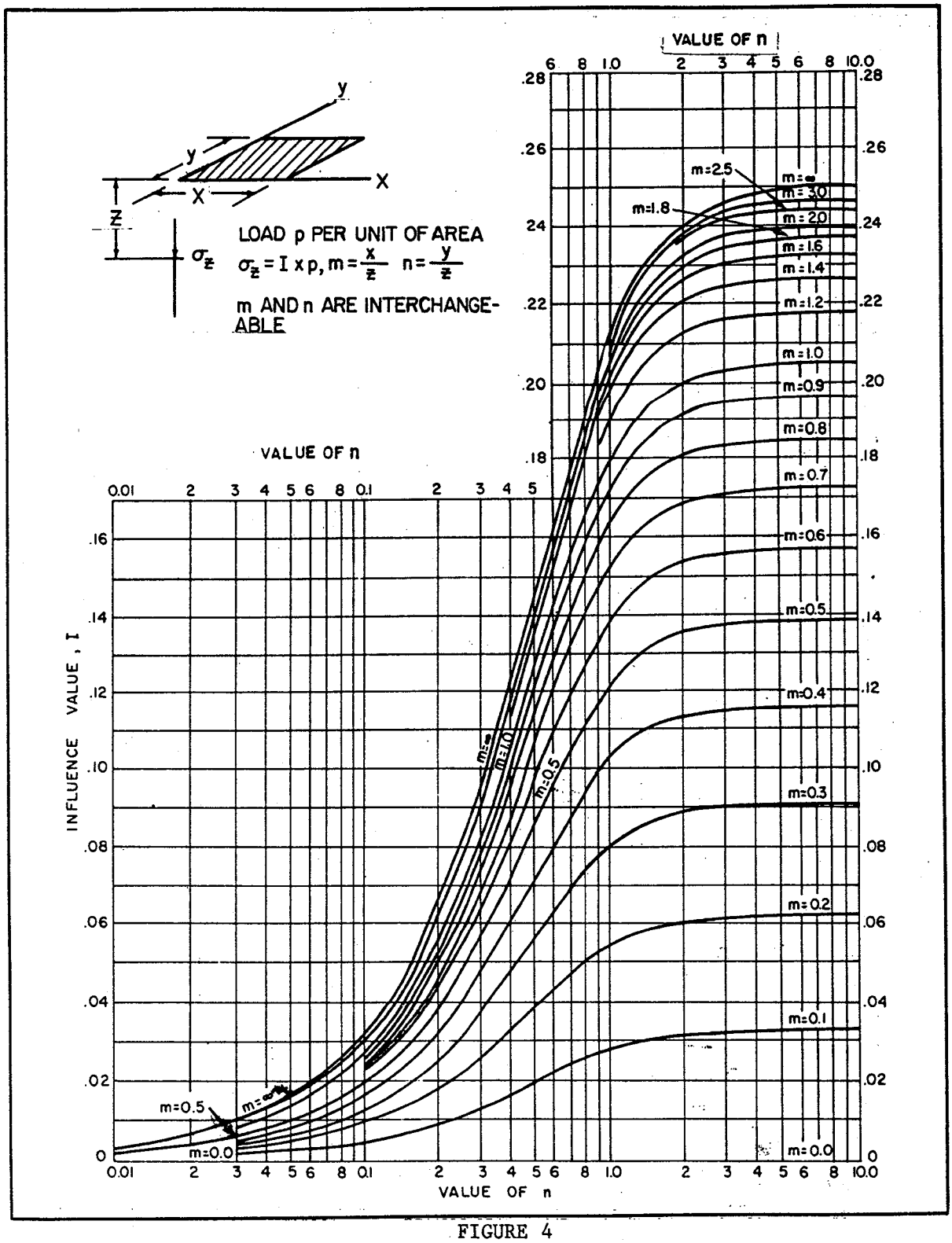

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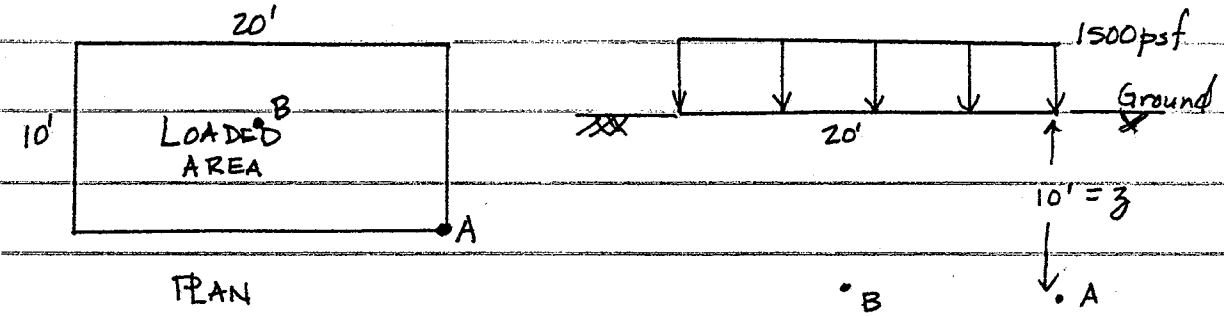

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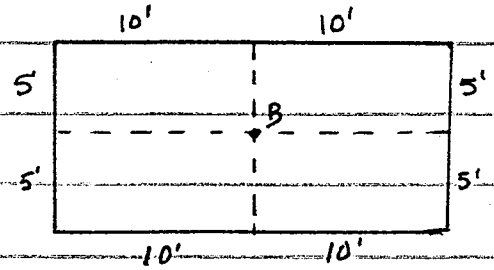
from chart, for $n = 2.0$ (bottom axis) & $m = 1.0$ (curves)

$I = 0.20$ (left axis)

$\therefore \Delta\sigma = 0.20\,(1500psf) = 300psf.$

2.) Under the <u>center</u> of the loaded area.

Use superposition (Boussinesq is a linear elastic theory) and treat as <u>4</u> rectangles, each 5' x 10'



Choose $x = 10'$ $\quad y = 5'$

$m = \frac{x}{z} = \frac{10'}{10'} = 1.0$

$n = \frac{y}{z} = \frac{5}{10} = 0.5$

$\therefore I = .125$ for each rectangle

one for each ▭

So $\Delta\sigma = (.125 + .125 + .125 + .125)(1500psf)$

$= 750$ psf

(Any shape can be modeled by breaking into rect.'s)

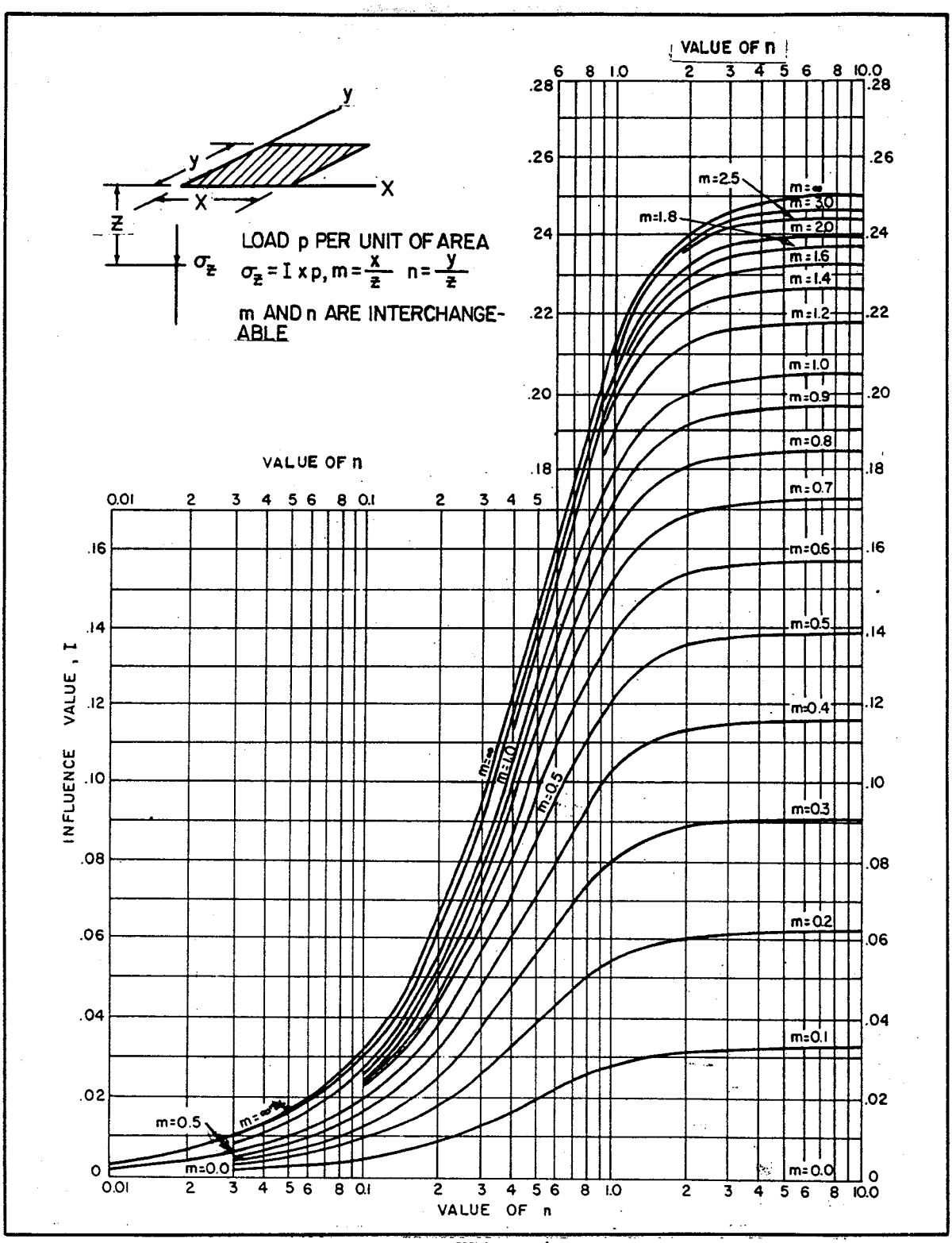
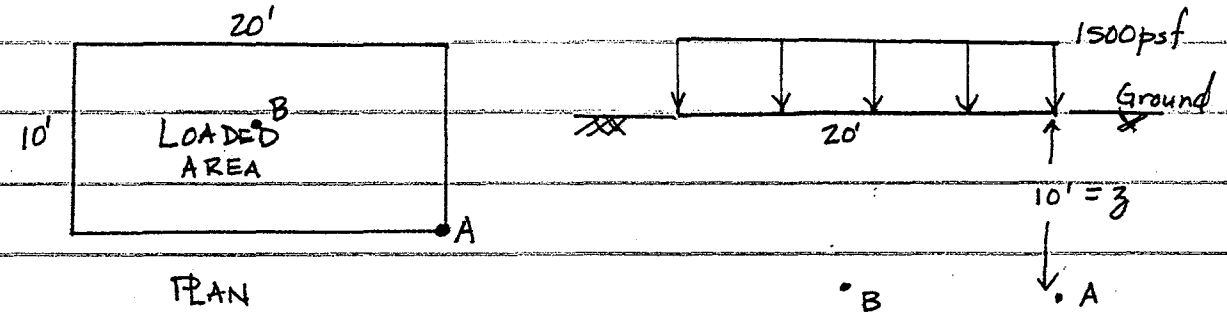


FIGURE 4
Influence Value for Vertical Stress Beneath a Corner of a Uniformly Loaded Rectangular Area (Boussinesq Case)

EXAMPLES

1.) Find the Δstress at a point 10 feet below the corner of a rectangular footing 20 feet long and 10 feet wide, carrying 1500psf. (POINT A)



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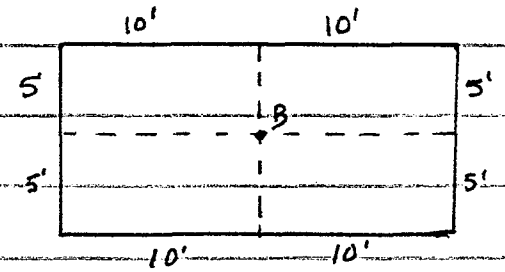

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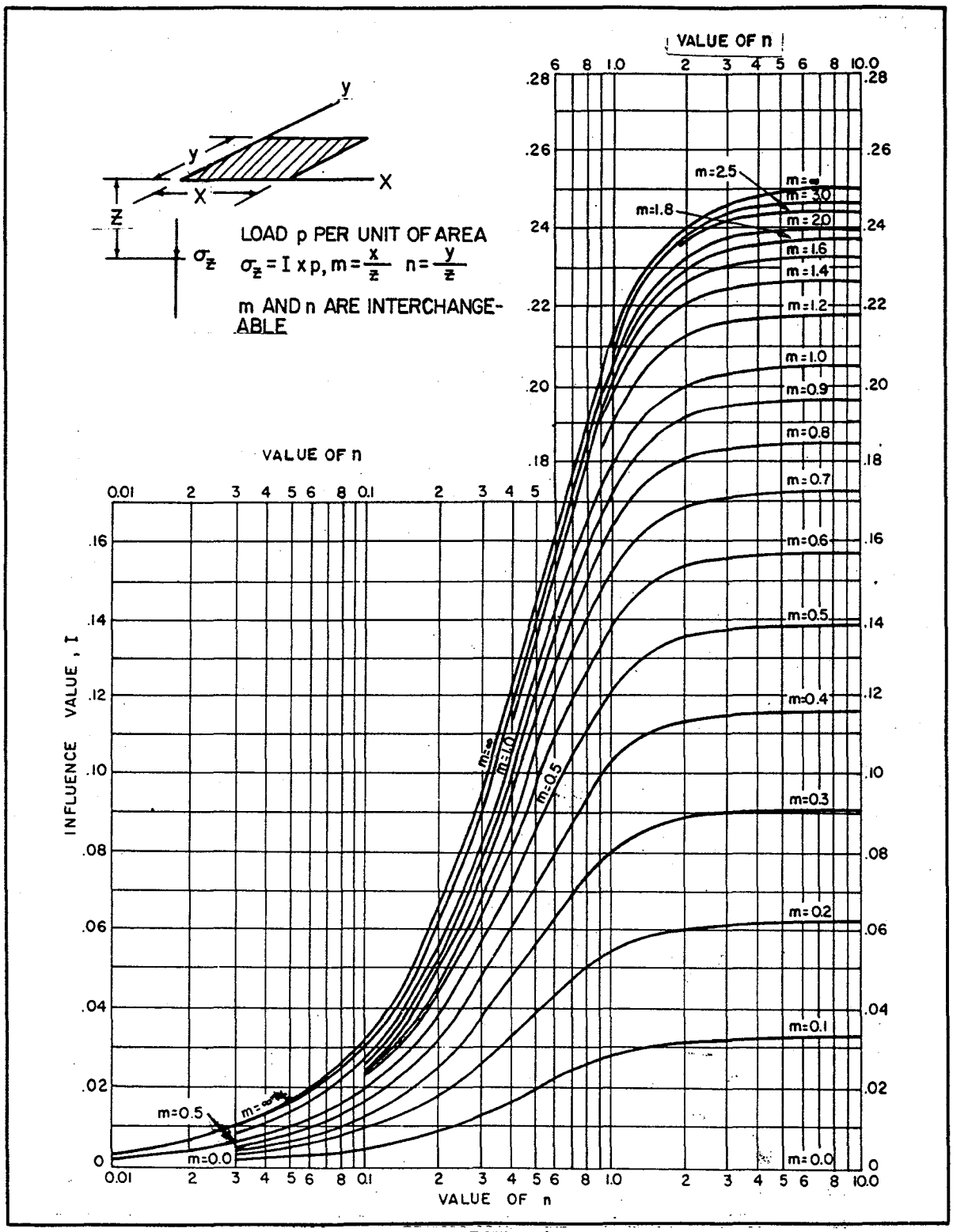

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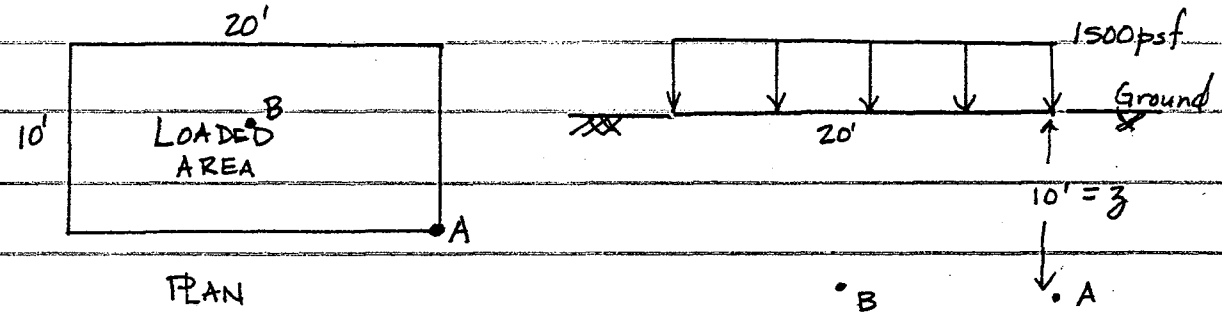

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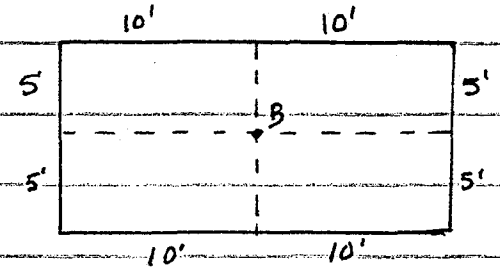

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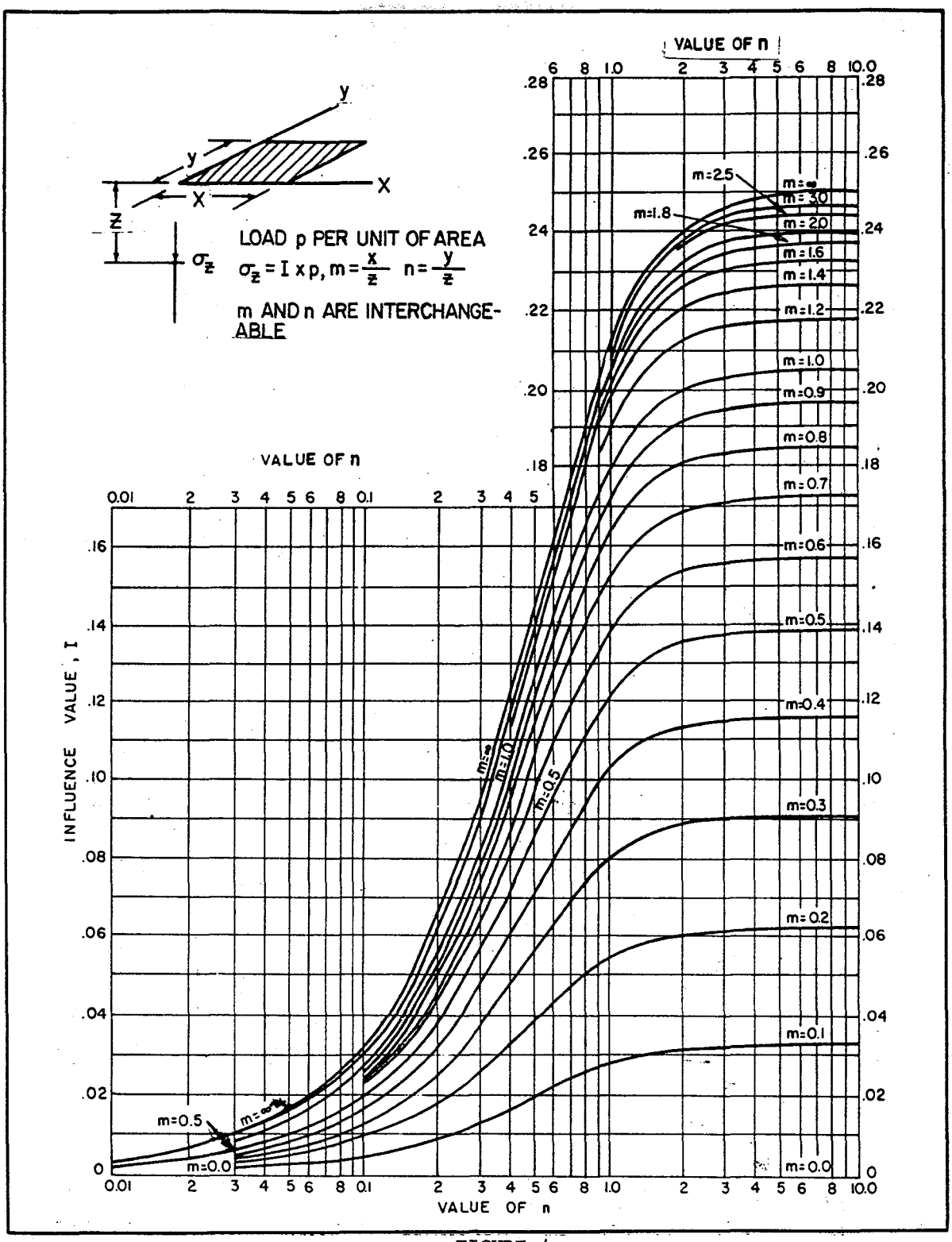

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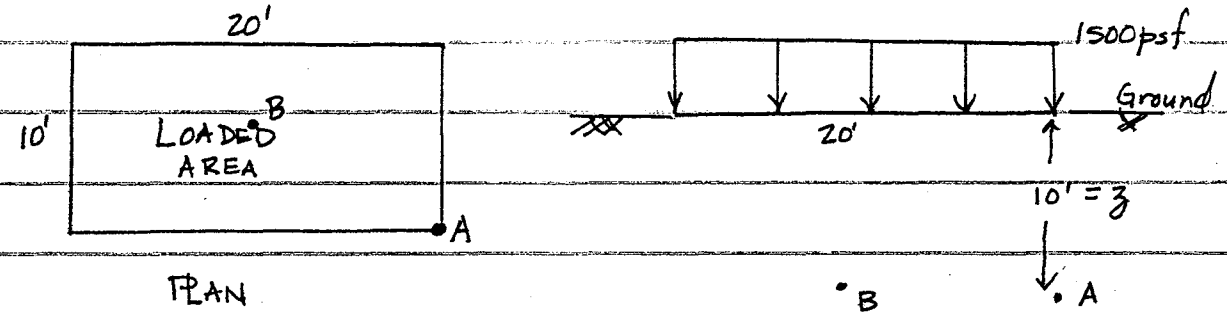

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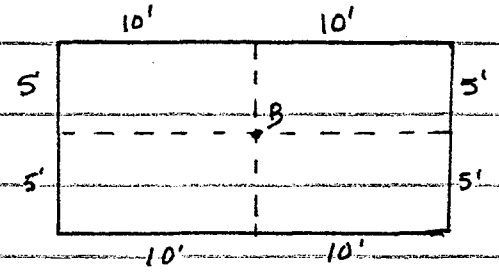

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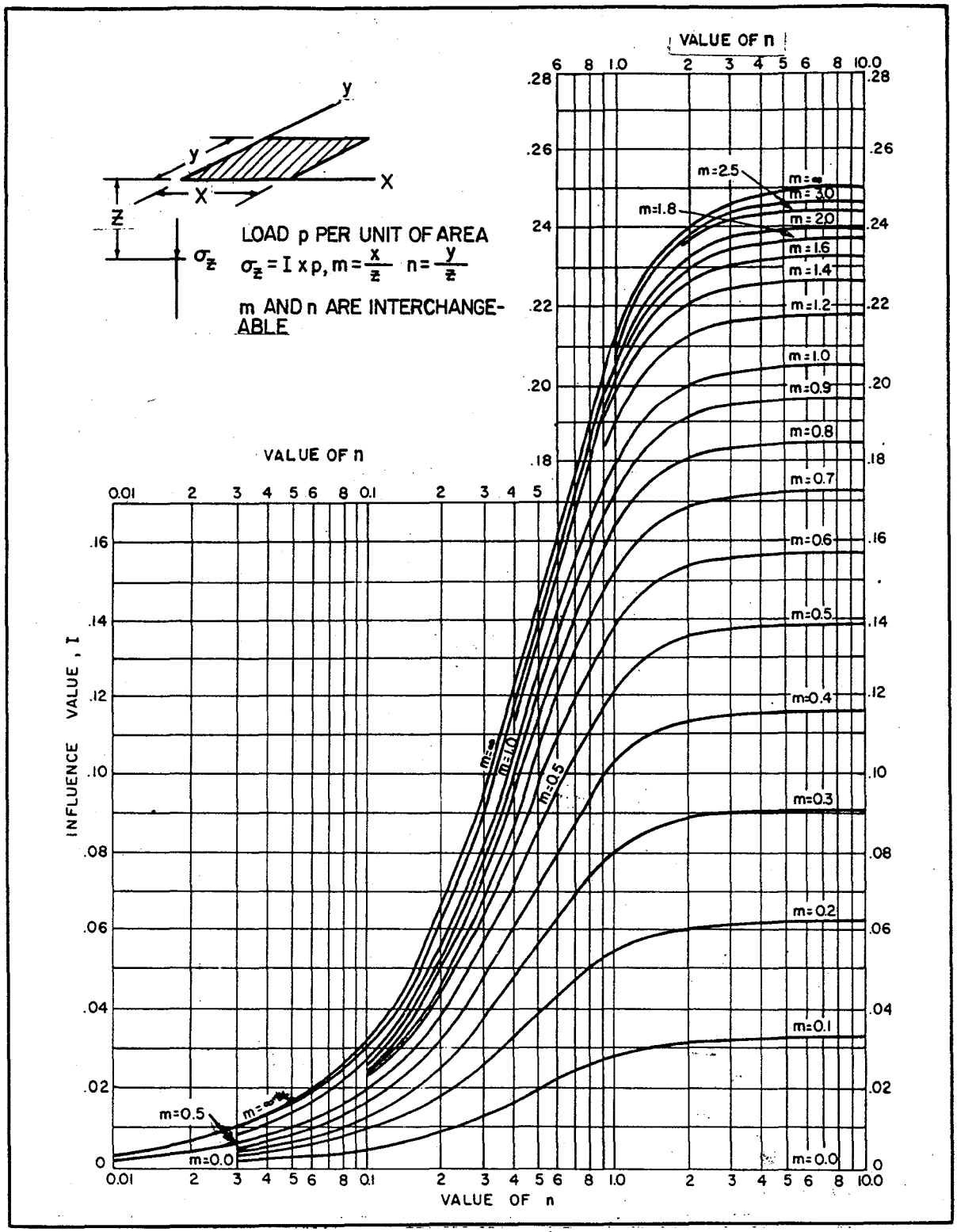

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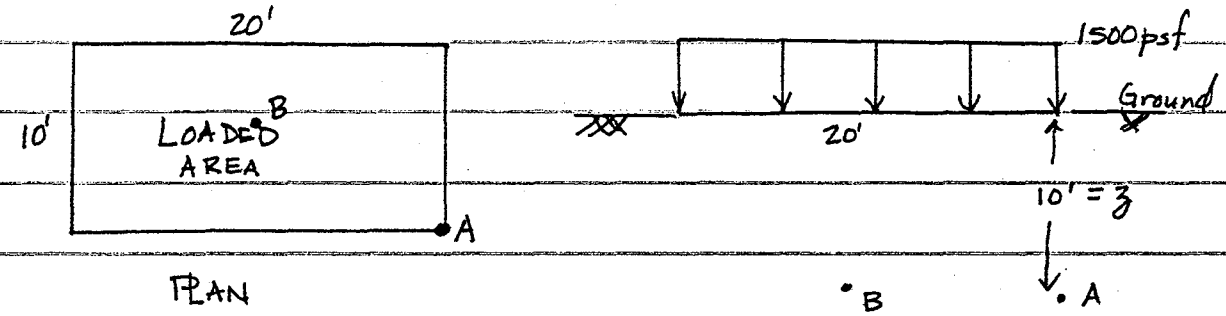

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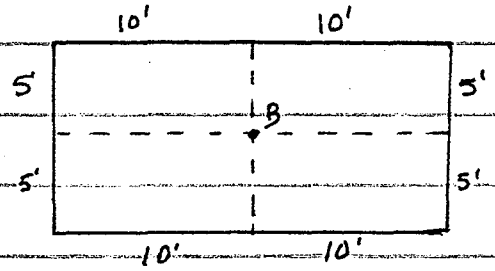

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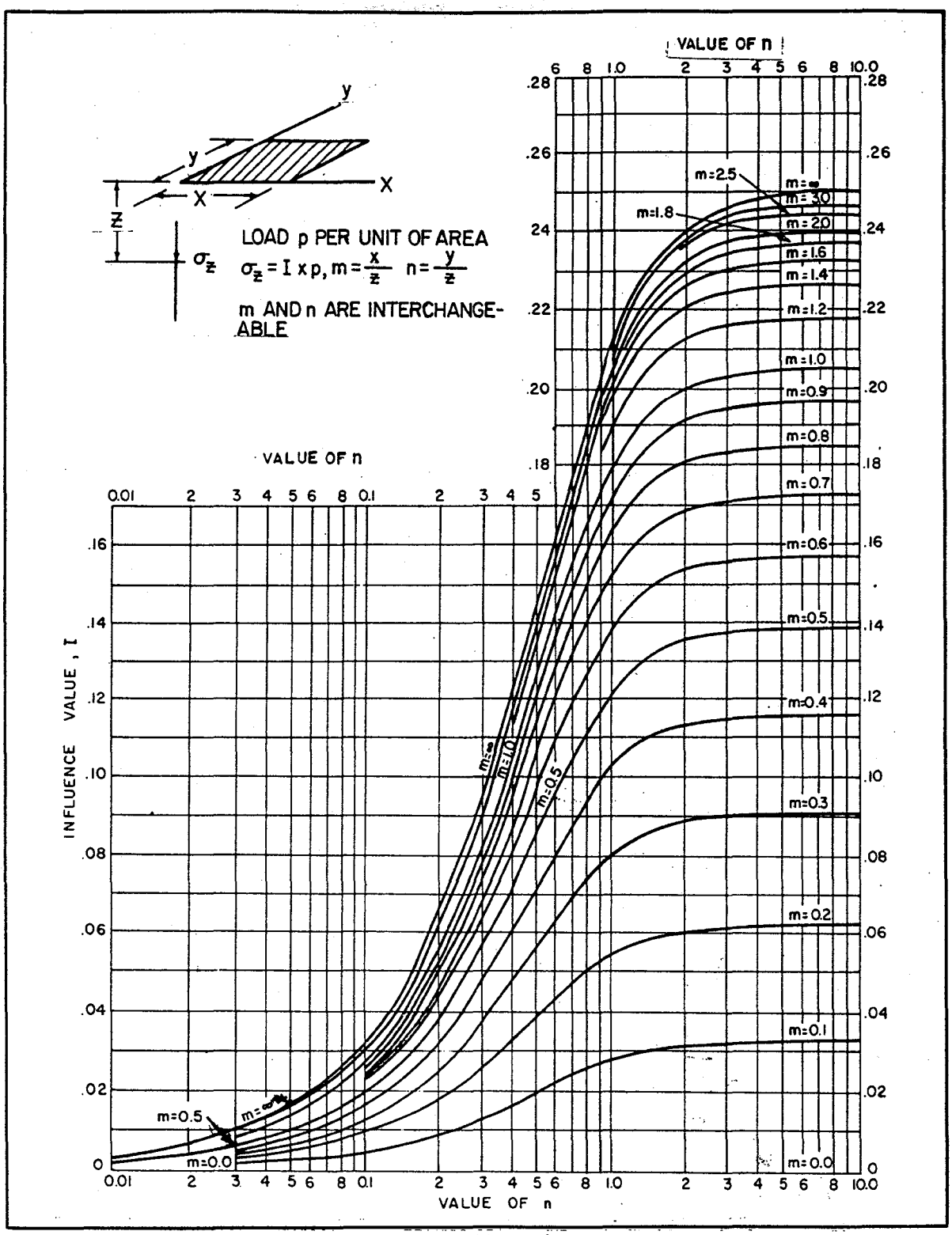

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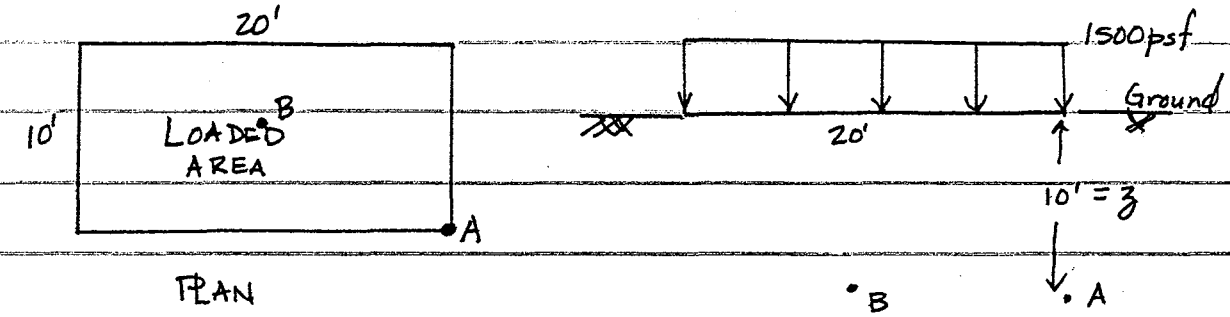

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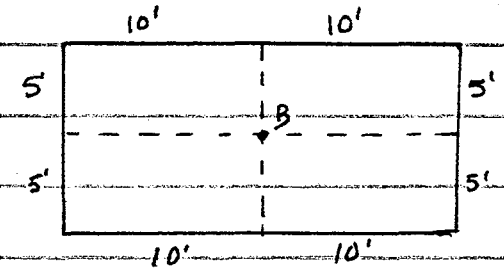

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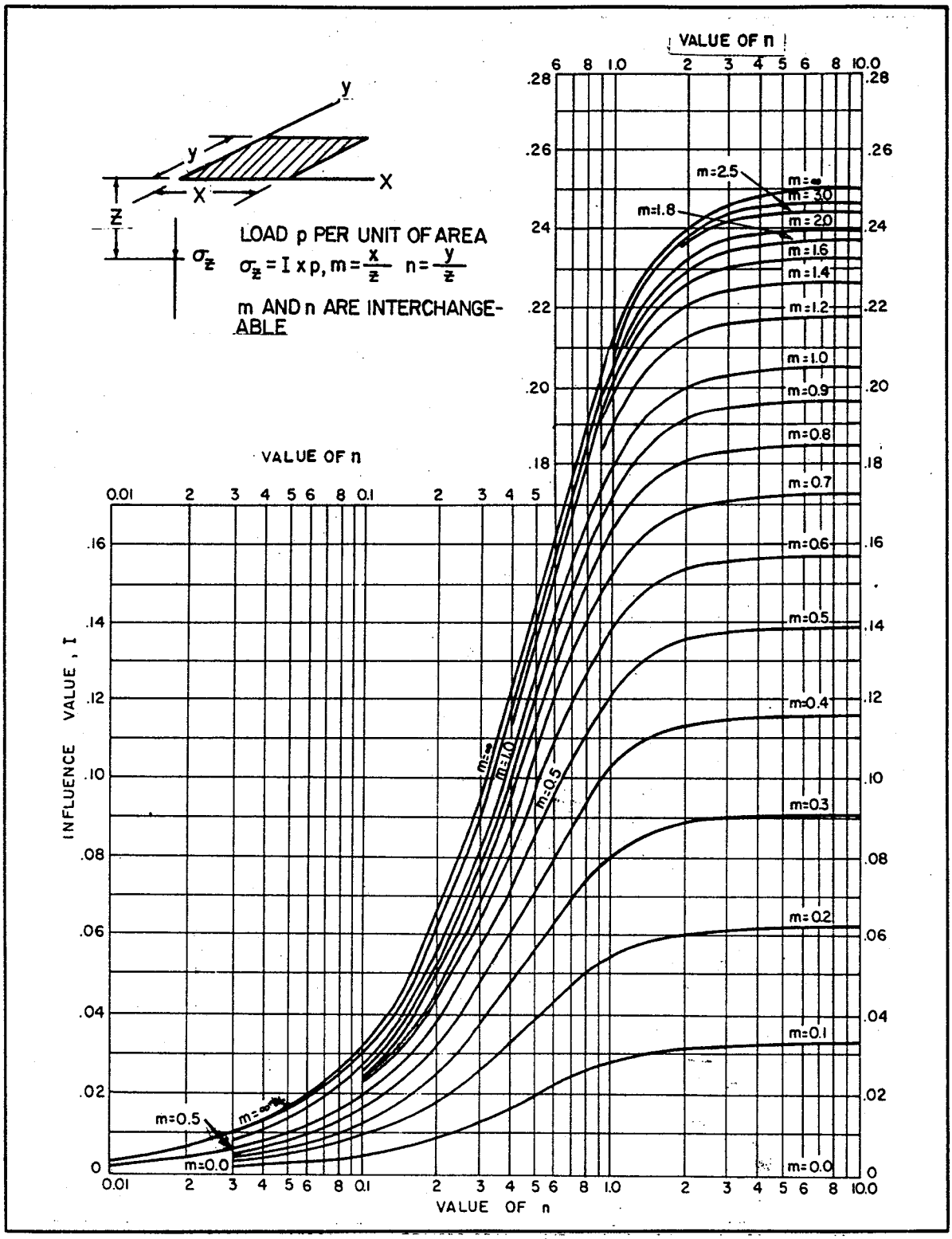

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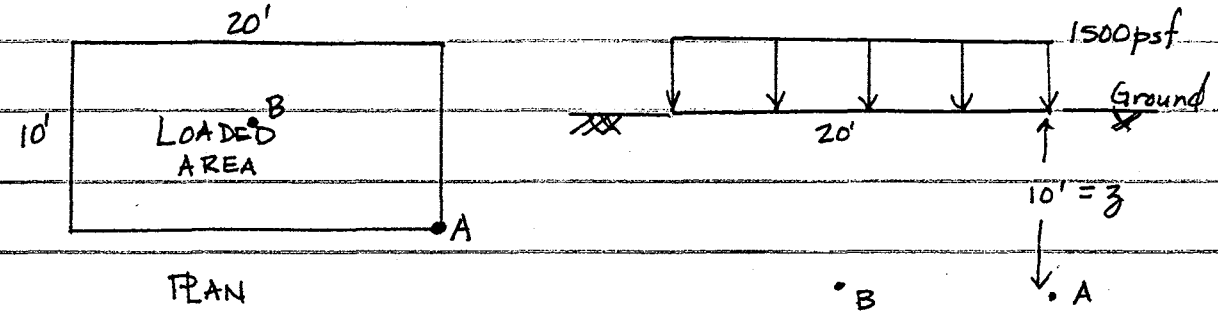

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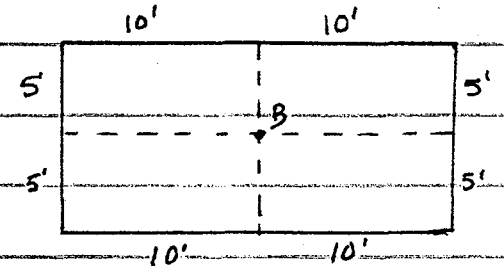

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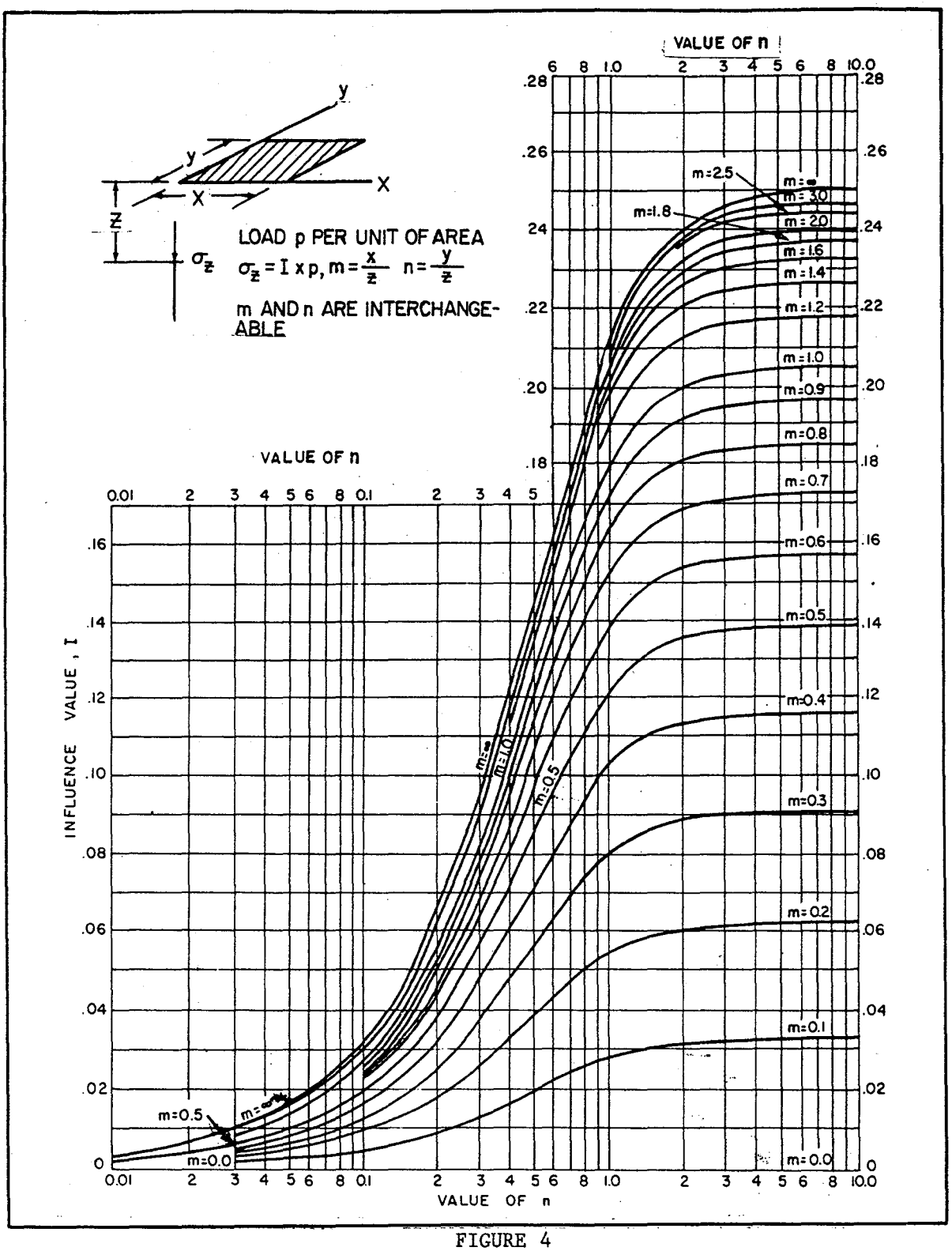

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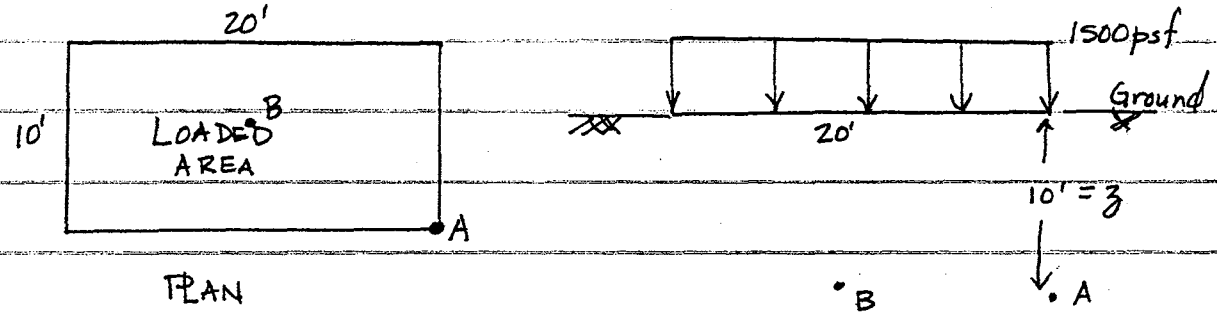

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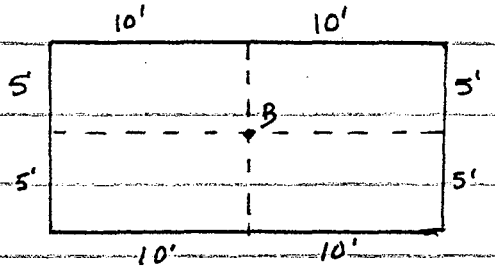

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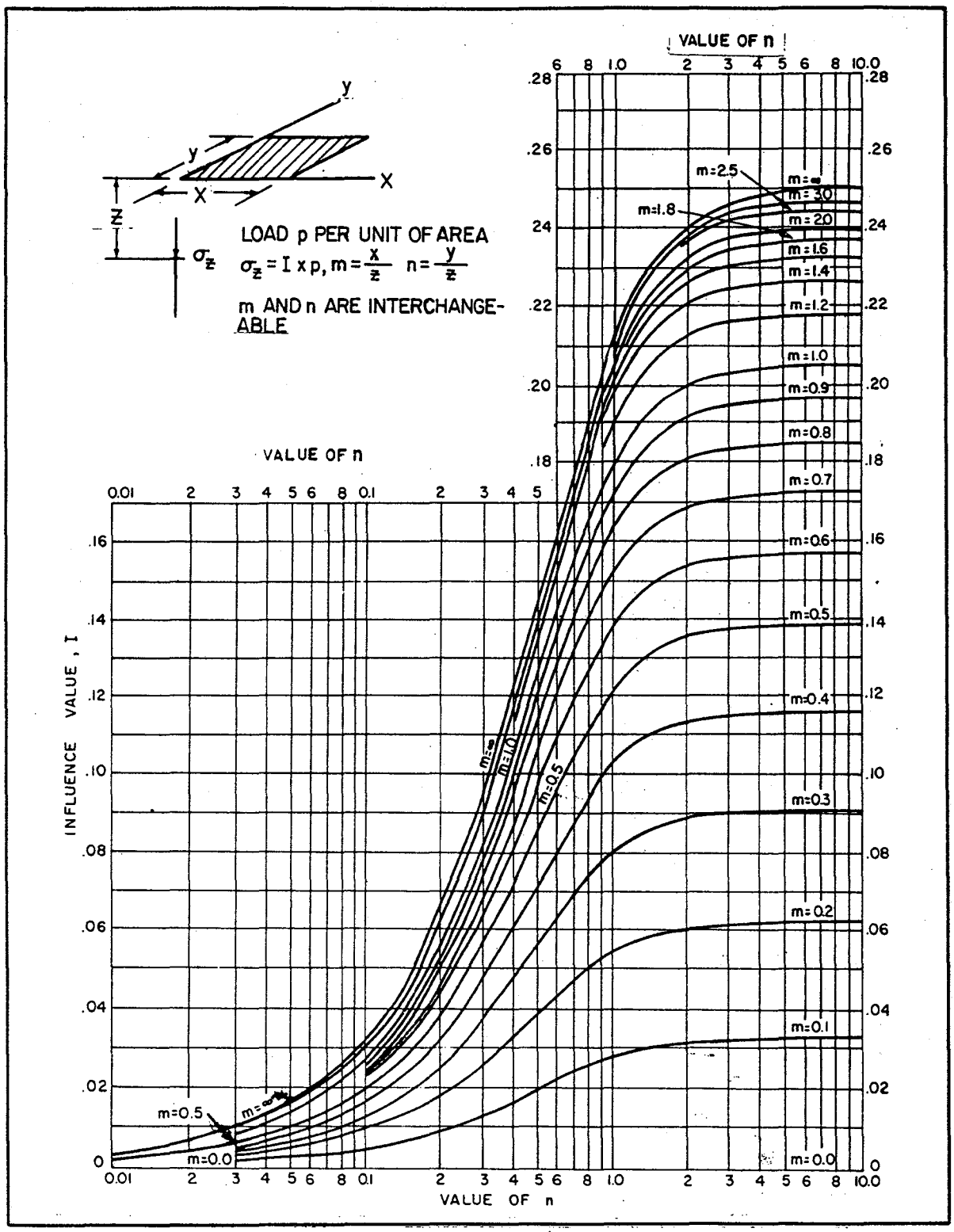

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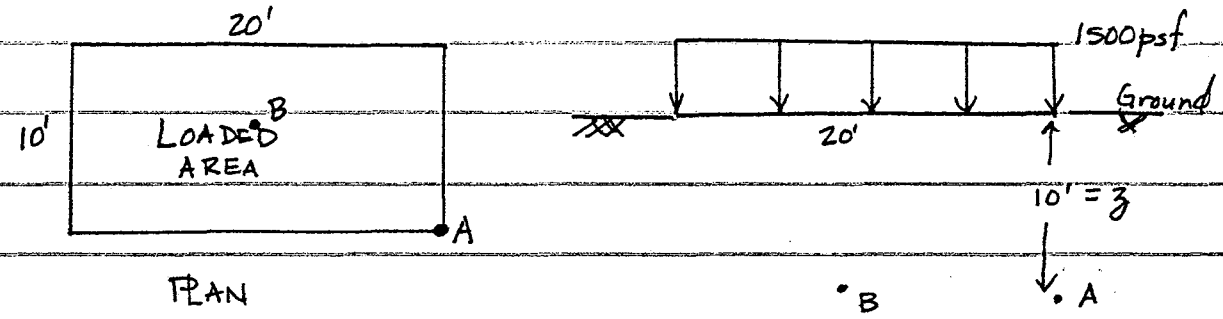

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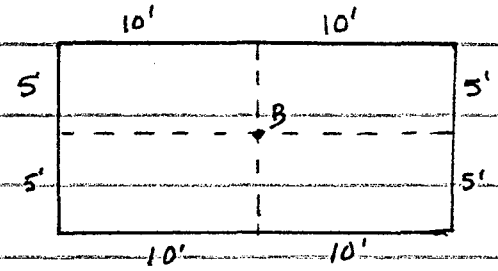

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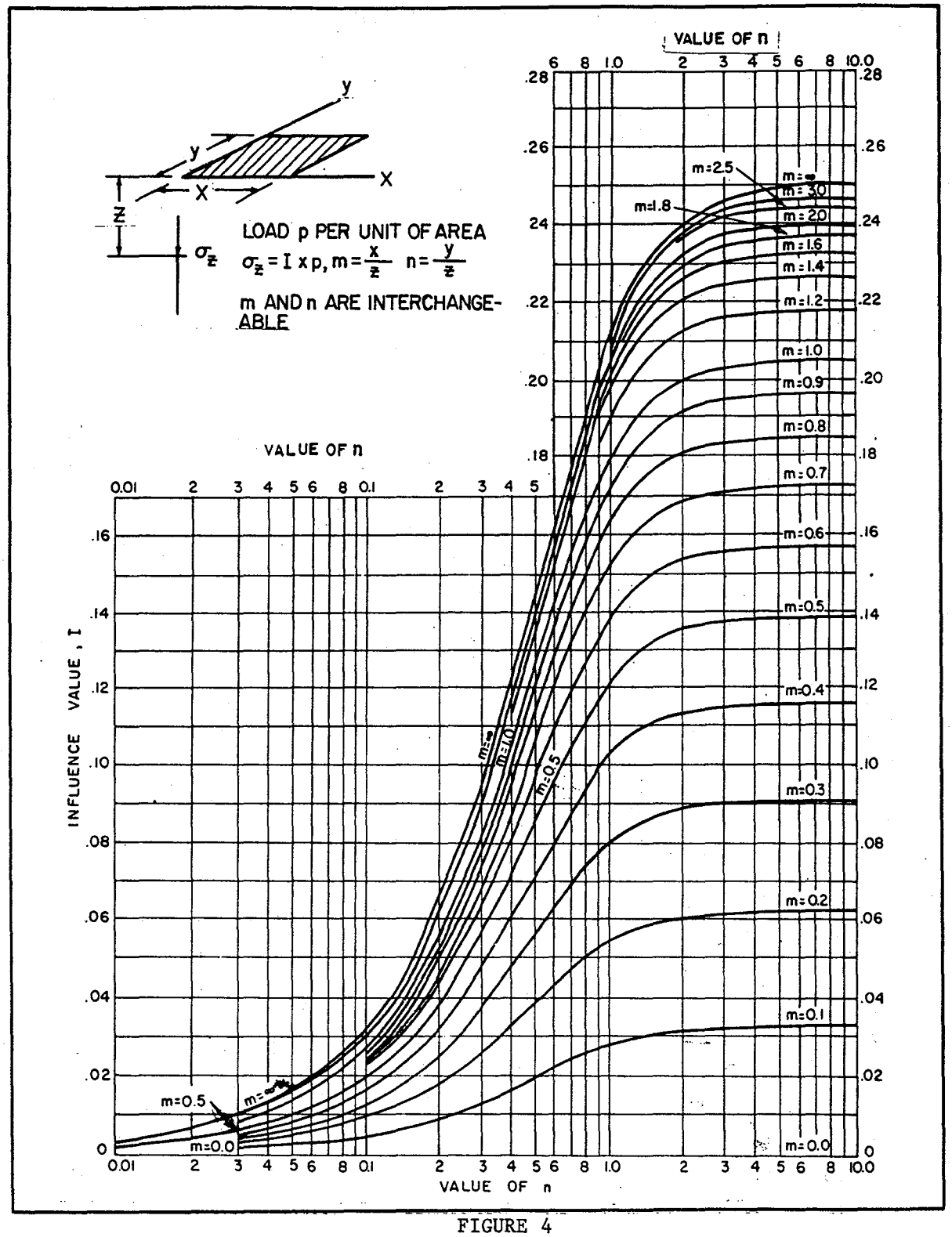

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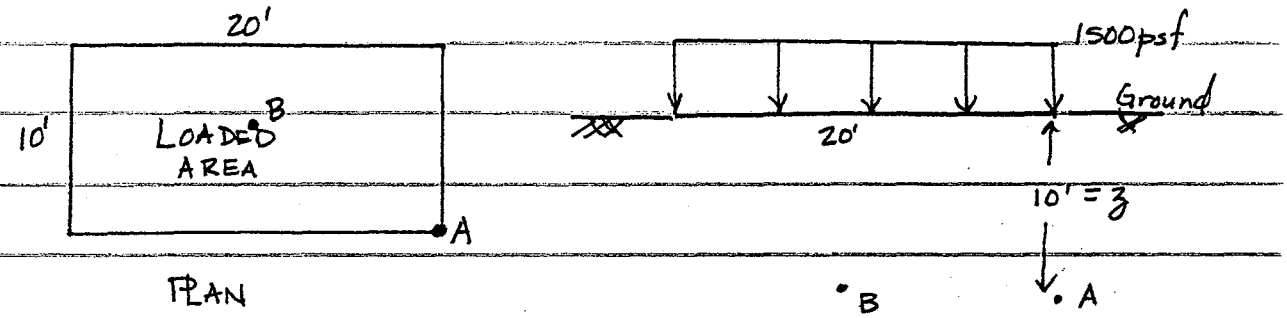

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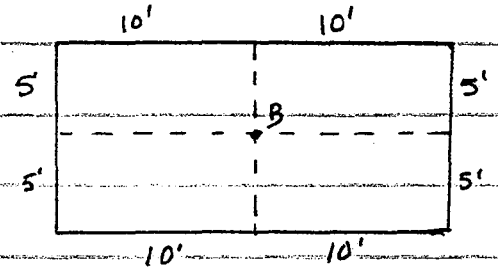

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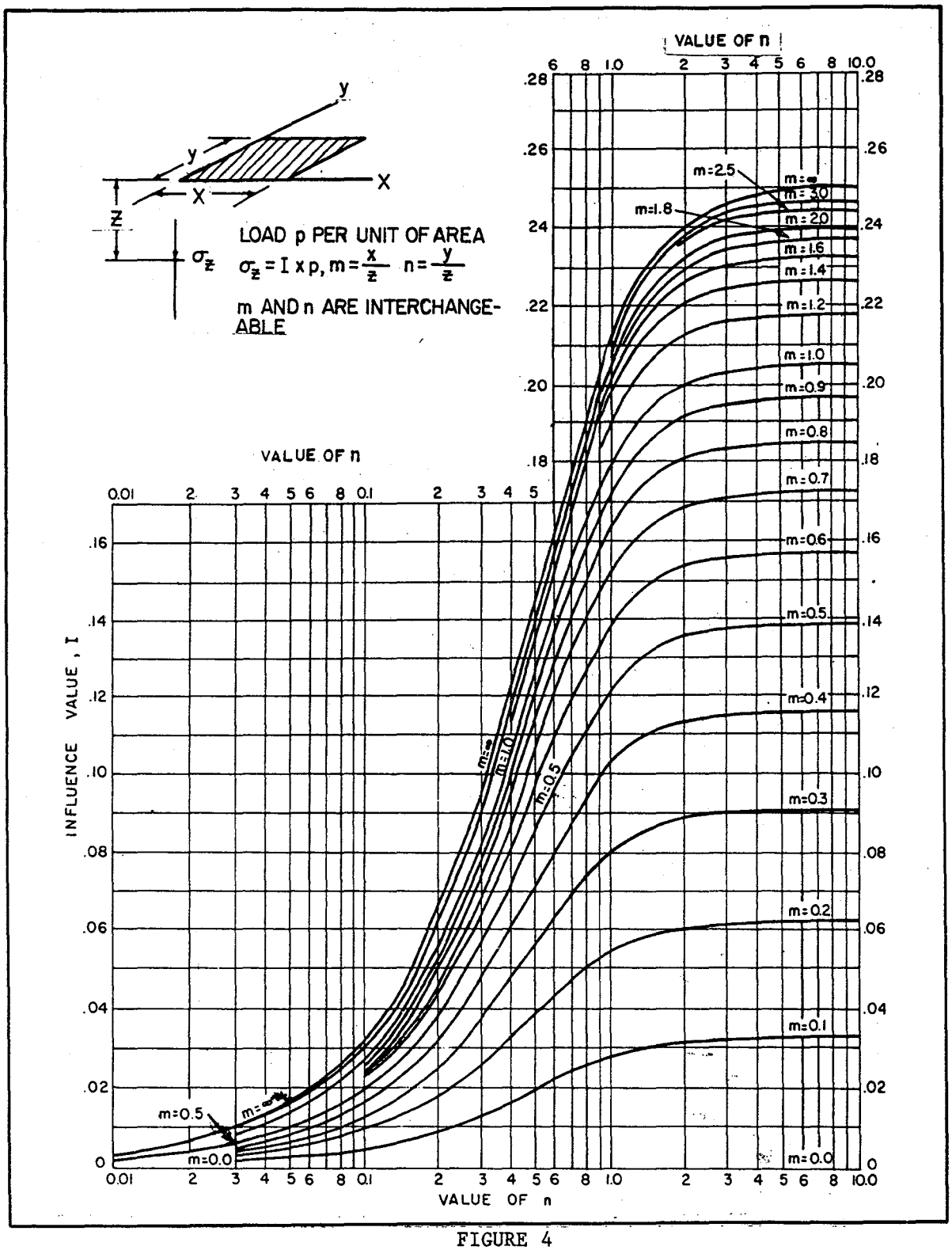

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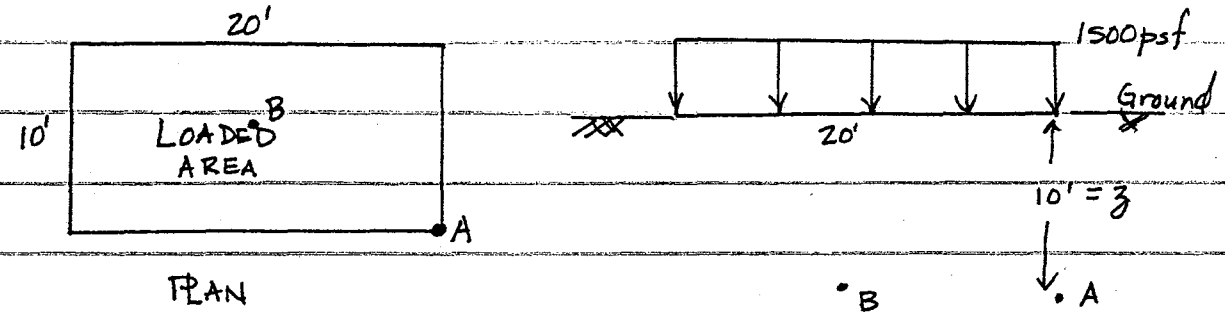

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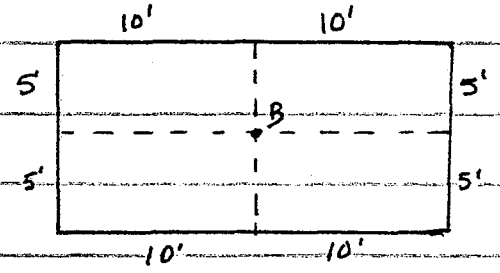

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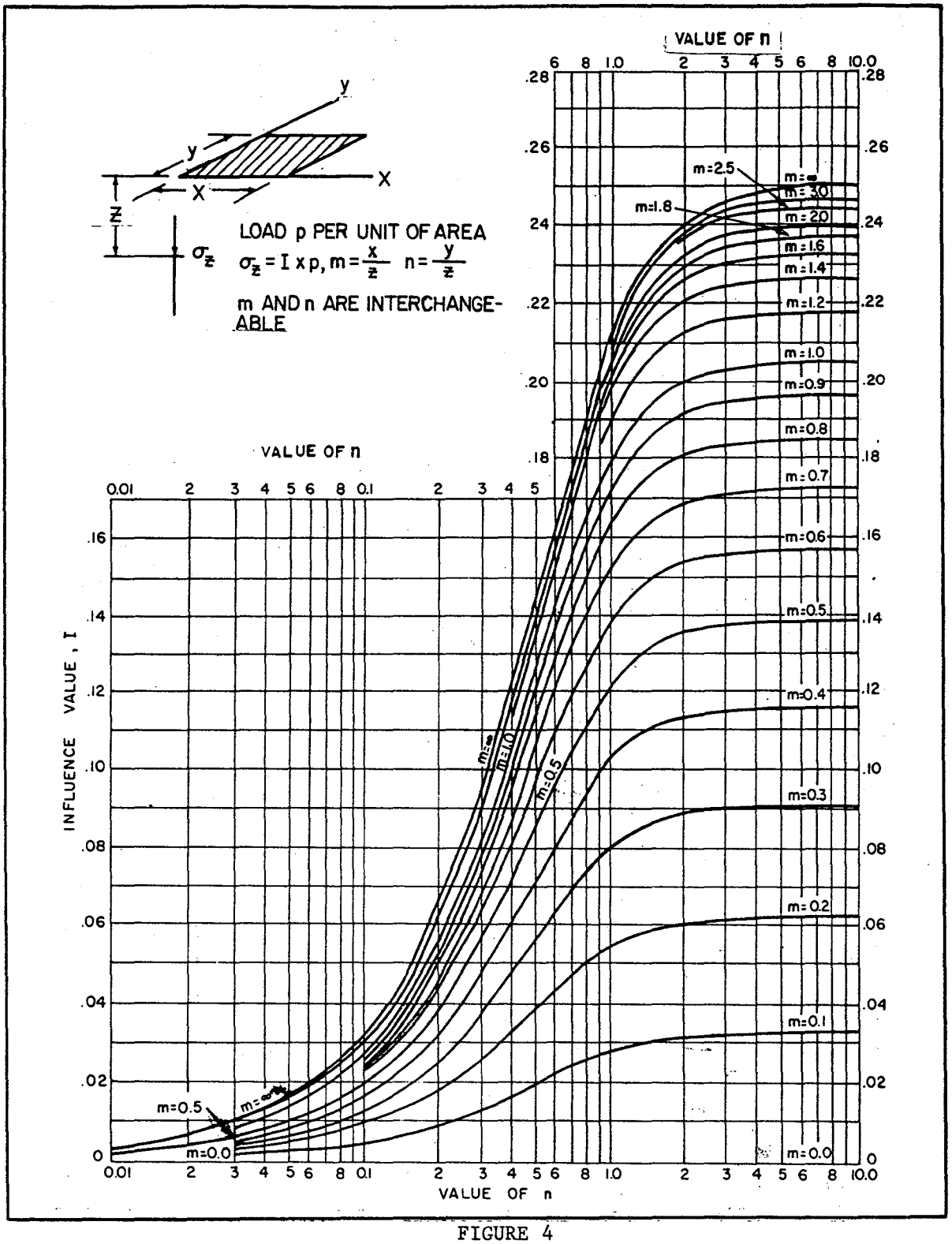
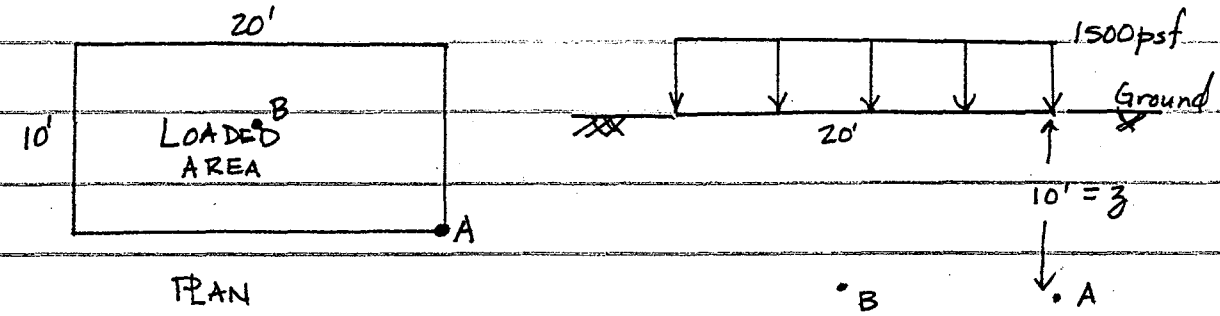


FIGURE 4
Influence Value for Vertical Stress Beneath a Corner of a Uniformly Loaded Rectangular Area (Boussinesq Case)

EXAMPLES

1.) Find the Δstress at a point 10 feet below the corner of a rectangular footing 20 feet long and 10 feet wide, carrying 1500psf. (POINT A)



Choose $x = 10'$, $y = 20'$

$m = \frac{x}{z} = \frac{10}{10} = 1.0$ $\quad n = \frac{y}{z} = \frac{20}{10} = 2.0$

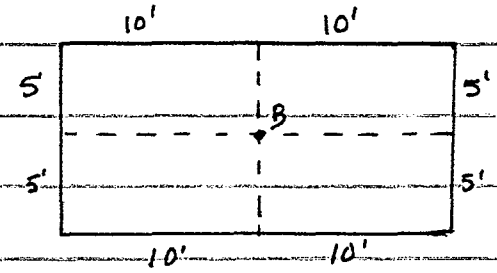
from chart, for $n = 2.0$ (bottom axis) & $m = 1.0$ (curves)

$I = 0.20$ (left axis)

$\therefore \Delta\sigma = 0.20\,(1500psf) = 300psf.$

2.) Under the <u>center</u> of the loaded area.

Use superposition (Boussinesq is a linear elastic theory) and treat as <u>4</u> rectangles, each 5' x 10'



Choose $x = 10'$ $\quad y = 5'$

$m = \frac{x}{z} = \frac{10'}{10'} = 1.0$

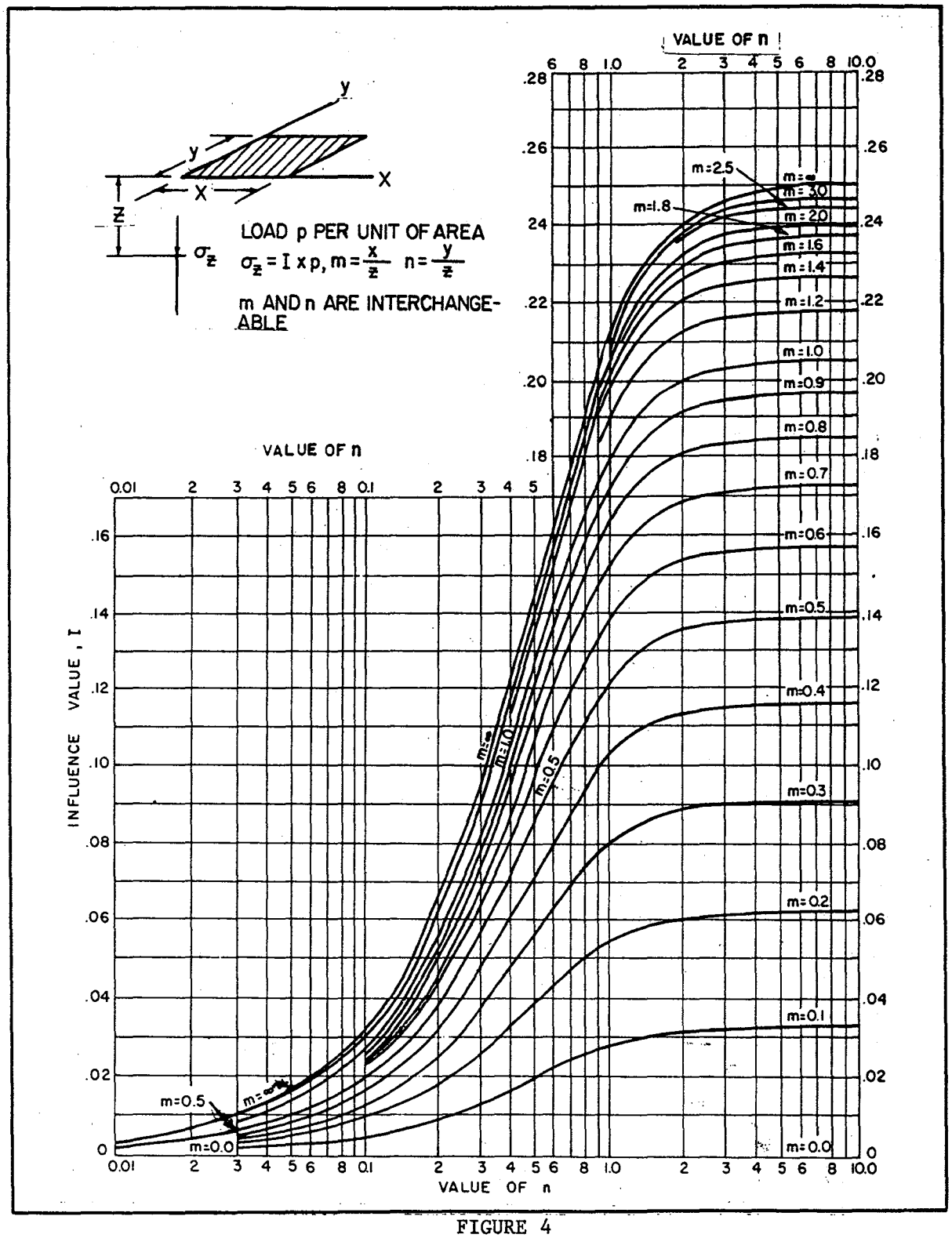
$n = \frac{y}{z} = \frac{5}{10} = 0.5$

$\therefore I = .125$ for each rectangle

(Any shape can be modeled by breaking into rect.'s)

So $\Delta\sigma = (.125 + .125 + .125 + .125)(1500psf)$ ← one for each ▭

$= 750$ psf



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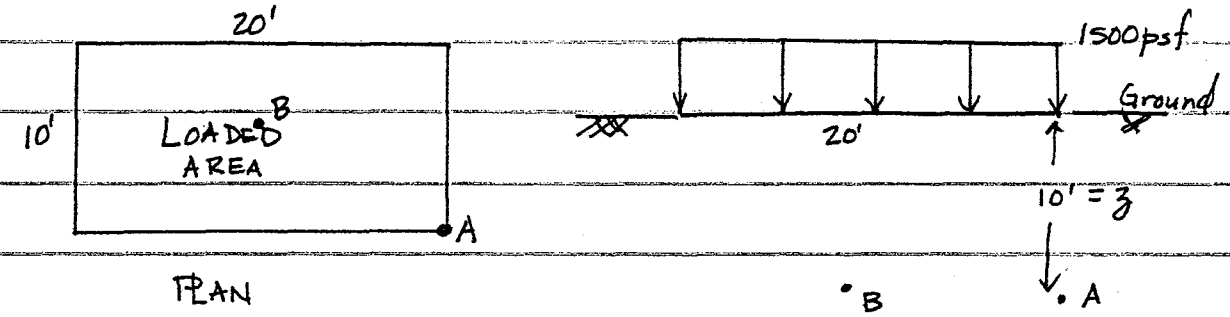

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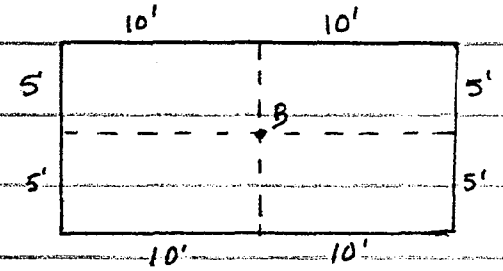

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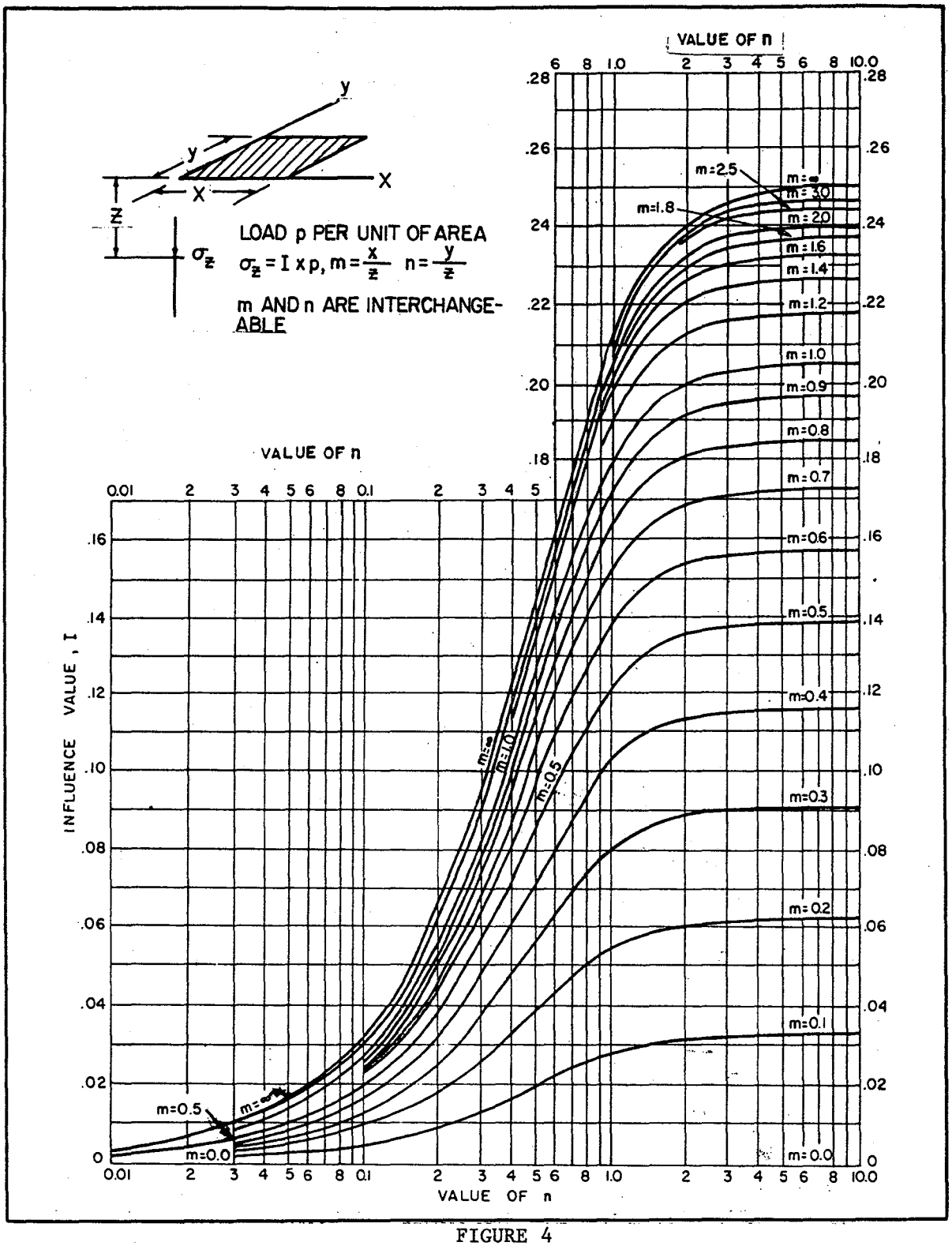

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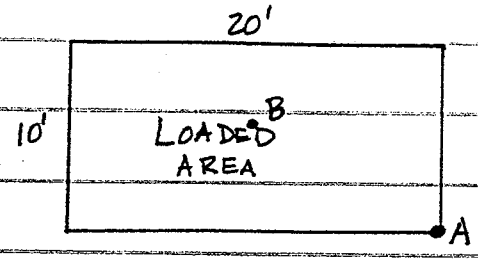

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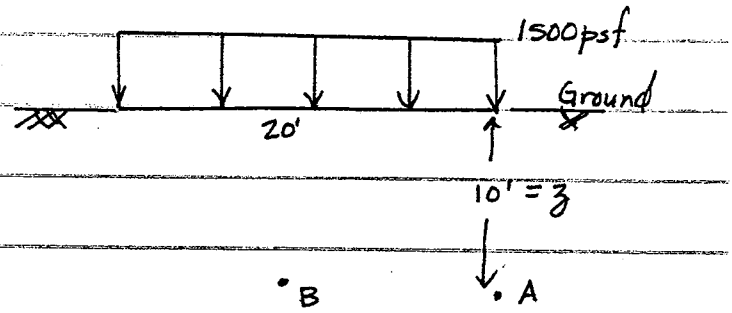

ELEV

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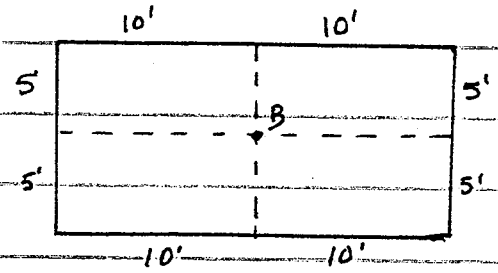

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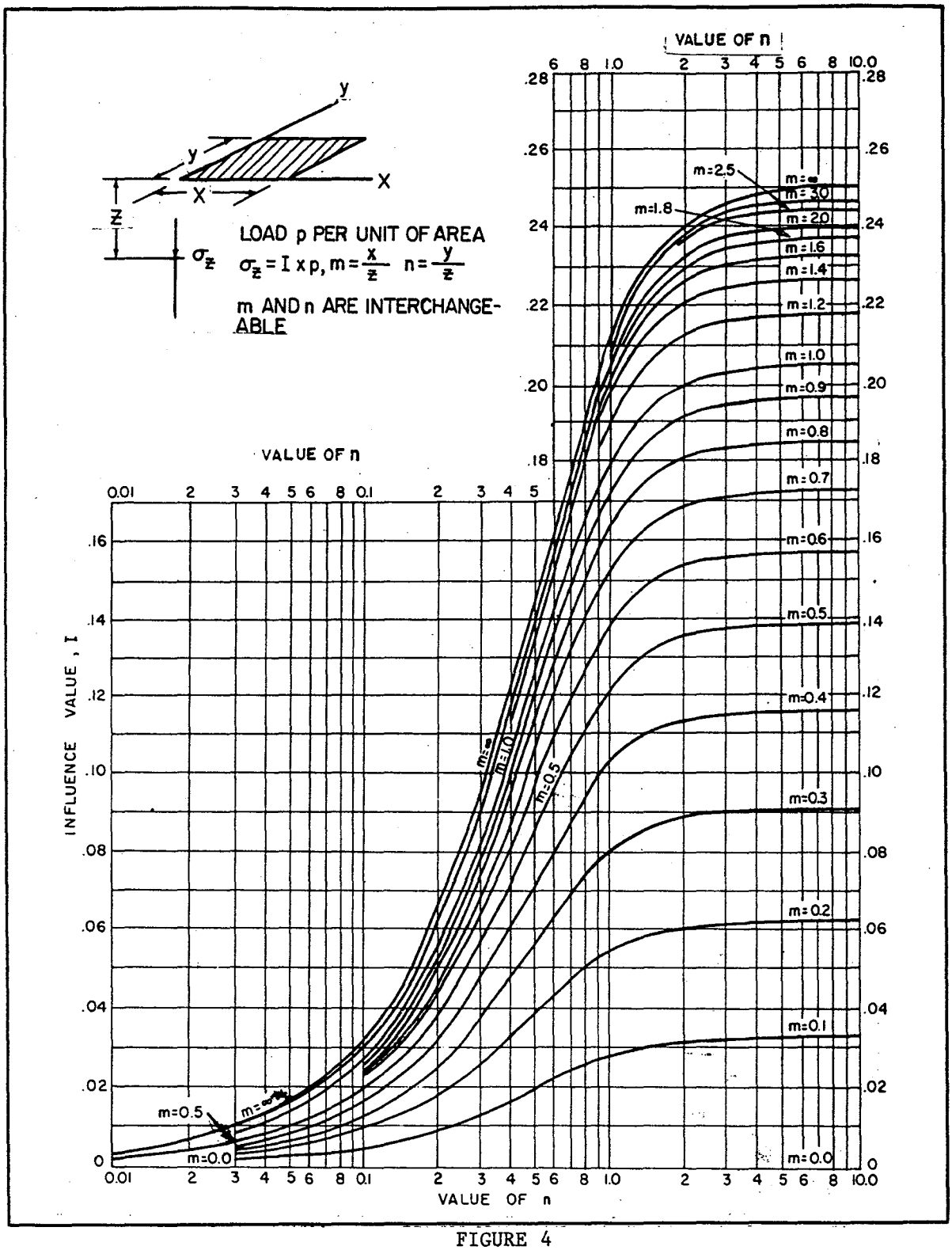

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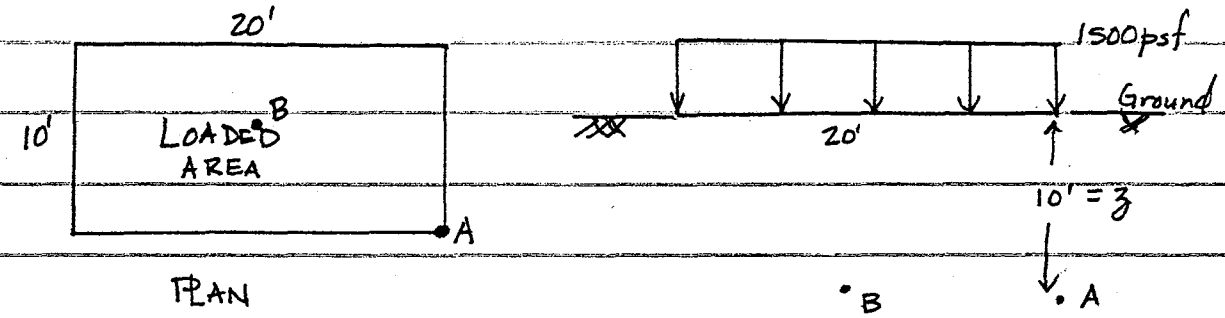

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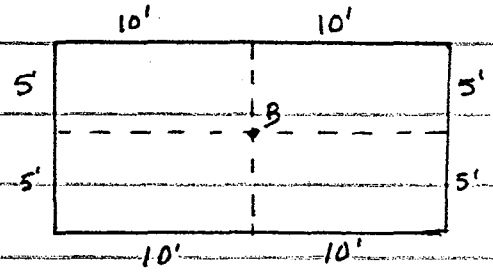

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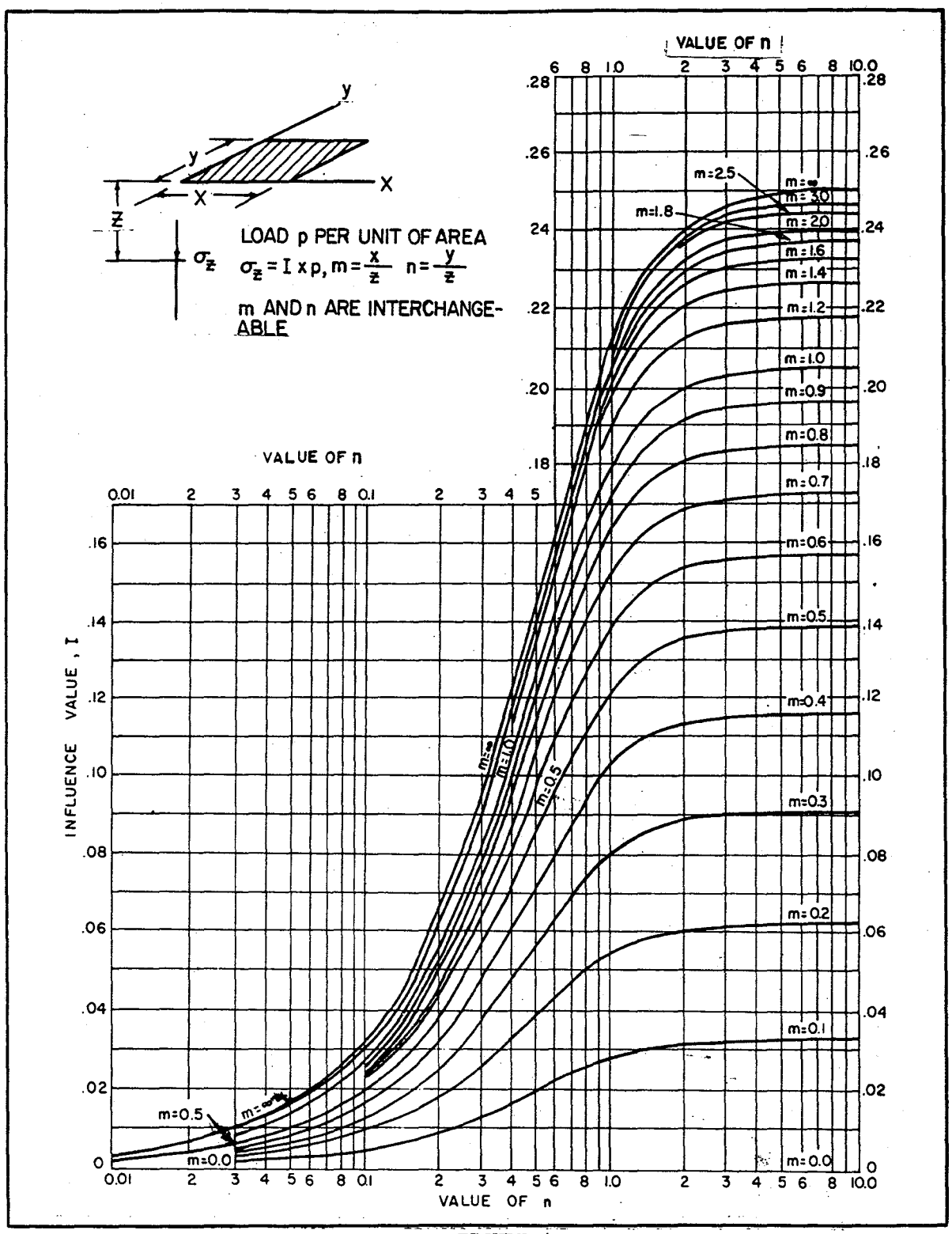

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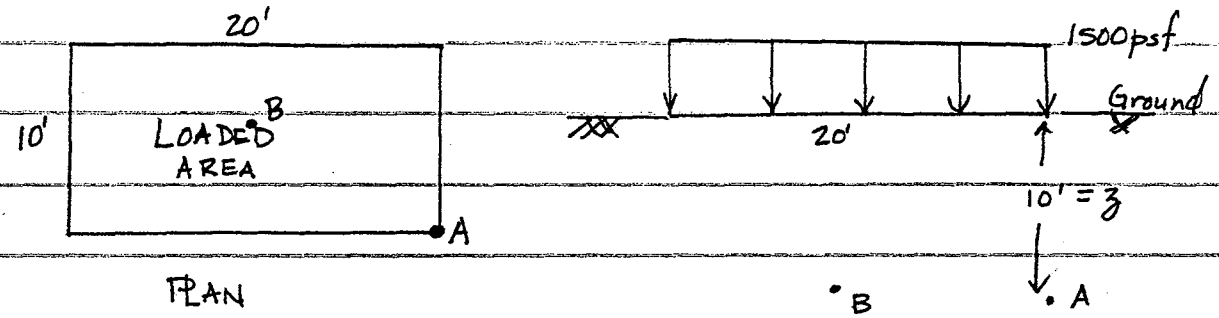

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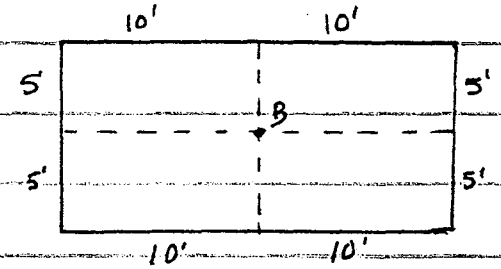

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