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A PRESENTATION OF *ARIZONA* STATE PLANE GRID *COORDINATES*

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CAD – TECHNOLOGY & SURVEYING SOLUTIONS

Friday, JULY 19th, 2002
Hawthorn University Center
Hawthorn Hotel
3353 East University Drive & I-10
Phoenix, Arizona

Instructor:
Gregory ("Gregg") M. Tuttle, AZ/PRLS
Manager, Land-Surveys Division,
SRP (Salt River Project)

RLS member of the
Arizona State Board of Technical Registration,
Chair, Legislative & Rules Committee

RLS member of the
Arizona Geographic Information Council,
Chair, Parcel Identification Committee,
Chair, Geodetic Control Committee

Gregory M. Tuttle, AZ-PRLS; FACSM

Since 1-1990, Gregg has been the manager-in-charge of all professional surveying, (both engineering and boundary), for both the water and the electric-power departments of the Salt River Project (SRP), the nation's third largest municipal power district. The annual budgetary expenditures for this surveying operation are in excess of $6.5M annually.

He has been an Arizona Registered Land Surveyor, (AZ-RLS, # 11121), since February, 1977. He has over thirty+ years of surveying experience, including 20 years in the Federal sector, concluding as the Chief-of-Surveys for the United States Department of the Interior's Bureau of Reclamation's Central Arizona Project, from 1984 through 1989. As a federal surveyor manager, he was responsible for geodetic, engineering-surveys, and land surveys in several western states, including all of Arizona, southeastern California, and western New Mexico.

Gregg is a member of National Society of Professional Surveyors (NSPS), the American Association of Geodetic Surveyors, (AAGS), the Cartography and Geographic Information Society (CaGIS), the Geographic and Land Information Society (GLIS). and also is an honored Fellow of the American Congress of Surveying and Mapping (F-ACSM) and has been a member of ACSM since 1975.

He is also an associate member of the California Land Surveyors Associations, (CLSA), the Professional Land Surveyors of Colorado, (PLSC), the New Mexico Professional Surveyors, the Nevada Association of Land Surveyors (NALS) and the Texas Society of Professional Surveyors (TSPS). He was also a long time associate member of both the Montana and Wyoming state societies of professional surveyors.
He is a past state Chair of the Arizona Professional Land Surveyors Association, (APLS), from 11/94 through 02/97.

Since 7-1997, by gubernatorial appointment, Gregg has been the Registered Land Surveyor (RLS) member, on the Arizona State Board of Technical Registration (AZ/BTR), and was its Board Chair from 7-1999 through 6-2001. He is now the Chair its Legislative and Rule Committee. He is the AZ-BTR RLS representative to the National Council of Examiners of Engineering and Surveying (NCEES). He is the Chair of the Land Surveyors' examination review workshop and evaluation committee for Arizona. From 1995 - 1997 he was actively involved on the BTR Enforcement Advisory committee, as a case assessor, and committee chair for land survey investigations. His most recent, major accomplishment was as Chair of the combined AZ/BTR & APLS committees, (11/99 – 04/2001), to revise the Arizona State Minimum Standards for Land Boundary Surveying, official as of 2/14/2002.

By gubernatorial appointment, in 4-2001, Gregg is the professional surveyor, member on the Arizona Geographic Information Council (AGIC), and has been previously active with AGIC since 1993. Gregg is currently the AGIC Chair of the Parcel Identification Committee, and responsible for the Geodetic Control Working Group.

Gregg is the Survey Chair of the Arizona - Kachina Chapter (# 28) of the International Right of Way Association (IRWA), and writes the monthly "Surveyor's Corner" series of popular articles.

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The RLS member of the AZ- Board of Technical Registration - http://www.btr.state.az.us/
The RLS member of the AZ Geographic Information Council - http://www.land.state.az.us/agic/agichome.html

How 'long' (as measured in feet) is an "arc-second" of Latitude, and of Longitude?

Use the simple formula: $L = 2\pi R\Delta/360°$

L = length of the arc (in feet)

π = PI = 3.141592654 (for this example).

R = Radius of the Earth; (we used Clarke's Spheroid 1866 – in FEET)

Δ = ANGLE (in degrees) as subtended; (in this case: 000° 00' 01.00")

$L = [2\pi(20{,}906{,}000\text{-ft})] \times (000° 00' 01''/360°)$

L = 101.3551-ft.

Use 101-ft. (+/-) as a good *"approximator"* for one-arc-second of Latitude.

REMEMBER: *@ the EQUATOR* BOTH LATITUDINAL & LONGITUDINAL DISTANCES, for one (1) arc-second, are *Equal*.

And, Remember, that Latitudinal distances (N-S distances) remain constant.

However, the Longitudinal distances DO NOT remain constant.

At the Equator they still equal about 101-ft (+/-) for each arc-second.

However, at Latitude N 33.4° they are approximately 85-ft (+/-) for each arc-second

To *"APPROXIMATE"* this, multiple the cosine of the latitude by 101-ft.

At Latitude 60° there only 50+ ft (+/-) per each arc-second of Longitude.

CLARKE, Alexander Ross

born Dec. 16, 1828 , Reading, Berkshire, Eng.
died Feb. 11, 1914 , Reigate, Surrey, Eng.

English geodesist whose calculations of the size and shape of the Earth were the first to approximate accepted modern values with respect to both polar flattening and equatorial radius.

The figures from his second determination (1866) became a standard reference for U.S. geodesy, *even after the acceptance of other figures by the International Union of Geodesy and Geophysics in 1924.*

Appointed to the Royal Engineers (1874), he remained with the British ordnance survey at Southampton from 1850 to 1881, except for a period between 1851 and 1854.

He was responsible for the principal triangulation (long-distance trigonometrical surveying) of the British Isles and published the results of the first geodetic survey of Great Britain in 1861.

Entrusted with comparing the standards of length for measuring an international arc of triangulation from Ireland to Russia, he published results in 1866.

His GEODESY (1880) has remained one of the best textbooks on the subject.

GREAT surveying websites:
(w/ an AZ bias)

http://www.btr.state.az.us/ AZ/BTR

http://www.azpls.org/ APLS

http://www.igc.topica.com/lists/AZSURVEY AZSURVEY

http://www.land.state.az.us/agic/agichome.html AZ Geographic Info Council

http://www.gci-net.com/~users/S/sowens/ APLS - Saguaro Chapter

http://www.dot.co.pima.az.us/gis/maps/control/index.htm Pima County GIS

http://www.dot.co.pima.az.us/gis/maps/control/index.htm
Pima County DOT - City of Tucson DOT Geodetic Control Points

http://www.mcdot.maricopa.gov/ Maricopa County DOT (* which will have a link to the M/C Geodetic Data and Cadastral Survey * within a few weeks.)

http://azwww.az.blm.gov/plats/cadhome.html BLM - AZ Cadastral offices

http://www.acea.org/ Arizona Consulting Engineers Association (ACEA)

http://www.srpnet.com/ Salt River Project (SRP)

(w/o the AZ bias)

http://www.wfps.org/ Western Federation of Professional Surveyors

http://www.survmap.org/acsmintro.html ACSM (NSPS & others)

http://www.ncees.org/ NCEES (examinations)

http://www.ca.blm.gov/webmanual/id2.htm BLM - the "Manual"

http://www.or.blm.gov/NILS/casebook/ BLM - the "Casebook"

http://www.i-boards.com/bnp/pob/ POB (formerly RPLS) messaging

http://www.forumboard.net/1635/ the 'other' messaging board

In the more than 67 years since the development of the STATE Plane Coordinate SYSTEMS, most States have passed legislation permitting their use in defining property boundaries and for other purposes.
BUT, most land boundary surveyors do not utilize these systems.
During this 65+ years there have been many papers presented which described the value of the STATE Grids and provide, (often in great detail), instructions on how to employ these systems, including the computations involved.
And, although recently, more surveyors have shown some increasing interest in STATE Plane Grid Coordinates, the numbers are still a small factor of the entire surveying community.

We will attempt to review and explain both the system based on the NAD-1927 geodetic positions and also the system based on the NAD-1983 positions, and also we will note, that due to refinements constantly being made, such as the use of Global Positioning Systems (GPS), surveyors should be aware of and understand *why three sets of state plane Grid coordinates are possible for the same point*, and most importantly, how to differentiate between them.

This system has been much misunderstood and little utilized by surveyors since its inception. With geodetic quality densification of control and improved accuracies in the system now afforded by continuous adjustments of the horizontal control network and the increasing use of GPS the emphasis to increase usage of the AZSPGCS will undoubtedly increase.
However, as many are becoming increasingly aware, the many changes made since by the new adjustments with the advent of GPS have created added confusion to what was already a confused system for most surveyors.

It is the attempted goal of this presentation to act as an introduction which will assist Arizona surveyors in overcoming their reluctance of employing Arizona State Plane Grid Coordinates.

We will give a history of the PLANE GRID Coordinate Systems, and then focus on those PLANE COORDINATE Systems most often used in ARIZONA.

We believe that the systems are fair easier to understand and use than most surveyor believe.

As a new generation of surveyors is assuming its place in our chosen profession, we hope that the AZSPGCS will take its place a very special and helpful tool, especially for the professional Arizona land boundary surveyors.

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NOAA Manual NOS NGS 5 James E. Stern, Rockville, MD 1/1989

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U.S. Department of Commerce, Coast and Geodetic Survey, Special Publication No. 257, US/GPO Washington, 1951

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U.S. Dept. of Commerce, NOAA NOS
NGS by Joseph F. Dracup, Rockville, MA 9/ 1974

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U.S. Dept. of Commerce, NOAA NOS
NGS by Joseph F. Dracup, Rockville, MA - Preprint of 1/1997

Methods for Converting Earlier Datum Plane Coordinates to North American Datum of 1983
Surveying and Land Information Systems, Vol. 51, No.2, 1991, pp. 75-82
by: Joseph F. Dracup, NGS retired.

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U.S. Department of Commerce, Coast and Geodetic Survey
Special Publication No. 193, by Oscar S. Adams & Charles N. Claire, US/GPO Washington, 1948, reprinted 9/1971

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U.S. Department of Commerce, Coast and Geodetic Survey
Special Publication No. 235, by Hugh C. Mitchell & Lansing G. Simmons
US/GPO, Washington 1945

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U.S. Dept. of Commerce, Environmental Science Services Administration
USC&GS Report No. 36, by Lansing G. Simmons, Washington, 1/1968

A manual on Astronomic and Grid Azimuth
R. B. Buckner, Ph.D., LS, PE, Planner, Landmark Enterprises
Rancho Cordova, CA 1st Edition, 1984

State Plane Coordinates in Modern Surveying Practice
by R. B. Buckner, Ph.D. LS, PE, Planner, Copyrighted 1993

STATE PLANE COORDINATES IN MODERN SURVEYING PRACTICE by R. B. Buckner, Ph.D. LS, PE, Planner 1994
125 pp. Regular price $45; ACSM member price $41. Order No. S312
on-line at: http://www.survmap.org/bsurveying.html

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article by by R. B. Buckner, Ph.D. LS in the Jan/Feb 1996 Issue of the ACSM Bulletin (pp. 22 - 29)

GEODESY for the LAYMAN (DMA Technical Report)
DMA TR 80-003 The Defense Mapping Agency
U.S. Naval Observatory, Washington 3/1984
(available on line at: http://164.214.2.59/GandG/pubs.html)

The UNIVERSAL GRIDS: Universal Transverse Mercator (UTM) -and- Universal Polar Stereographic (UPS)
Defense Mapping Agency Technical Manual DMATM 8358.2
Fairfax, VA 9/1989 (on-line at: http://164.214.2.59/GandG/pubs.html)

WORLD GEODETIC SYSTEM 1984 - its Definition and Relationship with Local Geodetic Systems
Dept. of Defense, NIMA TR8350.2 3rd Edition, 7/1997
(on-line at: http://164.214.2.59/GandG/pubs.html)

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Fairfax VA, 9/1900 (on-line at: http://164.214.2.59/GandG/pubs.html)

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H. W. Stoughton, Editor 1983: ACSM; 48 pp. Regular price $13; ACSM member price $10. Order No. S281 (order on-line at: http://www.survmap.org/bacsmbest.html) A collection of papers describing the planning and implementation of the readjustment of the North American horizontal network.

INTRODUCTION TO GEODESY: THE HISTORY AND CONCEPTS OF MODERN GEODESY by: James R. Smith
1997: John Wiley & Sons, Inc. (softcover); 240 pp., ISBN 0471-16660-X. Regular price $50; ACSM member price $45. Order No. S29 on-line at: http://www.survmap.org/bsurveying.html

Basic Geodesy: Introduction to the History and Concepts of Modern Geodesy by: James Smith
1988: Landmark Enterprises (softcover); 151pp., ISBN 0-910845-33-6. Regular price $45; ACSM member price $41. Order No. S295 on-line at: http://www.survmap.org/bsurveying.html

GPS & EDM Measurements - Why Don't They Match?
by: Jesse Kozlowski, PLS, GPS Project Manager at Taylor Wiseman & Taylor - as presented at the New Jersey Society of Professional Land Surveyors, Atlantic City, NJ 02/2000
available on-line in PDF at: http://www.profsurv.com/pdf/GPS_Vs_EDM.pdf

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BRIEF HISTORY OF THE STATE PLANE COORDINATE SYSTEM

For the last one hundred years and more, surveyors, engineers and cartographers have been seeking ways in which to coordinate various surveys and maps together. They have created surveys of increasing accuracy and precision, with superior monumentation, whose stations serve as starting points for resurveys and new surveys.

The backbone of this concept has been and continues to be the geodetic control station monuments of the National Geodetic Surveys, NGS (and before 1970, the U.S. Coast and Geodetic Survey, USC&GS). Designed originally to provide control for federal surveys and maps, this system comprises a national-wide, continental network connecting hundreds-of-thousands of marked points whose geodetic positions (latitude and longitude) control a myriad of lesser surveys, and are the basis for a variety of mapping products such as the USGS 7 1/2 minute series quadrangles.

But, the geodetic survey data (latitude and longitude, "Lats & Longs") requires a degree of training and usage with which the land surveyor, engineer and cartographer is ordinarily unacquainted. So, means were sought for making geodetic survey data available, without requiring knowledge of the tedious and involved procedures of geodetics. This was accomplished by transforming the "lats & longs" within a limited area into rectangular coordinates on a plane tangent to the area near its central point.

In 1933 this need was submitted to the USC&GS by a state highway engineer, who sought a method of utilizing the geodetic data over an entire state, which would involve only the formulas of plane surveying. This brought about the establishment of the North Carolina Coordinate System, by means of which the geodetic positions (lats & longs) could be transformed into plane-rectangular (x and y) coordinates on a single grid, and surveys in all parts of the state referenced thereto, so that survey monuments and landmarks could be accurately described by the stating their coordinates REFERRED TO A COMMON ORIGIN OF THE GRID.

Within a year, a similar system had been devised for each of the states. For some of the states a single origin and meridian of reference were sufficient. Other states, because of their large size, were each divided into several zones, each zone having its own origin and reference meridian. It is now the practice of the NGS to compute and publish the coordinates on the appropriate state system, of stations for which it determines the adjusted positions on the standard geodetic datum (the North American Datum of 1983).

One result of the growing use of the State Plane Grid Coordinate systems in engineering and land surveying, is the formal recognition given the systems in various state legislatures, by approving model legislation in adopting these systems in the statutes and codes of the various states (see ARS-33-121 TO ARS-33-128).

STATE PLANE COORDINATES - ADVANTAGES AND DISADVANTAGES

DISADVANTAGES:

Confusing, misunderstood, too complex.
Grid distances are not the same as ground distances.
Grid coordinates derived from sources that are questionable.

ADVANTAGES:

Measurement tool - if an entire area is referenced to the system at accuracies approaching second order, all new land subdivisions, mapping, utility, street planning and retracement work can be done much easier. This is true because the coordinate values provide a method of indirect measurements. By inverse calculations for length and direction, system users have a reliable measurement and this can avoid the necessity for direct measurements between unrelated structures.

Preserving Evidence of Monuments - Mathematical monuments are indestructible. They can be better than ground references, and can be a highly important part of retracement evidence.

Features and facilities properly located and referenced via Arizona State Plane Coordinates are also referenced to the National Geodetic network, as state plane coordinates can be converted to latitude and longitude, and as such, can be referenced to any other cartographic mapping system that is also based on latitude and longitude.

Integrated Land Systems -- If all cadastral, engineering, natural resource, taxation, and other land-related information is coordinated and mapped using a common grid; all planning and surveying in the future can be conducted more efficiently. The Arizona State Plane System provides a common base for building a common future database. The more it is properly used, the easier it will become to use and develop modern land systems and GIS'. Arizona State Plane Grid Coordinates are currently used on all federal projects (within Arizona) and by ADOT, MCDOT, and the various municipalities, (Phoenix, Scottsdale, Tempe, Mesa, etc.) for their various engineering and mapping projects.

WHY EMPLOY THE STATE PLANE COORDINATE SYSTEM?

- No point can ever be considered legally lost since it can be repositioned to original accuracy
- All surveys correlated to a single reference framework
- Bordering and overlapping mapping projects are consistent
- Few blunders will go undetected if two or more coordinated points are used to control a survey

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

ADDED THOUGHTS ABOUT THE STATE PLANE COORDINATE SYSTEM

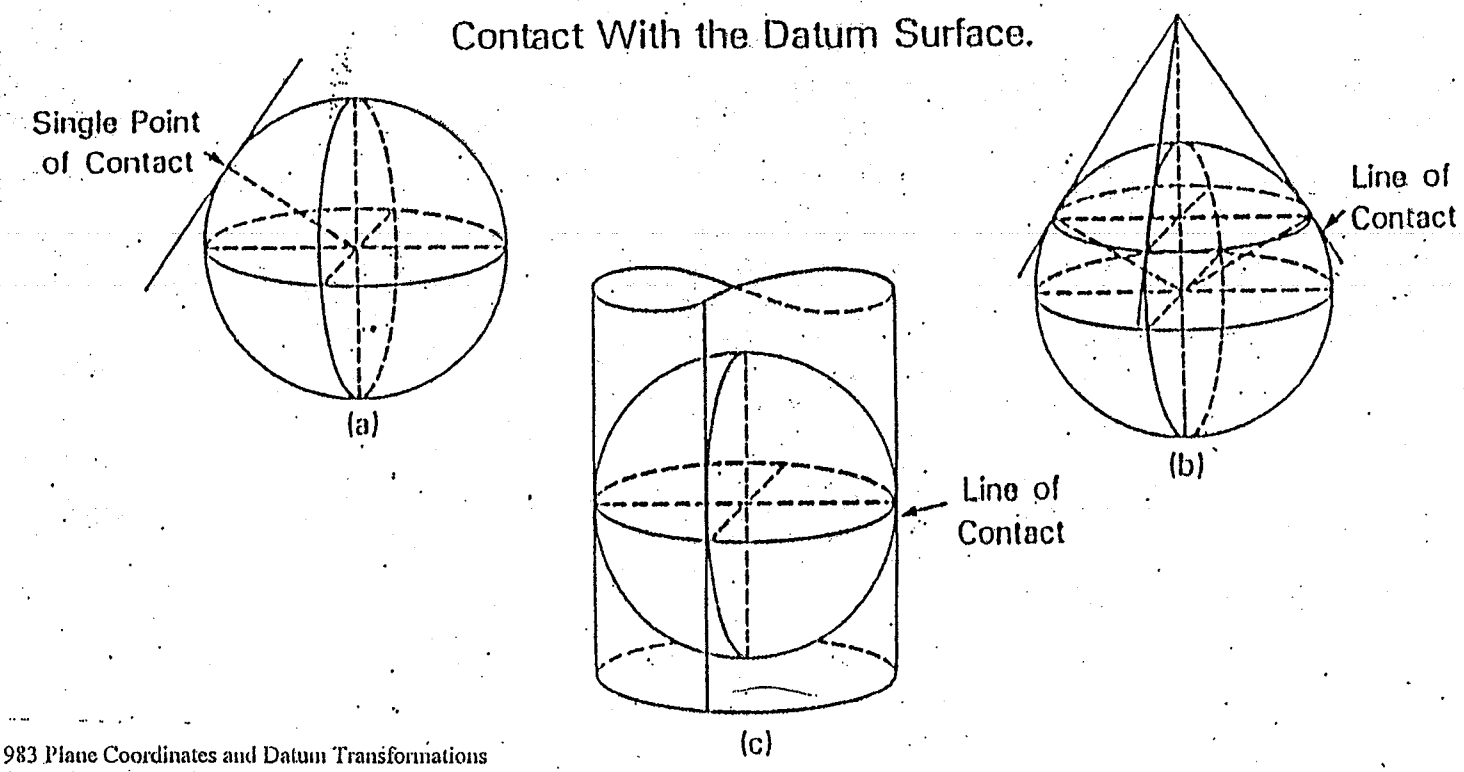
- THE ACCURACY OF A SURVEY HAS NOTHING TO DO WITH WHETHER OR NOT STATE PLANE COORDINATES ARE USED

- COMPUTATION TIME IS NEGLIGIBLE! ADDITIONAL COSTS ARE DUE TO FIELD OPERATIONS NECESSARY TO CONNECT SURVEY TO POINTS OF KNOWN COORDINATES

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

The Three Basic Projection Surfaces

(a) Plane, (b) Cone, (c) Cylinder
Showing the Respective Extent of
Contact With the Datum Surface.



Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
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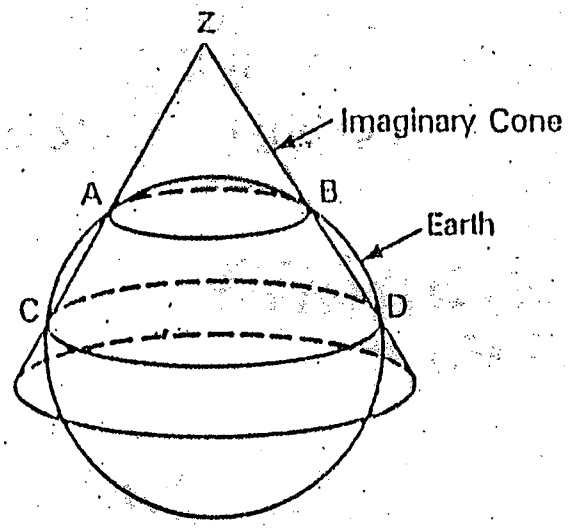
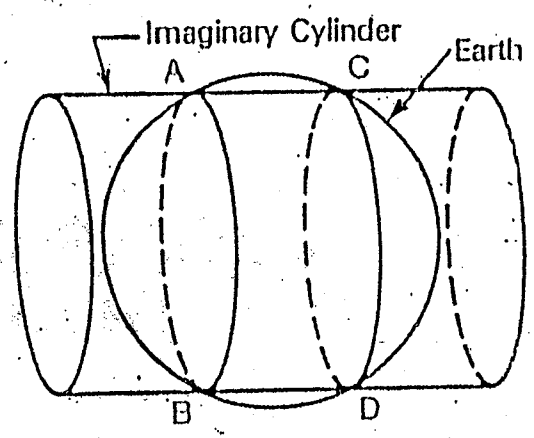
Surfaces Used In State Plane Coordinate Systems



Lambert Conformal Conic Projection



Transverse Mercator Projection

NAD PROJECT

STATE PLANE COORDINATE SYSTEM

PROJECTIONS:
1. Transverse Mercator
2. Lambert Conformal Conic

UNITS: Metric System

MODEL: Equations Support Millimeter Precision

SUPPORT:
1. Computer Programs
2. Publication Describing System
3. Special Projections

WHAT YOU NEED TO USE THE STATE PLANE COORDINATE SYSTEMS

- N & E State Plane Coordinates For Control Points

- Azimuths

  - Conversion from Geodetic Azimuth to Grid Azimuth (Mapping Angle)

  - Second Term Correction (t-T) for Long Lines (i.e. Generally greater than 2 miles)

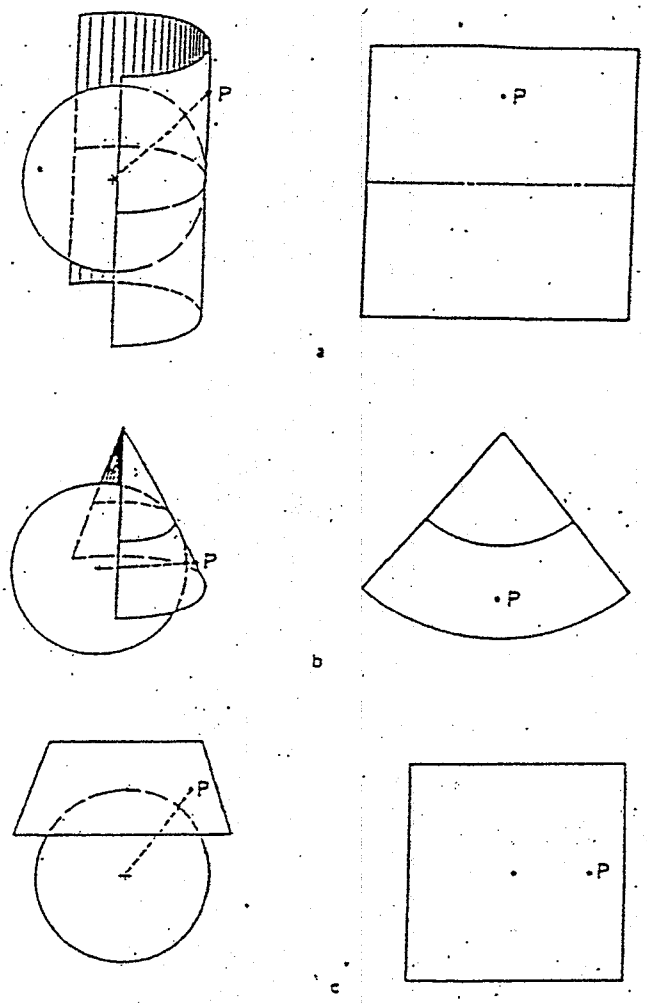
- Distance Observations

  - Reduction from Horizontal to Ellipsoidal "Sea Level Reduction/Correction Factor"

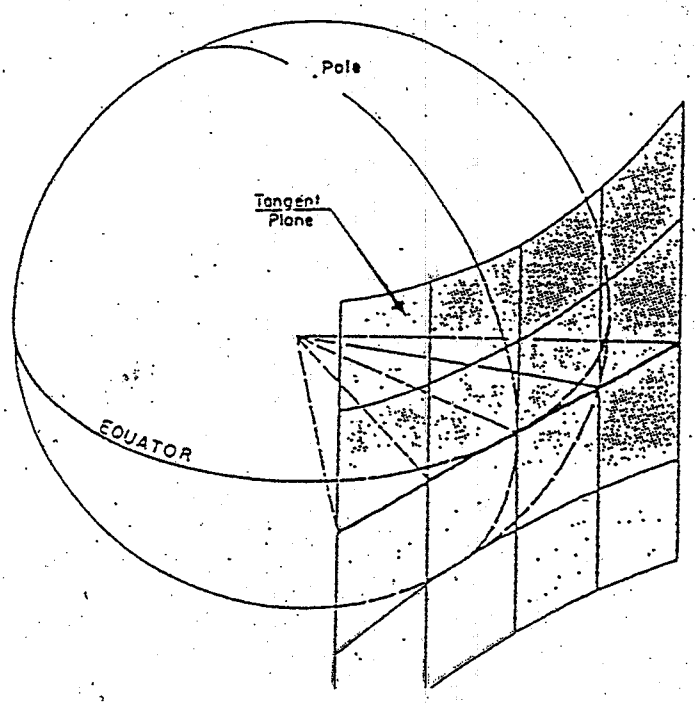
  - Correction for Grid Scale Factor

  - Combined Factor
    Ellipsoidal Reduction Factor x Grid Scale Factor

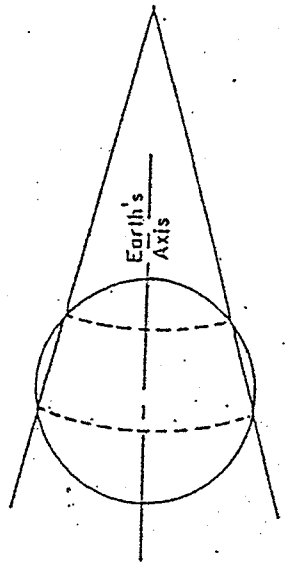
Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996



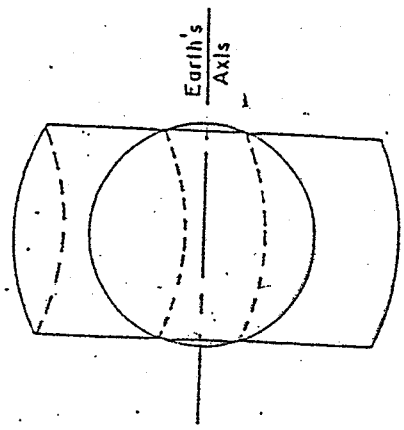
Developable projection surfaces. a. Cylinder tangent to a globe at the Equator, and the developed or "unrolled" map. b. Cone tangent to a globe along a parallel, and the developed or "unrolled" map. c. Plane tangent to a globe at the north pole, and part of the resulting map.



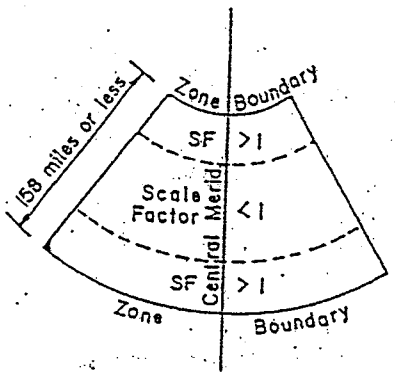
State Plane Projections



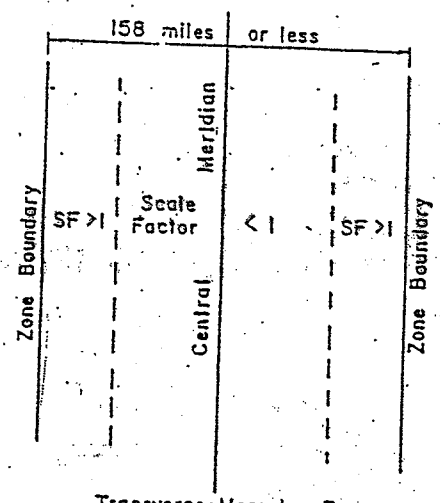
Lambert Projection
(Conical)



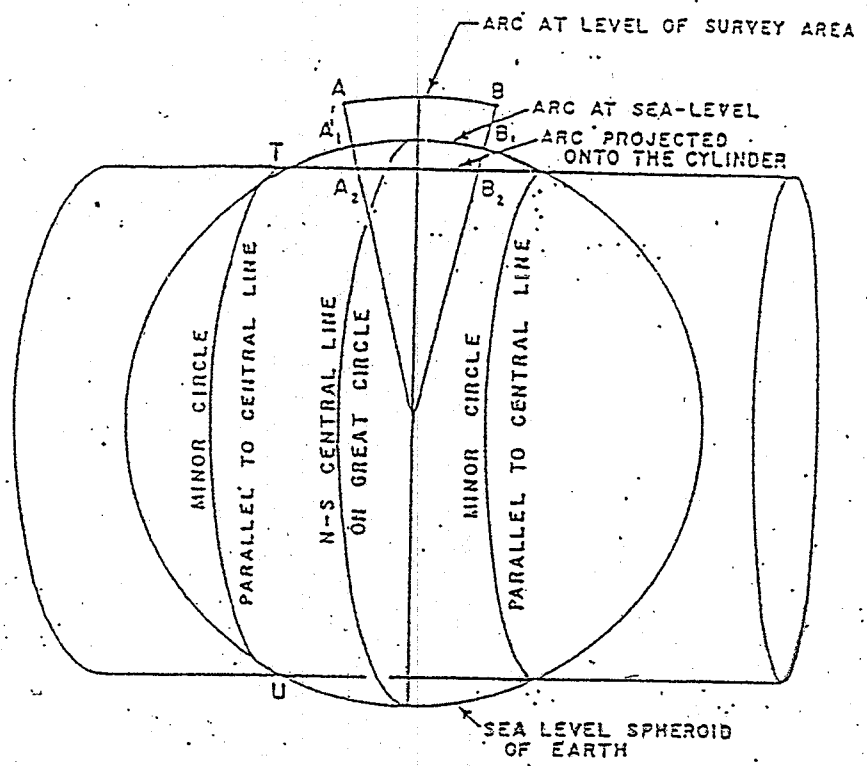
Transverse Mercator Projection
(Cylindrical)



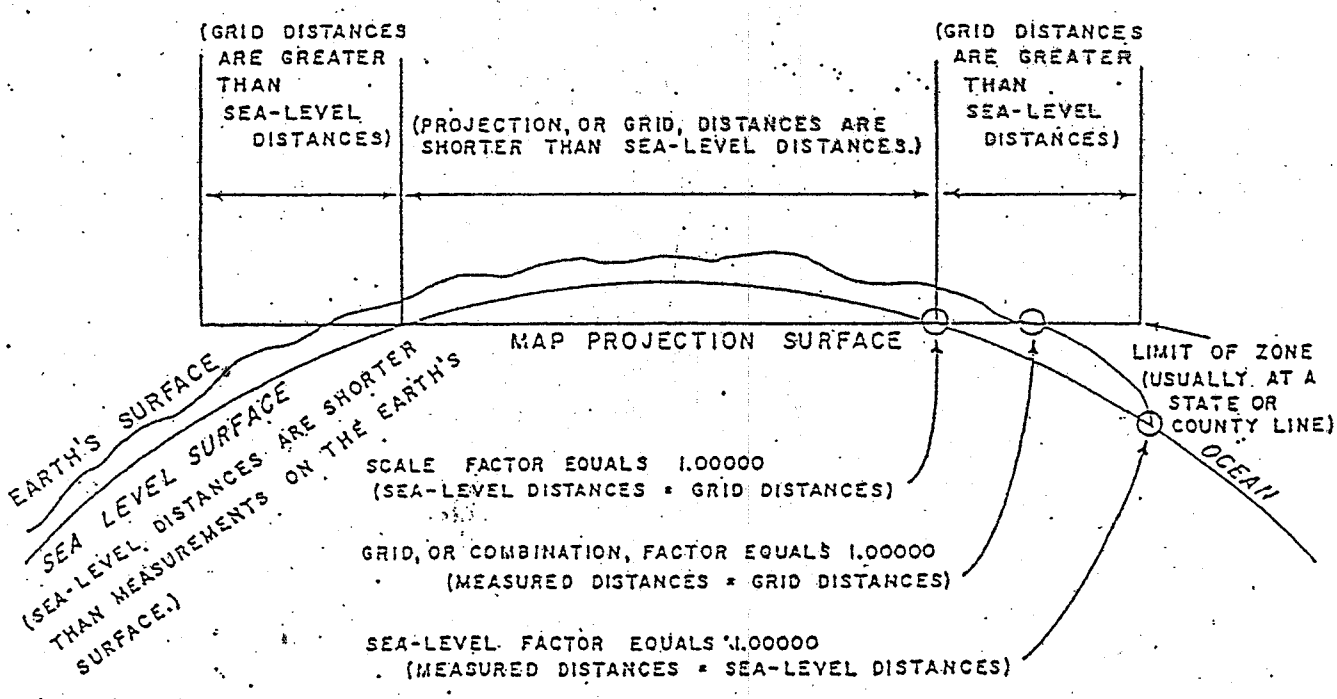
Lambert Zone
(cone developed into plane)



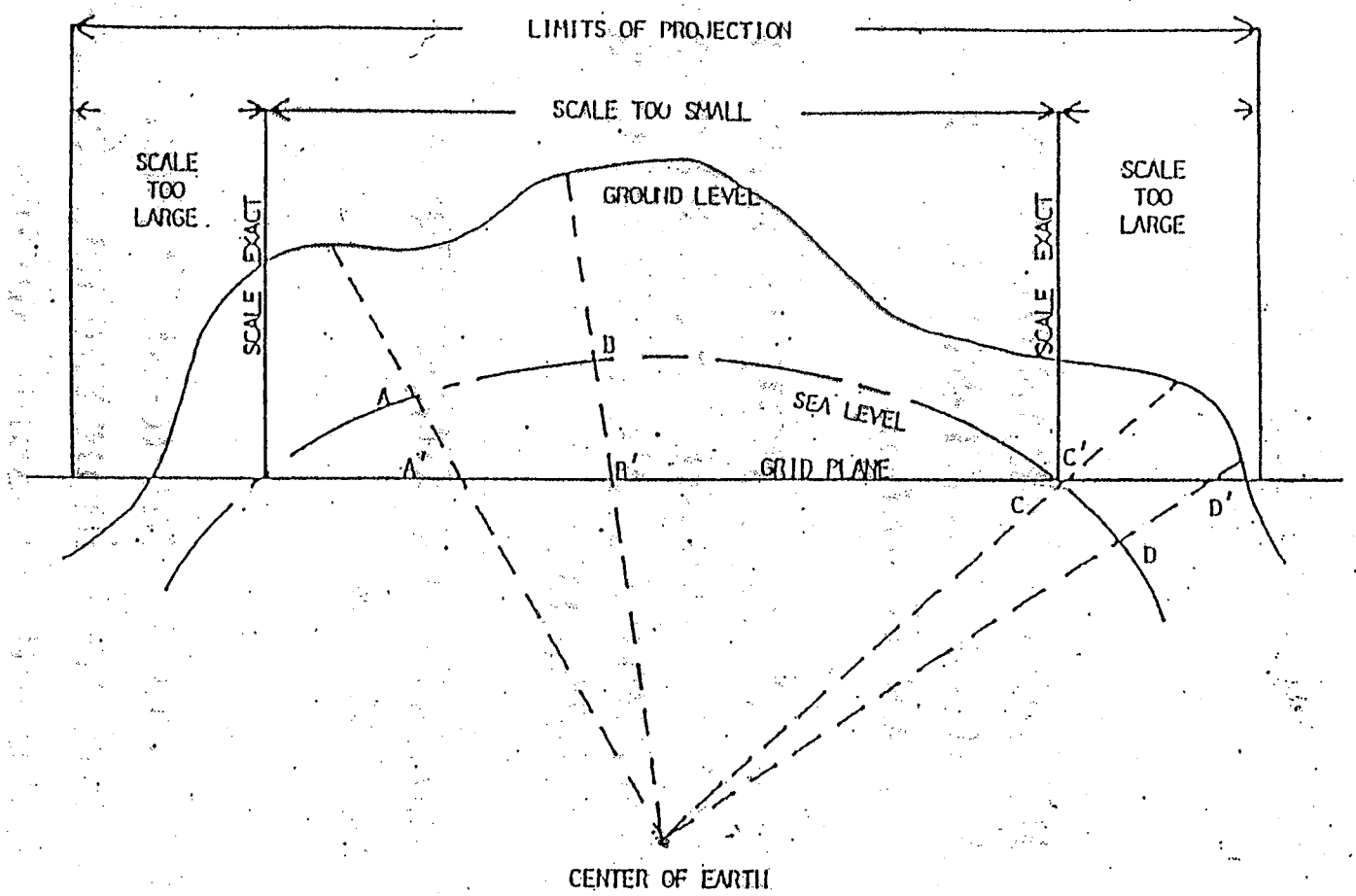
Transverse Mercator Zone
(cylinder developed into plane)



Elements of the Transverse Mercator Projection



Cross Section of a Secant Projection



GRID DISTANCE A' TO B' IS SMALLER THAN GEODETIC DISTANCE A TO B

GRID DISTANCE C' TO D' IS LARGER THAN GEODETIC DISTANCE C TO D

RELATIONSHIP OF GROUND, SEA LEVEL, AND GRID DISTANCE.

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

NAD 27

- DEFINITION BASED UPON "LOCAL" BEST-FIT ELLIPSOID (CLARKE 1866)
- REFERENCE POINTS BASED UPON "LOCAL" ADJUSTMENTS AND PIECE-WISE NETWORK ADDITIONS
- CLASSICAL TERRESTRIAL SURVEYS

- NAD 27 served the local US land surveyors well, in providing reference control points for small surveys.
- NAD 27 served the US navigational community well, as long as systems earth-base and relative.

DEFINITIONS
NAD - 1927 SPCS

TRANSVERSE MERCATOR PROJECTION = The SPCS used in those states extending primarily north to south -- (Arizona, New Mexico, Nevada).

LAMBERT PROJECTION = The SPCS used in those states whose major dimensions are primarily east to west -- (Colorado, Utah)

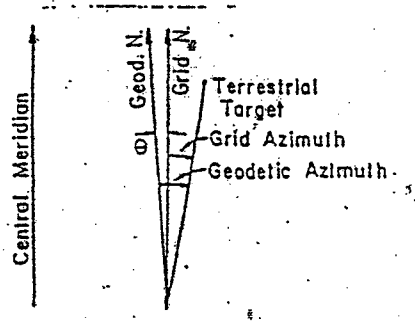
ZONE = A subdivision of the SPCS within a state. These subdivision follow county boundaries. In order to maintain a maximum scale reduction in the center of a particular system at 1:10,000, the limits cannot exceed 158 miles (east to west for the Transverse Mercator projection). There are some zones where for no reason or another the maximum is larger than 158 miles (AZ West Zone), and therefore the scale reduction is worse than 1:10,000 at their centers.

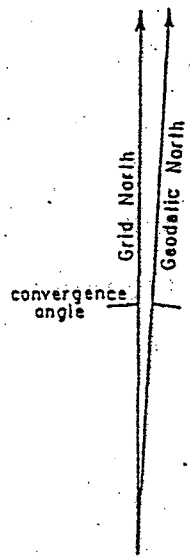
X = The plane coordinate value in feet (in the NAD 1927) perpendicular to the "Y" or north-south axis (center) of the system. In land surveying terms, "X" would be the Easting or "E" and consist of C +/- X′ where "C" is a constant of sufficient numerical size to keep the "X" value positive.

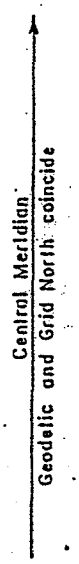
X′ = The distance in feet (NAD-1927) east or west of the "Y" or north-south axis of the system. When east of "Y" axis the values are added to the "C" and subtracted from "C" when west of the "Y" axis. This value may be considered the Departure measured from the "Y" axis.

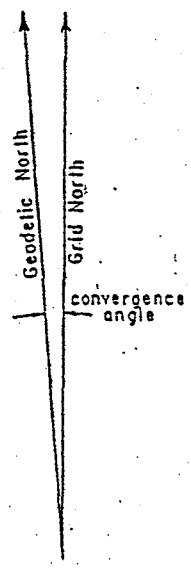
Y = The plane coordinate value in the feet along the "Y" or north-south axis, north from the origin to its intersection with the "X" coordinate. In plane surveying terminology, this quantity may be considered the Northing, or "N", or the Latitude measured from the origin.

C.M. = CENTRAL MERIDIAN = The meridian (Longitude) or "Y" (north-south) axis usually located near the center of the plane coordinate system which separates the positive and negative "X′" quantities.









ρ = phi = latitude

λ = lambda = longitude

Δ = delta = difference

α = alpha = geodetic azimuth

αg = alpha sub g = grid azimuth

αa = alpha sub a = astronomic azimuth

S = geodetic distance

Sg = grid distance

ΔX = Departure

ΔY = Latitude

$\Delta\alpha$ = DELTA ALPHA = The CONVERGENCE ANGLE, (also known as the"mapping angle"), in a transverse mercator projection. This value is applied to grid azimuths to obtain very close approximations to geodetic azimuths. The angle is considered positive when the point is east of the "Y" axis (C.M.) and negative west of this axis. This term is also used to describe the difference between forward and backward geodetic azimuths. It is often used as well in plane surveying to define the differences, convergence between "true" and grid bearings.

Grid Azimuth = Geodetic Azimuth - $\Delta\alpha$ - 2nd term.

The 2nd term relates to aspects of the spherical shape of the earth and the projection system. It is usually less than 1" arc unless survey lines are 25,000 ft. or greater, are oriented east-west, and are near the zone boundary edges. The 2nd term is negligible for survey work least than 1st order.

$\Delta\alpha = \Delta\lambda \sin\phi + g \qquad \Delta\lambda = \Delta_{CM} - \lambda$

λ_{CM} = longitude of the central meridian

λ = longitude of the occupied point.

ϕ = latitude of the occupied point.

The "g" term is a function of latitude and longitude; and it's like the "2nd term", with a maximum value of less than 1" and this maximum only occurs for very long lines at the extreme edges of a zone, and then only needs to be considered for 1st order survey work.

Use the formula as $\Delta\alpha = \Delta\lambda \sin\phi$ The algebraic sign of $\Delta\alpha$ is determined by the sign of $\Delta\lambda$. The values for ϕ and λ can be scaled from a USGS topographic map.

Example: NGS APOLAND = 34° 27′ 29.54104″ N, 112° 16′ 07.57859″
111° 55′ 00″ - 112° 16′ 07.57859″= - 00° 21′ 07.57859″
- 00° 21′ 07.57859″ (sin 34° 27′ 29.54104″) = - 00° 11′ 57″
$\Delta\alpha$ = - 00° 11′ 57″ @ Δ POLAND

(FOR SECOND & THIRD ORDER SURVEYING)

AZIMUTHS IN ARIZONA STATE PLANE COORDINATES

[$\Delta\alpha$ = DELTA-ALPHA]

TRUE (GEODETIC) AZIMUTH = GRID AZIMUTH + ($\pm\Delta\alpha$) "alpha"

GRID AZIMUTH = TRUE AZIMUTH − ($\pm\Delta\alpha$)

1. $\Delta\alpha'' = (\Delta\lambda'')\sin\phi$ Lambda Phi

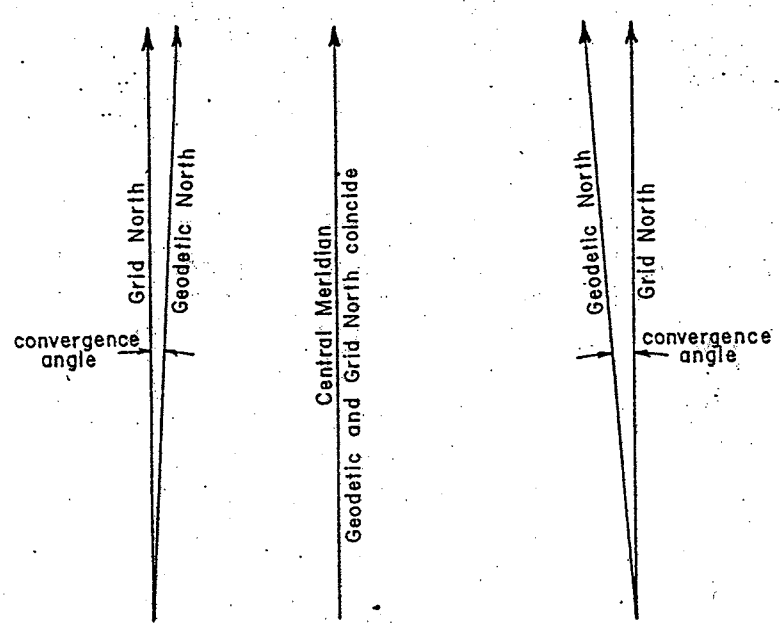
where: $\Delta\lambda''$ = distance from Central Meridian (in seconds)

ϕ = latitude

2. $\Delta\alpha = \pm(M)(X')$ alpha

where: X' is the same value used in the formula to calculate the SCALE FACTOR.

$\Delta\alpha$ is in seconds, and is <u>minus</u> (−) when the mean X coordinate value is <u>less</u> than 500,000; and is plus (+) when mean coordinate value is <u>more</u> than 500,000. (M IS GIVEN IN TABLE)



TRANSVERSE MERCATOR PROJECTION

Arizona

$$\Delta\alpha = Mx' - e$$

y	East and central zones		West zone	
	M	ΔM	M	ΔM
0	0.005 9177	644	0.005 9175	644
100,000	0.005 9821	648	0.005 9819	648
200,000	0.006 0469	652	0.006 0467	652
300,000	0.006 1121	656	0.006 1119	655
400,000	0.006 1777	659	0.006 1774	660
500,000	0.006 2436	664	0.006 2434	663
600,000	0.006 3100	667	0.006 3097	668
700,000	0.006 3767	672	0.006 3765	672
800,000	0.006 4439	676	0.006 4437	675
900,000	0.006 5115	680	0.006 5112	680
1,000,000	0.006 5795	684	0.006 5792	685
1,100,000	0.006 6479	689	0.006 6477	689
1,200,000	0.006 7168	693	0.006 7166	693
1,300,000	0.006 7861	698	0.006 7859	698
1,400,000	0.006 8559	703	0.006 8557	702
1,500,000	0.006 9262	707	0.006 9259	707
1,600,000	0.006 9969	712	0.006 9966	712
1,700,000	0.007 0681	717	0.007 0678	717
1,800,000	0.007 1398	722	0.007 1395	722
1,900,000	0.007 2120	727	0.007 2117	727
2,000,000	0.007 2847	732	0.007 2844	732
2,100,000	0.007 3579	737	0.007 3576	737
2,200,000	0.007 4316	742	0.007 4313	742
2,300,000	0.007 5058		0.007 5055	

e

y \ x'	200,000	300,000	400,000	500,000
0	0.0	0.1	0.3	0.6
1,000,000	0.0	0.1	0.4	0.8
2,000,000	0.0	0.2	0.5	1.0

(FOR SECOND & THIRD ORDER SURVEYING)

AZIMUTHS IN ARIZONA STATE PLANE COORDINATES
[$\Delta\alpha$ = DELTA-ALPHA]

TRUE (GEODETIC) AZIMUTH = GRID AZIMUTH + ($\pm\Delta\alpha$)

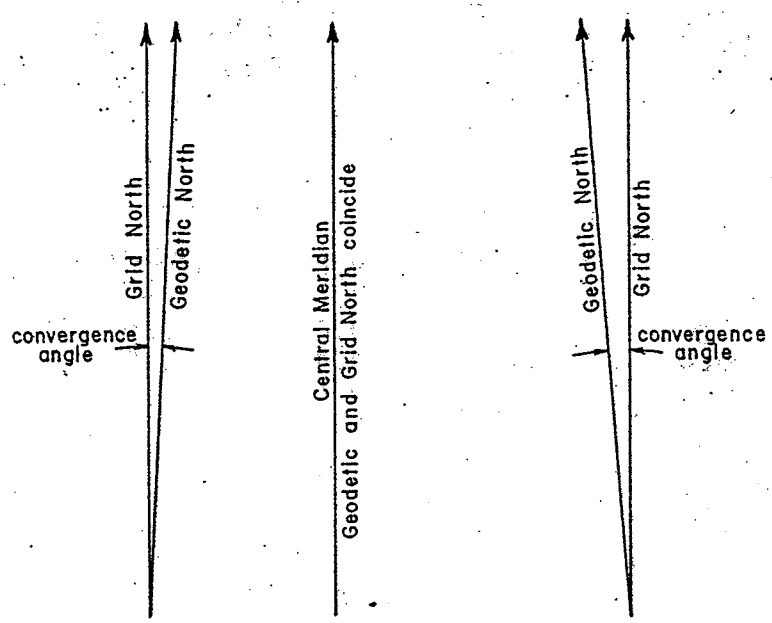
GRID AZIMUTH = TRUE AZIMUTH − ($\pm\Delta\alpha$)

1. $\Delta\alpha'' = (\Delta\lambda'')\sin\phi$

where: $\Delta\lambda''$ = distance from Central Meridian (in seconds)
ϕ = latitude

2. $\Delta\alpha = \pm(M)(X')$

where: X' is the same value used in the formula to calculate the SCALE FACTOR.
$\Delta\alpha$ is in seconds, and is <u>minus</u> (−) when the mean X coordinate value is <u>less</u> than 500,000; and is plus (+) when mean coordinate value is <u>more</u> than 500,000. (M IS GIVEN IN TABLE)



TRANSVERSE MERCATOR PROJECTION

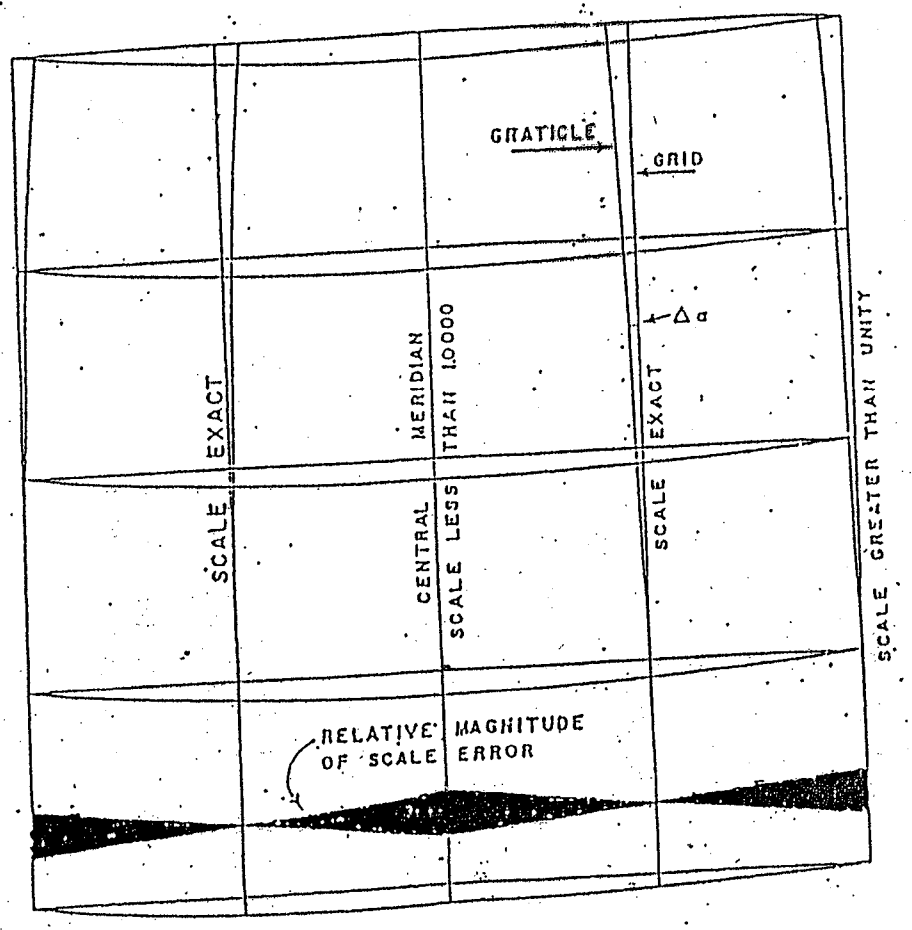
Arizona

$$\Delta\alpha = Mx' - e$$

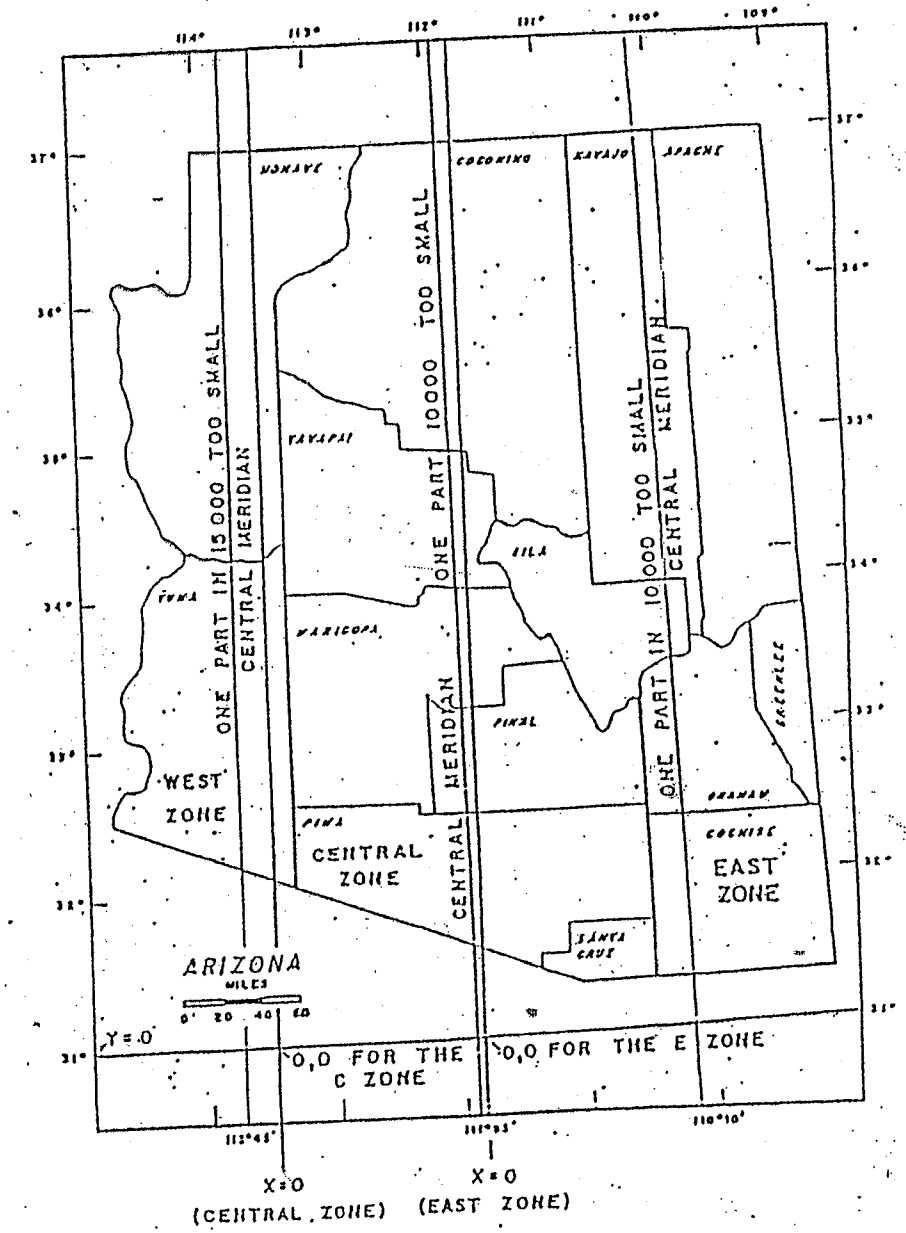
y	East and central zones		West zone	
	M	ΔM	M	ΔM
0	0.005 9177	644	0.005 9175	644
100,000	0.005 9821	648	0.005 9819	648
200,000	0.006 0469	652	0.006 0467	652
300,000	0.006 1121	656	0.006 1119	655
400,000	0.006 1777	659	0.006 1774	660
500,000	0.006 2436	664	0.006 2434	663
600,000	0.006 3100	667	0.006 3097	668
700,000	0.006 3767	672	0.006 3765	672
800,000	0.006 4439	676	0.006 4437	675
900,000	0.006 5115	680	0.006 5112	680
1,000,000	0.006 5795	684	0.006 5792	685
1,100,000	0.006 6479	689	0.006 6477	689
1,200,000	0.006 7168	693	0.006 7166	693
1,300,000	0.006 7861	698	0.006 7859	698
1,400,000	0.006 8559	703	0.006 8557	702
1,500,000	0.006 9262	707	0.006 9259	707
1,600,000	0.006 9969	712	0.006 9966	712
1,700,000	0.007 0681	717	0.007 0678	717
1,800,000	0.007 1398	722	0.007 1395	722
1,900,000	0.007 2120	727	0.007 2117	727
2,000,000	0.007 2847	732	0.007 2844	732
2,100,000	0.007 3579	737	0.007 3576	737
2,200,000	0.007 4316	742	0.007 4313	742
2,300,000	0.007 5058		0.007 5055	

e

y \ x'	200,000	300,000	400,000	500,000
0	0.0	0.1	0.3	0.6
1,000,000	0.0	0.1	0.4	0.8
2,000,000	0.0	0.2	0.5	1.0



Development of the Transverse Mercator Projection.



Plane Coordinate Zones in Arizona

ARIZONA STATE PLANE GRID COORDINATES
NAD - 1927 (U.S. SURVEY FOOT)

X' = The distance from the center meridian and a point at "x". This distance is determined by taking the current coordinate value at "x" minus the coordinate value at the central meridian (500,000'), or the reverse - which ever will leave a position value.

Examples:	x = 220,000	C.M. = 500,000	X' = 280,000 feet
	x = 640,500	C.M. = 500,000	X' = 140,500 feet

SCALE FACTORS (SF)

EAST or *CENTRAL* ZONE	= 0.9999	+	$[(X')^2 \times (1.145 \times 10^{-15})]$
WEST ZONE	= 0.9999333	+	$[(X')^2 \times (1.145 \times 10^{-15})]$

SEA LEVEL FACTOR (SLF):

Based on the radius of the earth *as determined by the Clark Spheroid of 1866*, as = 20,906,000 feet.
SLF = [20,906,000 / (20,906,000 + elevation of point)]

COMBINED FACTOR = (SCALE FACTOR X SEA LEVEL FACTOR)
GROUND DISTANCE -to- GRID = (GROUND DISTANCE)*(COMBINED FACTOR)
GRID DISTANCE -to- GROUND = ((GRID DISTANCE) / (COMBINED FACTOR))

EXAMPLE:

Assume the average easting coordinate is 751,251.31 feet

$X' = (751{,}251.31 - 500{,}000) = 251{,}251.31$

EAST ZONE *Scale Factor* = $0.9999 + [(X')^2 * (1.145 \times 10^{-15})]$

$0.9999 + [(251{,}251.31)^2 * (1.145 \times 10^{-14})]$
$0.9999 + [(6.31^{10}) * (1.145^{-15})]$
$0.9999 + 0.000072281$
0.999972281 = East Zone *Scale Factor* @ 751,251.31

Assume the average elevation is 4,521.31 feet

Sea Level Factor = [20,906,000 / (20,906,000 + 4,521.31)]
= [20,906,000 / 20,910,521.31]
= 0.999783778

Combined Factor = (SF)*(SLF) = (0.999972281)*(0.999783778) = 0.99975607

Assume Horizontal Ground Distance = 5,751.41 feet
Grid Distance = (Ground Distance)*(Combined Factor)
= (5,751.41)*(0.99975607) = 5,750.01 feet

Ground Distance = Grid Distance / Combined Factor
= 5750.01 / 0.99975607 = 5,751.41 feet

NGS Data Sheet By State and Designation

Data Sheets can be retrieved by specifying the designation or name of the control point and the state in which it is located. The designation does not need to be exact. If there are less than one hundred control points identified then datasheets will be produced. If between one hundred and two hundred control points are identified a listing of designations will be produced. Use this list of designations as input. If more than two hundred control points are identified a message will be produced requesting that a more exact designation be entered.

```
Ex.  To retrieve a single datasheet for the control point,
       PTS 57 M, in the state of Kentucky.

       Enter State:  KENTUCKY

       Enter Designation: PTS 57 M

Ex.  A listing of control points with designations beginning
       with "PTS" will be produced for:

       Enter State:  Kentucky

       Enter Designation: PTS
```

Enter State:
ALABAMA
ALASKA
AMERICAN SAMOA
ANGUILLA

Enter Designation: DEWEY

Submit query | Reset

The NGS Data Sheet

```
DATABASE = Sybase ,PROGRAM = datasheet, VERSION = 6.27
Starting Datasheet Retrieval...
1        National Geodetic Survey,   Retrieval Date = SEPTEMBER 14, 2000
 ET0862 ***********************************************************************
 ET0862  DESIGNATION -  DEWEY
 ET0862  PID         -  ET0862
 ET0862  STATE/COUNTY-  AZ/YAVAPAI
 ET0862  USGS QUAD   -  HUMBOLDT (1973)
 ET0862
 ET0862                          *CURRENT SURVEY CONTROL
 ET0862
 ET0862* NAD 83(1992)-  34 31 49.10511(N)    112 13 55.44243(W)     ADJUSTED
 ET0862* NAVD 88     -      1397.6    (meters)    4585.     (feet)  VERTCON
 ET0862
 ET0862  LAPLACE CORR-           1.55  (seconds)                    DEFLEC99
 ET0862  GEOID HEIGHT-         -26.09  (meters)                     GEOID99
 ET0862
 ET0862  HORZ ORDER  -  SECOND
 ET0862
 ET0862.The horizontal coordinates were established by classical geodetic methods
 ET0862.and adjusted by the National Geodetic Survey in August 1993.
 ET0862
 ET0862.The NAVD 88 height was computed by applying the VERTCON shift value to
 ET0862.the NGVD 29 height (displayed under SUPERSEDED SURVEY CONTROL.)
 ET0862
 ET0862.The Laplace correction was computed from DEFLEC99 derived deflections.
 ET0862
 ET0862.The geoid height was determined by GEOID99.
 ET0862
 ET0862;                 North         East     Units   Scale       Converg.
 ET0862;SPC AZ C     -   391,518.601   184,406.937   MT  0.99991033 -0 10 43.6
 ET0862;UTM  12      - 3,821,647.086   386,930.026   MT  0.99975758 -0 41 54.5
 ET0862
 ET0862:                 Primary Azimuth Mark                      Grid Az
 ET0862:SPC AZ C     -   ROUND                                     105 16 37.2
 ET0862:UTM  12      -   ROUND                                     105 47 48.1
 ET0862
 ET0862|-----------------------------------------------------------------------|
 ET0862| PID    Reference Object                    Distance       Geod. Az    |
 ET0862|                                                           dddmmss.s   |
 ET0862| ET0866 ROUND                             APPROX.  4.2 KM 1050553.6   |
 ET0862|        DEWEY RM 1                         14.176 METERS 17037        |
 ET0862| ET0919 HUMBOLDT SMOKE STACK              APPROX.  3.6 KM 1802627.0   |
 ET0862|        DEWEY RM 2                         13.445 METERS 27856        |
 ET0862|-----------------------------------------------------------------------|
 ET0862
 ET0862                          SUPERSEDED SURVEY CONTROL
 ET0862
 ET0862  NAD 83(1986)-  34 31 49.10126(N)    112 13 55.44054(W) AD(       ) 2
 ET0862  NAD 27      -  34 31 49.01237(N)    112 13 52.85943(W) AD(       ) 2
 ET0862  NGVD 29     -      1396.8    (m)        4583.     (f) VERT ANG
 ET0862
 ET0862.Superseded values are not recommended for survey control.
 ET0862.NGS no longer adjusts projects to the NAD 27 or NGVD 29 datums.
 ET0862.See file dsdata.txt to determine how the superseded data were derived.
 ET0862
 ET0862_MARKER: DD = SURVEY DISK
 ET0862_SETTING: 15 = METAL ROD DRIVEN INTO GROUND. SEE TEXT FOR ADDITIONAL
 ET0862+WITH SETTING: INFORMATION.
```

```
ET0862
ET0862  HISTORY     - Date      Condition        Recov. By
ET0862  HISTORY     - 1969      MONUMENTED       USGS
ET0862  HISTORY     - 1980      GOOD             USGS
ET0862  HISTORY     - 19971112  GOOD             USPSQD
ET0862
ET0862                          STATION DESCRIPTION
ET0862
ET0862'DESCRIBED BY US GEOLOGICAL SURVEY 1969 (FLA)
ET0862'LOCATED ABOUT 13 MI. E. OF PRESCOTT.  0.6 MI. E. OF DEWEY.  ON LEVEL
ET0862'GROUND AT A FENCE CORNER.
ET0862'
ET0862'TO REACH FROM THE JUNCTION OF STATE HIGHWAY 69 AND RD. E. TO DEWEY,
ET0862'DRIVE E. ACROSS THE RR. TRACKS FOR 0.6 MI. TO A N.-S. FENCE, TURN
ET0862'LEFT AND DRIVE N. ALONG THE W. SIDE OF THE FENCE FOR 300 FT. TO A
ET0862'FENCE CORNER AND STATION MARK.
ET0862'
ET0862'STATION MARK--A STANDARD USGS TABLET STAMPED DEWEY ET 1969 CRIMPED
ET0862'ON THE END OF A COPPERWELD ROD THAT HAS BEEN DRIVEN TO REFUSAL,
ET0862'LEVEL WITH THE GROUND.
ET0862'
ET0862'REFERENCE MARK NO. 1--A STANDARD USGS REFERENCE MARK TABLET STAMPED
ET0862'1 1969, CRIMPED ON THE END OF A COPPERWELD ROD THAT HAS BEEN DRIVEN
ET0862'TO REFUSAL, LEVEL WITH THE GROUND, AND 2 FT. BELOW THE STATION
ET0862'MARK.
ET0862'
ET0862'REFERENCE MARK NO. 2--A STANDARD USGS REFERENCE MARK TABLET STAMPED
ET0862'2 1969, CRIMPED ON THE END OF A COPPERWELD ROD THAT HAS BEEN DRIVEN
ET0862'TO REFUSAL, LEVEL WITH THE GROUND, AND 1 FT. ABOVE THE STATION.
ET0862'
ET0862'HUMBOLDT SMOKE STACK--A TALL MASONRY SMELTER STACK ABOUT 2.0 MI.
ET0862'FROM THE STATION MARK.
ET0862
ET0862                          STATION RECOVERY (1980)
ET0862
ET0862'RECOVERY NOTE BY US GEOLOGICAL SURVEY 1980 (GH)
ET0862'THE STATION MARK AND REFERENCE MARK 2 WERE RECOVERED AS DESCRIBED.
ET0862'REFERENCE MARK 1 HAS BEEN DESTROYED.  A NEW ROUTE TO THE
ET0862'STATION FOLLOWS.
ET0862'
ET0862'TO REACH THE STATION FROM THE POST OFFICE IN DEWEY, TRAVEL EAST ALONG
ET0862'ARIZONA HIGHWAY 169 FOR 0.7 MILE TO A SIDE ROAD LEFT.  TURN LEFT
ET0862'CROSSING OVER A CATTLE GUARD AND THEN BEAR LEFT ON A BLADED ROAD FOR
ET0862'0.18 MILE TO THE STATION ON THE LEFT, JUST PAST A RIGHT HAND CURVE IN
ET0862'THE ROAD.
ET0862
ET0862                          STATION RECOVERY (1997)
ET0862
ET0862'RECOVERY NOTE BY US POWER SQUADRON 1997
ET0862'RECOVERED IN GOOD CONDITION.
Elapsed Time = 00:00:07
```

The NGS Data Sheet

```
DATABASE = Sybase ,PROGRAM = datasheet, VERSION = 6.27
Starting Datasheet Retrieval...
1         National Geodetic Survey,   Retrieval Date = SEPTEMBER 14, 2000
 ET0938 ***********************************************************************
 ET0938  DESIGNATION -  POLAND
 ET0938  PID         -  ET0938
 ET0938  STATE/COUNTY-  AZ/YAVAPAI
 ET0938  USGS QUAD   -  POLAND JUNCTION (1975)
 ET0938
 ET0938                        *CURRENT SURVEY CONTROL
 ET0938
 ET0938* NAD 83(1992)-  34 27 29.64066(N)    112 16 10.16850(W)     ADJUSTED
 ET0938* NAVD 88     -      1527.2    (meters)    5010.    (feet)   VERTCON
 ET0938
 ET0938  LAPLACE CORR-          -1.61  (seconds)                    DEFLEC99
 ET0938  GEOID HEIGHT-         -26.32  (meters)                     GEOID99
 ET0938
 ET0938  HORZ ORDER  -  SECOND
 ET0938
 ET0938.The horizontal coordinates were established by classical geodetic methods
 ET0938.and adjusted by the National Geodetic Survey in August 1993.
 ET0938
 ET0938.The NAVD 88 height was computed by applying the VERTCON shift value to
 ET0938.the NGVD 29 height (displayed under SUPERSEDED SURVEY CONTROL.)
 ET0938
 ET0938.The Laplace correction was computed from DEFLEC99 derived deflections.
 ET0938
 ET0938.The geoid height was determined by GEOID99.
 ET0938
 ET0938;                  North         East     Units   Scale       Converg.
 ET0938;SPC AZ C     -   383,535.495   180,943.613   MT  0.99991295 -0 11 58.7
 ET0938;UTM  12      - 3,813,696.878   383,395.030   MT  0.99976759 -0 43 06.1
 ET0938
 ET0938:                Primary Azimuth Mark                      Grid Az
 ET0938:SPC AZ C     -  DEWEY                                     023 27 09.2
 ET0938:UTM  12      -  DEWEY                                     023 58 16.6
 ET0938
 ET0938|-----------------------------------------------------------------------|
 ET0938| PID    Reference Object                    Distance       Geod. Az    |
 ET0938|                                                           dddmmss.s   |
 ET0938| ET0862 DEWEY                              APPROX.  8.7 KM 0231510.5   |
 ET0938|        POLAND RM 2                         14.902 METERS 03358        |
 ET0938|        POLAND RM 1                          7.975 METERS 29700        |
 ET0938|-----------------------------------------------------------------------|
 ET0938
 ET0938                        SUPERSEDED SURVEY CONTROL
 ET0938
 ET0938  NAD 83(1986)-  34 27 29.63687(N)    112 16 10.16728(W) AD(       ) 2
 ET0938  NAD 27      -  34 27 29.54104(N)    112 16 07.57859(W) AD(       ) 2
 ET0938  NGVD 29     -      1526.4    (m)         5008.     (f) VERT ANG
 ET0938
 ET0938.Superseded values are not recommended for survey control.
 ET0938.NGS no longer adjusts projects to the NAD 27 or NGVD 29 datums.
 ET0938.See file dsdata.txt to determine how the superseded data were derived.
 ET0938
 ET0938_MARKER: DD = SURVEY DISK
 ET0938_SETTING: 15 = METAL ROD DRIVEN INTO GROUND. SEE TEXT FOR ADDITIONAL
 ET0938+WITH SETTING: INFORMATION.
 ET0938
```

```
ET0938  HISTORY     - Date     Condition        Recov: By
ET0938  HISTORY     - 1969     MONUMENTED       USGS
ET0938  HISTORY     - 1980     GOOD             AZDT
ET0938
ET0938                          STATION DESCRIPTION
ET0938
ET0938'DESCRIBED BY US GEOLOGICAL SURVEY 1969 (WFM)
ET0938'LOCATED ABOUT 13 MI. (AIRLINE) SE. OF PRESCOTT.  ABOUT 4 MI. NNW.
ET0938'OF MAYER.
ET0938'
ET0938'TO REACH FROM THE MAYER P.O. DRIVE N. 1.1 MI. TO ARIZONA HIGHWAY
ET0938'69.  GO N. 2.8 MI. TO A DRIVEWAY ON THE RIGHT.  TURN RIGHT, CROSS
ET0938'CATTLEGUARD THEN LEFT (N.) ON TRACK RD. ON E. SIDE OF HIGHWAY
ET0938'FENCE, FOR 0.6 MI. TO AN E.-W. FENCE AND END OF TRUCK TRAVEL.
ET0938'STATION IS 150 FT. NE. OF FENCE ON TOP.
ET0938'
ET0938'STATION MARK--A STANDARD USGS BRASS TABLET STAMPED POLAND ET 1969,
ET0938'CRIMPED TO A COPPERWELD ROD DRIVEN TO REFUSAL WITH A CEMENT COLLAR
ET0938'8 INCHES IN DIAMETER.
ET0938'
ET0938'REFERENCE MARK NO. 1--STANDARD USGS REFERENCE MARK TABLET STAMPED
ET0938'NO 1 1969 CEMENTED IN DRILL HOLE IN ROCK OUTCROP, ABOUT 0.6 FT.
ET0938'BELOW STATION.
ET0938'
ET0938'REFERENCE MARK NO. 2--STANDARD USGS REFERENCE MARK TABLET STAMPED
ET0938'NO 2 1969 CEMENTED IN DRILL HOLE IN ROCK OUTCROP, ABOUT 1.5 FT.
ET0938'BELOW STATION MARK.
ET0938
ET0938                          STATION RECOVERY (1980)
ET0938
ET0938'RECOVERY NOTE BY ARIZONA DEPARTMENT OF TRANSPORTATION 1980 (GH)
ET0938'THE STATION MARK AND REFERENCE MARKS 1 AND 2 WERE RECOVERED AS
ET0938'DESCRIBED IN 1969 AND FOUND IN GOOD CONDITION.  A NEW ROUTE TO THE
ET0938'STATION FOLLOWS.
ET0938'
ET0938'TO REACH THE STATION FROM THE POST OFFICE IN DEWEY WHICH IS ON ARIZONA
ET0938'HIGHWAY 69, TRAVEL SOUTHEAST ALONG SR 69 FOR 6.2 MILES TO A BLADED
ET0938'ROAD ON THE LEFT.  TURN LEFT CROSSING OVER A CATTLE GUARD, THEN LEFT
ET0938'AT FORK AND TRAVEL NORTH 0.5 MILE TO AN EAST WEST FENCELINE AND END OF
ET0938'TRUCK TRAVEL.  CROSS THE FENCE AND PACK NORTH UP THE HILL FOR ABOUT
ET0938'200 FEET TO THE STATION.
Elapsed Time = 00:00:10
```

ARIZONA STATE PLANE GRID CORDINATES
CENTRAL ZONE - NAD 1927

	"NORTHINGS"	"EASTINGS"
△ POLAND	1,258,265.90	393,861.64
△ DEWEY	1,284,457.18	405,223.72

from POLAND to DEWEY: N 23° 27′ 06″E 28,549.606′ (GRID)

ELEVATION @ POLAND = 1,526.4M = 5,008 FT.
ELEVATION @ DEWEY = 1,396.8M = 4,582 FT.
AVE. ELEVATION = 1,461.6M = 4,795.FT.

SEA LEVEL FACTOR = (20,906,000)/(20,906,000 + 4795) = 0.999770693

AVE. EASTING (POLAND to DEWEY) = 399,542.68
X' = (500,000 - 399,542.68) = 100,457.32

SCALE FACTOR =
$[(1.145 \times 10^{-15}) \times (100{,}457.32)^2] + 0.9999 = 0.999911555$

COMBINED FACTOR = (0.999770693) X (0.999911555) = 0.999682268

(28,549.606′/0.999682268) = 28,558.68′ = "GROUND" HORIZONTAL DIST.
BETWEEN POLAND AND DEWEY

("GROUND" HORIZONTAL DIST. IS DISTANCE BASED ON THE ELLIPSOID OF THE CLARKE SPHERIOD OF 1866).

4601
4602
3601
3602
1103
2500
3301
3302
4001
4002
1102
4904
4903
4902
4901
0401
2703
2701
4301
2600
0402
0501
4302
0403
2702
0502
1501
4303
0404
0503
1502
0405
0203
3501
4201
3502
0406
0202
0201
3003
3002
3001
4202
4203
4204
4205

DEFINITIONS
NAD - 1927 SPCS

TRANSVERSE MERCATOR PROJECTION = The SPCS used in those states extending primarily north to south -- (Arizona, New Mexico, Nevada).

LAMBERT PROJECTION = The SPCS used in those states whose major dimensions are primarily east to west -- (Colorado, Utah)

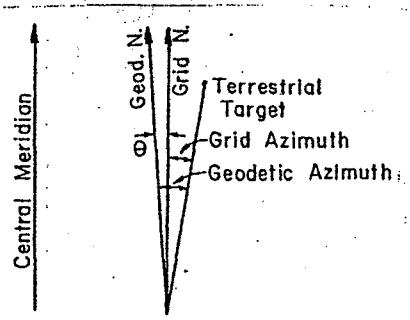
ZONE = A subdivision of the SPCS within a state. These subdivision follow county boundaries. In order to maintain a maximum scale reduction in the center of a particular system at 1:10,000, the limits cannot exceed 158 miles (east to west for the Transverse Mercator projection). There are some zones where for no reason or another the maximum is larger than 158 miles (AZ West Zone), and therefore the scale reduction is worse than 1:10,000 at their centers.

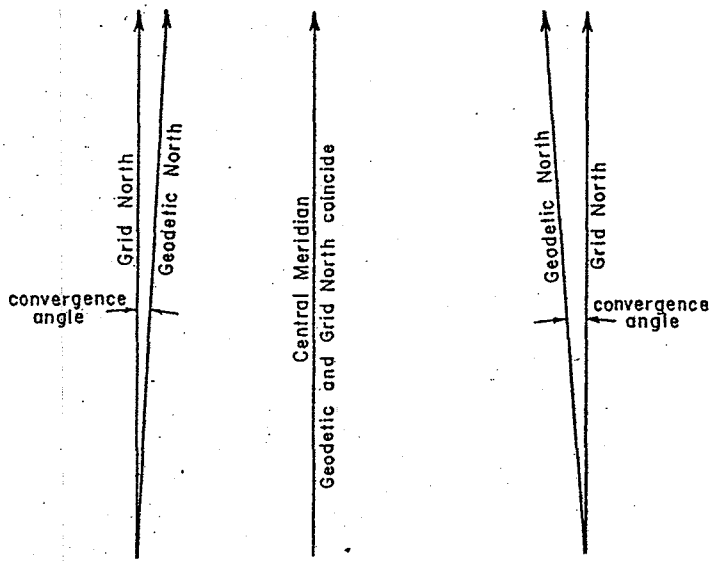
X = The plane coordinate value in feet (in the NAD 1927) perpendicular to the "Y" or north-south axis (center) of the system. In land surveying terms, "X" would be the Easting or "E" and consist of C +/- X′ where "C" is a constant of sufficient numerical size to keep the "X" value positive.

X′ = The distance in feet (NAD-1927) east or west of the "Y" or north-south axis of the system. When east of "Y" axis the values are added to the "C" and subtracted from "C" when west of the "Y" axis. This value may be considered the Departure measured from the "Y" axis.

Y = The plane coordinate value in the feet along the "Y" or north-south axis, north from the origin to its intersection with the "X" coordinate. In plane surveying terminology, this quantity may be considered the Northing, or "N", or the Latitude measured from the origin.

C.M. = CENTRAL MERIDIAN = The meridian (Longitude) or "Y" (north-south) axis usually located near the center of the plane coordinate system which separates the positive and negative "X′" quantities.





ARIZONA STATE PLANE GRID CORDINATES
CENTRAL ZONE - NAD 1927

	"NORTHINGS"	"EASTINGS"
▵ POLAND	1,258,265.90	393,861.64
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from POLAND to DEWEY: N 23° 27′ 06″E 28,549.606′ (GRID)

ELEVATION @ POLAND = 1,526.4M = 5,008 FT.
ELEVATION @ DEWEY = 1,396.8M = 4,582 FT.
AVE. ELEVATION = 1,461.6M = 4,795.FT.

1927 specific sea level reference

SEA LEVEL FACTOR = (20,906,000)/(20,906,000 + 4795) = 0.999770693

AVE. EASTING (POLAND to DEWEY) = 399,542.68
X' = (500,000 - 399,542.68) = 100,457.32

Important for 1927

SCALE FACTOR =
$[(1.145 \times 10^{-15}) \times (100{,}457.32)^2] + 0.9999 = 0.999911555$

COMBINED FACTOR = (0.999770693) X (0.999911555) = 0.999682268

(28,549.606'/0.999682268) = 28,558.68' = "GROUND" HORIZONTAL DIST. BETWEEN POLAND AND DEWEY

("GROUND" HORIZONTAL DIST. IS DISTANCE BASED ON THE ELLIPSOID OF THE CLARKE SPHERIOD OF 1866).

ARIZONA STATE PLANE GRID COORDINATES

ARIZONA STATE PLANE GRID COORDINATES
BASED ON THE CLARK SPHEROID OF 1866, AND
THE NORTH AMERICAN DATUM OF 1927 (NAD '27)

Three zones: EAST, CENTRAL and WEST.

Grid origins:	East =	Longitude	110° 10′ 00″
		Latitude	31° 00′ 00″
	Central	Longitude	111° 55′ 00″
		Latitude	31° 00′ 00″
	West	Longitude	113° 45′ 00″
		Latitude	31° 00′ 00″

The Central Meridian is equal to the Grid origin Longitude for each of the three zones, and the "EASTING" = 500,000 feet (U.S. Survey Foot)
(One Meter = 3.2808333333333333333333..... . . U.S. Survey Feet)
(1893 federal law stated a yard = 3600/3937m exactly = "Survey Foot")

==========

ARIZONA STATE PLANE GRID COORDINATES
BASED ON THE GEODETIC REFERENCE SYSTEM OF 1980, AND
THE NORTH AMERICAN DATUM OF 1983 (NAD '83)

There are the same three zones as the '27 system with the same three origins as mentioned above. (Please note: That the Latitude and Longitude are the same, **HOWEVER**, they are not at the same place on the surface of the earth because they are based on two different ellipsoid models of the earth, i.e., the Clark Spheroid of 1866 and the GRS of 1980.)

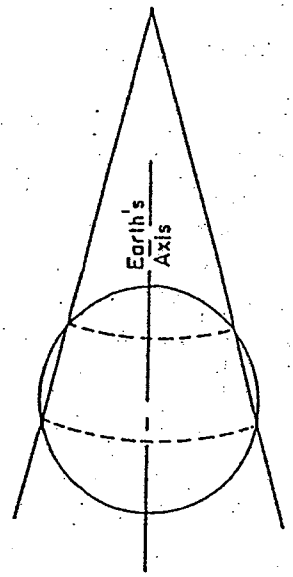
The Central Meridian for each zone is equal to the Grid Origin Longitude for each of the three zones. The "EASTING" for each central meridian is 700,000 feet (International Feet), this is by Arizona state statute.
(One Meter = 3.280839895 International Feet (to nine decimal places).
(1959 federal law stated a yard = 0.9144m exactly, = "International Foot")

==========

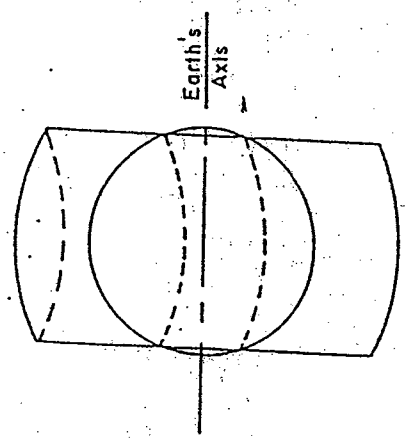
The mathematics necessary to operate in either the State Plan Coordinate System (SPCS) of '27 or of '83 are for most all applications (except those of extremely high (1st Order) precision) are the same.

Note: Arizona is the only state that enacted legislation defining their '83 state plane coordinate system based on the English ("foot") system of measurement. For conversion purposes the "700,000 ft." is considered to be equal to 213,360m -- by: The U.S. Department of Commerce
National Oceanic and Atmospheric Administration
National Ocean Survey
Charting and Geodetic Services
National Geodetic Survey
and the National Institute for Standards and Technology (NIST).

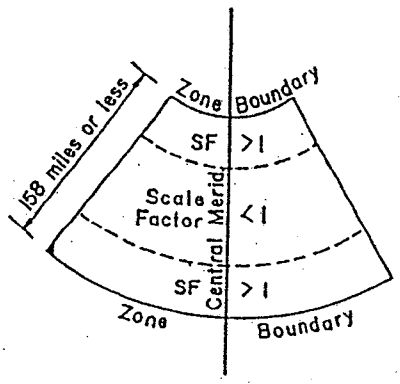
State Plane Projections



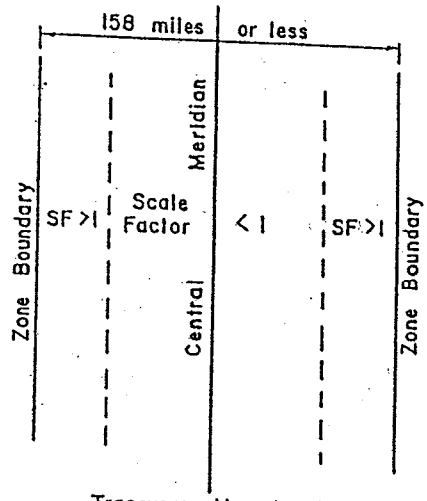
Lambert Projection
(Conical)



Transverse Mercator Projection
(Cylindrical)



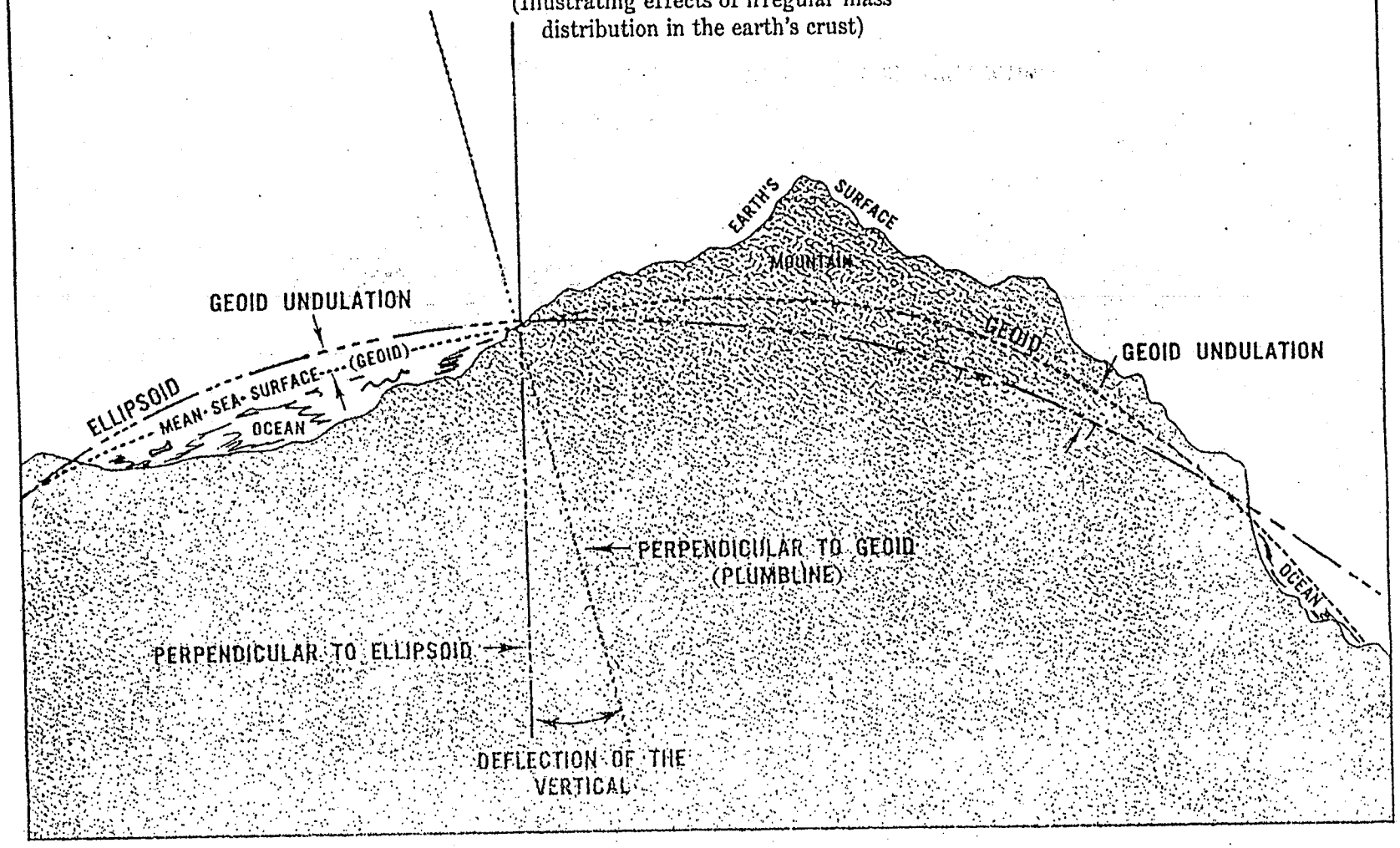
Lambert Zone
(cone developed into plane)

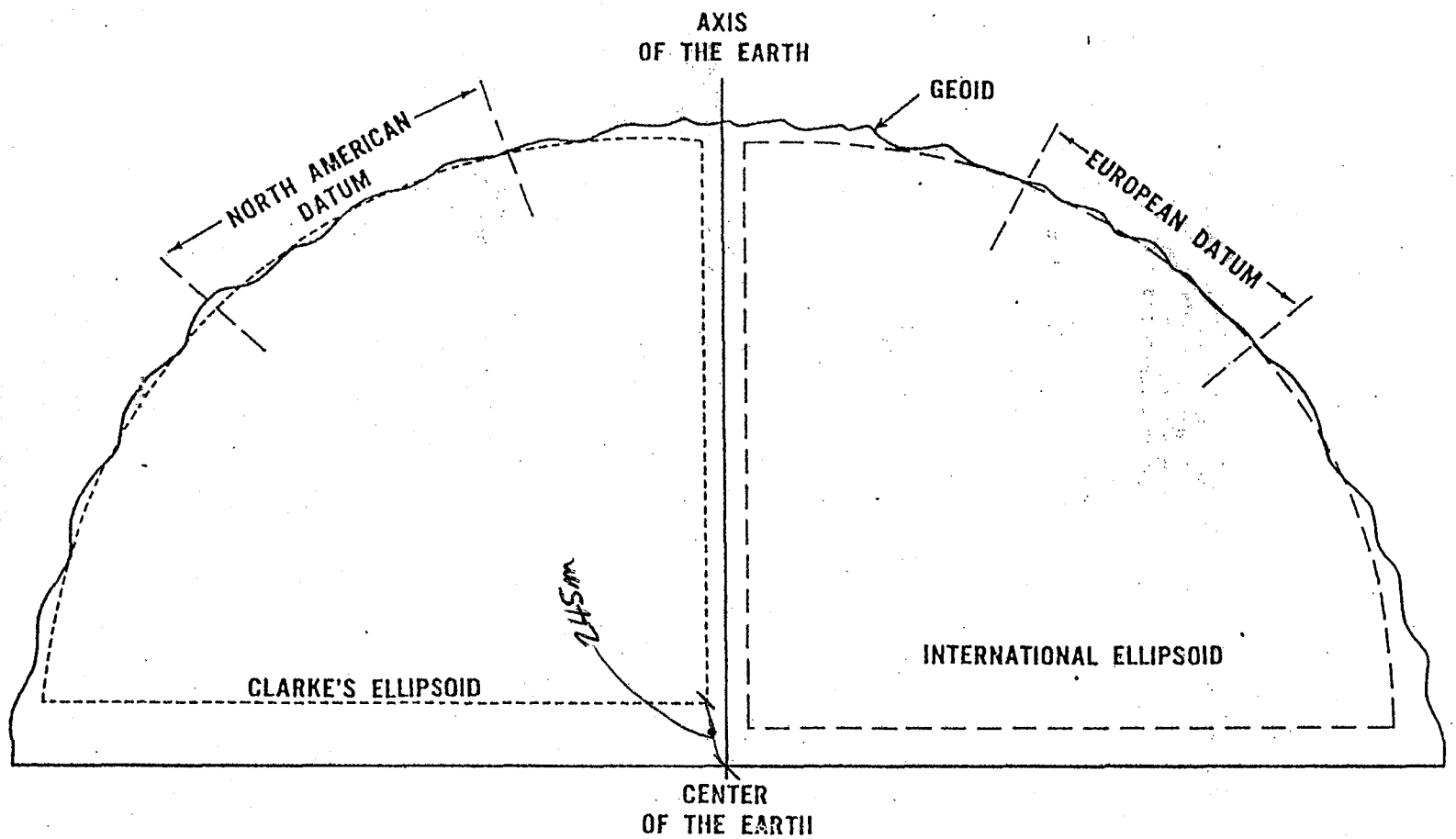


Transverse Mercator Zone
(cylinder developed into plane)

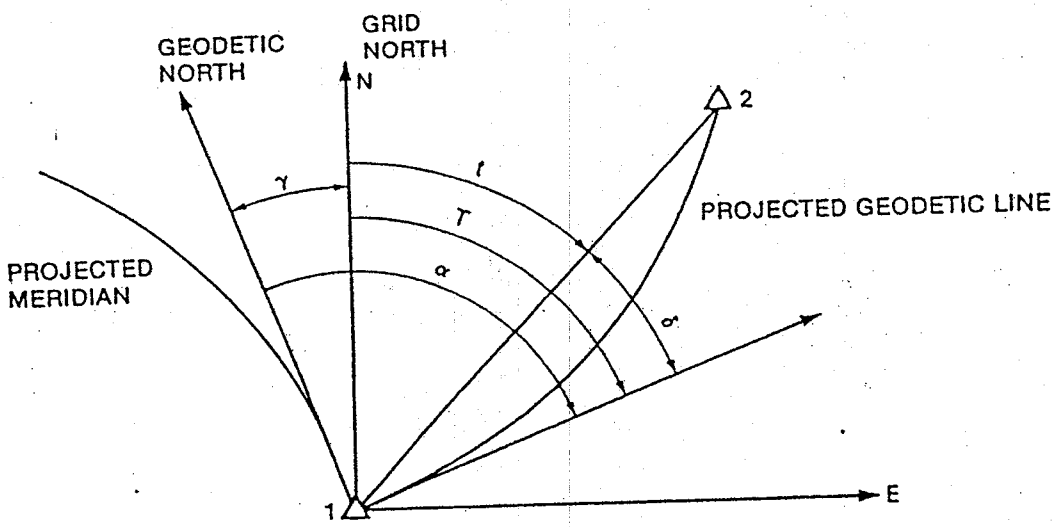
GEOID-ELLIPSOID RELATIONSHIPS

(Illustrating effects of irregular mass distribution in the earth's crust)





THE PREFERRED DATUMS ARE RELATIVE DATUMS

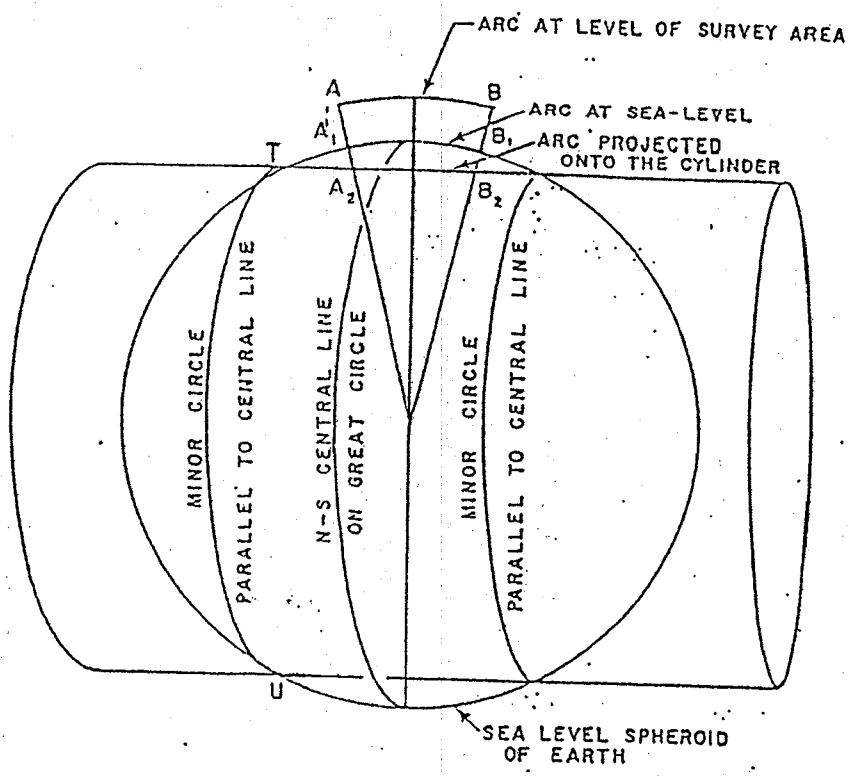


$$t = \alpha - \gamma + \delta$$

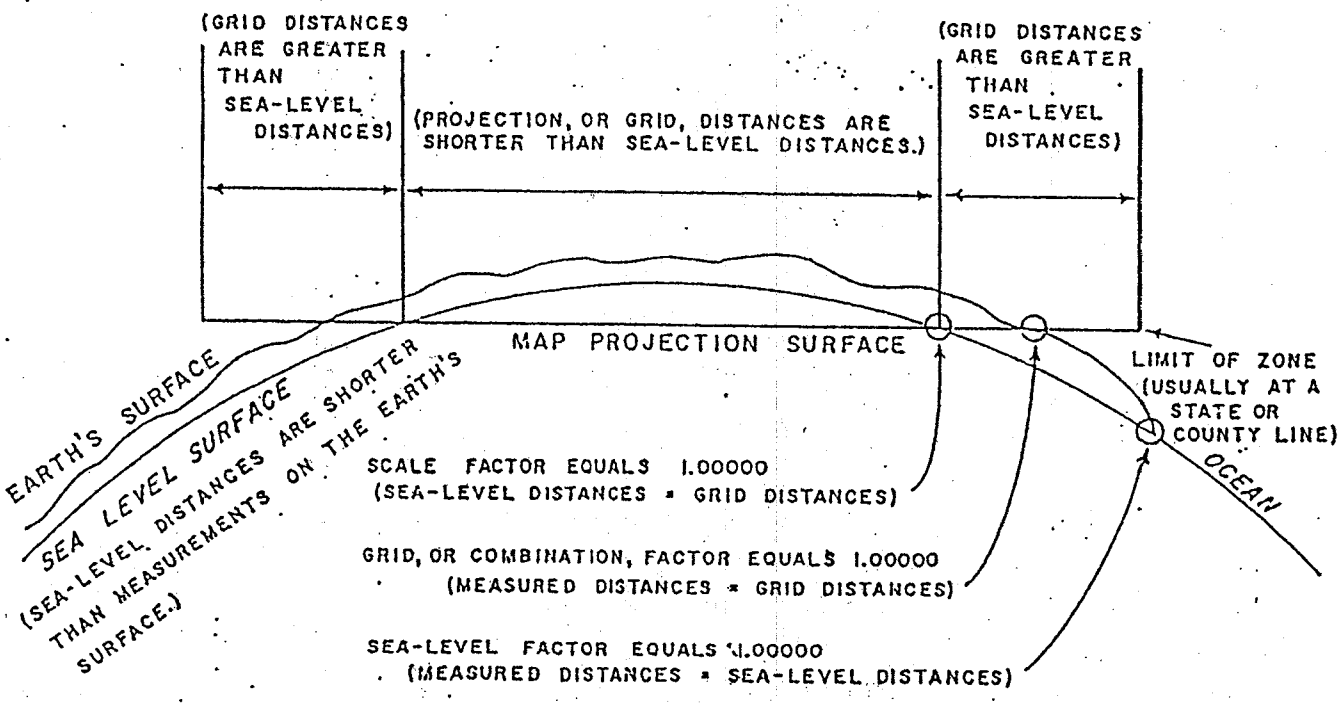
α = GEODETIC AZIMUTH RECKONED FROM NORTH
T = PROJECTED GEODETIC AZIMUTH
t = GRID AZIMUTH RECKONED FROM NORTH
γ = MAPPING ANGLE = CONVERGENCE ANGLE
δ = t-T = SECOND-TERM CORRECTION = ARC-TO-CHORD CORRECTION

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
Arizona	AZ					
East	E	0201	T.M.	110 10 1:10,000	110 10 31 00	213,360. 0
Central	C	0202	T.M.	111 55 1:10,000	111 55 31 00	213,360. 0
West	W	0203	T.M.	113 45 1:15,000	113 45 31 00	213,360. 0

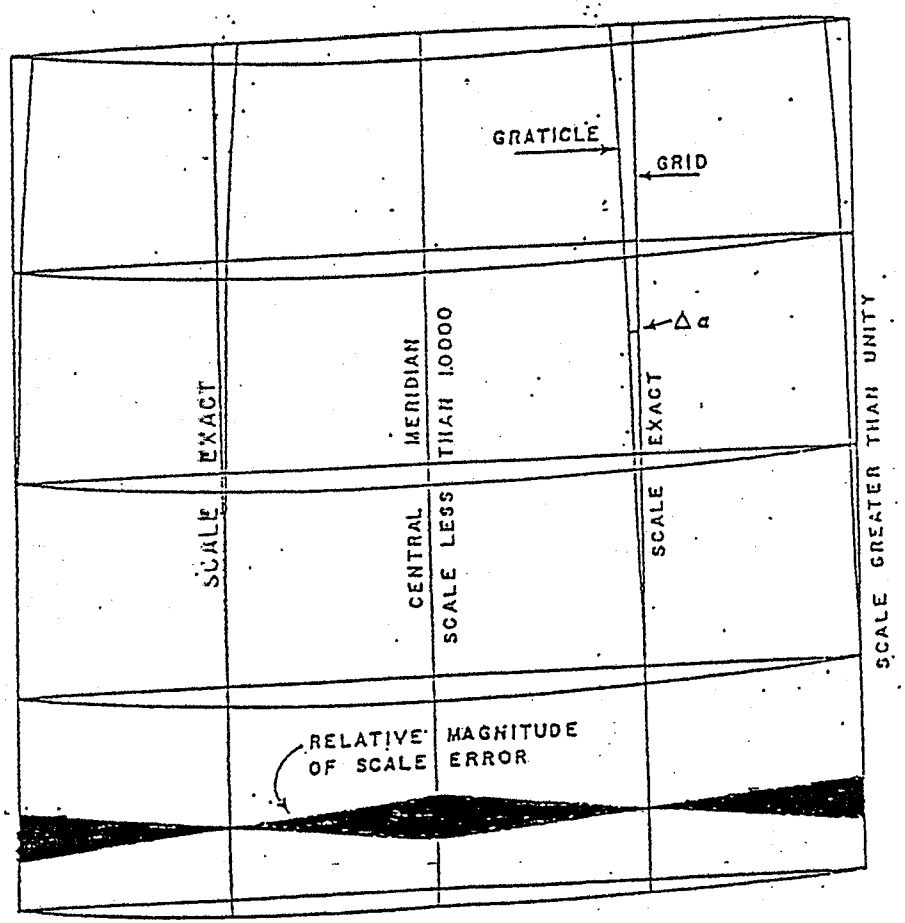
(State law defines the origin in International Feet)
(213,360M. = 700,000 International Feet)



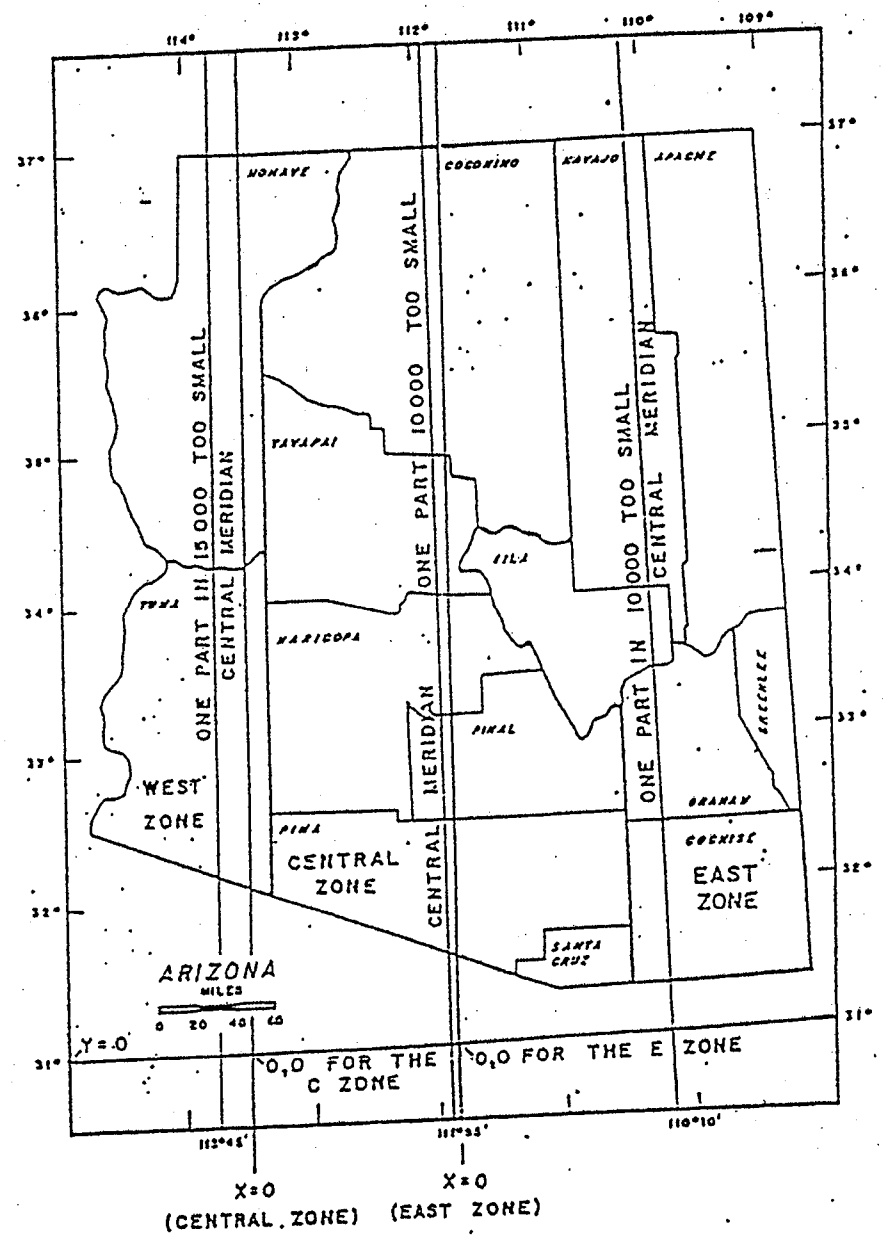
Elements of the Transverse Mercator Projection



Cross Section of a Secant Projection



Development of the Transverse Mercator Projection



Plane Coordinate Zones in Arizona

ARTICLE 2. ARIZONA COORDINATE SYSTEM

Article 2, consisting of §§ 33–121 to 33–128, is repealed by Laws 1985, Ch. 197, § 12, effective January 1, 1997.

Article 2, formerly Article 1.1 but renumbered Article 2 (see Reviser's Note, post), and consisting of §§ 33–121 to 33–128, was added by Laws 1972, Ch. 57, § 1, effective August 13, 1972.

Reviser's Note:
The article on the Arizona coordinate system as added by Laws 1972, Ch. 57 was renumbered as article 2 under the authority of § 41–1304.02.

§ 33–121. Arizona coordinate system; zones; composition

A. The Arizona coordinate system is the system of plane coordinates which has been established by the United States national geodetic survey for defining and stating the positions or locations of points on the surface of the earth within the state of Arizona.

B. The Arizona coordinate system shall contain three zones as follows:

1. The west zone, composed of La Paz, Mohave and Yuma counties.

2. The central zone, composed of Coconino, Maricopa, Pima, Pinal, Santa Cruz and Yavapai counties.

3. The east zone, composed of Apache, Cochise, Gila, Graham, Greenlee and Navajo counties.

C. In any land description in which the Arizona coordinate system is utilized the system shall be designated "Arizona coordinate system, _____ zone", with the name of the appropriate zone inserted, and the datum used, either 1927 or 1983.

Added by Laws 1972, Ch. 57, § 1. Amended by Laws 1983, Ch. 291, § 8, eff. April 27, 1983; Laws 1985, Ch. 197, § 6.

Repeal

This section is repealed by Laws 1985, Ch. 197, § 12, effective January 1, 1997.

Library References

Boundaries ⇐8.
WESTLAW Topic No. 59.
C.J.S. Boundaries § 14 et seq.

§ 33–122. Coordinates of system; zone definitions

A. The plane coordinates of a point on the earth's surface, to be used in expressing the position or location of such point in the appropriate zone of this system, shall consist of two distances, expressed in feet and decimals of a foot. One of these distances, to be known as the "X-coordinate", shall give the position in an east-and-west direction; the other, to be known as the "Y-coordinate", shall give the position in a north-and-south direction. These coor-

dinates shall be made to depend upon and conform to the coordinates, on the Arizona coordinate system, of the triangulation and traverse stations of the United States national geodetic survey within the state of Arizona, as those coordinates have been determined by such survey.

B. For the purpose of more precisely defining the Arizona coordinate system, the following definitions of the United States national geodetic survey are adopted:

1. The Arizona coordinate system, west zone, is a transverse mercator projection, having a central meridian 113° 45′ 00″ west of Greenwich, on which meridian the scale is set at one part in fifteen thousand too small. The origin of coordinates is at the intersection of the meridian 113° 45′ 00″ west of Greenwich and the parallel 31° 00′ 00″ north latitude. This origin is given the coordinates of "X" equals five hundred thousand feet, if using 1927 datum, or seven hundred thousand feet, if using 1983 datum, and "Y" equals zero feet in either case.

2. The Arizona coordinate system, central zone, is a transverse mercator projection, having a central meridian 111° 55′ 00″ west of Greenwich, on which meridian the scale is set at one part in ten thousand too small. The origin of coordinates is at the intersection of the meridian 111° 55′ 00″ west of Greenwich and the parallel 31° 00′ 00″ north latitude. This origin is given the coordinates of "X" equals five hundred thousand feet, if using 1927 datum, or seven hundred thousand feet, if using 1983 datum, and "Y" equals zero feet in either case.

3. The Arizona coordinate system, east zone, is a transverse mercator projection, having a central meridian 110° 10′ 00″ west of Greenwich, on which meridian the scale is set at one part in ten thousand too small. The origin of coordinates is at the intersection of the meridian 110° 10′ 00″ west of Greenwich and the parallel 31° 00′ 00″ north latitude. This origin is given the coordinates of "X" equals five hundred thousand feet, if using 1927 datum, or seven hundred thousand feet, if using 1983 datum, and "Y" equals zero feet in either case.

Added by Laws 1972, Ch. 57, § 1. Amended by Laws 1985, Ch. 197, § 7.

Repeal

This section is repealed by Laws 1985, Ch. 197, § 12, effective January 1, 1997.

Library References

Boundaries ⇐8.
WESTLAW Topic No. 59.
C.J.S. Boundaries § 14 et seq.

§ 33–123. Ground markings of system

The position of the Arizona coordinate system shall be as marked on the ground by triangulation or traverse stations established in conformity with standards adopted by the United States national geodetic survey for first-order

and second-order work, whose geodetic positions have been rigidly adjusted on the north American datum of either 1927 or 1983, and whose coordinates have been computed on the system defined in this article. Any such station may be used for establishing a survey connection with the Arizona coordinate system.

Added by Laws 1972, Ch. 57, § 1. Amended by Laws 1985, Ch. 197, § 8.

Repeal

This section is repealed by Laws 1985, Ch. 197, § 12, effective January 1, 1997.

Cross References

Recording prerequisites, see § 33–124.

Library References

Boundaries ⇐8, 54(1).
WESTLAW Topic No. 59.
C.J.S. Boundaries §§ 14 et seq., 89 et seq.

§ 33–124. Recording prerequisite; exception

No coordinates based on the Arizona coordinate system, purporting to define the position of a point on a land boundary, may be presented to be recorded in any public land records or deed records unless such point is within three statute miles of a triangulation or traverse station established in conformity with the standards prescribed in § 33–123, except that a six mile limitation may be substituted in isolated rural areas.

Added by Laws 1972, Ch. 57, § 1. Amended by Laws 1985, Ch. 197, § 9.

Repeal

This section is repealed by Laws 1985, Ch. 197, § 12, effective January 1, 1997.

Library References

Boundaries ⇐36(5).
WESTLAW Topic No. 59.
C.J.S. Boundaries § 114.

§ 33–125. Tract located in more than one zone; description

When any tract of land to be defined by a single description extends from one into another of the coordinate zones established by this article, the positions of all points on its boundaries may be referred to either of the two zones, the zone which is used being specifically named in the description.

Added by Laws 1972, Ch. 57, § 1. Amended by Laws 1985, Ch. 197, § 10.

Repeal

This section is repealed by Laws 1985, Ch. 197, § 12, effective January 1, 1997.

Library References

Boundaries ⇐11.
WESTLAW Topic No. 59.
C.J.S. Boundaries § 17 et seq.

§ 33–126. Public lands survey descriptions; conflicts; control

Where coordinates based on the Arizona coordinate system are used to describe any tract of land which in the same document is also described by reference to any subdivision, line or corner of the United States public land surveys, the description by coordinates shall be construed as supplemental to the basic description of such subdivision, line or corner contained in the official plats and field notes filed of record, and in the event of any conflict the description by reference to the subdivision, line or corner of the United States public land surveys shall prevail over the description by coordinates.
Added by Laws 1972, Ch. 57, § 1.

Repeal

This section is repealed by Laws 1985, Ch. 197, § 12, effective January 1, 1997.

Library References

Boundaries ⇐20(1).
WESTLAW Topic No. 59.
C.J.S. Boundaries § 35 et seq.

§ 33–127. Reliance of purchaser or mortgagee not required

Nothing contained in this article requires any purchaser or mortgagee to rely on a description, any part of which depends exclusively upon the Arizona coordinate system.
Added by Laws 1972, Ch. 57, § 1.

Repeal

This section is repealed by Laws 1985, Ch. 197, § 12, effective January 1, 1997.

§ 33–128. Limitation

The use of the term "Arizona coordinate system" on any map, report of survey or other document, shall be limited to coordinates based on the Arizona coordinate system as defined in this article.
Added by Laws 1972, Ch. 57, § 1.

Repeal

This section is repealed by Laws 1985, Ch. 197, § 12, effective January 1, 1997.

PHOENIX DEVELOPMENT
135 NORTH SECOND AVE., RM 520
PHOENIX, ARIZONA 85003

U. S. Department of Commerce
Charles Sawyer, *Secretary*

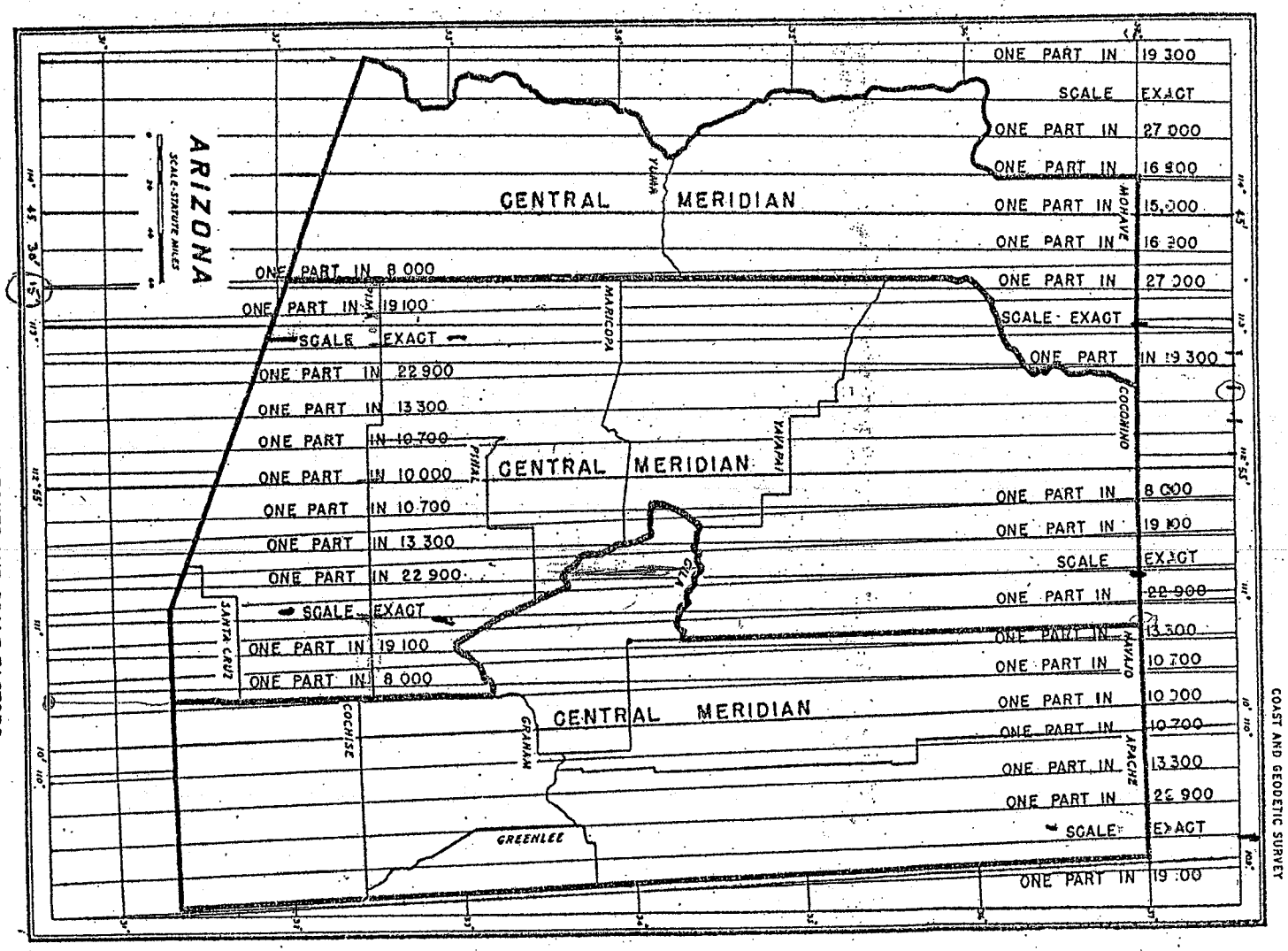
Coast and Geodetic Survey
Robert F. A. Studds, *Director*

Special Publication No. 257

PLANE COORDINATE PROJECTION TABLES
ARIZONA

UNITED STATES
GOVERNMENT PRINTING OFFICE
WASHINGTON : 1951

For sale by the Superintendent of Documents, U. S. Government Printing Office
Washington 25, D. C. - Price 20 cents (paper)



STATE PLANE-COORDINATE ZONES AND SCALE FACTORS

U. S. DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

Foreword

The plane coordinate system used in this State is based on the transverse Mercator projection using a reduced scale for the central meridian of the zone. The tables in this publication are to be used for the conversion of geographic positions to plane coordinates or plane coordinates to geographic positions. The constants of the projection are listed with the tables.

The methods of computation have been designed for machine calculation. All of the functions that are required are given in this publication.

The formulas and sample computations which follow show the general methods for computing either type of coordinates.

Plane coordinates from geographic positions

$$x = x' + 500{,}000$$

$$x' = H \cdot \Delta\lambda'' \pm a\,b$$

$$y = y_o + V\left(\frac{\Delta\lambda''}{100}\right)^2 \pm c$$

Grid azimuth = geodetic azimuth $-\Delta\alpha$ - second term

$$\Delta\alpha'' = \Delta\lambda'' \sin \phi + g$$

where

y_o, H, V, and a are based on the latitude of the geographic position,

and

b, c, and g are based on $\Delta\lambda''$.

$$\Delta\lambda'' = \text{Central Meridian} - \lambda$$

and

$\Delta\alpha''$ is the convergence of the meridian at the station with respect to the Central Meridian.

The second term for the reduction of geodetic to grid azimuths may be neglected for most work. However, for lines five miles or more in length if the same degree of accuracy is desired as is obtained by geographic computations, this term should be evaluated and used.

$$\text{Second term} = \frac{(y_2 - y_1)\ (2x_1' + x_2')}{(6\rho_o^2\ \sin 1'')_g}$$

<u>Geographic positions from plane coordinates</u>

$$P\ (x'/10{,}000)^2 + d = V\ (\Delta\lambda''/100)^2 + c$$

$$y_o = y - P\ (x'/10{,}000)^2 - d$$

Obtain the latitude from the table of y_o.

Use latitude to obtain H from the table.

$$x' = x - 500{,}000$$

approximate $\Delta\lambda'' = x' \div H.$

Determine <u>a</u> from latitude and <u>b</u> from approximate $\Delta\lambda$

then

$$\Delta\lambda'' = (x' \mp a\ b\ ) \div H$$

$$\Delta\alpha'' = Mx' - e$$

M is based on the y and e on the x and y of the plane coordinates.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form No. 744b

PLANE COORDINATES ON TRANSVERSE MERCATOR PROJECTION

(Condensed form for calculating-machine computation)

State Arizona Zone East Central meridian 110° 10′ 00.″000

	Station	Playa, 1935	Edge, 1936			
	ϕ	32 08 41.778	35 07 28.243			
	λ	109 45 13.226	110 30 34.948			
A	$\Delta\lambda$ = Central mer. − λ	+ 0° 24′ 46.″774	− 0° 20′ 34.″948			
	$\Delta\lambda''$	+1,486.774	−1,234.948			
	$\left(\frac{\Delta\lambda''}{100}\right)^2$	221.050	152.510			
	H	85.976 506	83.064 746			
	V	1.109 082	1.158 706			
	a / b	−0.912 / +4.969	−0.688 / +4.263			
	$x' = H \cdot \Delta\lambda \pm ab$	+127,823.10	−102,577.71			
	$V\left(\frac{\Delta\lambda''}{100}\right)^2 \pm c$	245.115	176.680			
	Tabular y	416,446.42	1,500,562.56			
	x	627,823.10	397,422.29			
	y	416,691.53	1,500,739.24			
	$\Delta\alpha''$	+791.07	−710.53			
	$\Delta\alpha$	+ 0 13 11.1	− 0 11 50.5			
	Geod. Az. to Az. Mk.	174 25 48.2	31 17 43.8			
	Grid Az. to Az. Mk.	174 12 37	31 29 34			

$x = x' + 500{,}000$

$y = \text{Tab. } y + V\left(\frac{\Delta\lambda''}{100}\right)^2 \pm c$

$\Delta\alpha'' = \Delta\lambda'' \sin\phi + g$

Grid Az. = Geod. Az. − $\Delta\alpha$

H and V = Tab. H and Tab. V.

When ab is $-$, decrease / $+$, increase $H \cdot \Delta\lambda$ numerically

g increases $\Delta\lambda'' \cdot \sin\phi$ numerically

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form No. 743a

6

GEODETIC POSITIONS FROM TRANSVERSE MERCATOR COORDINATES

(CALCULATING MACHINE COMPUTATION)

STATE ZONE Arizona - East

Station Pat, 1935 & 1945

X	683,147.83	Y	363,527.54
C	− 500,000.00	$P(\frac{X'}{10,000})^2 \div d$	− 500.37
X'	+ 183,147.83	Y_o	363,027.17
P	1.49155	Approx. $\Delta\lambda = X' \div H$	+ 2,127"
d	+0.06	$\Delta\lambda = (X' \mp ab) \div H$	+ 2,126."873
H	86.114 063	$\Delta\lambda$	+ 0° 35' 26."873
a | b	−0.924 | +6.324	Central Meridian	110 10 00.000
ϕ	31° 59' 53."103	λ = C.M. − $\Delta\lambda$	109° 34' 33."127

Station Ives, 1936

X	425,301.13	Y	1,515,853.43
C	− 500,000.00	$P(\frac{X'}{10,000})^2 + d$	− 93.84
X'	− 74,698.87	Y_o	1,515,759.59
P	1.68148	Approx. $\Delta\lambda = X' \div H$	−900"
d	+0.01	$\Delta\lambda = (X' \mp ab) \div H$	−899."771
H	83.022 323	$\Delta\lambda$	−0° 14' 59."771
a | b	−0.685 | +3.214³	Central Meridian	110 10 00.000
ϕ	35° 09' 58."568	λ = C.M. − $\Delta\lambda$	110° 24' 59."771

Station

X		Y	
C	−	$P(\frac{X'}{10,000})^2 + d$	−
X'		Y_o	
P		Approx. $\Delta\lambda = X' \div H$	"
d		$\Delta\lambda = (X' \mp ab) \div H$	"
H		$\Delta\lambda$	° ' "
a | b	|	Central Meridian	
ϕ	° ' "	λ = C.M. − $\Delta\lambda$	° ' "

Station

X		Y	
C	−	$P(\frac{X'}{10,000})^2 + d$	−
X'		Y_o	
P		Approx. $\Delta\lambda = X' \div H$	"
d		$\Delta\lambda = (X' \mp ab) \div H$	"
H		$\Delta\lambda$	° ' "
a | b	|	Central Meridian	
ϕ	° ' "	λ = C.M. − $\Delta\lambda$	° "

When ab is $\frac{+, \text{ decrease}}{-, \text{ increase}}$ X' numerically

M-2552-2

Constants for Arizona

Constant	Zone		
	East	Central	West
Central Meridian	110° 10' 00".000	111° 55' 00".000	113° 45' 00".000
log R	-434.3	-434.3	-289.5
Scale reduction (Central Meridian)	1 : 10,000	1 : 10,000	1 : 15,000
$\log\left(\frac{1}{6\rho_0^2}\right)_g$	4.581 6636 -20 $3.8164854 \times 10^{-16}$	4.581 6636 -20 $3.8164854 \times 10^{-16}$	4.581 6025 -20 $3.8159485 \times 10^{-16}$
$\log\left(\frac{1}{6\rho_0^2 \sin 1''}\right)_g$	9.896 0887 -20 $7.8720655 \times 10^{-11}$	9.896 0887 -20 $7.8720655 \times 10^{-11}$	9.896 0276 -20 $7.8709581 \times 10^{-11}$
$\left(\frac{1}{6\rho_0^2 \sin 1''}\right)_g$	0.7872×10^{-10}	0.7872×10^{-10}	0.7871×10^{-10}

TRANSVERSE MERCATOR PROJECTION

ARIZONA
EAST AND CENTRAL ZONES

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
31 00	0.00	101.026 67	87.033 908	252.41	1.086 800	5.62	-1.000
31 01	6 061.60	101.026 83	87.018 764	252.54	1.087 137	5.60	-.998
31 02	12 123.21	101.027 17	87.003 612	252.64	1.087 473	5.60	-.997
31 03	18 184.84	101.027 33	86.988 454	252.79	1.087 809	5.60	-.996
31 04	24 246.48	101.027 83	86.973 287	252.89	1.088 145	5.60	-.994
31 05	30 308.15	101.027 83	86.958 114	253.02	1.088 481	5.58	-.993
31 06	36 369.82	101.028 33	86.942 933	253.14	1.088 816	5.57	-.992
31 07	42 431.52	101.028 50	86.927 745	253.27	1.089 150	5.58	-.990
31 08	48 493.23	101.028 67	86.912 549	253.39	1.089 485	5.55	-.989
31 09	54 554.95	101.029 00	86.897 346	253.51	1.089 818	5.57	-.988
31 10	60 616.69	101.029 33	86.882 136	253.64	1.090 152	5.55	-.987
31 11	66 678.45	101.029 50	86.866 918	253.76	1.090 485	5.55	-.985
31 12	72 740.22	101.029 83	86.851 693	253.87	1.090 818	5.53	-.984
31 13	78 802.01	101.030 00	86.836 461	254.01	1.091 150	5.53	-.983
31 14	84 863.81	101.030 33	86.821 221	254.12	1.091 482	5.53	-.982
31 15	90 925.63	101.030 67	86.805 974	254.24	1.091 814	5.52	-.981
31 16	96 987.47	101.030 83	86.790 720	254.37	1.092 145	5.52	-.979
31 17	103 049.32	101.031 17	86.775 458	254.49	1.092 476	5.52	-.978
31 18	109 111.19	101.031 33	86.760 189	254.61	1.092 807	5.50	-.977
31 19	115 173.07	101.031 67	86.744 913	254.74	1.093 137	5.50	-.976
31 20	121 234.97	101.032 00	86.729 629	254.86	1.093 467	5.50	-.975
31 21	127 296.89	101.032 17	86.714 338	254.99	1.093 797	5.48	-.973
31 22	133 358.82	101.032 33	86.699 039	255.09	1.094 126	5.48	-.972
31 23	139 420.76	101.032 83	86.683 734	255.22	1.094 455	5.47	-.971
31 24	145 482.73	101.033 00	86.668 421	255.36	1.094 783	5.47	-.969
31 25	151 544.71	101.033 17	86.653 100	255.47	1.095 111	5.47	-.968
31 26	157 606.70	101.033 50	86.637 772	255.59	1.095 439	5.45	-.967
31 27	163 668.71	101.033 83	86.622 437	255.71	1.095 766	5.45	-.965
31 28	169 730.74	101.034 00	86.607 095	255.84	1.096 093	5.45	-.964
31 29	175 792.78	101.034 33	86.591 745	255.96	1.096 420	5.43	-.963
31 30	181 854.84	101.034 67	86.576 388	256.09	1.096 746	5.43	-.962
31 31	187 916.92	101.034 83	86.561 023	256.19	1.097 072	5.42	-.960
31 32	193 979.01	101.035 17	86.545 652	256.32	1.097 397	5.42	-.959
31 33	200 041.12	101.035 33	86.530 273	256.46	1.097 722	5.42	-.958
31 34	206 103.24	101.035 67	86.514 886	256.57	1.098 047	5.40	-.956
31 35	212 165.38	101.035 83	86.499 492	256.69	1.098 371	5.40	-.955
31 36	218 227.53	101.036 17	86.484 091	256.81	1.098 695	5.38	-.954
31 37	224 289.70	101.036 50	86.468 683	256.94	1.099 018	5.40	-.952
31 38	230 351.89	101.036 67	86.453 267	257.06	1.099 342	5.38	-.951
31 39	236 414.09	101.037 00	86.437 844	257.17	1.099 665	5.37	-.950
31 40	242 476.31	101.037 33	86.422 414	257.31	1.099 987	5.37	-.949
31 41	248 538.55	101.037 50	86.406 976	257.42	1.100 309	5.37	-.947
31 42	254 600.80	101.037 83	86.391 531	257.54	1.100 631	5.35	-.946
31 43	260 663.07	101.038 00	86.376 079	257.66	1.100 952	5.37	-.945
31 44	266 725.35	101.038 33	86.360 620	257.79	1.101 274	5.33	-.944
31 45	272 787.65	101.038 50	86.345 153	257.91	1.101 594	5.35	-.943
31 46	278 849.96	101.039 00	86.329 679	258.04	1.101 915	5.32	-.941
31 47	284 912.30	101.039 00	86.314 197	258.16	1.102 234	5.33	-.940
31 48	290 974.64	101.039 50	86.298 708	258.27	1.102 554	5.32	-.939
31 49	297 037.01	101.039 67	86.283 212	258.39	1.102 873	5.32	-.938
31 50	303 099.39	101.039 83	86.267 709	258.52	1.103 192	5.30	-.937
31 51	309 161.78	101.040 17	86.252 198	258.64	1.103 510	5.30	-.935
31 52	315 224.19	101.040 50	86.236 680	258.76	1.103 828	5.30	-.934
31 53	321 286.62	101.040 67	86.221 155	258.89	1.104 146	5.28	-.933
31 54	327 349.06	101.041 00	86.205 622	259.01	1.104 463	5.28	-.931
31 55	333 411.52	101.041 33	86.190 082	259.12	1.104 780	5.28	-.930
31 56	339 474.00	101.041 50	86.174 535	259.26	1.105 097	5.27	-.929
31 57	345 536.49	101.041 83	86.158 980	259.36	1.105 413	5.27	-.927
31 58	351 599.00	101.042 17	86.143 419	259.51	1.105 729	5.25	-.926
31 59	357 661.53	101.042 33	86.127 849	259.61	1.106 044	5.25	-.925
32 00	363 724.07		86.112 273		1.106 359		-.924

TRANSVERSE MERCATOR PROJECTION

ARIZONA
EAST AND CENTRAL ZONES

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
32 00	363 724.07	101.042 50	86.112 273	259.74	1.106 359	5.25	-.924
32 01	369 786.62	101.043 00	86.096 689	259.86	1.106 674	5.23	-.922
32 02	375 849.20	101.043 00	86.081 098	259.97	1.106 988	5.23	-.921
32 03	381 911.78	101.043 50	86.065 500	260.11	1.107 302	5.22	-.920
32 04	387 974.39	101.043 67	86.049 894	260.22	1.107 615	5.22	-.918
32 05	394 037.01	101.044 00	86.034 281	260.34	1.107 928	5.22	-.917
32 06	400 099.65	101.044 17	86.018 661	260.47	1.108 241	5.20	-.916
32 07	406 162.30	101.044 50	86.003 033	260.59	1.108 553	5.20	-.914
32 08	412 224.97	101.044 83	85.987 398	260.71	1.108 865	5.20	-.913
32 09	418 287.66	101.045 00	85.971 756	260.82	1.109 177	5.18	-.912
32 10	424 350.36	101.045 33	85.956 107	260.96	1.109 488	5.18	-.911
32 11	430 413.08	101.045 50	85.940 450	261.06	1.109 799	5.17	-.909
32 12	436 475.81	101.045 83	85.924 787	261.21	1.110 109	5.17	-.908
32 13	442 538.56	101.046 17	85.909 115	261.31	1.110 419	5.17	-.907
32 14	448 601.33	101.046 33	85.893 437	261.44	1.110 729	5.17	-.906
32 15	454 664.11	101.046 67	85.877 751	261.56	1.111 039	5.15	-.905
32 16	460 726.91	101.047 00	85.862 058	261.67	1.111 348	5.13	-.903
32 17	466 789.73	101.047 17	85.846 358	261.79	1.111 656	5.15	-.902
32 18	472 852.56	101.047 50	85.830 651	261.92	1.111 965	5.12	-.901
32 19	478 915.41	101.047 67	85.814 936	262.04	1.112 272	5.13	-.900
32 20	484 978.27	101.048 00	85.799 214	262.16	1.112 580	5.12	-.899
32 21	491 041.15	101.048 33	85.783 485	262.29	1.112 887	5.12	-.897
32 22	497 104.05	101.048 50	85.767 748	262.41	1.113 194	5.10	-.896
32 23	503 166.96	101.048 83	85.752 004	262.52	1.113 500	5.12	-.895
32 24	509 229.89	101.049 00	85.736 253	262.64	1.113 807	5.08	-.893
32 25	515 292.83	101.049 50	85.720 495	262.77	1.114 112	5.10	-.892
32 26	521 355.80	101.049 50	85.704 729	262.89	1.114 418	5.07	-.891
32 27	527 418.77	101.050 00	85.688 956	263.01	1.114 722	5.08	-.88[illegible]
32 28	533 481.77	101.050 17	85.673 176	263.12	1.115 027	5.07	-.88[illegible]
32 29	539 544.78	101.050 33	85.657 389	263.26	1.115 331	5.07	-.88[illegible]
32 30	545 607.80	101.050 67	85.641 594	263.37	1.115 635	5.05	-.886
32 31	551 670.84	101.051 00	85.625 792	263.49	1.115 938	5.05	-.884
32 32	557 733.90	101.051 33	85.609 983	263.62	1.116 241	5.05	-.883
32 33	563 796.98	101.051 50	85.594 166	263.72	1.116 544	5.03	-.882
32 34	569 860.07	101.051 83	85.578 343	263.86	1.116 846	5.03	-.881
32 35	575 923.18	101.052 00	85.562 512	263.97	1.117 148	5.03	-.880
32 36	581 986.30	101.052 33	85.546 674	264.11	1.117 450	5.02	-.878
32 37	588 049.44	101.052 67	85.530 828	264.21	1.117 751	5.02	-.877
32 38	594 112.60	101.052 83	85.514 976	264.34	1.118 052	5.00	-.876
32 39	600 175.77	101.053 17	85.499 116	264.46	1.118 352	5.00	-.875
32 40	606 238.96	101.053 50	85.483 249	264.57	1.118 652	5.00	-.874
32 41	612 302.17	101.053 67	85.467 375	264.71	1.118 952	4.98	-.872
32 42	618 365.39	101.054 00	85.451 493	264.81	1.119 251	4.98	-.871
32 43	624 428.63	101.054 17	85.435 605	264.94	1.119 550	4.97	-.870
32 44	630 491.88	101.054 50	85.419 709	265.06	1.119 848	4.97	-.868
32 45	636 555.15	101.054 83	85.403 806	265.19	1.120 146	4.97	-.867
32 46	642 618.44	101.055 00	85.387 895	265.31	1.120 444	4.95	-.866
32 47	648 681.74	101.055 33	85.371 977	265.41	1.120 741	4.95	-.864
32 48	654 745.06	101.055 67	85.356 053	265.56	1.121 038	4.95	-.863
32 49	660 808.40	101.055 83	85.340 120	265.66	1.121 335	4.93	-.862
32 50	666 871.75	101.056 17	85.324 181	265.79	1.121 631	4.93	-.861
32 51	672 935.12	101.056 33	85.308 234	265.91	1.121 927	4.92	-.859
32 52	678 998.50	101.056 67	85.292 280	266.02	1.122 222	4.92	-.858
32 53	685 061.90	101.057 00	85.276 319	266.14	1.122 517	4.92	-.857
32 54	691 125.32	101.057 33	85.260 351	266.27	1.122 812	4.90	-.855
32 55	697 188.76	101.057 50	85.244 375	266.39	1.123 106	4.90	-.854
32 56	703 252.21	101.057 67	85.228 392	266.51	1.123 400	4.90	-.853
32 57	709 315.67	101.058 17	85.212 402	266.62	1.123 694	4.88	-.851
32 58	715 379.16	101.058 33	85.196 405	266.74	1.123 987	4.88	-.850
32 59	721 442.66	101.058 50	85.180 401	266.87	1.124 280	4.87	-.849
33 00	727 506.17		85.164 389		1.124 572		-.84[illegible]

TRANSVERSE MERCATOR PROJECTION

ARIZONA
EAST AND CENTRAL ZONES

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
33 00	727 506.17	101.058 83	85.164 389	266.99	1.124 572	4.87	-.848
33 01	733 569.70	101.059 33	85.148 370	267.11	1.124 864	4.87	-.846
33 02	739 633.26	101.059 33	85.132 344	267.22	1.125 156	4.85	-.845
33 03	745 696.82	101.059 67	85.116 311	267.36	1.125 447	4.85	-.844
33 04	751 760.40	101.060 00	85.100 270	267.46	1.125 738	4.85	-.843
33 05	757 824.00	101.060 17	85.084 223	267.59	1.126 029	4.83	-.842
33 06	763 887.61	101.060 50	85.068 168	267.71	1.126 319	4.83	-.840
33 07	769 951.24	101.060 83	85.052 106	267.84	1.126 609	4.82	-.839
33 08	776 014.89	101.061 17	85.036 036	267.94	1.126 898	4.82	-.838
33 09	782 078.56	101.061 33	85.019 960	268.07	1.127 187	4.82	-.837
33 10	788 142.24	101.061 50	85.003 876	268.19	1.127 476	4.80	-.836
33 11	794 205.93	101.062 00	84.987 785	268.31	1.127 764	4.80	-.834
33 12	800 269.65	101.062 17	84.971 687	268.44	1.128 052	4.78	-.833
33 13	806 333.38	101.062 33	84.955 581	268.54	1.128 339	4.80	-.832
33 14	812 397.12	101.062 67	84.939 469	268.67	1.128 627	4.77	-.830
33 15	818 460.88	101.063 00	84.923 349	268.79	1.128 913	4.78	-.829
33 16	824 524.66	101.063 33	84.907 222	268.91	1.129 200	4.75	-.828
33 17	830 588.46	101.063 50	84.891 088	269.04	1.129 485	4.77	-.826
33 18	836 652.27	101.063 83	84.874 946	269.14	1.129 771	4.75	-.825
33 19	842 716.10	101.064 00	84.858 798	269.27	1.130 056	4.75	-.824
33 20	848 779.94	101.064 33	84.842 642	269.39	1.130 341	4.73	-.823
33 21	854 843.80	101.064 67	84.826 479	269.51	1.130 625	4.73	-.821
33 22	860 907.68	101.065 00	84.810 309	269.64	1.130 909	4.73	-.820
33 23	866 971.58	101.065 17	84.794 131	269.74	1.131 193	4.72	-.819
33 24	873 035.49	101.065 33	84.777 947	269.87	1.131 476	4.72	-.817
25	879 099.41	101.065 83	84.761 755	269.99	1.131 759	4.72	-.816
26	885 163.36	101.066 00	84.745 556	270.11	1.132 042	4.70	-.815
27	891 227.32	101.066 17	84.729 350	270.22	1.132 324	4.70	-.813
28	897 291.29	101.066 67	84.713 137	270.34	1.132 606	4.68	-.812
29	903 355.29	101.066 83	84.696 917	270.47	1.132 887	4.68	-.811
33 30	909 419.30	101.067 00	84.680 689	270.59	1.133 168	4.68	-.810
33 31	915 483.32	101.067 33	84.664 454	270.71	1.133 449	4.67	-.808
33 32	921 547.36	101.067 67	84.648 212	270.82	1.133 729	4.67	-.807
33 33	927 611.42	101.068 00	84.631 963	270.94	1.134 009	4.65	-.806
33 34	933 675.50	101.068 17	84.615 707	271.06	1.134 288	4.65	-.805
33 35	939 739.59	101.068 50	84.599 444	271.19	1.134 567	4.65	-.804
33 36	945 803.70	101.068 83	84.583 173	271.31	1.134 846	4.63	-.802
33 37	951 867.83	101.069 00	84.566 895	271.42	1.135 124	4.63	-.801
33 38	957 931.97	101.069 33	84.550 610	271.54	1.135 402	4.63	-.800
33 39	963 996.13	101.069 50	84.534 318	271.66	1.135 680	4.62	-.799
33 40	970 060.30	101.069 83	84.518 019	271.79	1.135 957	4.62	-.798
33 41	976 124.49	101.070 17	84.501 712	271.89	1.136 234	4.60	-.796
33 42	982 188.70	101.070 50	84.485 399	272.02	1.136 510	4.60	-.795
33 43	988 252.93	101.070 67	84.469 078	272.14	1.136 786	4.60	-.794
33 44	994 317.17	101.070 83	84.452 750	272.26	1.137 062	4.58	-.792
33 45	1 000 381.42	101.071 33	84.436 415	272.39	1.137 337	4.58	-.791
33 46	1 006 445.70	101.071 50	84.420 072	272.49	1.137 612	4.58	-.790
33 47	1 012 509.99	101.071 83	84.403 723	272.61	1.137 887	4.57	-.788
33 48	1 018 574.30	101.072 00	84.387 367	272.74	1.138 161	4.57	-.787
33 49	1 024 638.62	101.072 33	84.371 003	272.86	1.138 435	4.55	-.786
33 50	1 030 702.96	101.072 67	84.354 632	272.97	1.138 708	4.55	-.785
33 51	1 036 767.32	101.073 00	84.338 254	273.09	1.138 981	4.55	-.783
33 52	1 042 831.70	101.073 17	84.321 869	273.21	1.139 254	4.53	-.782
33 53	1 048 896.09	101.073 33	84.305 477	273.34	1.139 526	4.52	-.781
33 54	1 054 960.49	101.073 83	84.289 077	273.44	1.139 797	4.53	-.779
33 55	1 061 024.92	101.074 00	84.272 671	273.57	1.140 069	4.52	-.778
33 56	1 067 089.36	101.074 33	84.256 257	273.69	1.140 340	4.50	-.777
33 57	1 073 153.82	101.074 50	84.239 836	273.81	1.140 610	4.52	-.775
33 58	1 079 218.29	101.074 83	84.223 408	273.92	1.140 881	4.50	-.774
33 59	1 085 282.78	101.075 17	84.206 973	274.04	1.141 151	4.48	-.773
00	1 091 347.29		84.190 531		1.141 420		-.772

TRANSVERSE MERCATOR PROJECTION

ARIZONA
EAST AND CENTRAL ZONES

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
34 00	1 091 347.29	101.075 33	84.190 531	274.16	1.141 420	4.48	-.772
34 01	1 097 411.81	101.075 67	84.174 082	274.29	1.141 689	4.48	-.770
34 02	1 103 476.35	101.076 00	84.157 625	274.41	1.141 958	4.47	-.769
34 03	1 109 540.91	101.076 17	84.141 161	274.52	1.142 226	4.47	-.768
34 04	1 115 605.48	101.076 50	84.124 690	274.62	1.142 494	4.45	-.767
34 05	1 121 670.07	101.076 83	84.108 213	274.77	1.142 761	4.45	-.766
34 06	1 127 734.68	101.077 00	84.091 727	274.87	1.143 028	4.45	-.764
34 07	1 133 799.30	101.077 33	84.075 235	274.99	1.143 295	4.43	-.763
34 08	1 139 863.94	101.077 67	84.058 736	275.11	1.143 561	4.43	-.762
34 09	1 145 928.60	101.077 83	84.042 230	275.24	1.143 827	4.43	-.761
34 10	1 151 993.27	101.078 17	84.025 716	275.36	1.144 093	4.42	-.760
34 11	1 158 057.96	101.078 50	84.009 195	275.47	1.144 358	4.42	-.758
34 12	1 164 122.67	101.078 83	83.992 667	275.57	1.144 623	4.42	-.757
34 13	1 170 187.40	101.079 00	83.976 133	275.71	1.144 888	4.40	-.756
34 14	1 176 252.14	101.079 17	83.959 591	275.84	1.145 152	4.38	-.754
34 15	1 182 316.89	101.079 67	83.943 041	275.94	1.145 415	4.40	-.753
34 16	1 188 381.67	101.079 83	83.926 485	276.06	1.145 679	4.38	-.752
34 17	1 194 446.46	101.080 00	83.909 922	276.19	1.145 942	4.37	-.750
34 18	1 200 511.26	101.080 50	83.893 351	276.29	1.146 204	4.37	-.749
34 19	1 206 576.09	101.080 67	83.876 774	276.42	1.146 466	4.37	-.748
34 20	1 212 640.93	101.080 83	83.860 189	276.54	1.146 728	4.35	-.747
34 21	1 218 705.78	101.081 33	83.843 597	276.66	1.146 989	4.35	-.745
34 22	1 224 770.66	101.081 50	83.826 998	276.77	1.147 250	4.35	-.744
34 23	1 230 835.55	101.081 83	83.810 392	276.89	1.147 511	4.33	-.743
34 24	1 236 900.46	101.082 00	83.793 779	277.01	1.147 771	4.33	-.742
34 25	1 242 965.38	101.082 33	83.777 159	277.14	1.148 031	4.32	-.741
34 26	1 249 030.32	101.082 67	83.760 531	277.24	1.148 290	4.32	-.739
34 27	1 255 095.28	101.083 00	83.743 897	277.37	1.148 549	4.32	-.738
34 28	1 261 160.26	101.083 17	83.727 255	277.47	1.148 808	4.30	-.737
34 29	1 267 225.25	101.083 33	83.710 607	277.61	1.149 066	4.30	-.736
34 30	1 273 290.25	101.083 83	83.693 951	277.72	1.149 324	4.28	-.735
34 31	1 279 355.28	101.084 00	83.677 288	277.84	1.149 581	4.28	-.733
34 32	1 285 420.32	101.084 33	83.660 618	277.96	1.149 838	4.28	-.732
34 33	1 291 485.38	101.084 50	83.643 941	278.07	1.150 095	4.27	-.731
34 34	1 297 550.45	101.084 83	83.627 257	278.19	1.150 351	4.27	-.729
34 35	1 303 615.54	101.085 17	83.610 566	278.31	1.150 607	4.27	-.728
34 36	1 309 680.65	101.085 50	83.593 868	278.44	1.150 863	4.25	-.727
34 37	1 315 745.78	101.085 67	83.577 162	278.54	1.151 118	4.25	-.725
34 38	1 321 810.92	101.086 00	83.560 450	278.66	1.151 373	4.23	-.724
34 39	1 327 876.08	101.086 33	83.543 731	278.79	1.151 627	4.23	-.723
34 40	1 333 941.26	101.086 50	83.527 004	278.91	1.151 881	4.23	-.722
34 41	1 340 006.45	101.086 83	83.510 270	279.01	1.152 135	4.22	-.720
34 42	1 346 071.66	101.087 00	83.493 530	279.14	1.152 388	4.22	-.719
34 43	1 352 136.88	101.087 50	83.476 782	279.26	1.152 641	4.20	-.718
34 44	1 358 202.13	101.087 67	83.460 027	279.37	1.152 893	4.22	-.717
34 45	1 364 267.39	101.087 83	83.443 265	279.49	1.153 146	4.18	-.716
34 46	1 370 332.66	101.088 33	83.426 496	279.61	1.153 397	4.20	-.714
34 47	1 376 397.96	101.088 50	83.409 720	279.72	1.153 649	4.17	-.713
34 48	1 382 463.27	101.088 83	83.392 937	279.86	1.153 899	4.18	-.712
34 49	1 388 528.60	101.089 00	83.376 146	279.96	1.154 150	4.17	-.711
34 50	1 394 593.94	101.089 33	83.359 349	280.07	1.154 400	4.17	-.710
34 51	1 400 659.30	101.089 67	83.342 545	280.21	1.154 650	4.15	-.708
34 52	1 406 724.68	101.089 83	83.325 733	280.31	1.154 899	4.15	-.707
34 53	1 412 790.07	101.090 17	83.308 915	280.44	1.155 148	4.13	-.706
34 54	1 418 855.48	101.090 50	83.292 089	280.56	1.155 396	4.13	-.704
34 55	1 424 920.91	101.090 83	83.275 256	280.66	1.155 644	4.13	-.703
34 56	1 430 986.36	101.091 00	83.258 417	280.79	1.155 892	4.12	-.702
34 57	1 437 051.82	101.091 33	83.241 570	280.91	1.156 139	4.12	-.700
34 58	1 443 117.30	101.091 50	83.224 716	281.02	1.156 386	4.12	-.699
34 59	1 449 182.79	101.091 83	83.207 855	281.14	1.156 633	4.10	-.698
35 00	1 455 248.30		83.190 987		1.156 879		-.69

TRANSVERSE MERCATOR PROJECTION

ARIZONA
EAST AND CENTRAL ZONES

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
35 00	1 455 248.30	101.092 33	83.190 987	281.26	1.156 879	4.10	-.697
35 01	1 461 313.84	101.092 33	83.174 112	281.37	1.157 125	4.08	-.695
35 02	1 467 379.38	101.092 67	83.157 230	281.49	1.157 370	4.10	-.694
35 03	1 473 444.94	101.093 00	83.140 341	281.62	1.157 616	4.07	-.693
35 04	1 479 510.52	101.093 33	83.123 444	281.72	1.157 860	4.08	-.692
35 05	1 485 576.12	101.093 50	83.106 541	281.84	1.158 105	4.05	-.691
35 06	1 491 641.73	101.093 83	83.089 631	281.97	1.158 348	4.07	-.689
35 07	1 497 707.36	101.094 17	83.072 713	282.07	1.158 592	4.05	-.688
35 08	1 503 773.01	101.094 50	83.055 789	282.21	1.158 835	4.05	-.687
35 09	1 509 838.68	101.094 67	83.038 857	282.31	1.159 078	4.03	-.686
35 10	1 515 904.36	101.095 00	83.021 919	282.44	1.159 320	4.03	-.685
35 11	1 521 970.06	101.095 17	83.004 973	282.54	1.159 562	4.02	-.683
35 12	1 528 035.77	101.095 50	82.988 021	282.67	1.159 803	4.02	-.682
35 13	1 534 101.50	101.095 83	82.971 061	282.79	1.160 044	4.02	-.681
35 14	1 540 167.25	101.096 17	82.954 094	282.89	1.160 285	4.00	-.679
35 15	1 546 233.02	101.096 33	82.937 121	283.02	1.160 525	4.00	-.678
35 16	1 552 298.80	101.096 67	82.920 140	283.14	1.160 765	4.00	-.677
35 17	1 558 364.60	101.096 83	82.903 152	283.24	1.161 005	3.98	-.675
35 18	1 564 430.41	101.097 33	82.886 158	283.37	1.161 244	3.98	-.674
35 19	1 570 496.25	101.097 50	82.869 156	283.49	1.161 483	3.97	-.673
35 20	1 576 562.10	101.097 83	82.852 147	283.61	1.161 721	3.97	-.672
35 21	1 582 627.97	101.098 00	82.835 131	283.72	1.161 959	3.97	-.670
35 22	1 588 693.85	101.098 33	82.818 108	283.82	1.162 197	3.95	-.669
35 23	1 594 759.75	101.098 67	82.801 079	283.96	1.162 434	3.93	-.668
35 24	1 600 825.67	101.099 00	82.784 042	284.07	1.162 670	3.95	-.667
35 25	1 606 891.61	101.099 17	82.766 998	284.19	1.162 907	3.93	-.666
35 26	1 612 957.56	101.099 50	82.749 947	284.31	1.163 143	3.92	-.664
35 27	1 619 023.53	101.099 83	82.732 889	284.42	1.163 378	3.93	-.663
35 28	1 625 089.52	101.100 00	82.715 824	284.54	1.163 614	3.92	-.662
35 29	1 631 155.52	101.100 33	82.698 752	284.66	1.163 849	3.90	-.661
35 30	1 637 221.54	101.100 67	82.681 673	284.77	1.164 083	3.90	-.660
35 31	1 643 287.58	101.100 83	82.664 587	284.89	1.164 317	3.90	-.658
35 32	1 649 353.63	101.101 17	82.647 494	285.01	1.164 551	3.88	-.657
35 33	1 655 419.70	101.101 50	82.630 394	285.12	1.164 784	3.88	-.656
35 34	1 661 485.79	101.101 67	82.613 287	285.24	1.165 017	3.88	-.654
35 35	1 667 551.89	101.102 17	82.596 173	285.37	1.165 250	3.87	-.653
35 36	1 673 618.02	101.102 33	82.579 051	285.47	1.165 482	3.85	-.652
35 37	1 679 684.16	101.102 50	82.561 923	285.59	1.165 713	3.87	-.650
35 38	1 685 750.31	101.102 83	82.544 788	285.71	1.165 945	3.85	-.649
35 39	1 691 816.48	101.103 17	82.527 646	285.82	1.166 176	3.83	-.648
35 40	1 697 882.67	101.103 50	82.510 497	285.94	1.166 406	3.83	-.647
35 41	1 703 948.88	101.103 67	82.493 341	286.06	1.166 636	3.83	-.645
35 42	1 710 015.10	101.104 17	82.476 178	286.19	1.166 866	3.82	-.644
35 43	1 716 081.35	101.104 33	82.459 007	286.29	1.167 095	3.82	-.643
35 44	1 722 147.61	101.104 50	82.441 830	286.41	1.167 324	3.80	-.642
35 45	1 728 213.88	101.104 83	82.424 646	286.52	1.167 552	3.80	-.641
35 46	1 734 280.17	101.105 17	82.407 455	286.64	1.167 780	3.80	-.639
35 47	1 740 346.48	101.105 50	82.390 257	286.76	1.168 008	3.80	-.638
35 48	1 746 412.81	101.105 67	82.373 052	286.87	1.168 236	3.77	-.637
35 49	1 752 479.15	101.106 00	82.355 840	286.99	1.168 462	3.78	-.636
35 50	1 758 545.51	101.106 33	82.338 621	287.11	1.168 689	3.77	-.635
35 51	1 764 611.89	101.106 50	82.321 395	287.22	1.168 915	3.77	-.633
35 52	1 770 678.28	101.106 83	82.304 162	287.34	1.169 141	3.75	-.632
35 53	1 776 744.69	101.107 17	82.286 922	287.46	1.169 366	3.75	-.631
35 54	1 782 811.12	101.107 50	82.269 675	287.57	1.169 591	3.75	-.629
35 55	1 788 877.57	101.107 67	82.252 421	287.69	1.169 816	3.73	-.628
35 56	1 794 944.03	101.108 00	82.235 160	287.81	1.170 040	3.73	-.627
35 57	1 801 010.51	101.108 33	82.217 892	287.91	1.170 264	3.72	-.625
35 58	1 807 077.01	101.108 50	82.200 618	288.04	1.170 487	3.72	-.624
35 59	1 813 143.52	101.108 83	82.183 336	288.16	1.170 710	3.72	-.623
36 00	1 819 210.05		82.166 047		1.170 933		-.622

TRANSVERSE MERCATOR PROJECTION

ARIZONA
EAST AND CENTRAL ZONES

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
36 00	1 819 210.05	101.109 17	82.166 047	288.27	1.170 933	3.70	-.622
36 01	1 825 276.60	101.109 33	82.148 751	288.39	1.171 155	3.70	-.620
36 02	1 831 343.16	101.109 83	82.131 448	288.49	1.171 377	3.68	-.619
36 03	1 837 409.75	101.110 00	82.114 139	288.62	1.171 598	3.68	-.618
36 04	1 843 476.35	101.110 17	82.096 822	288.74	1.171 819	3.68	-.617
36 05	1 849 542.96	101.110 67	82.079 498	288.84	1.172 040	3.67	-.616
36 06	1 855 609.60	101.110 83	82.062 168	288.97	1.172 260	3.67	-.614
36 07	1 861 676.25	101.111 00	82.044 830	289.07	1.172 480	3.65	-.613
36 08	1 867 742.91	101.111 50	82.027 486	289.21	1.172 699	3.65	-.612
36 09	1 873 809.60	101.111 67	82.010 134	289.31	1.172 918	3.65	-.611
36 10	1 879 876.30	101.112 00	81.992 776	289.42	1.173 137	3.63	-.610
36 11	1 885 943.02	101.112 33	81.975 411	289.56	1.173 355	3.63	-.608
36 12	1 892 009.76	101.112 50	81.958 038	289.66	1.173 573	3.62	-.607
36 13	1 898 076.51	101.112 83	81.940 659	289.77	1.173 790	3.62	-.606
36 14	1 904 143.28	101.113 17	81.923 273	289.89	1.174 007	3.62	-.605
36 15	1 910 210.07	101.113 33	81.905 880	290.01	1.174 224	3.60	-.604
36 16	1 916 276.87	101.113 67	81.888 480	290.12	1.174 440	3.60	-.602
36 17	1 922 343.69	101.114 00	81.871 073	290.24	1.174 656	3.58	-.601
36 18	1 928 410.53	101.114 33	81.853 659	290.36	1.174 871	3.58	-.600
36 19	1 934 477.39	101.114 50	81.836 238	290.47	1.175 086	3.58	-.599
36 20	1 940 544.26	101.114 83	81.818 810	290.59	1.175 301	3.57	-.598
36 21	1 946 611.15	101.115 17	81.801 375	290.71	1.175 515	3.57	-.596
36 22	1 952 678.06	101.115 33	81.783 933	290.81	1.175 729	3.57	-.595
36 23	1 958 744.98	101.115 83	81.766 485	290.94	1.175 943	3.55	-.594
36 24	1 964 811.93	101.115 83	81.749 029	291.04	1.176 156	3.55	-.592
36 25	1 970 878.88	101.116 33	81.731 567	291.17	1.176 369	3.53	-.591
36 26	1 976 945.86	101.116 50	81.714 097	291.27	1.176 581	3.53	-.590
36 27	1 983 012.85	101.116 83	81.696 621	291.41	1.176 793	3.52	-.58[illegible]
36 28	1 989 079.86	101.117 17	81.679 137	291.51	1.177 004	3.52	-.58[illegible]
36 29	1 995 146.89	101.117 50	81.661 647	291.62	1.177 215	3.52	-.58[illegible]
36 30	2 001 213.94	101.117 67	81.644 150	291.74	1.177 426	3.50	-.585
36 31	2 007 281.00	101.118 00	81.626 646	291.86	1.177 636	3.50	-.583
36 32	2 013 348.08	101.118 17	81.609 135	291.97	1.177 846	3.50	-.582
36 33	2 019 415.17	101.118 67	81.591 617	292.09	1.178 056	3.48	-.581
36 34	2 025 482.29	101.118 83	81.574 092	292.21	1.178 265	3.47	-.580
36 35	2 031 549.42	101.119 17	81.556 560	292.32	1.178 473	3.48	-.579
36 36	2 037 616.57	101.119 33	81.539 021	292.44	1.178 682	3.47	-.577
36 37	2 043 683.73	101.119 67	81.521 475	292.54	1.178 890	3.45	-.576
36 38	2 049 750.91	101.120 00	81.503 923	292.67	1.179 097	3.45	-.575
36 39	2 055 818.11	101.120 33	81.486 363	292.77	1.179 304	3.45	-.574
36 40	2 061 885.33	101.120 50	81.468 797	292.89	1.179 511	3.43	-.573
36 41	2 067 952.56	101.120 83	81.451 224	293.01	1.179 717	3.43	-.571
36 42	2 074 019.81	101.121 17	81.433 644	293.12	1.179 923	3.43	-.570
36 43	2 080 087.08	101.121 50	81.416 057	293.24	1.180 129	3.42	-.569
36 44	2 086 154.37	101.121 67	81.398 463	293.36	1.180 334	3.42	-.568
36 45	2 092 221.67	101.122 00	81.380 862	293.47	1.180 539	3.42	-.567
36 46	2 098 288.99	101.122 33	81.363 254	293.59	1.180 744	3.40	-.565
36 47	2 104 356.33	101.122 50	81.345 639	293.69	1.180 948	3.38	-.564
36 48	2 110 423.68	101.122 83	81.328 018	293.82	1.181 151	3.38	-.563
36 49	2 116 491.05	101.123 17	81.310 389	293.92	1.181 354	3.38	-.562
36 50	2 122 558.44	101.123 50	81.292 754	294.04	1.181 557	3.37	-.561
36 51	2 128 625.85	101.123 67	81.275 112	294.17	1.181 759	3.37	-.559
36 52	2 134 693.27	101.124 00	81.257 462	294.27	1.181 961	3.37	-.558
36 53	2 140 760.71	101.124 33	81.239 806	294.39	1.182 163	3.35	-.557
36 54	2 146 828.17	101.124 50	81.222 143	294.49	1.182 364	3.35	-.556
36 55	2 152 895.64	101.125 00	81.204 474	294.62	1.182 565	3.33	-.555
36 56	2 158 963.14	101.125 17	81.186 797	294.74	1.182 765	3.33	-.553
36 57	2 165 030.65	101.125 33	81.169 113	294.84	1.182 965	3.32	-.552
36 58	2 171 098.17	101.125 83	81.151 423	294.97	1.183 164	3.32	-.551
36 59	2 177 165.72	101.126 00	81.133 725	295.07	1.183 363	3.32	-.550
37 00	2 183 233.28		81.116 021		1.183 562		-.54[illegible]

TRANSVERSE MERCATOR PROJECTION

ARIZONA
EAST AND CENTRAL ZONES

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
37 00	2 183 233.28	101.126 33	81.116 021	295.19	1.183 562	3.30	-.549
37 01	2 189 300.86	101.126 67	81.098 310	295.31	1.183 760	3.30	-.547
37 02	2 195 368.46	101.126 83	81.080 592	295.42	1.183 958	3.30	-.546
37 03	2 201 436.07	101.127 17	81.062 867	295.54	1.184 156	3.28	-.545
37 04	2 207 503.70	101.127 50	81.045 135	295.66	1.184 353	3.28	-.544
37 05	2 213 571.35	101.127 67	81.027 396	295.76	1.184 550	3.27	-.543
37 06	2 219 639.01	101.128 00	81.009 651	295.89	1.184 746	3.27	-.541
37 07	2 225 706.69	101.128 33	80.991 898	295.99	1.184 942	3.25	-.540
37 08	2 231 774.39	101.128 67	80.974 139	296.11	1.185 137	3.25	-.539
37 09	2 237 842.11	101.129 00	80.956 373	296.22	1.185 332	3.25	-.538
37 10	2 243 909.85	101.129 17	80.938 600	296.34	1.185 527	3.23	-.537
37 11	2 249 977.60	101.129 33	80.920 820	296.46	1.185 721	3.23	-.535
37 12	2 256 045.36	101.129 83	80.903 033	296.56	1.185 915	3.23	-.534
37 13	2 262 113.15	101.130 17	80.885 240	296.69	1.186 109	3.22	-.533
37 14	2 268 180.96	101.130 33	80.867 439	296.79	1.186 302	3.22	-.532
37 15	2 274 248.78	101.130 67	80.849 632	296.91	1.186 495	3.20	-.531
37 16	2 280 316.62	101.130 83	80.831 818	297.04	1.186 687	3.20	-.529
37 17	2 286 384.47	101.131 17	80.813 996	297.14	1.186 879	3.18	-.528
37 18	2 292 452.34	101.131 50	80.796 168	297.24	1.187 070	3.18	-.527
37 19	2 298 520.23	101.131 83	80.778 334	297.37	1.187 261	3.18	-.526
37 20	2 304 588.14		80.760 492		1.187 452		-.524

TRANSVERSE MERCATOR PROJECTION

ARIZONA
WEST ZONE

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
31 00	0.00	101.030 00	87.036 809	252.42	1.086 836	5.62	1.000
31 01	6 061.80	101.030 33	87.021 664	252.54	1.087 173	5.60	-.998
31 02	12 123.62	101.030 50	87.006 512	252.66	1.087 509	5.60	-.997
31 03	18 185.45	101.030 67	86.991 353	252.79	1.087 845	5.60	-.996
31 04	24 247.29	101.031 17	86.976 186	252.91	1.088 181	5.60	-.994
31 05	30 309.16	101.031 33	86.961 012	253.02	1.088 517	5.58	-.993
31 06	36 371.04	101.031 50	86.945 831	253.16	1.088 852	5.57	-.992
31 07	42 432.93	101.031 83	86.930 642	253.27	1.089 186	5.58	-.990
31 08	48 494.84	101.032 17	86.915 446	253.39	1.089 521	5.55	-.989
31 09	54 556.77	101.032 33	86.900 243	253.52	1.089 854	5.57	-.988
31 10	60 618.71	101.032 67	86.885 032	253.64	1.090 188	5.55	-.987
31 11	66 680.67	101.032 83	86.869 814	253.77	1.090 521	5.55	-.985
31 12	72 742.64	101.033 17	86.854 588	253.89	1.090 854	5.53	-.984
31 13	78 804.63	101.033 50	86.839 355	254.01	1.091 186	5.53	-.983
31 14	84 866.64	101.033 67	86.824 115	254.12	1.091 518	5.53	-.982
31 15	90 928.66	101.034 00	86.808 868	254.26	1.091 850	5.52	-.981
31 16	96 990.70	101.034 17	86.793 613	254.37	1.092 181	5.52	-.979
31 17	103 052.75	101.034 50	86.778 351	254.51	1.092 512	5.52	-.978
31 18	109 114.82	101.034 83	86.763 081	254.62	1.092 843	5.50	-.977
31 19	115 176.91	101.035 00	86.747 804	254.74	1.093 173	5.50	-.976
31 20	121 239.01	101.035 33	86.732 520	254.87	1.093 503	5.50	-.975
31 21	127 301.13	101.035 50	86.717 228	254.99	1.093 833	5.48	-.973
31 22	133 363.26	101.035 83	86.701 929	255.11	1.094 162	5.48	-.972
31 23	139 425.41	101.036 17	86.686 623	255.22	1.094 491	5.48	-.971
31 24	145 487.58	101.036 33	86.671 310	255.36	1.094 820	5.47	-.969
31 25	151 549.76	101.036 67	86.655 989	255.49	1.095 148	5.47	-.968
31 26	157 611.96	101.036 83	86.640 660	255.59	1.095 476	5.45	-.967
31 27	163 674.17	101.037 17	86.625 325	255.72	1.095 803	5.45	-.965
31 28	169 736.40	101.037 33	86.609 982	255.84	1.096 130	5.45	-.964
31 29	175 798.64	101.037 67	86.594 632	255.97	1.096 457	5.43	-.963
31 30	181 860.90	101.038 00	86.579 274	256.09	1.096 783	5.43	-.962
31 31	187 923.18	101.038 17	86.563 909	256.21	1.097 109	5.42	-.960
31 32	193 985.47	101.038 50	86.548 537	256.34	1.097 434	5.42	-.959
31 33	200 047.78	101.038 83	86.533 157	256.46	1.097 759	5.42	-.958
31 34	206 110.11	101.039 00	86.517 770	256.57	1.098 084	5.40	-.956
31 35	212 172.45	101.039 33	86.502 376	256.71	1.098 408	5.40	-.955
31 36	218 234.81	101.039 50	86.486 974	256.82	1.098 732	5.38	-.954
31 37	224 297.18	101.039 83	86.471 565	256.94	1.099 055	5.40	-.952
31 38	230 359.57	101.040 17	86.456 149	257.06	1.099 379	5.38	-.951
31 39	236 421.98	101.040 33	86.440 726	257.19	1.099 702	5.37	-.950
31 40	242 484.40	101.040 50	86.425 295	257.31	1.100 024	5.37	-.949
31 41	248 546.83	101.041 00	86.409 857	257.44	1.100 346	5.37	-.947
31 42	254 609.29	101.041 17	86.394 411	257.54	1.100 668	5.35	-.946
31 43	260 671.76	101.041 33	86.378 959	257.67	1.100 989	5.37	-.945
31 44	266 734.24	101.041 67	86.363 499	257.81	1.101 311	5.33	-.944
31 45	272 796.74	101.042 00	86.348 031	257.91	1.101 631	5.35	-.943
31 46	278 859.26	101.042 17	86.332 557	258.04	1.101 952	5.32	-.941
31 47	284 921.79	101.042 50	86.317 075	258.17	1.102 271	5.33	-.940
31 48	290 984.34	101.042 83	86.301 585	258.27	1.102 591	5.32	-.939
31 49	297 046.91	101.043 00	86.286 089	258.41	1.102 910	5.32	-.938
31 50	303 109.49	101.043 33	86.270 585	258.52	1.103 229	5.30	-.937
31 51	309 172.09	101.043 50	86.255 074	258.66	1.103 547	5.30	-.935
31 52	315 234.70	101.043 83	86.239 555	258.77	1.103 865	5.30	-.934
31 53	321 297.33	101.044 17	86.224 029	258.89	1.104 183	5.28	-.933
31 54	327 359.98	101.044 33	86.208 496	259.01	1.104 500	5.28	-.931
31 55	333 422.64	101.044 67	86.192 956	259.14	1.104 817	5.28	-.930
31 56	339 485.32	101.044 83	86.177 408	259.26	1.105 134	5.27	-.929
31 57	345 548.01	101.045 17	86.161 853	259.37	1.105 450	5.27	-.927
31 58	351 610.72	101.045 50	86.146 291	259.51	1.105 766	5.25	-.926
31 59	357 673.45	101.045 67	86.130 721	259.62	1.106 081	5.25	-.925
32 00	363 736.19		86.115 144		1.106 396		-.924

TRANSVERSE MERCATOR PROJECTION

ARIZONA
WEST ZONE

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
32 00	363 736.19	101.046 00	86.115 144	259.76	1.106 396	5.25	-.924
32 01	369 798.95	101.046 33	86.099 559	259.86	1.106 711	5.23	-.922
32 02	375 861.73	101.046 50	86.083 968	259.99	1.107 025	5.23	-.921
32 03	381 924.52	101.046 67	86.068 369	260.12	1.107 339	5.22	-.920
32 04	387 987.32	101.047 17	86.052 762	260.22	1.107 652	5.22	-.918
32 05	394 050.15	101.047 33	86.037 149	260.36	1.107 965	5.22	-.917
32 06	400 112.99	101.047 50	86.021 528	260.47	1.108 278	5.20	-.916
32 07	406 175.84	101.047 83	86.005 900	260.61	1.108 590	5.20	-.914
32 08	412 238.71	101.048 17	85.990 264	260.71	1.108 902	5.20	-.913
32 09	418 301.60	101.048 50	85.974 622	260.84	1.109 214	5.18	-.912
32 10	424 364.51	101.048 67	85.958 972	260.96	1.109 525	5.18	-.911
32 11	430 427.43	101.048 83	85.943 315	261.07	1.109 836	5.17	-.909
32 12	436 490.36	101.049 33	85.927 651	261.21	1.110 146	5.17	-.908
32 13	442 553.32	101.049 50	85.911 979	261.32	1.110 456	5.17	-.907
32 14	448 616.29	101.049 67	85.896 300	261.44	1.110 766	5.17	-.906
32 15	454 679.27	101.050 00	85.880 614	261.57	1.111 076	5.15	-.905
32 16	460 742.27	101.050 33	85.864 920	261.67	1.111 385	5.13	-.903
32 17	466 805.29	101.050 50	85.849 220	261.81	1.111 693	5.15	-.902
32 18	472 868.32	101.050 83	85.833 512	261.92	1.112 002	5.12	-.901
32 19	478 931.37	101.051 17	85.817 797	262.06	1.112 309	5.13	-.900
32 20	484 994.44	101.051 33	85.802 074	262.17	1.112 617	5.12	-.899
32 21	491 057.52	101.051 67	85.786 344	262.29	1.112 924	5.12	-.897
32 22	497 120.62	101.051 83	85.770 607	262.41	1.113 231	5.10	-.896
32 23	503 183.73	101.052 17	85.754 863	262.54	1.113 537	5.12	-.895
32 24	509 246.86	101.052 50	85.739 111	262.66	1.113 844	5.08	-.893
32 25	515 310.01	101.052 83	85.723 352	262.77	1.114 149	5.10	-.892
32 26	521 373.18	101.053 00	85.707 586	262.89	1.114 455	5.07	-.891
32 27	527 436.36	101.053 17	85.691 813	263.02	1.114 759	5.08	-.889
32 28	533 499.55	101.053 50	85.676 032	263.14	1.115 064	5.07	-.888
32 29	539 562.76	101.053 83	85.660 244	263.26	1.115 368	5.07	-.887
32 30	545 625.99	101.054 17	85.644 449	263.37	1.115 672	5.05	-.886
32 31	551 689.24	101.054 33	85.628 647	263.51	1.115 975	5.05	-.884
32 32	557 752.50	101.054 50	85.612 837	263.62	1.116 278	5.05	-.883
32 33	563 815.77	101.055 00	85.597 020	263.74	1.116 581	5.03	-.882
32 34	569 879.07	101.055 17	85.581 196	263.86	1.116 883	5.03	-.881
32 35	575 942.38	101.055 33	85.565 365	263.99	1.117 185	5.03	-.880
32 36	582 005.70	101.055 83	85.549 526	264.11	1.117 487	5.02	-.878
32 37	588 069.05	101.055 83	85.533 680	264.22	1.117 788	5.02	-.877
32 38	594 132.40	101.056 33	85.517 827	264.34	1.118 089	5.00	-.876
32 39	600 195.78	101.056 50	85.501 967	264.47	1.118 389	5.00	-.875
32 40	606 259.17	101.056 83	85.486 099	264.59	1.118 689	5.00	-.874
32 41	612 322.58	101.057 00	85.470 224	264.71	1.118 989	4.98	-.872
32 42	618 386.00	101.057 33	85.454 342	264.82	1.119 288	4.98	-.871
32 43	624 449.44	101.057 67	85.438 453	264.96	1.119 587	4.97	-.870
32 44	630 512.90	101.057 83	85.422 556	265.07	1.119 885	4.97	-.868
32 45	636 576.37	101.058 17	85.406 652	265.19	1.120 183	4.97	-.867
32 46	642 639.86	101.058 50	85.390 741	265.31	1.120 481	4.95	-.866
32 47	648 703.37	101.058 67	85.374 823	265.42	1.120 778	4.95	-.864
32 48	654 766.89	101.059 00	85.358 898	265.56	1.121 075	4.95	-.863
32 49	660 830.43	101.059 17	85.342 965	265.67	1.121 372	4.93	-.862
32 50	666 893.98	101.059 50	85.327 025	265.79	1.121 668	4.93	-.861
32 51	672 957.55	101.059 83	85.311 078	265.92	1.121 964	4.92	-.859
32 52	679 021.14	101.060 00	85.295 123	266.02	1.122 259	4.92	-.858
32 53	685 084.74	101.060 33	85.279 162	266.16	1.122 554	4.92	-.857
32 54	691 148.36	101.060 67	85.263 193	266.27	1.122 849	4.90	-.855
32 55	697 212.00	101.060 83	85.247 217	266.41	1.123 143	4.90	-.854
32 56	703 275.65	101.061 17	85.231 233	266.51	1.123 437	4.88	-.853
32 57	709 339.32	101.061 33	85.215 243	266.64	1.123 730	4.90	-.851
32 58	715 403.00	101.061 83	85.199 245	266.76	1.124 024	4.88	-.850
32 59	721 466.71	101.062 00	85.183 240	266.87	1.124 317	4.87	-.849
33 00	727 530.43		85.167 228		1.124 609		-.848

TRANSVERSE MERCATOR PROJECTION

ARIZONA
WEST ZONE

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
33 00	727 530.43	101.062 17	85.167 228	266.99	1.124 689	4.87	-.848
33 01	733 594.16	101.062 50	85.151 209	267.12	1.124 901	4.87	-.846
33 02	739 657.91	101.062 83	85.135 182	267.24	1.125 193	4.87	-.845
33 03	745 721.68	101.063 00	85.119 148	267.36	1.125 485	4.85	-.844
33 04	751 785.46	101.063 33	85.103 107	267.47	1.125 776	4.85	-.843
33 05	757 849.26	101.063 67	85.087 059	267.59	1.126 067	4.83	-.842
33 06	763 913.08	101.063 83	85.071 004	267.72	1.126 357	4.83	-.840
33 07	769 976.91	101.064 17	85.054 941	267.84	1.126 647	4.82	-.839
33 08	776 040.76	101.064 50	85.038 871	267.96	1.126 936	4.82	-.838
33 09	782 104.63	101.064 67	85.022 794	268.07	1.127 225	4.82	-.837
33 10	788 168.51	101.065 00	85.006 710	268.21	1.127 514	4.80	-.836
33 11	794 232.41	101.065 17	84.990 618	268.32	1.127 802	4.80	-.834
33 12	800 296.32	101.065 67	84.974 519	268.44	1.128 090	4.78	-.833
33 13	806 360.26	101.065 67	84.958 413	268.56	1.128 377	4.80	-.832
33 14	812 424.20	101.066 17	84.942 300	268.67	1.128 665	4.77	-.830
33 15	818 488.17	101.066 33	84.926 180	268.81	1.128 951	4.78	-.829
33 16	824 552.15	101.066 67	84.910 052	268.92	1.129 238	4.75	-.828
33 17	830 616.15	101.066 83	84.893 917	269.04	1.129 523	4.77	-.826
33 18	836 680.16	101.067 17	84.877 775	269.16	1.129 809	4.75	-.825
33 19	842 744.19	101.067 50	84.861 626	269.27	1.130 094	4.75	-.824
33 20	848 808.24	101.067 67	84.845 470	269.39	1.130 379	4.73	-.823
33 21	854 872.30	101.068 00	84.829 307	269.52	1.130 663	4.73	-.821
33 22	860 936.38	101.068 33	84.813 136	269.64	1.130 947	4.73	-.820
33 23	867 000.48	101.068 50	84.796 958	269.76	1.131 231	4.72	-.819
33 24	873 064.59	101.068 83	84.780 773	269.87	1.131 514	4.72	-.817
33 25	879 128.72	101.069 00	84.764 581	270.01	1.131 797	4.72	-.816
33 26	885 192.86	101.069 50	84.748 381	270.11	1.132 080	4.70	-.815
33 27	891 257.03	101.069 67	84.732 175	270.24	1.132 362	4.70	-.3
33 28	897 321.21	101.069 83	84.715 961	270.36	1.132 644	4.68	[illegible]
33 29	903 385.40	101.070 17	84.699 740	270.47	1.132 925	4.68	[illegible]
33 30	909 449.61	101.070 50	84.683 512	270.59	1.133 206	4.68	-.810
33 31	915 513.84	101.070 83	84.667 277	270.72	1.133 487	4.67	-.808
33 32	921 578.09	101.071 00	84.651 034	270.84	1.133 767	4.67	-.807
33 33	927 642.35	101.071 17	84.634 784	270.94	1.134 047	4.65	-.806
33 34	933 706.62	101.071 67	84.618 528	271.07	1.134 326	4.65	-.805
33 35	939 770.92	101.071 83	84.602 264	271.21	1.134 605	4.65	-.804
33 36	945 835.23	101.072 17	84.585 992	271.31	1.134 884	4.63	-.802
33 37	951 899.56	101.072 33	84.569 714	271.44	1.135 162	4.63	-.801
33 38	957 963.90	101.072 67	84.553 428	271.54	1.135 440	4.63	-.800
33 39	964 028.26	101.073 00	84.537 136	271.67	1.135 718	4.62	-.799
33 40	970 092.64	101.073 17	84.520 836	271.79	1.135 995	4.62	-.798
33 41	976 157.03	101.073 50	84.504 529	271.91	1.136 272	4.60	-.796
33 42	982 221.44	101.073 83	84.488 215	272.04	1.136 548	4.60	-.795
33 43	988 285.87	101.074 00	84.471 893	272.14	1.136 824	4.60	-.794
33 44	994 350.31	101.074 33	84.455 565	272.27	1.137 100	4.58	-.792
33 45	1 000 414.77	101.074 67	84.439 229	272.39	1.137 375	4.58	-.791
33 46	1 006 479.25	101.074 83	84.422 886	272.49	1.137 650	4.58	-.790
33 47	1 012 543.74	101.075 17	84.406 537	272.62	1.137 925	4.57	-.788
33 48	1 018 608.25	101.075 50	84.390 180	272.76	1.138 199	4.57	-.787
33 49	1 024 672.78	101.075 67	84.373 815	272.86	1.138 473	4.55	-.786
33 50	1 030 737.32	101.076 00	84.357 444	272.97	1.138 746	4.55	-.785
33 51	1 036 801.88	101.076 33	84.341 066	273.11	1.139 019	4.55	-.783
33 52	1 042 866.46	101.076 50	84.324 680	273.22	1.139 292	4.53	-.782
33 53	1 048 931.05	101.076 83	84.308 287	273.34	1.139 564	4.52	-.781
33 54	1 054 995.66	101.077 17	84.291 887	273.46	1.139 835	4.53	-.779
33 55	1 061 060.29	101.077 33	84.275 480	273.57	1.140 107	4.52	-.778
33 56	1 067 124.93	101.077 67	84.259 066	273.69	1.140 378	4.50	-.777
33 57	1 073 189.59	101.078 00	84.242 645	273.82	1.140 648	4.52	-.775
33 58	1 079 254.27	101.078 17	84.226 216	273.92	1.140 919	4.50	-.774
33 59	1 085 318.96	101.078 50	84.209 781	274.06	1.141 189	4.48	-.773
34 00	1 091 383.67		84.193 338		1.141 458		[illegible]

TRANSVERSE MERCATOR PROJECTION

ARIZONA
WEST ZONE

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
34 00	1 091 383.67	101.078 67	84.193 338	274.17	1.141 450	4.48	-.772
34 01	1 097 448.39	101.079 17	84.176 888	274.29	1.141 727	4.48	-.770
34 02	1 103 513.14	101.079 33	84.160 431	274.42	1.141 996	4.47	-.769
34 03	1 109 577.90	101.079 50	84.143 966	274.52	1.142 264	4.47	-.768
34 04	1 115 642.67	101.079 83	84.127 495	274.66	1.142 532	4.45	-.767
34 05	1 121 707.46	101.080 17	84.111 016	274.76	1.142 799	4.45	-.766
34 06	1 127 772.27	101.080 50	84.094 531	274.89	1.143 066	4.45	-.764
34 07	1 133 837.10	101.080 67	84.078 038	275.01	1.143 333	4.43	-.763
34 08	1 139 901.94	101.081 00	84.061 538	275.12	1.143 599	4.43	-.762
34 09	1 145 966.80	101.081 33	84.045 031	275.24	1.143 865	4.43	-.761
34 10	1 152 031.68	101.081 50	84.028 517	275.36	1.144 131	4.42	-.760
34 11	1 158 096.57	101.081 83	84.011 996	275.47	1.144 396	4.42	-.758
34 12	1 164 161.48	101.082 17	83.995 468	275.61	1.144 661	4.42	-.757
34 13	1 170 226.41	101.082 33	83.978 932	275.71	1.144 926	4.40	-.756
34 14	1 176 291.35	101.082 67	83.962 390	275.84	1.145 190	4.38	-.754
34 15	1 182 356.31	101.082 83	83.945 840	275.96	1.145 453	4.40	-.753
34 16	1 188 421.28	101.083 33	83.929 283	276.07	1.145 717	4.38	-.752
34 17	1 194 486.28	101.083 33	83.912 719	276.19	1.145 980	4.37	-.750
34 18	1 200 551.28	101.083 83	83.896 148	276.31	1.146 242	4.37	-.749
34 19	1 206 616.31	101.084 00	83.879 570	276.42	1.146 504	4.37	-.748
34 20	1 212 681.35	101.084 33	83.862 985	276.54	1.146 766	4.35	-.747
34 21	1 218 746.41	101.084 67	83.846 393	276.67	1.147 027	4.35	-.745
34 22	1 224 811.49	101.084 83	83.829 793	276.79	1.147 288	4.35	-.744
34 23	1 230 876.58	101.085 17	83.813 186	276.91	1.147 549	4.33	-.743
34 24	1 236 941.69	101.085 50	83.796 572	277.01	1.147 809	4.33	-.742
34 25	1 243 006.82	101.085 67	83.779 952	277.14	1.148 069	4.32	-.741
34 26	1 249 071.96	101.086 00	83.763 324	277.26	1.148 328	4.32	-.739
34 27	1 255 137.12	101.086 33	83.746 689	277.37	1.148 587	4.32	-.738
34 28	1 261 202.30	101.086 50	83.730 047	277.51	1.148 846	4.30	-.737
34 29	1 267 267.49	101.086 83	83.713 397	277.61	1.149 104	4.30	-.736
34 30	1 273 332.70	101.087 17	83.696 741	277.72	1.149 362	4.28	-.735
34 31	1 279 397.93	101.087 33	83.680 078	277.86	1.149 619	4.28	-.733
34 32	1 285 463.17	101.087 67	83.663 407	277.97	1.149 876	4.28	-.732
34 33	1 291 528.43	101.088 00	83.646 729	278.07	1.150 133	4.27	-.731
34 34	1 297 593.71	101.088 17	83.630 045	278.21	1.150 389	4.27	-.729
34 35	1 303 659.00	101.088 50	83.613 353	278.32	1.150 645	4.27	-.728
34 36	1 309 724.31	101.088 83	83.596 654	278.44	1.150 901	4.25	-.727
34 37	1 315 789.64	101.089 17	83.579 948	278.56	1.151 156	4.25	-.725
34 38	1 321 854.99	101.089 33	83.563 235	278.67	1.151 411	4.23	-.724
34 39	1 327 920.35	101.089 50	83.546 515	278.79	1.151 665	4.23	-.723
34 40	1 333 985.72	101.090 00	83.529 788	278.91	1.151 919	4.23	-.722
34 41	1 340 051.12	101.090 17	83.513 054	279.02	1.152 173	4.22	-.720
34 42	1 346 116.53	101.090 50	83.496 313	279.16	1.152 426	4.22	-.719
34 43	1 352 181.96	101.090 67	83.479 564	279.26	1.152 679	4.20	-.718
34 44	1 358 247.40	101.091 17	83.462 809	279.37	1.152 931	4.20	-.717
34 45	1 364 312.87	101.091 33	83.446 047	279.51	1.153 183	4.20	-.716
34 46	1 370 378.35	101.091 50	83.429 277	279.62	1.153 435	4.18	-.714
34 47	1 376 443.84	101.091 83	83.412 500	279.72	1.153 686	4.18	-.713
34 48	1 382 509.35	101.092 17	83.395 717	279.86	1.153 937	4.18	-.712
34 49	1 388 574.88	101.092 50	83.378 926	279.97	1.154 188	4.17	-.711
34 50	1 394 640.43	101.092 67	83.362 128	280.09	1.154 438	4.17	-.710
34 51	1 400 705.99	101.093 00	83.345 323	280.21	1.154 688	4.15	-.708
34 52	1 406 771.57	101.093 33	83.328 511	280.32	1.154 937	4.15	-.707
34 53	1 412 837.17	101.093 50	83.311 692	280.46	1.155 186	4.15	-.706
34 54	1 418 902.78	101.093 83	83.294 865	280.56	1.155 435	4.13	-.704
34 55	1 424 968.41	101.094 17	83.278 032	280.67	1.155 683	4.13	-.703
34 56	1 431 034.06	101.094 33	83.261 192	280.81	1.155 931	4.12	-.702
34 57	1 437 099.72	101.094 67	83.244 344	280.91	1.156 178	4.12	-.700
34 58	1 443 165.40	101.095 00	83.227 490	281.02	1.156 425	4.12	-.699
34 59	1 449 231.10	101.095 33	83.210 629	281.16	1.156 672	4.10	-.698
35	1 455 296.82		83.193 760		1.156 918		-.697

TRANSVERSE MERCATOR PROJECTION

ARIZONA
WEST ZONE

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
35 00	1 455 296.82	101.095 50	83.193 760	281.27	1.156 918	4.10	-.697
35 01	1 461 362.55	101.095 83	83.176 884	281.37	1.157 164	4.08	-.695
35 02	1 467 428.30	101.096 00	83.160 002	281.51	1.157 409	4.10	-.694
35 03	1 473 494.06	101.096 33	83.143 112	281.62	1.157 655	4.07	-.693
35 04	1 479 559.84	101.096 67	83.126 215	281.72	1.157 899	4.08	-.692
35 05	1 485 625.64	101.097 00	83.109 312	281.86	1.158 144	4.05	-.691
35 06	1 491 691.46	101.097 17	83.092 401	281.97	1.158 387	4.07	-.689
35 07	1 497 757.29	101.097 50	83.075 483	282.09	1.158 631	4.05	-.688
35 08	1 503 823.14	101.097 83	83.058 558	282.21	1.158 874	4.05	-.687
35 09	1 509 889.01	101.098 00	83.041 626	282.32	1.159 117	4.03	-.686
35 10	1 515 954.89	101.098 33	83.024 687	282.44	1.159 359	4.03	-.685
35 11	1 522 020.79	101.098 67	83.007 741	282.56	1.159 601	4.02	-.683
35 12	1 528 086.71	101.098 83	82.990 788	282.69	1.159 842	4.02	-.682
35 13	1 534 152.64	101.099 17	82.973 827	282.79	1.160 083	4.02	-.681
35 14	1 540 218.59	101.099 50	82.956 860	282.91	1.160 324	4.00	-.679
35 15	1 546 284.56	101.099 83	82.939 886	283.04	1.160 564	4.00	-.678
35 16	1 552 350.55	101.100 00	82.922 904	283.14	1.160 804	4.00	-.677
35 17	1 558 416.55	101.100 33	82.905 916	283.26	1.161 044	3.98	-.675
35 18	1 564 482.57	101.100 50	82.888 921	283.39	1.161 283	3.98	-.674
35 19	1 570 548.60	101.101 00	82.871 918	283.49	1.161 522	3.97	-.673
35 20	1 576 614.66	101.101 17	82.854 909	283.61	1.161 760	3.97	-.672
35 21	1 582 680.73	101.101 33	82.837 893	283.74	1.161 998	3.97	-.670
35 22	1 588 746.81	101.101 83	82.820 869	283.84	1.162 236	3.95	-.669
35 23	1 594 812.92	101.102 00	82.803 839	283.97	1.162 473	3.93	-.668
35 24	1 600 879.04	101.102 33	82.786 801	284.07	1.162 709	3.95	-.667
35 25	1 606 945.18	101.102 50	82.769 757	284.21	1.162 946	3.93	-.666
35 26	1 613 011.33	101.102 83	82.752 705	284.31	1.163 182	3.92	-.664
35 27	1 619 077.50	101.103 17	82.735 647	284.44	1.163 417	3.93	-.663
35 28	1 625 143.69	101.103 50	82.718 581	284.54	1.163 653	3.92	-.6[illegible]
35 29	1 631 209.90	101.103 67	82.701 509	284.67	1.163 888	3.90	-.[illegible]
35 30	1 637 276.12	101.103 83	82.684 429	284.79	1.164 122	3.90	-.660
35 31	1 643 342.35	101.104 33	82.667 342	284.89	1.164 356	3.90	-.658
35 32	1 649 408.61	101.104 67	82.650 249	285.02	1.164 590	3.88	-.657
35 33	1 655 474.89	101.104 83	82.633 148	285.12	1.164 823	3.88	-.656
35 34	1 661 541.18	101.105 00	82.616 041	285.26	1.165 056	3.88	-.654
35 35	1 667 607.48	101.105 50	82.598 926	285.36	1.165 289	3.87	-.653
35 36	1 673 673.81	101.105 67	82.581 805	285.49	1.165 521	3.85	-.652
35 37	1 679 740.15	101.106 00	82.564 676	285.61	1.165 752	3.87	-.650
35 38	1 685 806.51	101.106 17	82.547 540	285.71	1.165 984	3.85	-.649
35 39	1 691 872.88	101.106 67	82.530 398	285.84	1.166 215	3.83	-.648
35 40	1 697 939.28	101.106 83	82.513 248	285.96	1.166 445	3.83	-.647
35 41	1 704 005.69	101.107 00	82.496 091	286.07	1.166 675	3.83	-.645
35 42	1 710 072.11	101.107 50	82.478 927	286.17	1.166 905	3.82	-.644
35 43	1 716 138.56	101.107 67	82.461 757	286.31	1.167 134	3.82	-.643
35 44	1 722 205.02	101.107 83	82.444 579	286.42	1.167 363	3.80	-.642
35 45	1 728 271.49	101.108 33	82.427 394	286.52	1.167 591	3.80	-.641
35 46	1 734 337.99	101.108 50	82.410 203	286.66	1.167 819	3.80	-.639
35 47	1 740 404.50	101.108 83	82.393 004	286.77	1.168 047	3.80	-.638
35 48	1 746 471.03	101.109 00	82.375 798	286.87	1.168 275	3.77	-.637
35 49	1 752 537.57	101.109 33	82.358 586	287.01	1.168 501	3.78	-.636
35 50	1 758 604.13	101.109 67	82.341 366	287.12	1.168 728	3.77	-.635
35 51	1 764 670.71	101.110 00	82.324 139	287.22	1.168 954	3.77	-.633
35 52	1 770 737.31	101.110 17	82.306 906	287.36	1.169 180	3.75	-.632
35 53	1 776 803.92	101.110 67	82.289 665	287.46	1.169 405	3.75	-.631
35 54	1 782 870.56	101.110 67	82.272 418	287.59	1.169 630	3.75	-.629
35 55	1 788 937.20	101.111 17	82.255 163	287.69	1.169 855	3.73	-.628
35 56	1 795 003.87	101.111 33	82.237 902	287.82	1.170 079	3.73	-.627
35 57	1 801 070.55	101.111 67	82.220 633	287.92	1.170 303	3.72	-.625
35 58	1 807 137.25	101.111 83	82.203 358	288.06	1.170 526	3.72	-.624
35 59	1 813 203.96	101.112 33	82.186 075	288.16	1.170 749	3.72	-.623
36 00	1 819 270.70		82.168 786		1.170 972		-.622

TRANSVERSE MERCATOR PROJECTION

ARIZONA
WEST ZONE

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
36 00	1 819 270.70	101.112 50	82.168 786	288.27	1.170 972	3.70	-.622
36 01	1 825 337.45	101.112 83	82.151 490	288.41	1.171 194	3.70	-.620
36 02	1 831 404.22	101.113 00	82.134 186	288.51	1.171 416	3.68	-.619
36 03	1 837 471.00	101.113 33	82.116 876	288.64	1.171 637	3.68	-.618
36 04	1 843 537.80	101.113 67	82.099 558	288.74	1.171 858	3.68	-.617
36 05	1 849 604.62	101.114 00	82.082 234	288.86	1.172 079	3.67	-.616
36 06	1 855 671.46	101.114 17	82.064 903	288.97	1.172 299	3.67	-.614
36 07	1 861 738.31	101.114 50	82.047 565	289.09	1.172 519	3.65	-.613
36 08	1 867 805.18	101.114 83	82.030 220	289.21	1.172 738	3.65	-.612
36 09	1 873 872.07	101.115 00	82.012 868	289.32	1.172 957	3.65	-.611
36 10	1 879 938.97	101.115 33	81.995 509	289.44	1.173 176	3.63	-.610
36 11	1 886 005.89	101.115 67	81.978 143	289.56	1.173 394	3.63	-.608
36 12	1 892 072.83	101.115 83	81.960 770	289.66	1.173 612	3.62	-.607
36 13	1 898 139.78	101.116 33	81.943 391	289.79	1.173 829	3.62	-.606
36 14	1 904 206.76	101.116 50	81.926 004	289.91	1.174 046	3.62	-.605
36 15	1 910 273.75	101.116 83	81.908 610	290.01	1.174 263	3.60	-.604
36 16	1 916 340.76	101.117 00	81.891 210	290.14	1.174 479	3.60	-.602
36 17	1 922 407.78	101.117 33	81.873 802	290.24	1.174 695	3.58	-.601
36 18	1 928 474.82	101.117 67	81.856 388	290.37	1.174 910	3.58	-.600
36 19	1 934 541.88	101.117 83	81.838 966	290.47	1.175 125	3.58	-.599
36 20	1 940 608.95	101.118 33	81.821 538	290.59	1.175 340	3.57	-.598
36 21	1 946 676.05	101.118 50	81.804 103	290.72	1.175 554	3.57	-.596
36 22	1 952 743.16	101.118 67	81.786 660	290.82	1.175 768	3.57	-.595
36 23	1 958 810.28	101.119 17	81.769 211	290.94	1.175 982	3.55	-.594
36 24	1 964 877.43	101.119 33	81.751 755	291.06	1.176 195	3.55	-.592
36 [illegible]	1 970 944.59	101.119 67	81.734 292	291.17	1.176 408	3.53	-.591
[illegible]	1 977 011.77	101.119 83	81.716 822	291.29	1.176 620	3.53	-.590
36 27	1 983 078.96	101.120 33	81.699 345	291.41	1.176 832	3.52	-.588
36 28	1 989 146.18	101.120 50	81.681 861	291.52	1.177 043	3.52	-.587
36 29	1 995 213.41	101.120 67	81.664 370	291.64	1.177 254	3.52	-.586
36 30	2 001 280.65	101.121 17	81.646 872	291.76	1.177 465	3.50	-.585
36 31	2 007 347.92	101.121 33	81.629 367	291.87	1.177 675	3.50	-.583
36 32	2 013 415.20	101.121 67	81.611 855	291.97	1.177 885	3.50	-.582
36 33	2 019 482.50	101.121 83	81.594 337	292.11	1.178 095	3.48	-.581
36 34	2 025 549.81	101.122 17	81.576 811	292.21	1.178 304	3.47	-.580
36 35	2 031 617.14	101.122 50	81.559 279	292.34	1.178 512	3.48	-.579
36 36	2 037 684.49	101.122 83	81.541 739	292.44	1.178 721	3.47	-.577
36 37	2 043 751.86	101.123 17	81.524 193	292.56	1.178 929	3.45	-.576
36 38	2 049 819.25	101.123 33	81.506 640	292.67	1.179 136	3.45	-.575
36 39	2 055 886.65	101.123 67	81.489 080	292.79	1.179 343	3.45	-.574
36 40	2 061 954.07	101.123 83	81.471 513	292.91	1.179 550	3.43	-.573
36 41	2 068 021.50	101.124 33	81.453 939	293.02	1.179 756	3.43	-.571
36 42	2 074 088.96	101.124 50	81.436 358	293.12	1.179 962	3.43	-.570
36 43	2 080 156.43	101.124 83	81.418 771	293.26	1.180 168	3.42	-.569
36 44	2 086 223.92	101.125 00	81.401 176	293.36	1.180 373	3.42	-.568
36 45	2 092 291.42	101.125 33	81.383 575	293.49	1.180 578	3.42	-.567
36 46	2 098 358.94	101.125 67	81.365 966	293.59	1.180 783	3.40	-.565
36 47	2 104 426.48	101.126 00	81.348 351	293.71	1.180 987	3.38	-.564
36 48	2 110 494.04	101.126 17	81.330 729	293.82	1.181 190	3.38	-.563
36 49	2 116 561.61	101.126 50	81.313 100	293.94	1.181 393	3.38	-.562
36 50	2 122 629.20	101.126 83	81.295 464	294.06	1.181 596	3.37	-.561
36 51	2 128 696.81	101.127 17	81.277 821	294.17	1.181 798	3.37	-.559
36 52	2 134 764.44	101.127 33	81.260 171	294.27	1.182 000	3.37	-.558
36 53	2 140 832.08	101.127 67	81.242 515	294.41	1.182 202	3.35	-.557
36 54	2 146 899.74	101.128 00	81.224 851	294.51	1.182 403	3.33	-.556
36 55	2 152 967.42	101.128 17	81.207 181	294.64	1.182 603	3.35	-.555
36 56	2 159 035.11	101.128 50	81.189 503	294.74	1.182 804	3.33	-.553
36 57	2 165 102.82	101.128 83	81.171 819	294.86	1.183 004	3.32	-.552
36 58	2 171 170.55	101.129 17	81.154 128	294.97	1.183 203	3.32	-.551
36 [illegible]	2 177 238.30	101.129 33	81.136 430	295.09	1.183 402	3.32	-.550
100	2 183 306.06		81.118 725		1.183 601		-.549

TRANSVERSE MERCATOR PROJECTION

ARIZONA
WEST ZONE

Lat.	y_o feet	Δy_o per second	H	ΔH per second	V	ΔV per second	a
37 00	2 183 306.06	101.129 67	81.118 725	295.21	1.183 601	3.30	-.549
37 01	2 189 373.84	101.130 00	81.101 013	295.31	1.183 799	3.30	-.547
37 02	2 195 441.64	101.130 33	81.083 295	295.44	1.183 997	3.30	-.546
37 03	2 201 509.46	101.130 50	81.065 569	295.54	1.184 195	3.28	-.545
37 04	2 207 577.29	101.130 83	81.047 837	295.67	1.184 392	3.28	-.544
37 05	2 213 645.14	101.131 17	81.030 097	295.77	1.184 589	3.28	-.543
37 06	2 219 713.01	101.131 33	81.012 351	295.89	1.184 786	3.27	-.541
37 07	2 225 780.89	101.131 67	80.994 598	296.01	1.184 982	3.25	-.540
37 08	2 231 848.79	101.132 00	80.976 838	296.11	1.185 177	3.25	-.539
37 09	2 237 916.71	101.132 33	80.959 072	296.24	1.185 372	3.25	-.538
37 10	2 243 984.65	101.132 50	80.941 298	296.36	1.185 567	3.23	-.537
37 11	2 250 052.60	101.132 83	80.923 517	296.46	1.185 761	3.23	-.535
37 12	2 256 120.57	101.133 17	80.905 730	296.57	1.185 955	3.23	-.534
37 13	2 262 188.56	101.133 50	80.887 936	296.69	1.186 149	3.22	-.533
37 14	2 268 256.57	101.133 67	80.870 135	296.81	1.186 342	3.22	-.532
37 15	2 274 324.59	101.134 00	80.852 327	296.92	1.186 535	3.20	-.531
37 16	2 280 392.63	101.134 33	80.834 512	297.04	1.186 727	3.20	-.529
37 17	2 286 460.69	101.134 50	80.816 690	297.14	1.186 919	3.18	-.528
37 18	2 292 528.76	101.135 00	80.798 862	297.27	1.187 110	3.18	-.527
37 19	2 298 596.86	101.135 00	80.781 026	297.37	1.187 301	3.18	-.526
37 20	2 304 664.96		80.763 184		1.187 492		-.524

TRANSVERSE MERCATOR PROJECTION

ARIZONA

All Zones

$\Delta\lambda''$	b	Δb	c	$\Delta\lambda''$	b	Δb	c
0	0.000	+0.370	0.000				
100	+0.370	+0.370	0.000	3100	+6.697	-0.110	-0.133
200	+0.740	+0.367	-0.001	3200	+6.587	-0.141	-0.135
300	+1.107	+0.364	-0.002	3300	+6.446	-0.173	-0.136
400	+1.471	+0.360	-0.003	3400	+6.273	-0.205	-0.135
500	+1.831	+0.355	-0.005	3500	+6.068	-0.239	-0.133
600	+2.186	+0.350	-0.007	3600	+5.829	-0.273	-0.131
700	+2.536	+0.343	-0.010	3700	+5.556	-0.309	-0.128
800	+2.879	+0.335	-0.014	3800	+5.247	-0.346	-0.124
900	+3.214	+0.326	-0.018	3900	+4.901	-0.382	-0.120
1000	+3.540	+0.315	-0.022	4000	+4.519	-0.422	-0.115
1100	+3.855	+0.305	-0.027	4100	+4.097	-0.462	-0.109
1200	+4.160	+0.294	-0.032	4200	+3.635	-0.503	-0.101
1300	+4.454	+0.282	-0.038	4300	+3.132	-0.544	-0.091
1400	+4.736	+0.268	-0.043	4400	+2.588	-0.585	-0.078
1500	+5.004	+0.255	-0.049	4500	+2.003	-0.626	-0.063
1600	+5.259	+0.239	-0.055	4600	+1.377	-0.667	-0.045
1700	+5.498	+0.223	-0.061	4700	+0.710	-0.710	-0.025
1800	+5.721	+0.206	-0.067	4800	0.000	-0.755	0.000
1900	+5.927	+0.188	-0.073	4900	-0.755	-0.807	+0.026
2000	+6.115	+0.169	-0.079	5000	-1.562	-0.860	+0.053
2100	+6.284	+0.149	-0.085	5100	-2.422	-0.911	+0.084
2200	+6.433	+0.128	-0.091	5200	-3.333	-0.960	+0.117
2300	+6.561	+0.105	-0.096	5300	-4.293	-1.014	+0.153
2400	+6.666	+0.082	-0.101	5400	-5.307	-1.067	+0.191
2500	+6.748	+0.058	-0.106	5500	-6.374	-1.120	+0.232
2600	+6.806	+0.033	-0.111	5600	-7.494	-1.172	+0.275
2700	+6.839	+0.007	-0.116	5700	-8.666	-1.225	+0.321
2800	+6.846	-0.021	-0.121	5800	-9.891	-1.277	+0.371
2900	+6.825	-0.049	-0.125	5900	-11.168	-1.331	+0.426
3000	+6.776	-0.079	-0.130	6000	-12.499		+0.487

$F = 7.47 \times 10^{-13}$

TRANSVERSE MERCATOR PROJECTION

TABLE FOR g

$$\Delta\alpha'' = \sin\phi\,(\Delta\lambda'') + g$$

Latitude	$\Delta\lambda''$						
	0"	1000"	2000"	3000"	4000"	5000"	6000"
24°	0".00	0".00	0".02	0".07	0".17	0".33	0".58
25	0	0	0.02	0.07	0.17	0.34	0.59
26°	0.00	0.00	0.02	0.08	0.18	0.35	0.60
27	0	0	0.02	0.08	0.18	0.35	0.61
28	0	0	0.02	0.08	0.18	0.36	0.62
29	0	0	0.02	0.08	0.19	0.37	0.63
30	0	0	0.02	0.08	0.19	0.37	0.64
31°	0.00	0.00	0.02	0.08	0.19	0.37	0.64
32	0	0	0.02	0.08	0.19	0.38	0.65
33	0	0	0.02	0.08	0.19	0.38	0[illegible]
34	0	0	0.02	0.08	0.19	0.38	0[illegible]
35	0	0	0.02	0.08	0.19	0.38	0.65
36°	0.00	0.00	0.02	0.08	0.19	0.38	0.65
37	0	0	0.02	0.08	0.19	0.38	0.65
38	0	0	0.02	0.08	0.19	0.38	0.65
39	0	0	0.02	0.08	0.19	0.37	0.64
40	0	0	0.02	0.08	0.19	0.37	0.64
41°	0.00	0.00	0.02	0.08	0.19	0.37	0.63
42	0	0	0.02	0.08	0.18	0.36	0.63
43	0	0	0.02	0.08	0.18	0.36	0.62
44	0	0	0.02	0.08	0.18	0.35	0.61
45	0	0	0.02	0.08	0.18	0.35	0.60
46°	0.00	0.00	0.02	0.07	0.17	0.34	0.59
47	0	0	0.02	0.07	0.17	0.33	0.58
48	0	0	0.02	0.07	0.17	0.33	0.56
49	0	0	0.02	0.07	0.16	0.32	0.55
50	0.00	0.00	0.02	0.07	0.16	0.31	0.54

$$g = \left[\frac{C\,(\sin 1'')\cos^3\phi}{2A^2} + F\right](\Delta\lambda'')^3$$

A, C and F are position factors.

Y CORRECTION FOR COMPUTATION OF GEOGRAPHIC POSITIONS FROM PLANE COORDINATES TRANSVERSE MERCATOR PROJECTION, ARIZONA-EAST & CENTRAL ZONES

$$P(x'/10{,}000)^2 + d = V(\Delta y/100)^2 + c$$

P taken out for y-coordinate
d taken out for x'

y	P	ΔP
0	1.43434	1562
100,000	1.44996	1570
200,000	1.46566	1580
300,000	1.48146	1589
400,000	1.49735	1598
500,000	1.51333	1608
600,000	1.52941	1618
700,000	1.54559	1628
800,000	1.56187	1638
900,000	1.57825	1649
1,000,000	1.59474	1658
1,100,000	1.61132	1670
1,200,000	1.62802	1680
1,300,000	1.64482	1692
1,400,000	1.66174	1702
1,500,000	1.67876	1714
1,600,000	1.69590	1726
1,700,000	1.71316	1738
1,800,000	1.73054	1749
1,900,000	1.74803	1761

x'	d
0	0.00
50,000	+ 0.01
100,000	+ 0.02
150,000	+ 0.04
200,000	+ 0.07
250,000	+ 0.09
300,000	+ 0.10
350,000	+ 0.08
400,000	+ 0.03
450,000	- 0.07

y	P	ΔP
2,000,000	1.76564	1775
2,100,000	1.78339	1786
2,200,000	1.80125	1800
2,300,000	1.81925	

Y CORRECTION FOR COMPUTATION OF GEOGRAPHIC POSITIONS FROM PLANE COORDINATES TRANSVERSE MERCATOR PROJECTION, ARIZONA-WEST ZONE

$$P(x'/10{,}000)^2 + d = V(\Delta y/100)^2 + c$$

P taken out for y-coordinate
d taken out for x'

y	P	ΔP
0	1.43438	1561
100,000	1.44999	1571
200,000	1.46570	1580
300,000	1.48150	1589
400,000	1.49739	1598
500,000	1.51337	1608
600,000	1.52945	1618
700,000	1.54563	1628
800,000	1.56191	1638
900,000	1.57829	1649
1,000,000	1.59478	1658
1,100,000	1.61136	1670
1,200,000	1.62806	1680
1,300,000	1.64486	1692
1,400,000	1.66178	1702
1,500,000	1.67880	1714
1,600,000	1.69594	1726
1,700,000	1.71320	1737
1,800,000	1.73057	1749
1,900,000	1.74806	1762

x'	d
0	0.00
50,000	0.00
100,000	+ 0.01
150,000	+ 0.02
200,000	+ 0.03
250,000	+ 0.03
300,000	+ 0.02
350,000	- 0.04
400,000	- 0.13

y	P	ΔP
2,000,000	1.76568	1774
2,100,000	1.78342	1787
2,200,000	1.80129	1800
2,300,000	1.81929	

TRANSVERSE MERCATOR PROJECTION

ARIZONA

East and Central Zones

x' feet	Scale in units of 7th place of logs	Scale expressed as a ratio	x' feet	Scale in units of 7th place of logs	Scale expressed as a ratio
0	-434.3	0.9999000	175,000	-282.0	0.9999351
5,000	-434.2	0.9999000	180,000	-273.2	0.9999371
10,000	-433.8	0.9999001	185,000	-264.1	0.9999392
15,000	-433.2	0.9999003	190,000	-254.8	0.9999413
20,000	-432.3	0.9999005	195,000	-245.2	0.9999435
25,000	-431.2	0.9999007	200,000	-235.4	0.9999458
30,000	-429.8	0.9999010	205,000	-225.3	0.9999481
35,000	-428.2	0.9999014	210,000	-215.0	0.9999505
40,000	-426.4	0.9999018	215,000	-204.5	0.9999529
45,000	-424.3	0.9999023	220,000	-193.7	0.9999554
50,000	-421.9	0.9999029	225,000	-182.6	0.9999580
55,000	-419.3	0.9999035	230,000	-171.3	0.9999606
60,000	-416.4	0.9999041	235,000	-159.7	0.9999632
65,000	-413.3	0.9999048	240,000	-147.9	0.9999659
70,000	-410.0	0.9999056	245,000	-135.9	0.9999687
75,000	-406.4	0.9999064	250,000	-123.6	0.9999715
80,000	-402.5	0.9999073	255,000	-111.0	0.9999744
85,000	-398.4	0.9999083	260,000	- 98.2	0.9999774
90,000	-394.0	0.9999093	265,000	- 85.2	0.9999804
95,000	-389.4	0.9999103	270,000	- 71.9	0.9999834
100,000	-384.6	0.9999114	275,000	- 58.4	0.9999866
105,000	-379.5	0.9999126	280,000	- 44.6	0.9999897
110,000	-374.1	0.9999139	285,000	- 30.5	0.9999930
115,000	-368.5	0.9999151	290,000	- 16.2	0.9999963
120,000	-362.7	0.9999165	295,000	- 1.7	0.9999996
125,000	-356.6	0.9999179	300,000	+ 13.1	1.0000030
130,000	-350.3	0.9999193	305,000	+ 28.1	1.0000065
135,000	-343.7	0.9999209	310,000	+ 43.4	1.0000100
140,000	-336.8	0.9999224	315,000	+ 59.0	1.0000136
145,000	-329.7	0.9999241	320,000	+ 74.8	1.0000172
150,000	-322.4	0.9999258	325,000	+ 90.8	1.0000209
155,000	-314.8	0.9999275	330,000	+107.1	1.0000247
160,000	-307.0	0.9999293	335,000	+123.6	1.0000285
165,000	-298.9	0.9999312	340,000	+140.4	1.0000323
170,000	-290.6	0.9999331	345,000	+157.4	1.0000362

TRANSVERSE MERCATOR PROJECTION

Arizona

$$\Delta\alpha = Mx' - e$$

y	East and central zones		West zone	
	M	ΔM	M	ΔM
0	0.005 9177	644	0.005 9175	644
100,000	0.005 9821	648	0.005 9819	648
200,000	0.006 0469	652	0.006 0467	652
300,000	0.006 1121	656	0.006 1119	655
400,000	0.006 1777	659	0.006 1774	660
500,000	0.006 2436	664	0.006 2434	663
600,000	0.006 3100	667	0.006 3097	668
700,000	0.006 3767	672	0.006 3765	672
800,000	0.006 4439	676	0.006 4437	675
900,000	0.006 5115	680	0.006 5112	680
1,000,000	0.006 5795	684	0.006 5792	685
1,100,000	0.006 6479	689	0.006 6477	689
1,200,000	0.006 7168	693	0.006 7166	693
1,300,000	0.006 7861	698	0.006 7859	698
1,400,000	0.006 8559	703	0.006 8557	702
1,500,000	0.006 9262	707	0.006 9259	707
1,600,000	0.006 9969	712	0.006 9966	712
1,700,000	0.007 0681	717	0.007 0678	717
1,800,000	0.007 1398	722	0.007 1395	722
1,900,000	0.007 2120	727	0.007 2117	727
2,000,000	0.007 2847	732	0.007 2844	732
2,100,000	0.007 3579	737	0.007 3576	737
2,200,000	0.007 4316	742	0.007 4313	742
2,300,000	0.007 5058		0.007 5055	

e

y \ x'	200,000	300,000	400,000	500,000
0	0.0	0.1	0.3	0.6
1,000,000	0.0	0.1	0.4	0.8
2,000,000	0.0	0.2	0.5	1.0

TRANSVERSE MERCATOR PROJECTION

ARIZONA

East and Central Zones

x' feet	Scale in units of 7th place of logs	Scale expressed as a ratio	x' feet	Scale in units of 7th place of logs	Scale expressed as a ratio
0	-434.3	0.9999000	175,000	-282.0	0.9999351
5,000	-434.2	0.9999000	180,000	-273.2	0.9999371
10,000	-433.8	0.9999001	185,000	-264.1	0.9999392
15,000	-433.2	0.9999003	190,000	-254.8	0.9999413
20,000	-432.3	0.9999005	195,000	-245.2	0.9999435
25,000	-431.2	0.9999007	200,000	-235.4	0.9999458
30,000	-429.8	0.9999010	205,000	-225.3	0.9999481
35,000	-428.2	0.9999014	210,000	-215.0	0.9999505
40,000	-426.4	0.9999018	215,000	-204.5	0.9999529
45,000	-424.3	0.9999023	220,000	-193.7	0.9999554
50,000	-421.9	0.9999029	225,000	-182.6	0.9999580
55,000	-419.3	0.9999035	230,000	-171.3	0.9999606
60,000	-416.4	0.9999041	235,000	-159.7	0.9999632
65,000	-413.3	0.9999048	240,000	-147.9	0.9999659
70,000	-410.0	0.9999056	245,000	-135.9	0.9999687
75,000	-406.4	0.9999064	250,000	-123.6	0.9999715
80,000	-402.5	0.9999073	255,000	-111.0	0.9999744
85,000	-398.4	0.9999083	260,000	- 98.2	0.9999774
90,000	-394.0	0.9999093	265,000	- 85.2	0.9999804
95,000	-389.4	0.9999103	270,000	- 71.9	0.9999834
100,000	-384.6	0.9999114	275,000	- 58.4	0.9999866
105,000	-379.5	0.9999126	280,000	- 44.6	0.9999897
110,000	-374.1	0.9999139	285,000	- 30.5	0.9999930
115,000	-368.5	0.9999151	290,000	- 16.2	0.9999963
120,000	-362.7	0.9999165	295,000	- 1.7	0.9999996
125,000	-356.6	0.9999179	300,000	+ 13.1	1.0000030
130,000	-350.3	0.9999193	305,000	+ 28.1	1.0000065
135,000	-343.7	0.9999209	310,000	+ 43.4	1.0000100
140,000	-336.8	0.9999224	315,000	+ 59.0	1.0000136
145,000	-329.7	0.9999241	320,000	+ 74.8	1.0000172
150,000	-322.4	0.9999258	325,000	+ 90.8	1.0000209
155,000	-314.8	0.9999275	330,000	+107.1	1.0000247
160,000	-307.0	0.9999293	335,000	+123.6	1.0000285
165,000	-298.9	0.9999312	340,000	+140.4	1.0000323
170,000	-290.6	0.9999331	345,000	+157.4	1.0000362

TRANSVERSE MERCATOR PROJECTION

ARIZONA

East and Central Zones

x' feet	Scale in units of 7th place of logs	Scale expressed as a ratio
350,000	+174.7	1.0000402
355,000	+192.2	1.0000443
360,000	+210.0	1.0000484
365,000	+228.0	1.0000525
370,000	+246.3	1.0000567
375,000	+264.8	1.0000610
380,000	+283.6	1.0000653
385,000	+302.6	1.0000697
390,000	+321.8	1.0000741
395,000	+341.3	1.0000786
400,000	+361.1	1.0000831
405,000	+381.1	1.0000878
410,000	+401.4	1.0000924
415,000	+421.9	1.0000971
420,000	+442.6	1.0001019
425,000	+463.6	1.0001067
430,000	+484.9	1.0001117
435,000	+506.4	1.0001166
440,000	+528.2	1.0001216
445,000	+550.2	1.0001267
450,000	+572.4	1.0001318
455,000	+594.9	1.0001370
460,000	+617.6	1.0001422
465,000	+640.6	1.0001475
470,000	+663.9	1.0001529
475,000	+687.4	1.0001583
480,000	+711.1	1.0001637
485,000	+735.1	1.0001693
490,000	+759.4	1.0001749
495,000	+783.9	1.0001805
500,000	+808.6	1.0001862
505,000	+833.6	1.0001919
510,000	+858.8	1.0001977
515,000	+884.3	1.0002036
520,000	+910.0	1.0002095
525,000	+936.0	1.0002155

TRANSVERSE MERCATOR PROJECTION

ARIZONA

West Zone

x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio
350,000	+319.5	1.0000736
355,000	+337.0	1.0000776
360,000	+354.8	1.0000817
365,000	+372.8	1.0000858
370,000	+391.1	1.0000901
375,000	+409.6	1.0000943
380,000	+428.4	1.0000986
385,000	+447.4	1.0001030
390,000	+466.6	1.0001074
395,000	+486.1	1.0001119
400,000	+505.9	1.0001165

CORRECTIONS TO NATURAL SCALE RATIOS*
(in units of the 7th decimal place)

For Lambert Projection

Δø' as argument			
Δø'	Corr'n (Plus)	Δø'	Corr'n (Plus)
1	0	31	34
2	0	32	36
3	0	33	38
4	1	34	40
5	1	35	43
6	1	36	45
7	2	37	48
8	2	38	51
9	3	39	53
10	4	40	56
11	4	41	59
12	5	42	62
13	6	43	65
14	7	44	68
15	8	45	71
16	9	46	74
17	10	47	77
18	11	48	81
19	13	49	84
20	14	50	88
21	15	51	91
22	17	52	95
23	19	53	98
24	20	54	102
25	22	55	106
26	24	56	110
27	26	57	114
28	27	58	118
29	29	59	122
30	32	60	126

Δø' is the difference in latitude in minutes of the ends of the line.

For Lambert or transverse Mercator Projection

Δy or Δx	Corr'n (Plus)
10,000	0
20,000	0
30,000	1
40,000	2
50,000	2
60,000	3
70,000	5
80,000	6
90,000	8
100,000	10
110,000	11
120,000	14
130,000	16
140,000	19
150,000	21
160,000	24
170,000	27
180,000	31
190,000	34
200,000	38
210,000	42
220,000	46
230,000	50
240,000	55
250,000	59
260,000	64
270,000	69
280,000	74
290,000	80
300,000	86
310,000	91
320,000	97
330,000	103
340,000	110
350,000	116

*Scale ratio interpolated for mean latitude or mean x' of the ends of a line and corrected by the above table is a true mean value accurate to within one in the seventh decimal place.

U. S. GOVERNMENT PRINTING OFFICE : 1952 O—979350

GPO 864-970

7-1654 (7-73)
Bureau of Reclamation

COMPUTATION SHEET

*U.S. Government Printing Office: 1973-784-219/1142 Region 8

BY	DATE	PROJECT	SHEET ____ OF ____
CHKD BY	DATE	FEATURE	

DETAILS GEODETIC POSITIONS FROM COORDINATES

STATION		
X	1	GIVEN
C	2	GIVEN (Y COORDINATE OF 18)
X'	3	1 - 2
$(X' \div 10{,}000)^2$	4	$(3 \div 10{,}000)^2$
P	5	⊕ USE page 25 OR 26 USING 3
$(X' \div 10{,}000)^2 \times P$	6	5 × 4
d	7	⊕ page 25 OR 26 USING 3
$(X' \div 10{,}000)^2 \times P + d$	8	6 + 7
Y	9	GIVEN
$Y_o = Y - (X' \div 10{,}000)^2 \times P + d$	10	9 - 8
H	11	⊕ page 9- OR 16- USING 10
a	12	⊕ page 9- OR 16- USING 10
APPROX. $\Delta Y = X' \div H$	13	3 ÷ 11
b	14	⊕ page 23 USING 13
a × b	15	12 × 14
* $X' \pm ab$	16	3 ± 15
$\Delta Y''$	17	16 ÷ 11
CENTRAL MERIDIAN	18	GIVEN
ΔY	19	17 IN degrees, mins. & secs.
λ = LONGITUDE	20	18 - 19
ϕ = LATITUDE	21	⊕ page 9- OR 16- USING 10

⊕ INTERPOLATION FROM: U.S.C.&G.S. PLANE COORDINATE PROJECTION TABLES
TRANVERSE MERCATOR SPECIAL PUBLICATION NO. 257
ARIZONA

X = NORTHING - STATE PLANE GRID COORDINATE
Y = EASTING - " " " " "

CENTRAL MERIDIAN		
EAST	CENTRAL	WEST
110° 10' 00.000"	111° 55' 00.000"	113° 45' 00.000"

* When axb is $\frac{+, \text{ decrease}}{-, \text{ increase}}$ X' numerically

7-1654 (7-73)
Bureau of Reclamation

COMPUTATION SHEET

*U.S. Government Printing Office: 1973-784-219/1142 Region 8

BY	DATE	PROJECT	SHEET ____ OF ____
CHKD BY	DATE	FEATURE	
DETAILS	GEODETIC POSITIONS FROM COORDINATES		

STATION		
X	1	GIVEN
C	2	GIVEN (Y COORDINATE OF 18)
X'	3	1 - 2
$(X' \div 10{,}000)^2$	4	$(3 \div 10{,}000)^2$
P	5	USE page 25 OR 26 USING 3
$(X' \div 10{,}000)^2 \times P$	6	5 × 4
d	7	page 25 OR 26 USING 3
$(X' \div 10{,}000)^2 \times P + d$	8	6 + 7
Y	9	GIVEN
$Y_o = Y - (X' \div 10{,}000)^2 \times P + d$	10	9 - 8
H	11	page 9 OR 16 USING 10
a	12	page 9 OR 16 USING 10
APPROX. $\Delta Y = X' \div H$	13	3 ÷ 11
b	14	page 23 USING 13
a × b	15	12 × 14
* $X' \pm ab$	16	3 ± 15
$\Delta Y''$	17	16 ÷ 11
CENTRAL MERIDIAN	18	GIVEN
ΔY	19	17 IN degrees, mins. & secs.
λ = LONGITUDE	20	18 - 19
ϕ = LATITUDE	21	page 9 OR 16 USING 10

INTERPOLATION FROM: U.S.C.&G.S. PLANE COORDINATE PROJECTION TABLES
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X = NORTHING - STATE PLANE GRID COORDINATE
Y = EASTING - " " " " "

CENTRAL MERIDIAN		
EAST	CENTRAL	WEST
110° 10' 00.000"	111° 55' 00.000"	113° 45' 00.000"

* When a×b is $\frac{+,\ \text{decrease}}{-,\ \text{increase}}$ X' numerically

STATION _____

CENTRAL MERIDIAN	_____	1	
ϕ LATITUDE	_____	2	GIVEN
λ LONGITUDE	_____	3	GIVEN
$\Delta\lambda$	_____	4	1 - 3
$\Delta\lambda''$ (IN SECONDS)	_____	5	4 changed to seconds by X 3600
$(\Delta\lambda'' \div 100)^2$	_____	6	$(5 \div 100)^2$
Y'	_____	7	⌟ Y_0 - page 9 OR 16 USING 2
H	_____	8	⌟ H - " " " " "
V	_____	9	⌟ V - " " " " "
a	_____	10	⌟ a - " " " " "
b	_____	11	⌟ b - page 23 USING 5
* c	_____	12	⌟ c - page 23 USING 5
$H \times \Delta\lambda''$	_____	13	8 × 5
** $a \times b$	_____	14	10 × 11
** $X' = (H \times \Delta\lambda'') + ab$	_____	15	13 ± 14
* $V(\Delta\lambda'' \div 100)^2 \pm C$	_____	16	9 × 6 ± 12
$X = X' + 500{,}000$	_____	17	15 + 500,000
$Y = Y' + V(\Delta\lambda'' \div 100)^2 \pm C$	_____	18	7 + 16

⌟ INTERPOLATION FROM: U.S.C.&G.S. PLANE COORDINATES PROJECTION TABLES
TRANVERSE MERCATOR SPECIAL PUBLICATION NO. 257
ARIZONA

CENTRAL MERIDIAN

EAST	CENTRAL	WEST
110°10′00.000″	111°55′00.000′	113°45′00.000″

* when c is (+ add); (- subtract) to $V(\Delta\lambda \div 100)^2$

** When ab is $\frac{-\text{, decrease}}{+\text{, increase}}$ $(H \times \Delta\lambda'')$ numerically

COMPUTATION SHEET

DETAILS: COORDINATES FROM GEODETIC POSITIONS

BY | DATE | PROJECT | SHEET ____ OF ____

CHKD BY | DATE | FEATURE

7-1654 (7-73)
Bureau of Reclamation

*U.S. Government Printing Office: 1973-784-219/1142 Region 8

STATION _____

CENTRAL MERIDIAN	_____	①	
ϕ LATITUDE	_____	②	GIVEN
λ LONGITUDE	_____	③	GIVEN
$\Delta\lambda$	_____	④	① - ③
$\Delta\lambda''$ (IN SECONDS)	_____	⑤	④ changed to seconds by X 3600
$(\Delta\lambda'' \div 100)^2$	_____	⑥	⑤ ÷ 100)²
Y'	_____	⑦	⌟ Y_0 - page 9 OR 16 USING ②
H	_____	⑧	⌟ H - " " " " "
V	_____	⑨	⌟ V - " " " " "
a	_____	⑩	⌟ a - " " " " "
b	_____	⑪	⌟ b - page 23 USING ⑤
* c	_____	⑫	⌟ c - page 23 USING ⑤
$H \times \Delta\lambda''$	_____	⑬	⑧ × ⑤
** $a \times b$	_____	⑭	⑩ × ⑪
** $X' = (H \times \Delta\lambda'') + ab$	_____	⑮	⑬ ± ⑭
* $V(\Delta\lambda'' \div 100)^2 \pm C$	_____	⑯	⑨ × ⑥ ± ⑫
$X = X' + 500{,}000$	_____	⑰	⑮ + 500,000
$Y = Y' + V(\Delta\lambda'' \div 100)^2 \pm C$	_____	⑱	⑦ + ⑯

⌟ INTERPOLATION FROM: U.S.C.&G.S. PLANE COORDINATES PROJECTION TABLES
TRANVERSE MERCATOR SPECIAL PUBLICATION NO. 257
ARIZONA

CENTRAL MERIDIAN

EAST	CENTRAL	WEST
110°10′00.000″	111°55′00.000′	113°45′00.000″

* when c is (+ add); (− subtract) to $V(\Delta\lambda \div 100)^2$

** When ab is $\frac{-\text{, decrease}}{+\text{, increase}}$ $(H \times \Delta\lambda'')$ numerically

7-1654 (7-73)
Bureau of Reclamation

COMPUTATION SHEET

*U.S. Government Printing Office: 1973-784-219/1142 Region 8

BY	DATE	PROJECT	SHEET ____ OF ____
CHKD BY	DATE	FEATURE	
DETAILS COORDINATES FROM GEODETIC POSITIONS			

EXAMPLE

USC&GS

STATION CALL

X	616,126.24	(1)	GIVEN
C	500,000.00	(2)	GIVEN (Y COORDINATE OF (18))
X'	116,126.24	(3)	(1) − (2)
$(X' \div 10{,}000)^2$	134.853	(4)	$((3) \div 10{,}000)^2$
P	1.584	(5)	◻ USE page 25 OR 26 USING (3)
$(X' \div 10{,}000)^2 \times P$	213.607	(6)	(5) × (4)
d	0.01	(7)	◻ page 25 OR 26 USING (3)
$(X' \div 10{,}000)^2 \times P + d$	213.617	(8)	(6) + (7)
Y	938,836.09	(9)	GIVEN
$Y_0 = Y - (X' \div 10{,}000)^2 \times P + d$	938,622.47	(10)	(9) − (8)
H	84.6053 4406	(11)	◻ page 9 OR 16 USING (10)
a	− 0.804	(12)	◻ page 9 OR 16 USING (10)
APPROX. $\Delta Y = X' \div H$	1372.5639'	(13)	(3) ÷ (11)
b	+ 4.659	(14)	◻ page 23 USING (3)
a × b	− 3.745	(15)	(12) × (14)
$X' \pm ab$	116,129.985	(16)	(3) ± (15)
$\Delta Y''$	1372.6081	(17)	(16) ÷ (11)
CENTRAL MERIDIAN	113°45'00.000"	(18)	GIVEN
ΔY	00° 22' 52.6081	(19)	(17) IN degrees, mins. & secs.
λ = LONGITUDE	113° 22' 07.3919"	(20)	(18) − (19)
ϕ = LATITUDE	33° 34' 48.637"	(21)	◻ page 9 OR 16 USING (10)

◻ INTERPOLATION FROM: U.S.C.&G.S. PLANE COORDINATE PROJECTION TABLES
TRANVERSE MERCATOR SPECIAL PUBLICATION NO. 257
ARIZONA

		CENTRAL MERIDIAN	
X = NORTHING - STATE PLANE GRID COORDINATE	EAST	CENTRAL	WEST
Y = EASTING - " " " " "	110° 10' 00.000"	111° 55' 00.000"	113° 45' 00.000"

COMPUTATION SHEET
GEODETIC POSITIONS FROM COORDINATES

BY | DATE | PROJECT | SHEET ___ OF ___
CHKD BY | DATE | FEATURE
DETAILS

7-1664 (7-73)
Bureau of Reclamation
☆U.S. Government Printing Office: 1973-784-219/1142 Region 8

EXAMPLE

USC&GS

STATION CALL

X	616,126.24	(1)	GIVEN
C	500,000.00	(2)	GIVEN (Y COORDINATE OF (18))
X'	116,126.24	(3)	(1) − (2)
$(X' \div 10{,}000)^2$	134.853	(4)	$((3) \div 10{,}000)^2$
P	1.584	(5)	⊕ USE page 25 OR 26 USING (3)
$(X' \div 10{,}000)^2 \times P$	213.607	(6)	(5)×(4)
d	0.01	(7)	⊕ page 25 OR 26 USING (3)
$(X' \div 10{,}000)^2 \times P + d$	213.617	(8)	(6)+(7)
Y	938,836.09	(9)	GIVEN
$Y_0 = Y - (X' \div 10{,}000)^2 \times P + d$	938,622.47	(10)	(9)−(8)
H	84.60534406	(11)	⊕ page 9- OR 16- USING (10)
a	−0.804	(12)	⊕ page 9- OR 16- USING (10)
APPROX. $\Delta Y = X' \div H$	1372.5639'	(13)	(3) ÷ (11)
b	+4.659	(14)	⊕ page 23 USING (13)
a×b	−3.745	(15)	(12) × (14)
$X' \pm ab$	116,129.985	(16)	(3) ± (15)
$\Delta Y''$	1372.6081	(17)	(16) ÷ (11)
CENTRAL MERIDIAN	113°45'00.000"	(18)	GIVEN
ΔY	00° 22' 52.6081	(19)	(17) IN degrees, mins. & secs.
λ = LONGITUDE	113° 22' 07.3919"	(20)	(18)−(19)
φ = LATITUDE	33° 34' 48.637"	(21)	⊕ page 9- OR 16- USING (10)

⊕ INTERPOLATION FROM: U.S.C.&G.S. PLANE COORDINATE PROJECTION TABLES
TRANVERSE MERCATOR SPECIAL PUBLICATION NO. 257
ARIZONA

X = NORTHING - STATE PLANE GRID COORDINATE
Y = EASTING - " " " " "

EAST	CENTRAL MERIDIAN CENTRAL	WEST
110° 10' 00.000"	111° 55' 00.000"	113° 45' 00.000"

COMPUTATION SHEET
GEODETIC POSITIONS FROM COORDINATES

DETAILS	BY	DATE	PROJECT	SHEET ___ OF ___
	CHKD BY	DATE	FEATURE	

7-1654 (7-73)
Bureau of Reclamation
*U.S. Government Printing Office: 1973-784-219/1142 Region 8

USC&GS

STATION CALL

CENTRAL MERIDIAN	111° 55′ 00.000″	(1)	
ϕ LATITUDE	33° 34′ 48.637″	(2)	GIVEN
λ LONGITUDE	113° 22′ 07.392″	(3)	GIVEN
$\Delta\lambda$	01° 27′ 07.392″	(4)	(1) − (3)
$\Delta\lambda''$ (IN SECONDS)	5227.392″	(5)	(4) changed to seconds by × 3600
$(\Delta\lambda'' \div 100)^2$	2732.563″	(6)	$(5 \div 100)^2$
Y'	938,591.152	(7)	⌟ Y_0 – page 9 OR 16 USING (2)
H	84.602523	(8)	⌟ H – " " " " "
V	1.134515	(9)	⌟ V – " " " " "
a	−.804	(10)	⌟ a – " " " " "
b	−3.596	(11)	⌟ b – page 23 USING (5)
* c	+0.127	(12)	⌟ c – page 23 USING (5)
$H \times \Delta\lambda''$	442250.55	(13)	(8) × (5)
** a × b	2.89	(14)	(10) × (11)
** $X' = (H \times \Delta\lambda'') + ab$	442,253.44	(15)	(13) ± (14)
* $V(\Delta\lambda'' \div 100)^2 \pm C$	3100.261	(16)	(9) × (6) ± (12)
$X = X' + 500{,}000$	57,746.56 (.59)	(17)	(15) + 500,000
$Y = Y' + V(\Delta\lambda'' \div 100)^2 \pm C$	941,691.41 (34)	(18)	(7) + (16)

⌟ INTERPOLATION FROM: U.S.C.&G.S. PLANE COORDINATES PROJECTION TABLES
TRANVERSE MERCATOR SPECIAL PUBLICATION NO. 257
ARIZONA

CENTRAL MERIDIAN

EAST	CENTRAL	WEST
110° 10′ 00.000″	111° 55′ 00.000″	113° 45′ 00.000″

* when c is (+ add); (− subtract) to $V(\Delta\lambda \div 100)^2$

** When ab is −, decrease / +, increase $(H \times \Delta\lambda'')$ numerically

COMPUTATION SHEET

COORDINATES FROM GEODETIC POSITIONS

DETAILS | CHKD BY | DATE | FEATURE | BY | DATE | PROJECT | SHEET ___ OF ___

7-1654 (7-73)
Bureau of Reclamation
U.S. Government Printing Office: 1973-784-219/1142 Region 5

USC&GS

STATION CALL

CENTRAL MERIDIAN	111° 55′ 00.000″	(1)	
ϕ LATITUDE	33° 34′ 48.637″	(2)	GIVEN
λ LONGITUDE	113° 22′ 07.392″	(3)	GIVEN
$\Delta\lambda$	01° 27′ 07.392″	(4)	(1) − (3)
$\Delta\lambda''$ (IN SECONDS)	5227.392″	(5)	(4) changed to seconds by × 3600
$(\Delta\lambda'' \div 100)^2$	2732.563″	(6)	$((5) \div 100)^2$
Y'	938,591.152	(7)	⌟ Y_0 – page 9 OR 16 USING (2)
H	84.602.523	(8)	⌟ H – " " " " "
V	1.134515	(9)	⌟ V – " " " " "
a	−.804	(10)	⌟ a – " " " " "
b	−3.596	(11)	⌟ b – page 23 USING (5)
* c	+0.127	(12)	⌟ c – page 23 USING (5)
$H \times \Delta\lambda''$	442250.55	(13)	(8) × (5)
** $a \times b$	2.89	(14)	(10) × (11)
** $X' = (H \times \Delta\lambda'') + ab$	442,253.44	(15)	(13) ± (14)
* $V(\Delta\lambda'' \div 100)^2 \pm C$	3100.261	(16)	(9) × (6) ± (12)
$X = X' + 500{,}000$	57,746.56 (.59)	(17)	(15) + 500,000
$Y = Y' + V(\Delta\lambda'' \div 100)^2 \pm C$	941,691.41 (34)	(18)	(7) + (16)

⌟ INTERPOLATION FROM: U.S.C.&G.S. PLANE COORDINATES PROJECTION TABLES
TRANVERSE MERCATOR SPECIAL PUBLICATION NO. 257
ARIZONA

CENTRAL MERIDIAN

EAST	CENTRAL	WEST
110° 10′ 00.000″	111° 55′ 00.000″	113° 45′ 00.000″

* when c is (+ add); (− subtract) to $V(\Delta\lambda \div 100)^2$

** When ab is −, decrease / +, increase $(H \times \Delta\lambda'')$ numerically

COMPUTATION SHEET
U.S. Government Printing Office: 1973-784-219/1142 Region 8

BY	DATE	PROJECT	SHEET ___ OF ___
CHKD BY	DATE	FEATURE	
DETAILS COORDINATES FROM GEODETIC POSITIONS			

7-1654 (7-73)
Bureau of Reclamation

GEODETIC CONTROL

SERVES AS

THE BASIC FRAMEWORK FOR:

MULTIPURPOSE CADASTRES

LAND RECORDS PROGRAMS

AUTOMATED MAPPING PROJECTS

NETWORK OF MONUMENTED POINTS

GEODETIC CONTROL:

PRECISELY MEASURED IN ACCORDANCE WITH STANDARD PROCEDURES

MEET ACCURACY SPECIFICATIONS

ADJUSTED TO TIE TOGETHER

DOCUMENTED FOR MULTIPLE USE

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

NAD 83 NETWORK PROBLEMS

NOT "GPSABLE" - POOR ACCESSIBILITY
IRREGULARLY SPACED & LOWER ACCURACY

HIGH ACCURACY REFERENCE NETWORKS (HARNs)

"GPSABLE" Clear Horizons for Satellite Signal Acquisition
EASY ACCESSIBILITY
Few Special Vehicle or Property Entrance Requirements
REGULARLY SPACED Always within 15-100 Kilometers
HIGH ACCURACY (HORIZONTAL)
A-Order (5mm + 1:10,000,000)
B-Order (8mm + 1:1,000,000)

FEDERAL BASE NETWORK

NGS Funded
NGS Mark Maintenance
100 Kilometer Station Spacing
A and B-Order Horizontal Accuracy
First-Order Class II Orthometric Heights
Second-Order Class I Orthometric Heights
Second-Order Class I Ellipsoidal Heights
50 microgals Gravity Accuracy
1 mm/year Horizontal and Vertical
Crustal Motion Relative Accuracy
Revisited every 3 to 5 years

COOPERATIVE BASE NETWORK

State Funded
State Mark Maintenance
25 to 30 kilometer Station Spacing
B-Order Horizontal Accuracy
Second-Order Class I Orthometric Heights
Third-Order Class I Ellipsoidal Heights
50 microgals Gravity Accuracy
1 mm/year Horizontal and Vertical
Crustal Motion Relative Accuracy
Revisited as Needed

USER DENSIFICATION NETWORK

Federal, State and Local Funding Station Spacing as Required
GPS Observations Only (Horizontal) First-Order Horizontal Accuracy
NO Vertical Requirements
Second-Order Class II Vertical
(Conventional Leveling)
NO Gravity Requirements
NO Velocity Predictions
Revisited as Needed

```
        National Geodetic Survey,
DU0876 ***********************************************************************
DU0876  DESIGNATION -  GUN
DU0876  PID         -  DU0876
DU0876  STATE/COUNTY-  AZ/MARICOPA
DU0876  USGS QUAD   -  TEMPE (1986)
DU0876
DU0876  HORZ DATUM  -  NAD 83 (1992)
DU0876  VERT DATUM  -  NAVD 88
DU0876
DU0876  POSITION    -  33 27 42.16658(N)   111 57 07.89249(W)   ADJUSTED
DU0876  92 minus 83 -      +00.00551            -00.00515       ADJUSTED
DU0876  83 minus 27 -      +00.15877            +02.52624       ADJUSTED
DU0876
DU0876
DU0876  HEIGHT      -      401.461  (meters)    1317.12   (feet) ADJUSTED
DU0876  88 minus 29 -       +0.658                               ADJ UNCH
DU0876  DY minus 88 -       -0.465                               COMPUTED
DU0876. (NOTE - For assistance in applying shifts see file readme.dat)
DU0876 ***********************************************************************
DU0876
DU0876  LAPLACE CORR-          0.62                              DEFLEC93
DU0876  GEOID HEIGHT-        -29.57                              GEOID93
DU0876  MODELED GRAV-     979,467.7                              NAVD88
DU0876
DU0876
DU0876  HORZ ORDER  -  SECOND
DU0876  VERT ORDER  -  SECOND    CLASS 0
DU0876
DU0876
DU0876.The horizontal position was established by classical geodetic methods
DU0876.and adjusted by the National Geodetic Survey in August 1993.
DU0876
DU0876.The orthometric height was determined by differential leveling
DU0876.and adjusted by the National Geodetic Survey in November 1993.
DU0876
DU0876.The dynamic height is computed by dividing the NAVD 88
DU0876.geopotential number by the normal gravity value computed on the
DU0876.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
DU0876.degrees latitude (G = 980.6199 gals.).
DU0876
DU0876.The Laplace correction was computed from DEFLEC93 derived deflections.
DU0876
DU0876.The geoid height was determined by GEOID93.
DU0876
DU0876.The modeled gravity was interpolated from observed gravity values.
DU0876
DU0876;                    North        East       Scale          Converg.
DU0876;SPC AZ C     -    272,954.910  210,057.740 0.99990013     -0 01 10.5  MT
DU0876;UTM  12      -  3,702,879.816  411,514.730 0.99969653     -0 31 30.2  MT
DU0876
DU0876:                Primary Azimuth Mark                      Grid Az
DU0876:SPC AZ C     -  GOVERNOR HUNTS TOMB                       146 17 18.7
DU0876:UTM  12      -  GOVERNOR HUNTS TOMB                       146 47 38.4
DU0876
DU0876|-----------------------------------------------------------------------|
DU0876| Pid     Reference Object                     Distance      Geod. AZ   |
DU0876|                                                            dddmmss.s |
DU0876|
DU0876| DU0877 GUN RM 1                              5.327 METERS 02811      |
DU0876| DU2181 SCOTTSDALE MOTOROLA WATER TANK      APPROX. 4.5 KM 0875851.4 |
DU0876| DU2182 GOVERNOR HUNTS TOMB                 APPROX. 1.3 KM 1461608.2 |
DU0876| DU2177 TEMPE SALT R PROJECT PAD MAST       APPROX. 2.3 KM 1522259.3 |
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The NGS Data Sheet

```
DATABASE = Sybase ,PROGRAM = datasheet, VERSION = 6.27
Starting Datasheet Retrieval...
1        National Geodetic Survey,   Retrieval Date = SEPTEMBER 14, 2000
 DU0876 ***********************************************************************
 DU0876  DESIGNATION -  GUN
 DU0876  PID         -  DU0876
 DU0876  STATE/COUNTY-  AZ/MARICOPA
 DU0876  USGS QUAD   -  TEMPE (1982)
 DU0876
 DU0876                          *CURRENT SURVEY CONTROL
 DU0876  ___________________________________________________________________
 DU0876* NAD 83(1992)-  33 27 42.16658(N)    111 57 07.89249(W)     ADJUSTED
 DU0876* NAVD 88     -       401.461  (meters)    1317.13   (feet) ADJUSTED
 DU0876  ___________________________________________________________________
 DU0876  LAPLACE CORR-           1.06  (seconds)                   DEFLEC99
 DU0876  GEOID HEIGHT-         -29.65  (meters)                    GEOID99
 DU0876  DYNAMIC HT  -         400.996 (meters)    1315.60   (feet) COMP
 DU0876  MODELED GRAV-     979,467.7   (mgal)                      NAVD 88
 DU0876
 DU0876  HORZ ORDER  -  SECOND
 DU0876  VERT ORDER  -  SECOND    CLASS 0
 DU0876
 DU0876.The horizontal coordinates were established by classical geodetic methods
 DU0876.and adjusted by the National Geodetic Survey in August 1993.
 DU0876
 DU0876.The orthometric height was determined by differential leveling
 DU0876.and adjusted by the National Geodetic Survey in November 1993.
 DU0876
 DU0876.The Laplace correction was computed from DEFLEC99 derived deflections.
 DU0876
 DU0876.The geoid height was determined by GEOID99.
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 DU0876.The dynamic height is computed by dividing the NAVD 88
 DU0876.geopotential number by the normal gravity value computed on the
 DU0876.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
 DU0876.degrees latitude (g = 980.6199 gals.).
 DU0876
 DU0876.The modeled gravity was interpolated from observed gravity values.
 DU0876
 DU0876;                    North         East     Units   Scale      Converg.
 DU0876;SPC AZ C     -   272,954.910   210,057.740   MT  0.99990013 -0 01 10.5
 DU0876;UTM  12      - 3,702,879.816   411,514.730   MT  0.99969653 -0 31 30.2
 DU0876
 DU0876:                Primary Azimuth Mark                     Grid Az
 DU0876:SPC AZ C     -  GOVERNOR HUNTS TOMB                      146 17 18.7
 DU0876:UTM  12      -  GOVERNOR HUNTS TOMB                      146 47 38.4
 DU0876
 DU0876|-----------------------------------------------------------------------|
 DU0876| PID    Reference Object                     Distance       Geod. Az  |
 DU0876|                                                            dddmmss.s |
 DU0876| DU0877 GUN RM 1                               5.327 METERS 02811     |
 DU0876| DU2181 SCOTTSDALE MOTOROLA WATER TANK      APPROX. 4.5 KM 0875851.4  |
 DU0876| DU2182 GOVERNOR HUNTS TOMB                 APPROX. 1.3 KM 1461608.2  |
 DU0876| DU2177 TEMPE SALT R PROJECT RAD MAST       APPROX. 2.3 KM 1522259.3  |
 DU0876| DU2175 PHOENIX RADIO STA KOOL S MAST       APPROX. 3.3 KM 2862027.2  |
 DU0876| DV2065 PHOENIX RADIO KTAR S TWR OF 2       APPROX. 5.0 KM 2904656.7  |
 DU0876| DU0875 GUN RM 2                               7.563 METERS 30358     |
 DU0876| DU2184 PAPAGO                              APPROX. 0.6 KM 3283405.6  |
 DU0876|-----------------------------------------------------------------------|
```

Tuesday, Sept. 10, 1996

USC&gs (ngs) 1st Order Horizontal Control Station: "GUN"

	LATITUDE	LONGITUDE
NAD-83/92	N 33 27 42.16658	W 111 57 07.89249
93/92 - 83	+00.00551	-00.00515
NAD - 83	N 33 27 42.16107	W 111 57 07.89764
83 - 27	+00.15877	+02.52624
NAD - 27	N 33 27 42.00230	W 111 57 05.37140

SPC AZ C	272,954.910	210,057.740	NAD 83/92	METERS
	895,519.567	689,164.435	NAD 83/92	U.S. SURVEY FEET
	895,521.332	689,165.794	NAD 83/92	"INTERNATIONAL" FEET
SPC AZ C	272,954.740	210,057.607	NAD 83	METERS
	895,519.009	689,163.999	NAD 83	U.S. SURVEY FEET
	895,520.775	689,165.357	NAD 83	"INTERNATIONAL" FEET
SPC AZ C	895,474.582	489,379.190	NAD 83/92	"converted" to NAD - 27
	895,474.026	489,378.754	NAD 83	"converted" to NAD - 27
	895,474.109	489,379.163	NAD 27	(U.S. SURVEY FEET)

1 METER = 3.280833333333333333333333333333333333..... U.S. SURVEY FEET
1 METER = 3.2808398 (to 7th place right of decimal) INTERNATIONAL FEET

Index of On Line Publications

Publications on the NGS web site:

- An Accuracy Assessment of The Geoid96 Geoid Height Model for the State of OHIO.
- The CARIB97 high resolution geoid height model for the Caribbean Sea
- CD-ROM as Medium for Large Geographic Information Files
- Computing GPS-derived orthometric heights with the GEOID90 geoid height model
- The Contribution of Geodetic Data to the National Spatial Data Infrastructure
- Converting GPS Height into NAVD 88 Elevation with the GEOID96 Geoid Height Model
- Data Processing in the National Geodetic Survey
- Development of the National Spatial Reference System
- The EGM96 geoid: Subtle points on the use of a global geopotential model
- Evaluation of the EGM96 Model of the Geopotential in the United States
- Exchange File Format (for submitting data to the NGS Obstruction Chart Database) [WordPerfect format] [PDF format]
- Evaluation of the High Accuracy Reference Network relative to the Continuously Operating Reference Stations
- Evaluation of Preliminary Models of the Geopotential in the United States
- A Free-Fall Determination of the Newtonian Constant of Gravity
- Federal Register Notices (pdf format)
    - Use of the NAD 83 / WGS 84 Dataum Tag on Mapping Products (August 14, 1995)
    - Affirmation of Vertical Datum for Surveying and Mapping Activities (June 23, 1993; NAVD88)
    - Affirmation of Datum for Surveying and Mapping Activities (June 13, 1989; NAD83)
    - National Vertical Control Net (May 15, 1973; NGVD29)
    - Refinement of Values for the Yard and the Pound (June 30, 1959; Meters to Feet Conversion)
- FGCS Technical Publications (including the NGS **Blue Book**)
- The GEOID96 high resolution geoid height model for the United States
- Geodesy: Imagine the Possibilities (pdf, 765 KB)
- Geodetic Phase of NOS' San Francisco Bay Demonstration Project
- Geodetic Surveys in the United States The Beginning and the Next One Hundred Years 1807 - 1940
- Geodetic Surveying 1940 - 1990
- High Accuracy Reference Network for Michigan
- High Accuracy Reference Network for Nebraska
- High Accuracy Reference Network for Nevada
- High Accuracy Reference Network for Ohio
- High Accuracy Reference Network for Oklahoma
- High Accuracy Reference Network for Utah
- High Accuracy Reference Network for West Virginia

- Improving Geoid Undulations and Ocean Dynamic Topography Through Altimetry and A Modified Kalman Filter
- International Geoid Commission Activity Report 1991-1995 (Various countries including the USA) [postscript format]
- International Geoid Commission National Report for the United States of America
- National Ocean Service Shoreline Mapping Program
- New Coordinate Adjustment for Mississippi
- NOS NGS-58 - Guidelines for Establishing GPS-Derived Ellipsoid Heights
- Overview of Global Positioning System Continuously Operating Reference Stations
- Planning the Arkansas High Accuracy Reference Network
- Planning the Kansas High Accuracy Reference Network
- Report to IAG SSG 3.136: High Resolution Geoid Modeling and Evaluation
- Results of the General Adjustment of the North American Vertical Datum of 1988
- Surveying Support for Eastern Europe
- Where Freedom Stands (using GPS to determine the position the Statue of Freedom)

Articles of interest from other web sites:

- Darmstadt IGS AC Workshop Draft Documents [PDF format]
- NIMA Publications on Geodesy and Geophysics including "Geodesy for the Layman"

Return to NGS Home Page *Last modified: Dec 8, 1998*

Geodesy and Geophysics

G&G Publications

Welcome to the NIMA Geodesy and Geophysics publications listing. Many of the documents are available in Adobe's portable document format (PDF). The Adobe Acrobat viewer software is available free... Get Acrobat Reader

1. Geodesy for the Layman (HTML)
   - Geodesy for the Layman *(3,507 KB)*
2. DoD World Geodetic System 1984 - Its Definition and Relationships with Local Geodetic Systems - TR 8350.2
3. Datums, Ellipsoids, Grids and Grid Reference Systems - TM 8358.1 (HTML)
   - Datums, Ellipsoids and Grid Reference Systems - TM 8358.1
     - Chapters 1 - 5 *(6,418 KB)*
     - Chapters 6 - 7 *(7,747 KB)*
     - Chapters 8-11 *(6,824 KB)*
     - Appendix A - C *(286 KB)*
4. The Universal Grids: Universal Transverse Mercator (UTM) and Universal Polar Stereographic (UPS) - TM8358.2 *(2,883 KB)*
5. GEOTRANS v2.0.3 (GEOgraphic TRANSlator)
   - Revised version 2.0.3 posted April 14, 2000
   - Summary of Fixes from 2.0.1 to 2.0.2 to 2.0.3
   - A GEOTRANS username and password is required for this page. They can be obtained by completing this online registration form. Note: You must have an email extension of .mil or .gov to access this software.
6. WGS 84 Coordinate Validation and Improvement for the NIMA and Air Force GPS Tracking Stations (NSWCDD/TR-96/201)
7. Horizontal Datums
8. Vertical Datums
9. Publications of Historic Interest
10. Ordering instructions for Geodesy and Geophysics publications

Reference Ellipsoid Constants

Reference Ellipsoids	a (Meters)	f
Airy	6377563.396	1/299.3249646
Modified Airy	6377340.189	1/299.3249646
Australian National	6378160	1/298.25
Bessel 1841	6377397.155*	1/299.1528128
Clarke 1866	6378206.4	1/294.9786982
Clarke 1880	6378249.145	1/293.465
Everest	6377276.345	1/300.8017
Modified Everest	6377304.063	1/300.8017
Fischer 1960 (Mercury)	6378166	1/298.3
Modified Fischer 1960 (South Asia)	6378155	1/298.3
Fischer 1968	6378150	1/298.3
Geodetic Reference System 1967	6378160	1/298.247167427
Geodetic Reference System 1980	6378137	1/298.257222101
Helmert 1906	6378200	1/298.3
Hough	6378270	1/297
International	6378388	1/297
Krassovsky	6378245	1/298.3
South American 1969	6378160	1/298.25
WGS 60	6378165	1/298.3
WGS 66	6378145	1/298.25
WGS 72	6378135	1/298.26
WGS 84	6378137	1/298.257223563

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

NATIONAL SPATIAL REFERENCE SYSTEM

Definition:

The National Spatial Reference System (NSRS) is the name given to all Geodetic Control contained in the National Geodetic Survey (NGS) Data Base.

The NSRS includes:

- **A, B, First, Second, and Third-Order horizontal and vertical control observed by NGS as well as data submitted by other agencies (i.e. USGS, BLM, States, Counties, Cities and private surveying organizations.)**
- **Precise Astronomic Data**
- **Gravity Data**
- **Satellite Data**
- **Very Long Base Line Interferometry**

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

NATIONAL SPATIAL REFERENCE SYSTEM

ELLIPSOID DEFINITIONS

CLARKE 1866
GEODETIC REFERENCE SYSTEM 1980

DATUM DEFINITIONS - HORIZONTAL

NORTH AMERICAN DATUM OF 1927
NORTH AMERICAN DATUM OF 1983

DATUM DEFINITIONS - VERTICAL

MEAN SEA LEVEL DATUM OF 1929
NATIONAL GEODETIC VERTICAL DATUM OF 1929
NORTH AMERICAN VERTICAL DATUM OF 1988

GEOID HEIGHT MODELS

OSU 78
OSU 89B
GEOID 90
GEOID 93
GEOID 96
GEOID 99
GEOID 200?

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

NAD PROJECT ELEMENTS

READJUSTMENT - Mathematical computation to remove unpredictable distortions in the network

REDEFINITION - Define a new datum that is:

Worldwide - Not just best fitting North America

Earth-Centered - Center of the ellipsoid is at the mass center of the Earth

Uses the best values for the size and shape

- GRS 80 / WGS 84 -

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

COMPARISON OF DATUM ELEMENTS

	NAD 27	NAD 83
Reference Ellipsoid	Clarke 1866 a = 6,378,206.4 m 1/f = 294.9786982	GRS 80 a = 6,378,137. m 1/f = 298.257222101
Datum Point	Triangulation Station MEADES RANCH, KANSAS	NONE (Mass Center of the Earth)
Longitude Origin	Greenwich Meridian (BIH Zero Meridian)	Greenwich Meridian (BIH Zero Meridian)
Azimuth Orientation	From South	From North
Adjustment	25k Stations Several Hundred Base Lines Several Hundred Astro Azi.	250k Stations Approx. 30k EDMI Base Lines 5k Astro Azimuths Doppler Point Positions VLBI Vectors
Best Fitting	North America	World-Wide

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At the Annual TSPS Conference in San Antonio Texas, August 24, 1996

EFFECTS ON USERS OF GEODETIC DATA

- For new surveys beginning and closing on National Geodetic Reference System (NGRS) stations:

  - NO change to field procedures

  - Obtain NAD 83 coordinates from NGS

  - Perform computations as always

- For old surveys tied to and data bases referenced to NGRS:

  - A datum transformation is necessary

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996.

STATUS OF NAD 27 AND NAD 83 STATE PLANE COORDINATE LEGISLATION

(As of March, 1996)

NO SPCS LEGISLATION (4)	EXISTING NAD 27 SPCS LEGISLATION (4)	ENACTED NAD 83 SPCS LEGISLATION (44)	
District of Columbia	Alabama	Alaska	New Hampshire
Hawaii	Arkansas	Arizona (I)	New Jersey
Nebraska	Delaware	California (S)	New Mexico (S)
Puerto Rico	Illinois	Colorado (s)	New York (S)
		Connecticut (S)	North Carolina (S)
		Florida (S)	North Dakota (I)
		Georgia	Ohio
		Idaho (S)	Oklahoma (S)
		Indiana (S)	Oregon (I)
		Iowa	Pennsylvania (S)
		Kansas	Rhode Island
		Kentucky (S)	South Carolina (I)
		Louisiana	South Dakota
		Maine	Tennessee (S)
		Maryland (S)	Texas
		Massachusetts (S)	Utah (I)
		Michigan (I)	Vermont
		Minnesota	Virginia
		Mississippi (S)	Washington (S)
		Missouri	West Virginia
		Montana	Wisconsin (S)
		Nevada	Wyoming

UNITS
I = International Feet and Meters
S = U.S. Survey Feet and Meters
ALL OTHERS METERS ONLY

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

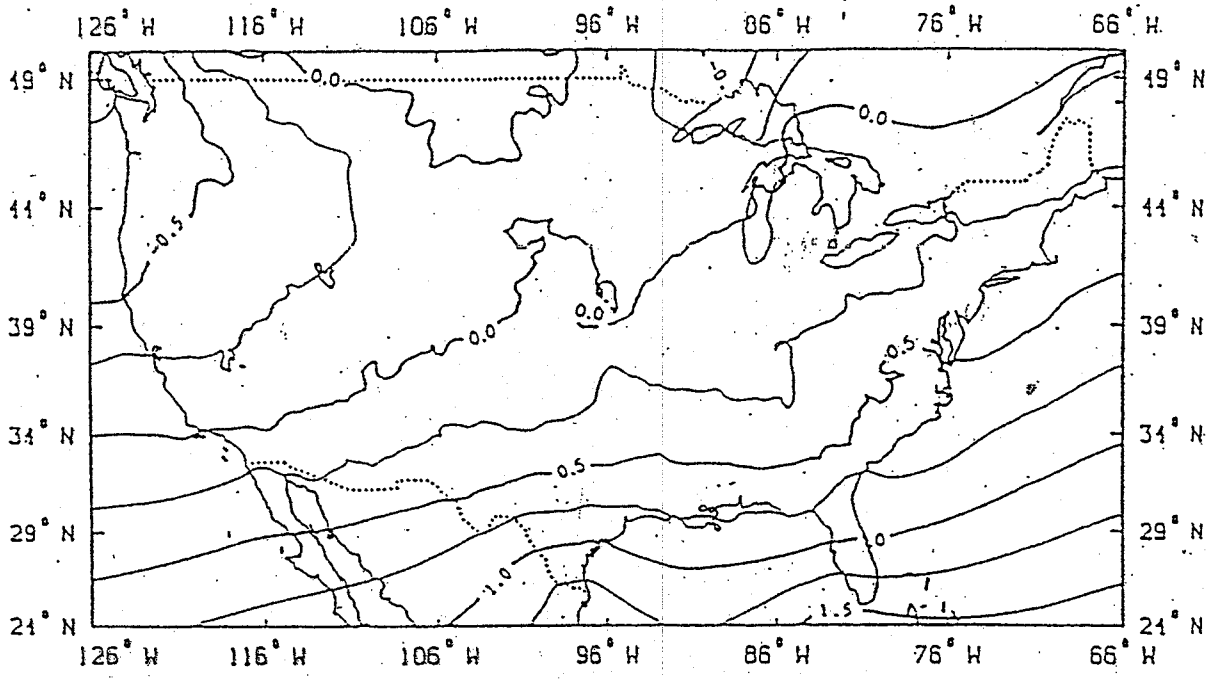


Figure 21.1. Latitude datum shift in the conterminous United States in seconds of arc (NAD 83 minus NAD 27).

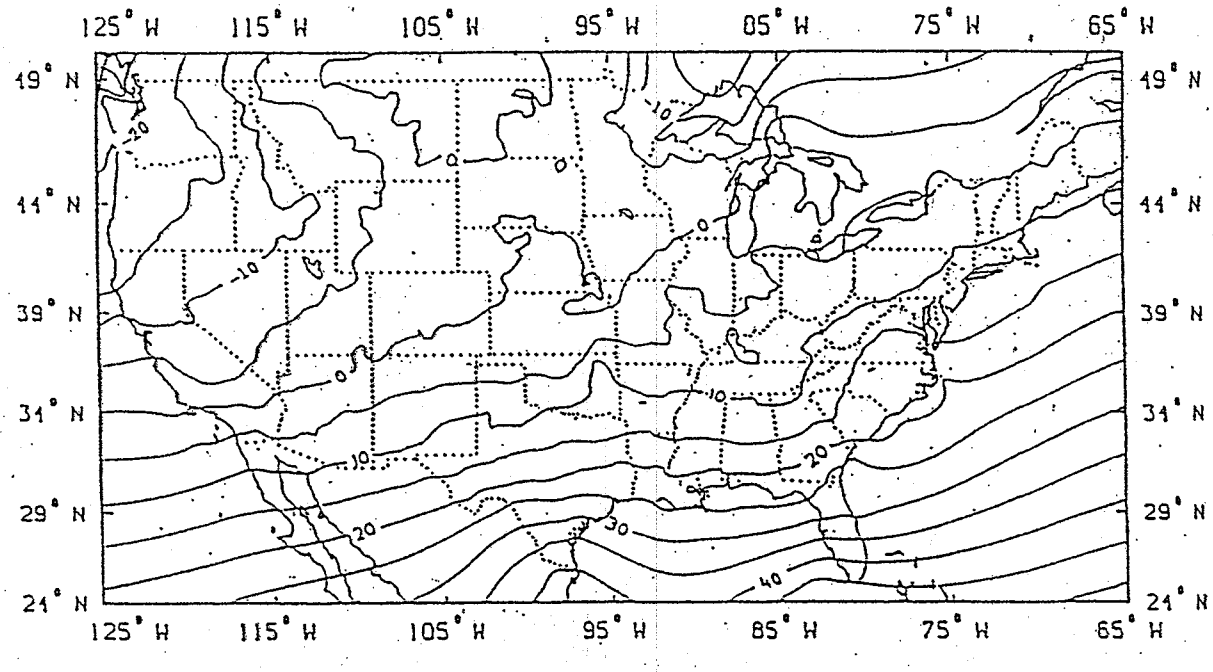


Figure 21.2. Latitude datum shift in the conterminous United States in meters (NAD 83 minus NAD 27).

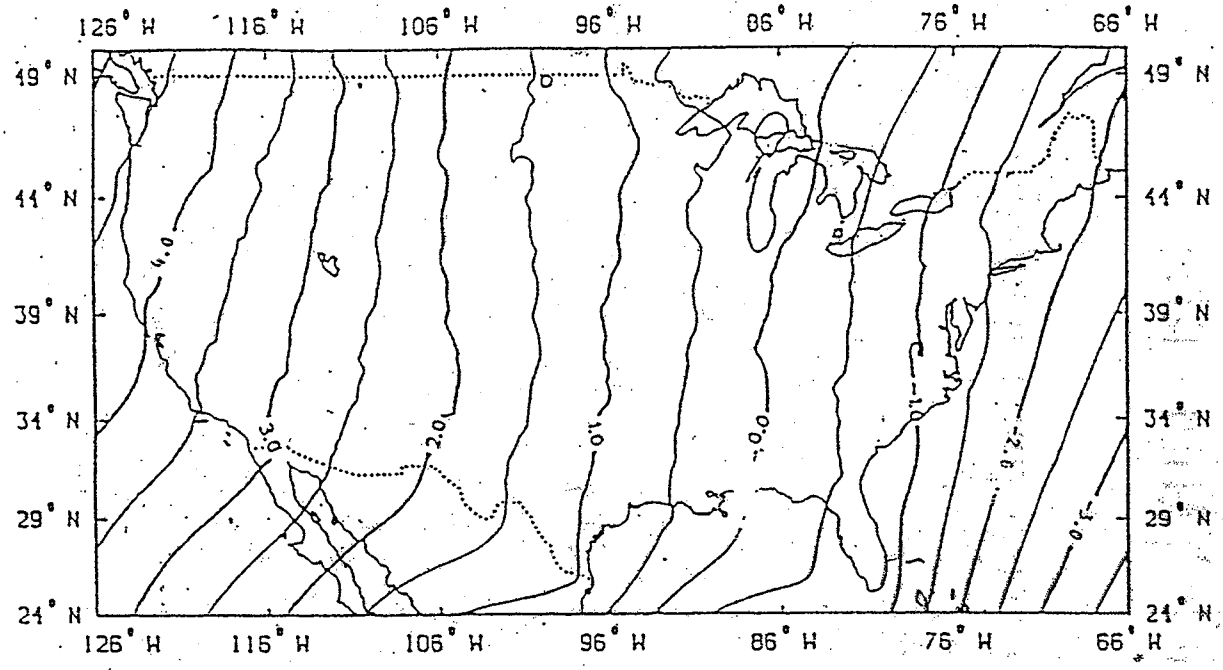


Figure 21.3. Longitude datum shift in the conterminous United States in seconds of arc (NAD 83 minus NAD 27).

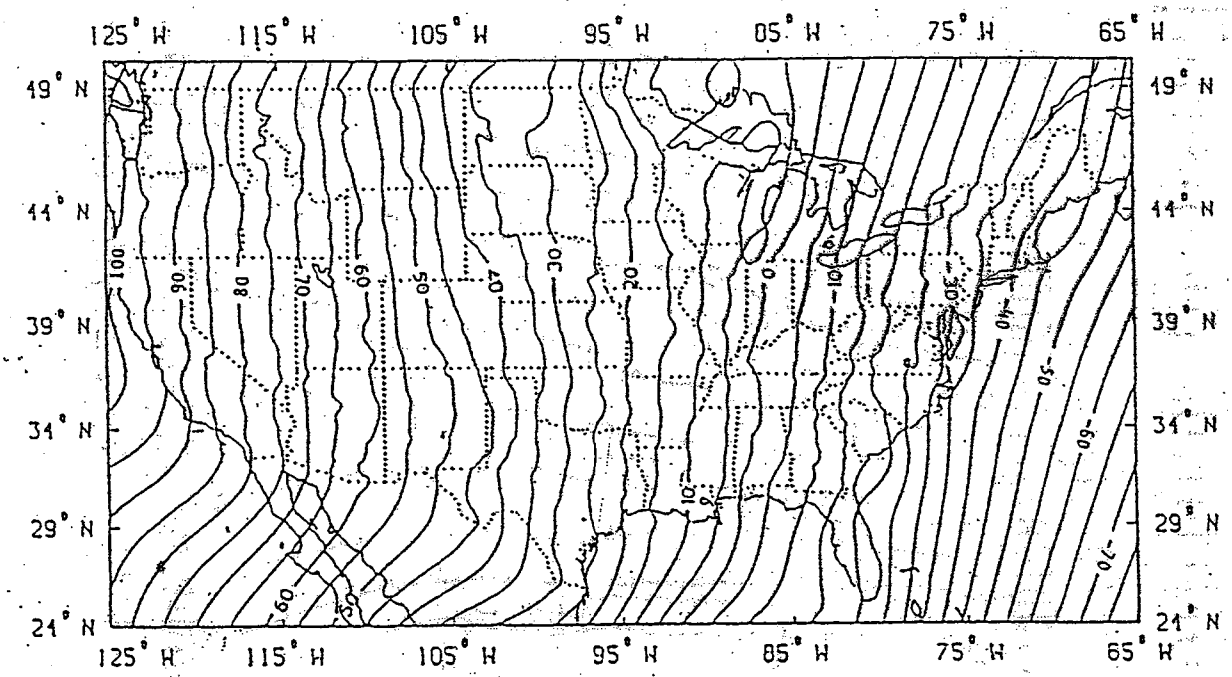
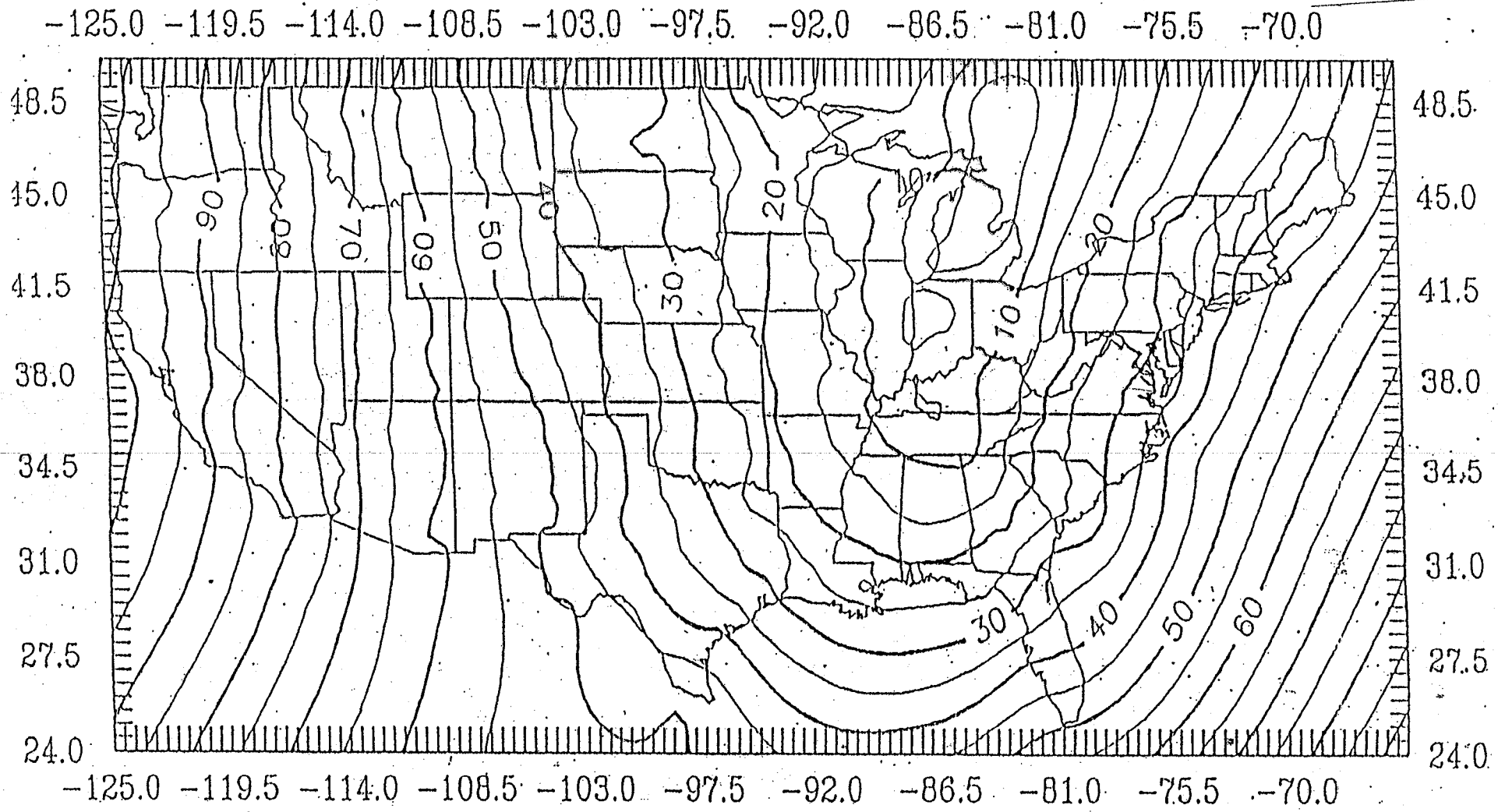


Figure 21.4. Longitude datum shift in the conterminous United States in meters (NAD 83 minus NAD 27).

MAGNITUDE OF DATUM SHIFT (METERS)



Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio-Texas, August 24, 1996

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

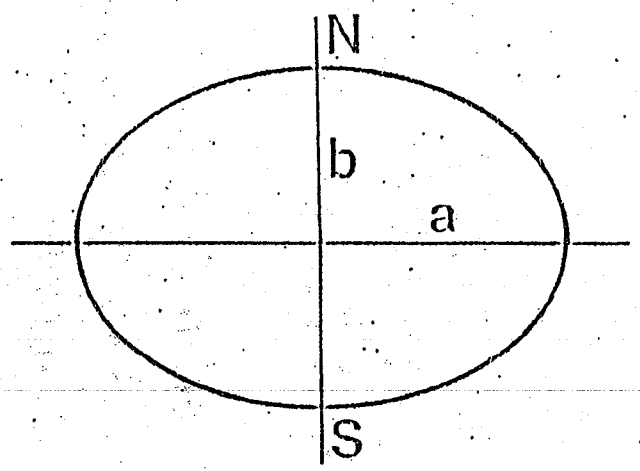
NAD 83

Geodetic Reference System

- O DEFINITION BASED UPON GRS 80 ELLIPSOID (GLOBAL, NOT LOCAL).
- O CONTROL POINTS BASE UPON SIMULTANEOUS ADJUSTMENT OF 1,785,772 OBSERVATIONS FOR 266,436 STATION (U.S. CANADA, MEXICO, CENTRAL AMERICA).
- O COMBINATION OF BOTH CLASSICAL TERRESTRIAL AND SPACE BASED DATA.

- O NAD 83 is intended to bridge the gap between traditional survey methodologies and future requirements, both mapping and navigation.
- O Navigation and mapping will largely be done using satellite technology in the future - GPS.

THE ELLIPSOID



a = Semi major axis

b = Semi minor axis

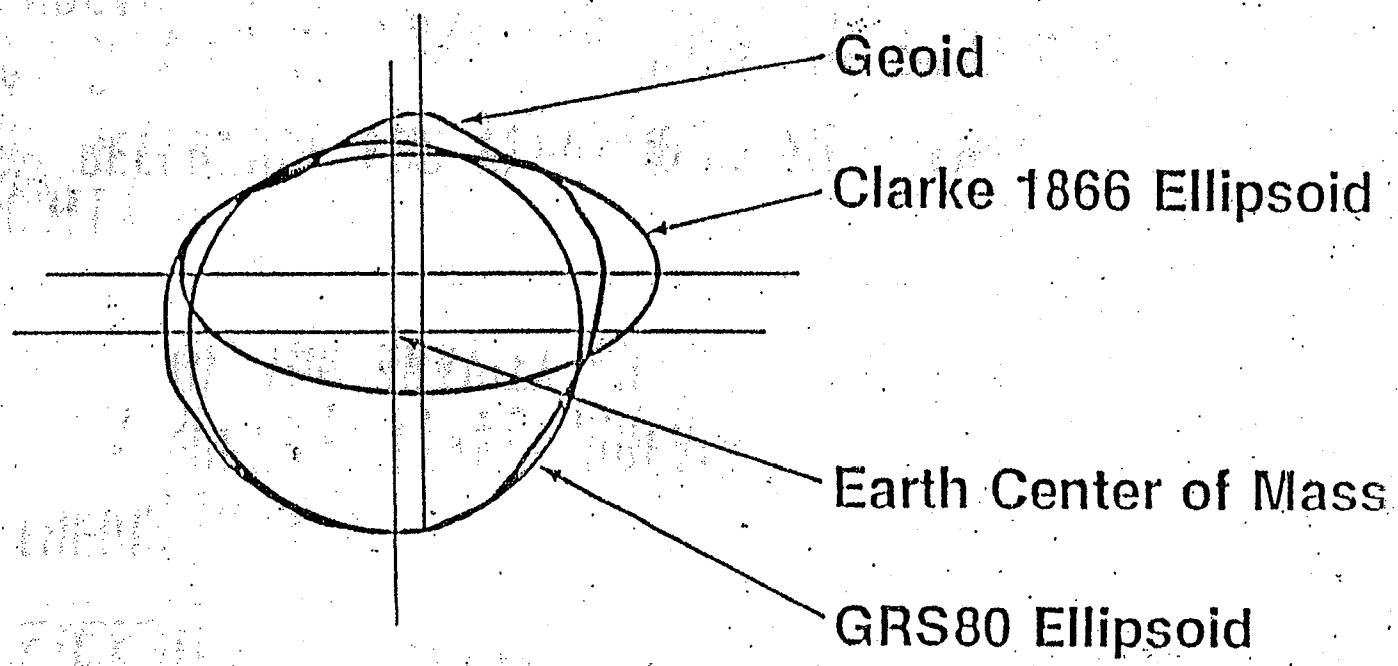
$f = \frac{a-b}{a}$ = Flattening

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas August 24, 1996

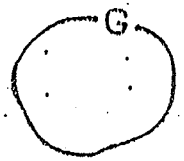
NAD PROJECT

THE GEOID AND TWO ELLIPSOIDS



THE GEOID --- WHAT IS IT ?

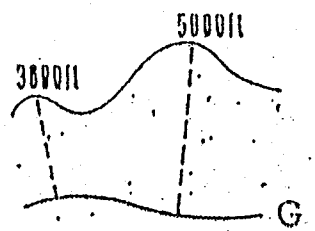
THEORETICALLY:

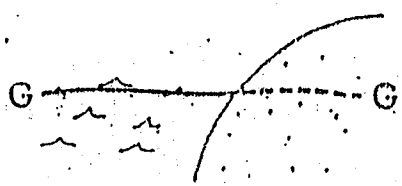


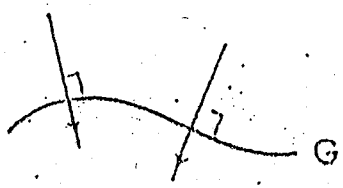
A SPECIFIC LEVEL SURFACE OF THE GRAVITY FIELD

PRACTICALLY:

- ZERO REFERENCE FOR ELEVATIONS AND DEPTHS
- MEAN SEA LEVEL CONTINUED INTO THE CONTINENTS
- A SURFACE ⊥ TO THE DIRECTION OF GRAVITY

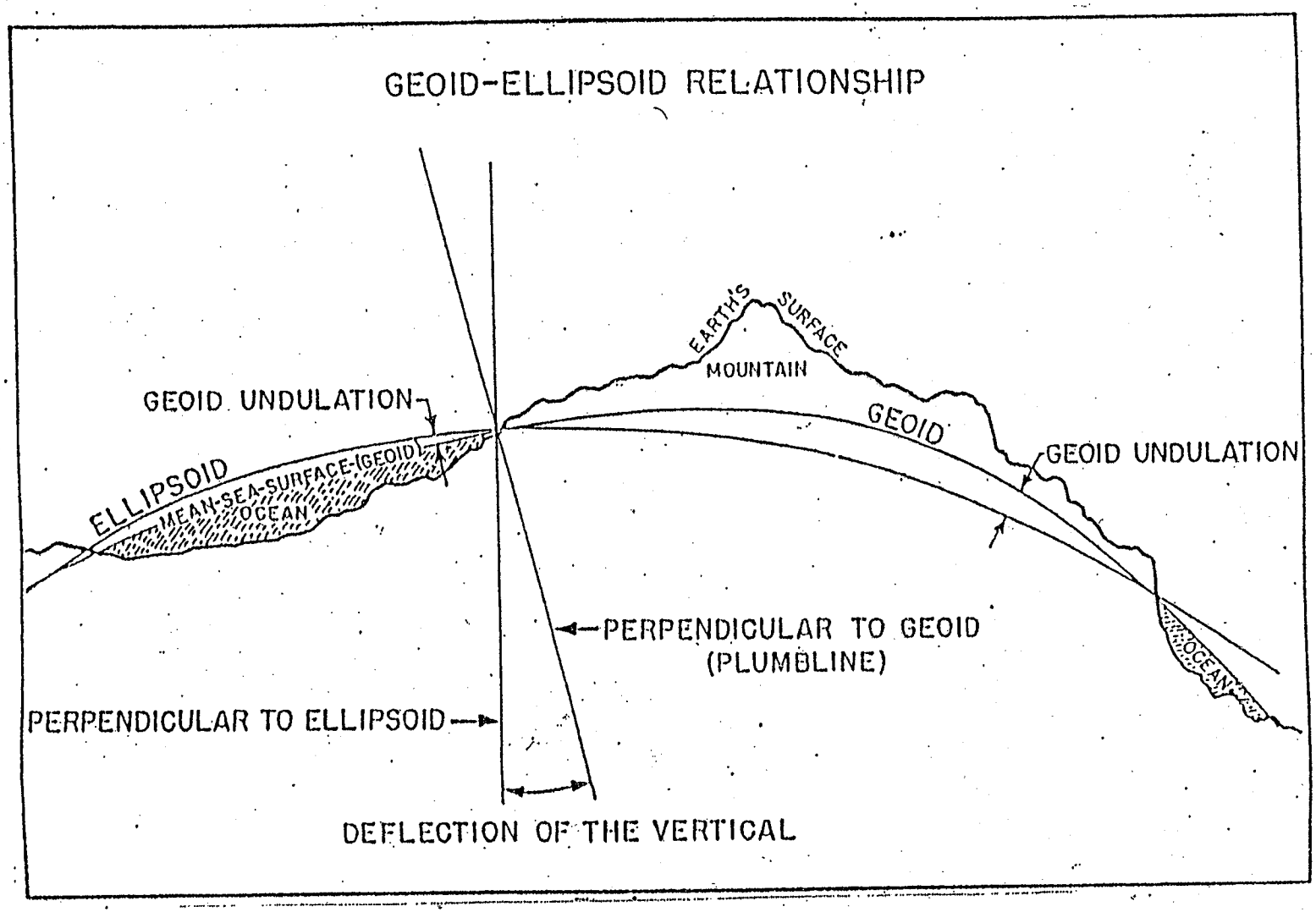






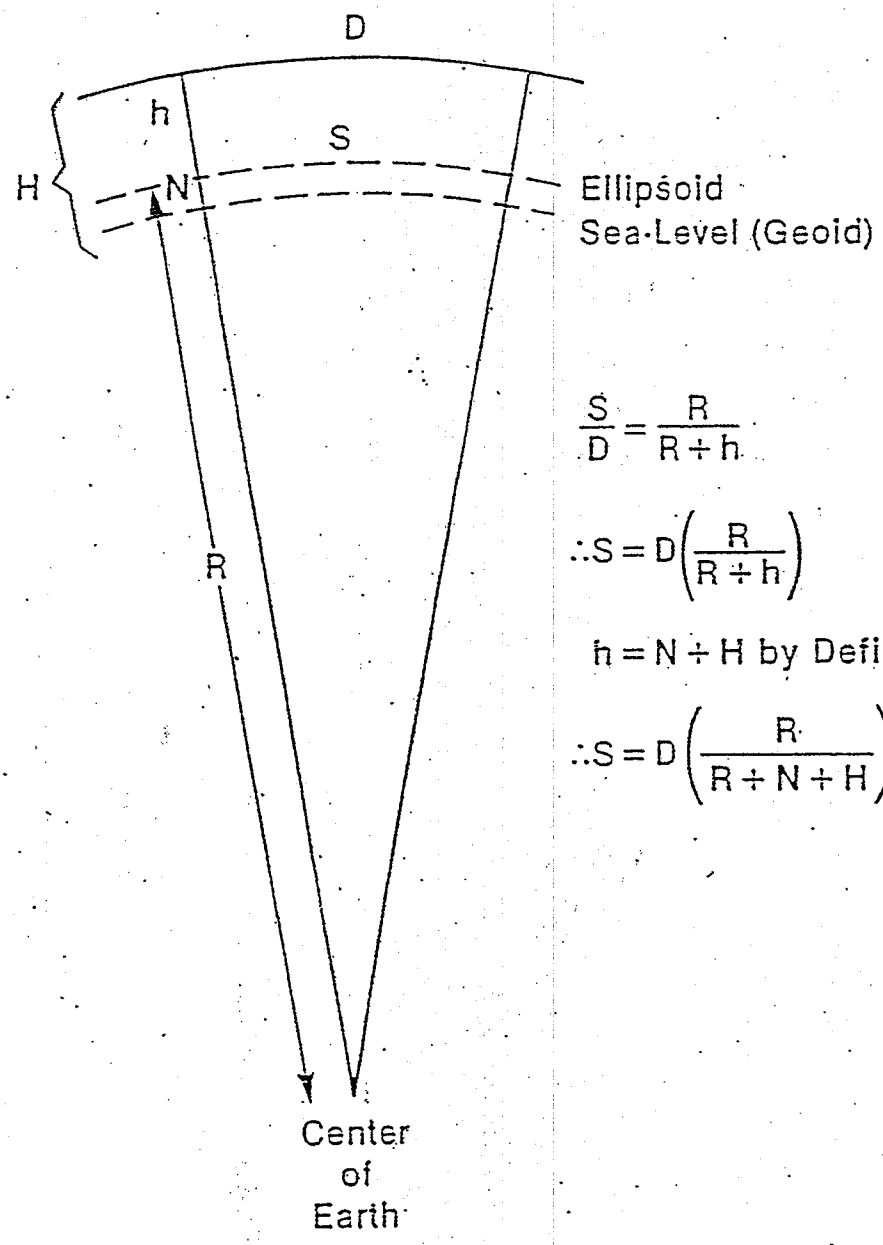
The geoid is defined as that equipotential (level) surface of the earth's gravity field (attraction and rotation) which, on the average, coincides with mean sea-level in the open undisturbed ocean.

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At the Annual TSPS Conference in San Antonio Texas, August 24, 1996



Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

REDUCTION TO THE ELLIPSOID
(ILLUSTRATED WITH A NEGATIVE GEOID HEIGHT)



$$\frac{S}{D} = \frac{R}{R + h}$$

$$\therefore S = D\left(\frac{R}{R + h}\right)$$

$h = N + H$ by Definition

$$\therefore S = D\left(\frac{R}{R + N + H}\right)$$

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

REDUCTION TO ELLIPSOID

$S = D \times [R / (R + h)]$

Where: $h = H + N$

S = Geodetic Distance

D = Horizontal Distance at Mean Elevation of Points

R = Radius of the Earth

H = Mean Elevation of Points

N = Mean Geoid Height of Points

h = Mean Ellipsoidal Height of Points

EXAMPLE: D = 1,500 ft. H = 581 ft. N = - 86 ft.
R = 20,906,000 ft. (Mean for Average Latitude of Contiguous U.S.)

WITHOUT GEOID HEIGHT

h = 581
S = 1,500.000 x (20,906,000/20,906,581)
S = 1,499.958

WITH GEOID HEIGHT

h = 581 + (-86) = 495
S = 1,500.000 x (20,906,000/20,906,495
S = 1499.964

Difference = 0.006 ft (1:250,000)

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At the Annual TSPS Conference in San Antonio Texas, August 24, 1996

82-3850 R. 2/92

COMPUTATION SHEET

BY GM Tuttle DATE 8-2-2001 SUBJECT SRP GRID Comps SHEET NO. 1 OF 1

CHKD. BY GMT DATE 8-2-2001 NAD-83 AZSPGC FACTOR JOB NO.

GIVEN:

COORDINATES IN INT. FEET

N 879,340. 2711' E 810,729. 1312'

S.I. CONVERSION = 3.280839895

N 268,022. 9146 M E 247,110. 2392 M

FORMULA: $K_p = 0.9999 + (E')^2 \times 1.23244 \times 10^{-14}$

$E' = (247{,}110.2392M - 213{,}360M) = 33{,}750.2392M = X'$

SCALE FACTOR $K_p = 0.9999 + (33{,}750M)^2 \times 1.23244 \times 10^{-14} = 0.99991404$

Ellipsoid Factor $R/(R+N+H)$ Diff. N from Sea Level Factor

$[6{,}370{,}000/(6{,}370{,}000M + \cancel{29M} - 525.04M)]$ (-29M)

E.F. $6{,}370{,}000/6370{,}\cancel{554}\,496 = 0.9999\cancel{1303}\,2214$

Combined Factor $= (0.99991404)(0.9999\cancel{1303}\,2214) = 0.9998\cancel{2708}\,3619$ GRID FACTOR

Beware the negative sign of the Geoid Separation!!

ARIZONA STATE PLANE GRID COORDINATES
CENTRAL ZONE --- NAD 1983

	"NORTHINGS"	"EASTINGS"
▵ POLAND	383,535.379	180,943.644
▵ DEWEY	391,518.482	184,406.985

from POLAND to DEWEY: N 23° 27′ 10.2″ E 8,701.992m (GRID)

ELEVATION @ POLAND = 1,526.4m
ELEVATION @ DEWEY = 1,396.8m
AVERAGE ELEVATION = 1,461.6m = (el. @ Poland + el. @ Dewey)/2

REDUCTION TO ELLIPSOID: $S = D \times [R / (R + N + H)]$

S = distance on the ellipsoid
D = horizontal distance on the surface
R = mean radius of the curvature of the ellipsoid = 6,370,000m
H = elevation (height above sea level)
N = Geoid Height = Height above ellipsoid (h) - Elevation (H)

$[R / (R + N + H)]$
$(6{,}370{,}000)/(6{,}370{,}000 + 1461.6 + [-30]) = 0.99977531$

AVERAGE "EASTING" OF THE LINE POLAND to DEWEY = 182,675.3145m
$E' = (213{,}360 - 182{,}675.3145) = 30{,}684.686$m

REDUCTION TO GRID

k = Grid Scale Factor $= 0.9999 + [(E')^2 \times 1.23244 \times 10^{-14}]$
$= 0.9999 + [(30.684.686)^2 \times 1.23244 \times 10^{-14}]$
$= 0.9999 + [941{,}549{,}924.2 \times 1.23244 \times 10^{-14}]$
$= 0.9999 + 0.00001160$
$= 0.99991160$

COMBINED FACTORS = (REDUCTION TO ELLIPSOID) x (REDUCTION TO GRID)
= (0.99977531) X (0.99991160) = (0.99968693)

(GRID DISTANCE / COMBINED FACTORS) = "GROUND DISTANCE"
(8,701.992m / 0.99968693) = 8704.717m = "GROUND DISTANCE"

8,704.717m x 3.28083333 = 28,558.73' (U.S. Survey Foot)
8,704.717m x 3.280839895 = 28,558.78' (International Foot)

ARIZONA STATE PLAN GRID COORDINATES

ARIZONA STATE PLAN GRID COORDINATES BASED ON THE CLARK SPHEROID OF 1866, AND THE NORTH AMERICAN DATUM OF 1927 (NAD '27)

Three (3) zones: EAST, CENTRAL AND WEST.

Grid origins:	**East** =	Longitude	110° 10' 00"
		Latitude	31° 00' 00"
	Central =	Longitude	111° 55' 00"
		Latitude	31° 00' 00"
	West =	Longitude	113° 45' 00"
		Latitude	31° 00' 00"

The Central Meridian is equal to the Grid origin Longitude for each of the three zones, and the "EASTING" = 500,000 feet (U.S. Survey Foot)

(One Meter = 3.2808333333333333333333.....U.S. Survey Feet)
(1893 federal law stated a yard = 3600/3937m exactly = "Survey Foot")

ARIZONA STATE PLAN GRID COORDINATES BASED ON THE GEODETIC REFERENCE SYSTEM OF 1980, AND THE NORTH AMERICAN DATUM OF 1983 (NAD '83)

There are the same three zones as the '27 system with the same three origins as mentioned above.
(**Please note:** That the Latitude and Longitude are the same, **HOWEVER**, *they are not at the same place on the surface of the earth because they are based on two different ellipsoid models of the earth, i.e., the Clark Spheroid of 1866 and the GRS of 1980.)*

The Central Meridian for each zone is equal to the Grid Origin Longitude for each of the three zones.
The "EASTING" for each central meridian is 700,000 feet (International Feet); *this is by Arizona state statute.*
(One Meter = 3.280839895 *International Feet* (to nine decimal places).
(1959 federal law stated a yard = 0.9144m exactly, = "International Foot")

The mathematics necessary to operate in either the State Plan Coordinate System (SPCS) of '27 or of '83 are for *most* applications (except those of extremely high (1st Order, and above) precision) the same.

Note: Arizona is the only state that enacted legislation defining their '83 state plane coordinate system based on the English ("foot") system of measurement. For conversion purposes the "700,000 foot" is considered to be equal to 213,360m --

by: The U.S. Department of Commerce
National Oceanic and Atmospheric Administration
National Ocean Survey
Charting and Geodetic Services
National Geodetic Survey
-and- the National Institute for Standards and Technology (NIST)

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

CONVERTING STATE PLANE COORDINATES TO LATITUDE AND LONGITUDE
(Also Grid Scale Factors and Mapping Angle)

PC SOFTWARE

GPPCGP (NAD 27)

SPCS 83 (NAD 83)

CORPSCON (NAD 27 & NAD 83)

HP-41 SOFTWARE

42-41 Lambert Projection (NAD 83)

54-41 Transverse Mercator Projection (NAD 83)

49-41 Alaska Zone 1 (NAD 83)

43-41 Alaska Zones 2-9 (NAD 83)

3/4-41 Transverse Mercator Projection (NAD 27)

5/6-41 Lambert Projection (NAD 27)

9/10-41 Alaska Zone 1 (NAD 27)

7-41 Alaska Zone 2-9 (PC to GP, NAD 27)

8-41 Alaska Zone 2-9 (GP to PC, NAD 27)

0-41 Guam

PUBLICATIONS

PROJECTION TABLES (Separate NAD 27 & NAD 83)

THE STATE COORDINATE SYSTEMS SP-235 (NAD 27)
(A Manual for Surveyors)

STATE PLANE COORDINATE SYSTEM OF 1983, NOS-5

STATE PLANE COORDINATES BY AUTOMATIC DATA PROCESSING
USC&GS 62-4 (NAD 27)

QUESTIONS TO ASK ABOUT DATUM TRANSFORMATIONS

1. What datum are the existing coordinates on?

2. What datum do I want the new coordinates on?

3. How large a geographical area do I want to convert at one time?

4. How many points are COMMON to both datums?

5. What is the distribution of the COMMON points?

6. How accurate are the existing coordinates?

   0.1 Foot ?
   1.0 Foot ?
   10. Feet ?

7. How accurate do I want the new coordinates?

NOTE: The answers to these questions will determine which transformation method to use.

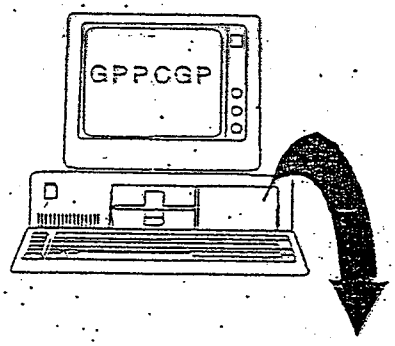
Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey

Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

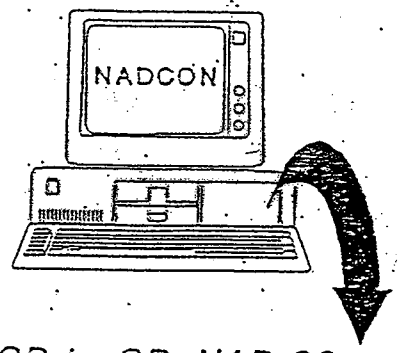
Performing Mathematical NAD 27-83 Transformations

Using NGS Programs

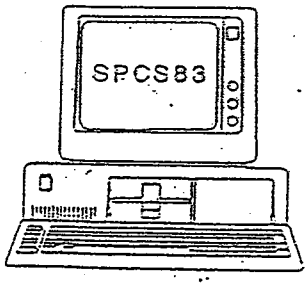
SP to GP, NAD 27



NAD 27 to NAD 83

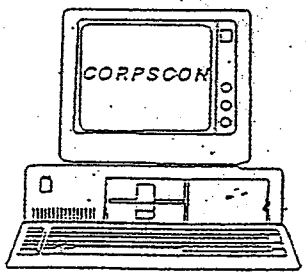


GP to SP, NAD 83



Using CORPSCON

SP-NAD 27 to SP-NAD 83



Source:
North American Datum of 1983 Plane Coordinates and Datum Transformations
Presented by: David R. Doyle, National Geodetic Survey
At: the Annual TSPS Conference in San Antonio Texas, August 24, 1996

COORDINATE COMPARISON

ADJUSTED VS. TRANSFORMED

Point: BUCKNER RESET

GEODETIC COORDINATES

Latitude	Longitude	
32° 47' 45.80090"	96° 40' 56.52223"	- NAD 83
32 47 45.80634	96 40 56.52875	- NADCON
0.00544"	0.00652"	
0.168 M.	0.169 M.	
0.55 FT.	0.55 FT.	

THIS CORRESPONDS TO A POSITIONAL DIFFERENCE OF 0.78 FT

-----Original Message-----
From: Marta Dent - FCDX [mailto:mld@mail.maricopa.gov]
Sent: Tuesday, October 02, 2001 7:26 AM
To: 'gmtuttle@srpnet.com'
Subject: Question

Good morning Greg. Last Saturday you mentioned that it is not always correct to use just the NAD27 coordinates and use the conversion programs to derive the NAD83 coordinates.

I would like to do some research and I was wondering if you could help me with that.

When I got the GIS routines that do the conversion for you, I wanted to test them to see how accurate they were. To do that I downloaded all NGS in the county from the NGS web site, their parameters and I use the programs to do the conversions.
Then I compared the projected coordinates with the published coordinates of NGS.
They match very well.
However it is possible that those points were used to actually create the routines on the first place.
When surveyors deliver data to the district in both datums, I run the conversion program to check how good the program is and the RMSE that I get is usually in the thousands of a foot.

However, they could also be using similar programs to do their conversions.

As you can see the experience that I have checking conversions is very limited and I would like to do more research to get a better handle of the conversion issues.

Do you think this would be something you would like to help me with?

If I misunderstood what you said, please disregard this message.
Thanks,
Marta

From: TUTTLE GREGORY M (GREGG)
Sent: Sunday, October 07, 2001 10:02 AM
To: 'Marta Dent - FCDX'
Subject: RE: Question

MARTA:
HI!
I am just now back from a week of vacation and am attempting to get caught up on my e-mail correspondence. Sorry for the delay in my reply.

Please let me response by quoting certain sections from
"STATE PLANE COORDINATES IN MODERN SURVEYING PRACTICE"
by R.B. Buckner, Ph.D, - self published in August 1993.
Page 106, Section 5-4 TRANSFORMATION BETWEEN NAD27 AND NAD83:

Introduction
General concept of transformations.
"A coordinate transformation changes the geodetic coordinates of a fixed point from one reference ellipsoid to another (i.e., Clarke Spheroid to GRS80 or vice versa). Transformations start from geodetic coordinates. After transforming the point from one ellipsoid to another, with a consequent change in its geodetic coordinates, the plane coordinates of the point may be determined by a coordinate conversion. Although there is a distinction between these two processes, surveyors often think of changing the SPC27 plane coordinates of a point to SPC83 plane coordinates as a transformation."

Transformation Method - Introduction.
"As with other surfaces, coordinate transformations can be made mathematically between different ellipsoids without any errors other than the usual computational (round-off) errors. Uncertainties much larger than round-off errors occur in transformations between NAD27 and NAD83 because of "distortions" in the readjustment (non-uniformity caused by the variations in weighting the data, relative accuracies of the data used, etc.). Except as the stations used in the readjustment, any transformation done without actually computing positions from survey data has uncertainties larger than allowed on most surveying projects. The best methods available cannot be executed without uncertainties of 10cm. or more for points located anywhere between control stations used in the readjustment."

"...the best way to arrive at SPC83 coordinates is to use the original field data (distances, angles, azimuths, etc.) that produced the SPC27 plane coordinates, and (re)calculate the SPC83 coordinates. The determination of the distances factors and convergence angles on the SPC83 system would modify distances and directions slightly. With this approach, coordinate

conversions may be required, if SPC83 plane coordinates of the reference station are not known. Transformation errors caused by distortions in the system are avoided when this method is used. For survey accuracy, this may be the best approach. For mapping and other purposes requiring less accuracy than land or
control surveys, some of the other transformation systems may also be useful."

The point that the good doctor and I are attempting to make is that if you have, initially, NAD-83 and then convert "backwards" to NAD-27 - you should be within +/- 10cm (or greater) to NAD-27 coordinates on points whom have been previously recomputed (transformed / converted) from NAD-27 "forward" into NAD-83.

The problems will arise when you have NAD-27 coordinates where the original survey data (i.e., angles and distances) are not available to re-compute the coordinates into NAD-83, but a 'simple' transformation conversion routine is followed without the best (direct re-computations) method, such as when a "modeling" transformation
system is used instead. Then, you can anticipate even larger 'distortional' errors.

This is a good example of knowing enough about the metadata of the original coordinates (NAD-27).
You used NGS coordinate pairs in your example of conversion analysis. Obviously NGS had the original survey metadata for their own points (NAD-27) and recomputed the original field measurements from the original surveys on NAD-27 to obtain a tight NAD-83 transformation/conversion. No problems there.
My guess about your example of dual data (NAD-27 & -83) received from private surveyors is that they transformed data originating in NAD-83 to -27 using the same program you use to verify their results.

My point is about using 'historic' NAD-27 data-sets that have no original field measurement meta-data available, and then plugging those coordinates into a transformation/conversion routine, and then verifying the NAD-83 results by references (GPS and/or conventional 'ties') to known NAD-83 coordinates which originated directly from NAD-83, (or have been correctly transformed/converted with use of original field measurement data be re-computing into -83).
This is where 'distortions' in the range exceeding 10 ft. and greater can be (hopefully) discovered.
We here at SRP have found +/- 2.5 ft. and greater in such examples examined and analyzed.
I hope this feedback is useful and insightful.
Thanks, Gregg Tuttle, @ SRP LAND-Surveys.

ARTICLE 3. ARIZONA COORDINATE SYSTEM, 1983

Article 3, consisting of §§ 33–131 to 33–138 was added by Laws 1985, Ch. 197, § 11, effective January 1, 1987.

Historical and Statutory Notes

Laws 1985, Ch. 197, § 13 provides:
"Sec. 13. Effective date
"Section 11 of this act is effective from and after December 31, 1986."

§ 33–131. Arizona coordinate system, 1983; zones; composition

A. The Arizona coordinate system, 1983, is the system of plane coordinates which has been established by the national geodetic survey for defining and stating the positions or locations of points on the surface of the earth in this state.

B. The Arizona coordinate system, 1983, contains three zones as follows:

1. The west zone, composed of La Paz, Mohave and Yuma counties.

2. The central zone, composed of Coconino, Maricopa, Pima, Pinal, Santa Cruz and Yavapai counties.

3. The east zone, composed of Apache, Cochise, Gila, Graham, Greenlee and Navajo counties.

C. In any land description in which the Arizona coordinate system, 1983, is utilized the system shall be designated "Arizona coordinate system, 1983, _____ zone", with the name of the appropriate zone inserted.

Added by Laws 1985, Ch. 197, § 11, eff. Jan. 1, 1987.

Historical and Statutory Notes

Laws 1985, Ch. 197, § 13 provides:

"Section 11 of this act is effective from and after December 31, 1986."

§ 33–132. Coordinates of system; zone definitions

A. The plane coordinates of a point on the earth's surface, to be used in the position or location of such point in the appropriate zone of the system, shall consist of two distances, expressed in feet and decimals of a foot (foot value 0.3048 meter exact). One of these distances, to be known as the "X-coordinate", shall give the position in an east-and-west direction, and the other, to be known as the "Y-coordinate", shall give the position in a north-and-south direction. These coordinates shall depend on and conform to the coordinates on the Arizona coordinate system, 1983, of the horizontal control stations of the national geodetic survey in this state, as these coordinates have been determined by the survey.

S.I. UNITS →

B. For the purpose of more precisely defining the Arizona coordinate system, the following definitions of the national geodetic survey are adopted:

1. The Arizona coordinate system, 1983, west zone, is a transverse mercator projection of the North American datum, 1983, having a central meridian

113° 45′ 00″ west of Greenwich, on which meridian the scale is set one part in fifteen thousand too small. The origin of the coordinates is at the intersection of the meridian 113° 45′ 00″ west of Greenwich and the parallel of 31° 00′ 00″ north latitude. This origin is given the coordinates of "X" equals seven hundred thousand feet and "Y" equals zero feet.

2. The Arizona coordinate system, 1983, central zone, is a transverse mercator projection of the North American datum, 1983, having a central meridian 111° 55′ 00″ west of Greenwich, on which meridian the scale is set at one part in ten thousand too small. The origin of the coordinates is at the intersection of the meridian 111° 55′ 00″ west of Greenwich and the parallel of 31° 00′ 00″ north latitude. This origin is given the coordinates of "X" equals seven hundred thousand feet and "Y" equals zero feet.

3. The Arizona coordinate system, 1983, east zone, is a transverse mercator projection of the North American datum, 1983, having a central meridian 110° 10′ 00″ west of Greenwich, on which meridian the scale is set at one part in ten thousand too small. The origin of coordinates is at the intersection of the meridian 110° 10′ 00″ west of Greenwich and the parallel of 31° 00′ 00″ north latitude. The origin is given the coordinates of "X" equals seven hundred thousand feet and "Y" equals zero feet.

Added by Laws 1985, Ch. 197, § 11, eff. Jan. 1, 1987.

Historical and Statutory Notes

For effective date provision of Laws 1985, Ch. 197, see Historical Note following § 33–131.

Library References

Boundaries ⇐8.
WESTLAW Topic No. 59.
C.J.S. Boundaries § 14 et seq.

§ 33–133. Ground markings of system; accuracy specifications; horizontal control stations

A. The position of the Arizona coordinate system shall be marked on the ground by horizontal control stations which have been established in conformity with standards adopted by the federal geodetic control committee for first order, second order class I or second order class II surveys or equivalent standards adopted by successors, at the time the surveys were made and computed on the North American datum, 1983.

B. A horizontal control station normally consists of, if practicable, a group of bronze or brass discs imbedded in concrete posts nearly flush with the ground surface or cemented into holes drilled into rock outcrops or ledges in such a configuration that the station is referenced by a subsurface mark in a precise vertical register with the surface mark, two reference marks, similar to the surface mark accurately located by azimuth and horizontal distance in respect to the horizontal control station and not more than one hundred fifty feet distant and an azimuth mark which may be similar to the horizontal

control station not less than one thousand feet distant, or optionally, an object not less than three thousand feet distant such as a church spire, water tank, radio or television transmitting antenna, by which azimuth mark subsequent surveys may be accurately oriented.

C. Horizontal control stations shall be established, if practicable, in proximity to road intersections, hill or mountain tops and similar locations as an aid in the field searches for the horizontal control stations.

Added by Laws 1985, Ch. 197, § 11, eff. Jan. 1, 1987.

Historical and Statutory Notes

For effective date provision of Laws 1985, Ch. 197, see Historical Note following § 33–131.

Library References

Boundaries ⇐8.
WESTLAW Topic No. 59.
C.J.S. Boundaries § 14 et seq.

§ 33–134. Tract located in more than one zone; description

If a tract of land to be defined by a single description extends from one into another of the coordinate zones established by this article, the position of all points on its boundaries may be referred to as either of the two zones, the zone which is used being specifically named in the description.

Added by Laws 1985, Ch. 197, § 11, eff. Jan. 1, 1987.

Historical and Statutory Notes

For effective date provision of Laws 1985, Ch. 197, see Historical Note following § 33–131.

Library References

Boundaries ⇐11.
WESTLAW Topic No. 59.
C.J.S. Boundaries § 17 et seq.

§ 33–135. Reliance of purchaser or mortgagee not required

Nothing contained in this article requires any purchaser or mortgagee to rely on any land description, any part of which depends exclusively on the Arizona coordinate system, 1983.

Added by Laws 1985, Ch. 197, § 11, eff. Jan. 1, 1987.

Historical and Statutory Notes

For effective date provision of Laws 1985, Ch. 197, see Historical Note following § 33–131.

Library References

Boundaries ⇐54(4).
WESTLAW Topic No. 59.
C.J.S. Boundaries § 89 et seq.

§ 33–136. Public lands survey descriptions; conflicts; control

If coordinates based on the Arizona coordinate system, 1983, are used to describe a tract of land which in the same document is also described by reference to a subdivision, line or corner of the United States public land surveys, the description by coordinates shall be construed as supplemental to the basic description of the subdivision, line or corner contained in the official field notes and plat filed of record, and in the event of a conflict the description by reference to the subdivision, line or corner of the United States public land surveys prevails over the description by coordinates.
Added by Laws 1985, Ch. 197, § 11, eff. Jan. 1, 1987.

Historical and Statutory Notes

For effective date provision of Laws 1985, Ch. 197, see Historical Note following § 33–131.

§ 33–137. Recording, filing, publishing extensions and densifications of the ground marking system

A. Extensions and densifications of the ground marking system of the Arizona coordinate system, 1983, shall be executed in conformity with the standards and specifications of the federal geodetic control committee for first order, second order class I and second order class II surveys and computed on the North American datum, 1983.

B. The results of these surveys shall be published by a competent department of the federal government, such as the national geodetic survey, or a surveyor qualified to practice in this state, in which case the survey results shall bear a certification to the effect that the specifications of the federal geodetic control committee have been followed.

C. Extensions and densifications of the ground marking system by first order, second order class I or second order class II methods only shall be used. Such extensions and densifications shall be integrated with previously established horizontal control stations of equal or higher order to form a matrix or network no part of which shall have positional errors exceeding those specified for class II second order. The spacing intervals shall not exceed three miles.

D. The results of these surveys are public domain and shall be duly recorded in the office of the county recorder of the county where the horizontal control station is situated.
Added by Laws 1985, Ch. 197, § 11, eff. Jan. 1, 1987.

Historical and Statutory Notes

For effective date provision of Laws 1985, Ch. 197, see Historical Note following § 33-131.

§ 33-138. Recording prerequisite

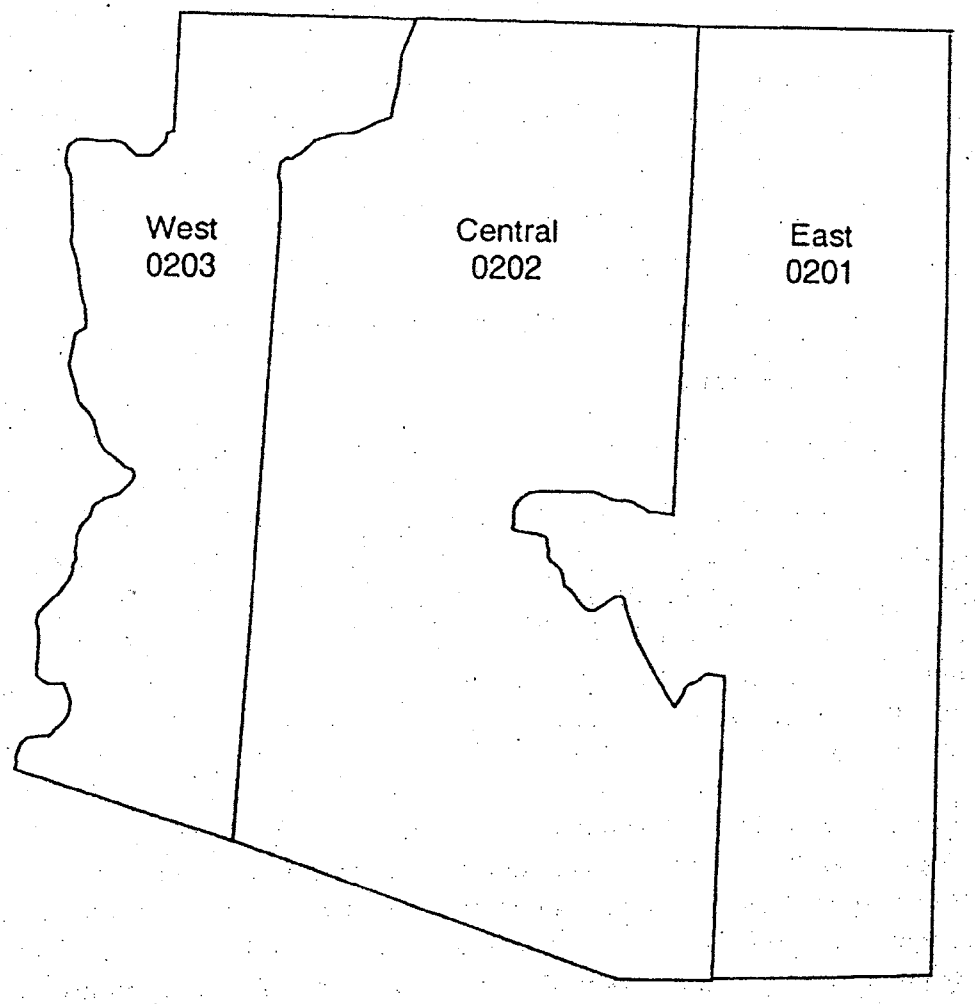
Coordinates based on the Arizona coordinate system, 1983, shall not be presented to be recorded in any public land records unless the recording document also contains the descriptions of not less than two horizontal control stations of first order, second order class I or second order class II positional accuracy which shall not exceed six miles from the nearest point or line of the land survey.

Added by Laws 1985, Ch. 197, § 11, eff. Jan. 1, 1987.

Historical and Statutory Notes

For effective date provision of Laws 1985, Ch. 197, see Historical Note following § 33-131.

The Arizona Plane Coordinate System
1983



R.L.Lawrence Jr.
Pima County Department of Transportation
Survey Section
June1990

THE ARIZONA PLANE COORDINATE SYSTEM, 1983

A Brief Background

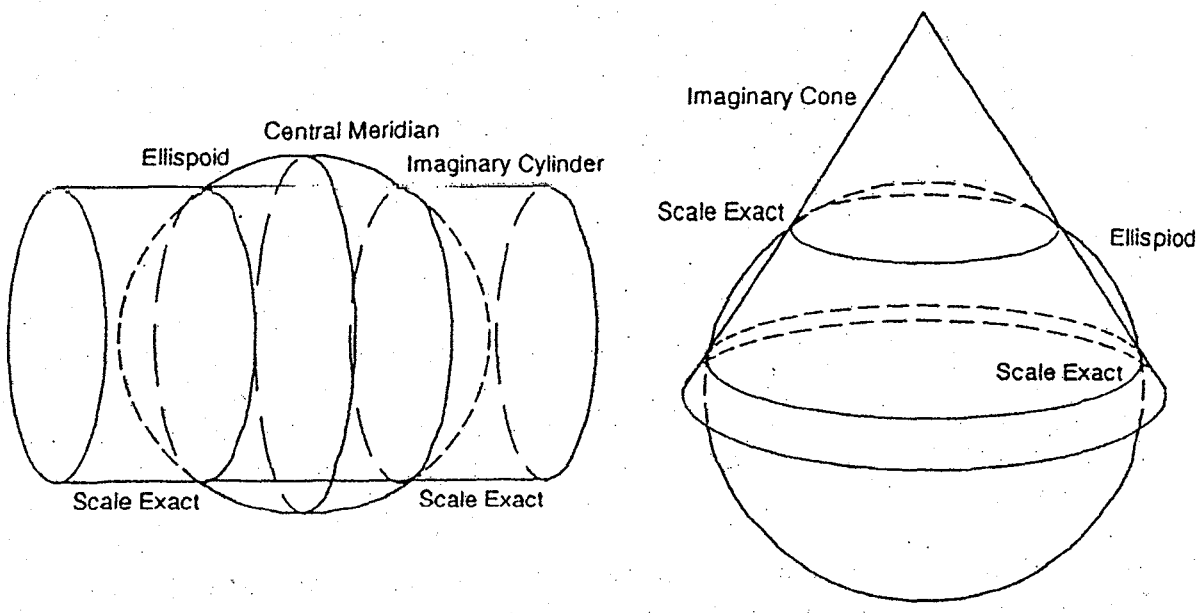
The State Plane Coordinate System (SPCS) was established in 1933 by the United States Coast and Geodetic Survey. This came about after a request from the North Carolina Highway Department for a plane coordinate system that would cover the entire state and involve mathematics no more complicated than those used in plane surveying. Within a few years similar systems were devised for all of the states. The SPCS provide plane coordinates for each of the horizontal control points in the National Geodetic Reference System. The criteria used in the design of the original state plane coordinate systems were:

- Use of a conformal map projection.
- Restrict the maximum scale distortion to less than one part in 10,000
- Cover an entire state with as few zones of a projection as possible.
- Define the boundaries of the zones as an aggregation of the counties.

A map projection is a mathematical function that defines the relationship between the coordinates of points on one curved surface to the coordinates of points on another curved surface. One of the mathematical surfaces traditionally employed by surveyors is the local tangent plane with few, if any, reductions made to the field data. The plane coordinates resulting from computations of this nature are sufficient for independent projects over a small extent. At the other extreme, field observations are reduced to an ellipsoid of revolution and subsequent computations are preformed using geodetic coordinates and ellipsoidal geometry.

Two conformal map projections were chosen as the reference planes for the vast majority of the State Plane Coordinate Systems. The Lambert Conformal Conic Projection was chosen for states longer in the east-west direction and the Transverse Mercator Projection was chosen for states longer in the north-south direction. In a conformal projection the scale varies over the surface of the map but is fixed at any point and the computation of its value is independent of azimuth. As a corollary the angle between two lines at a point on the surface will appear as the same angle on the map.

In the Transverse Mercator Projection, the one chosen for Arizona's SPCS, the first surface is the ellipsoid that approximates the surface of the earth and the second is a transverse secant cylinder (See figure 1). It is on the plane developed from this cylinder to which survey observations are reduced and where the plane coordinates are computed. The radius of this cylinder is such that the scale on the surface of the cylinder is one part in ten-thousand smaller along its defining central meridian than on the surface of the ellipsoid. The secant cylinder intersects the ellipsoid at two small ellipses that are equidistant from the central meridian. The scale along these ellipses is exact and increases with the distance from the central meridian. The specification of the latitude - longitude and the linear grid values of the grid origin along with the longitude and scale factor of the central meridian are the "defining constants" for a single zone of a projection.



Tranverse Mercator Projection

Lambert Conformal Conic Projection

Figure 1

The North American Datum of 1983

The State Plane Coordinate Systems of 1983 became necessary after the readjustment of the North American Datum was completed in 1983 (NAD 83). The project, which was begun in 1975, was undertaken because the quality of the horizontal control data was no longer meeting the requirements of the surveying and engineering communities. NAD 83 supplies the following improvements:

- Simultaneous adjustment of approximately 1.8 million geodetic observations, thus eliminating error propagation that occurs when projects must be assembled from smaller interdependent projects.
- The inclusion of data from the precise transcontinental traverse, satellite triangulation, Doppler positions, electronically measured baselines, and baselines established by very long baseline interferometry contributed to improving the internal consistency of the network.
- A new figure of the Earth, the Geodetic Reference System of 1980 (GRS 80), was chosen because it is a better approximation of the surface of the Earth than Clarke's Spheroid of 1866 which was used for NAD 27.
- The origin of the datum was moved from Meades Ranch in Kansas to the Earth's center of mass. This is for compatibility with satellite systems.

Not only will the published geodetic position of each control point change, but the State Plane Coordinates of each point will also change for the following reasons:

- The State Plane Coordinates are mathematically derived from the geodetic coordinates.
- The new figure of the Earth, the GRS 80 ellipsoid, has different ellipsoidal parameters than Clarke's spheroid and these parameters are used in the "mapping equations" to compute plane coordinates.

- The defining constants for several zones have been redefined by the states.
- The numeric value of the grid origin of each zone has been changed significantly to make the plane coordinates appear clearly different.
- The State Plane Coordinates published by NGS on NAD 83 will be in metric units.
- SPCS 83 is based on the Gauss-Kruger form of the transverse Mercator projection. SPCS 27 used the Gauss-Schreiber form of the equations.

SPCS 27 versus SPCS 83

In the mid-1970s NGS conducted a study to consider several alternatives to SPCS 83. Ultimately, it was decided to retain the basic SPCS 27 design. It has been in use for over 40 years, accepted by legislative action in 37 states, most surveyors and engineers are familiar with the definitions and procedures involved, and except for purely academic reason the philosophy behind SPCS 27 was fundamentally sound.

To meet the criteria outlined above, both the Arizona SPCS 27 and SPCS 83 use the Transverse Mercator Projection with the state divided into three zones, West, Center, and East. Each zone is defined by a central meridian, a central meridian scale factor, and a grid origin.

For SPCS 27 these definitions are as follows:

			Grid Origin			
	Central Meridian	Scale Factor	Longitude	Latitude	X	Y
East	110° 10' 00"	1:10,000	110° 10' 00"	31° 00' 00"	500,000	0
Central	111° 55' 00"	1:10,000	111° 55' 00"	31° 00' 00"	500,000	0
West	113° 45' 00"	1:15,000	113° 45' 00"	31° 00' 00"	500,000	0

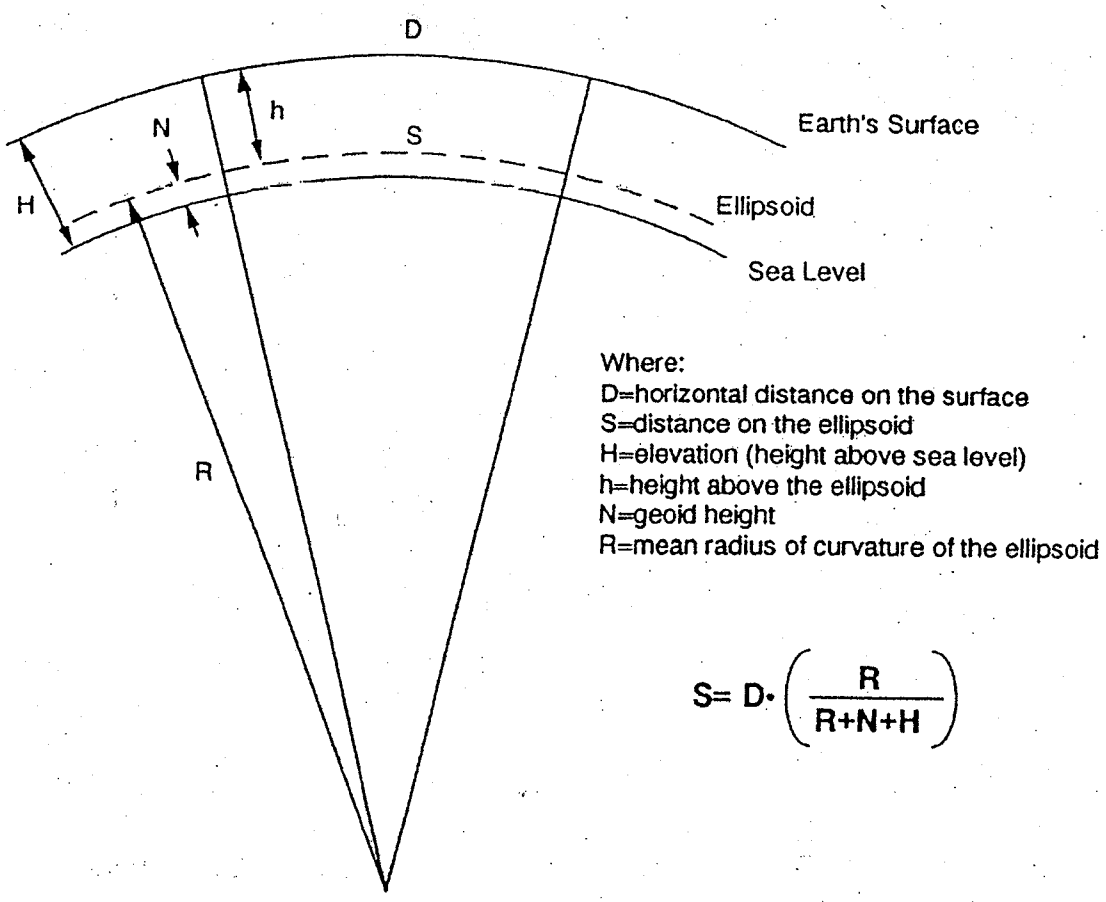
NOTE: the X and Y coordinates for SPCS 27 are in survey feet

For SPCS 83 these definition as are as follows:

			Grid Origin			
	Central Meridian	Scale Factor	Longitude	Latitude	North	East
East (0201)	110° 10' 00"	1:10,000	110° 10' 00"	31° 00' 00"	0	700,000
Central (0202)	111° 55' 00"	1:10,000	111° 55' 00"	31° 00' 00"	0	700,000
West (0203)	113° 45' 00"	1:15,000	113° 45' 00"	31° 00' 00"	0	700,000

NOTE: The Northings and Eastings are by statute expressed in International Feet

As can be seen the only difference in the parameters defining the zones is in the false eastings assigned to the central meridians. Special note should be made of the fact that the unit of linear measure specified for Arizona's SPCS 83 is the International Foot. The procedures for using either system (or any SPCS for that matter) are essentially the same. Unfortunately, it is not possible to easily translate coordinates from one system to those of the other. This is due, for the most part, to the adjustment of the geodetic positions of the horizontal control points in the NGRS. It would be very rare for the adjustment process to have resulted in the same correction in geodetic position being applied to all of the points used to control a survey. The only foolproof way to convert from SPCS 27 to SPCS 83 is to go back to the original field observations and recompute.



$$S = D \cdot \left(\frac{R}{R+N+H} \right)$$

Figure 2

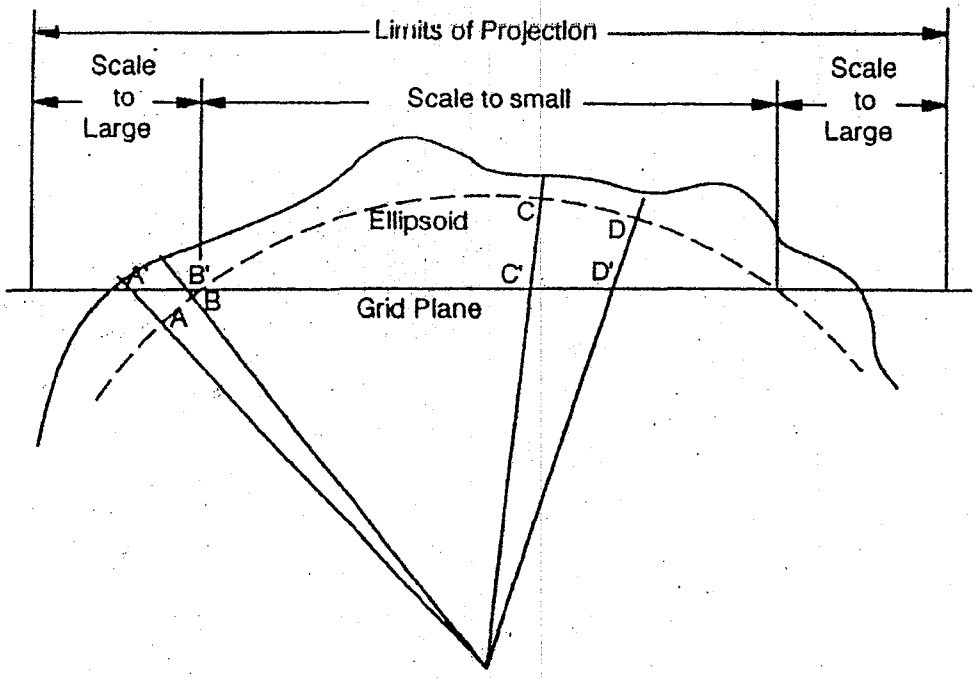
Reduction to the Ellipsoid

Knowledge of the radii of curvature on ellipsoids is basic to the reduction of distances measured on the surface of the earth to any other ellipsoidal surface. Ellipsoid radii are a function of both latitude and azimuth. Fortunately, for most surveys any reasonable mean radius of curvature will produce satisfactory results. Distance reductions are often preformed on a sphere with a radius equal to the mean radius of curvature for the contiguous states. For NAD 27 the radius used was 6,372,000 m or 20,906,000 ft. For NAD 83 this approximate radius serves equally well. For Arizona, a slightly smaller radius of 6,370,000 m (computed for latitude 34° N at an azimuth of 45°) is a slightly better approximation. For distances less than 10,000 m either radius and any of the several slope reduction procedures that take into account the earth's curvature and atmospheric refraction will produce essentially the same horizontal distance. If a more rigorous reduction is required the procedure can be found in NOAA Technical Memorandum NOS NGS-10 Use of Calibration Baselines appendix I.

The first step in using the SPCS is to reduce observed distances to the ellipsoid. In the past, because Clarke's spheroid of 1866 closely approximates the geoid in North America, distances were usually reduced to sea level. With the adoption of the worldwide datum of NAD 83 the difference between sea level and the ellipsoid becomes significant. This difference is referred to as the Geoid Height (N) and is defined as:

Geoid Height (N)=Height above the Ellipsoid (h)-Height above Sea Level (H)

A geoid height of -30 m will systematically affect the reduction of all horizontal distances by approximately a -5 ppm. Figure 2 illustrates the reduction of a horizontal distance measured on the surface to the ellipsoid.



The Grid Distance A' to B' is *Larger* than the Geodetic Distance A to B.

The Grid Distance C' to D' is *Smaller* than the Geodetic Distance C to D.

Figure 3

Reduction to the Grid

After the distances are reduced to the GRS 80 ellipsoid they must then be transformed to the grid. This is accomplished applying the grid scale factor. The grid scale factor represents the distortion in the length of a line on the ellipsoid caused by its projection on the grid plane. As was mentioned earlier, one of the properties of a conformal map projection is that the grid scale of a point is constant regardless of azimuth. The grid scale factor, symbolize by the letter "k", is the ratio of an incremental linear unit on the grid to its corresponding incremental unit on the ellipsoid. In the Arizona Plane Coordinate System the scale factor is defined as k_0=0.9999 at the central meridian of the central and east zones and 0.9999333... for the west zone and increases with the distance from the central meridian. (see Figure 3). The mapping equations for the Transverse Mercator Projection show that the grid scale factor is actually a function of both latitude and the distance from the central meridian. However, the following formula yields an approximate value that will meet most requirements when used within the limits of a zone.

For the central and east zones :

$$k_p = 0.9999 + (E')^2 \times 1.23244 \times 10^{-14}$$

For the west zone:

$$k_p = 0.9999 + (E')^2 \times 1.23236 \times 10^{-14}$$

where E'= the Easting of the point in meters minus 213,360 meters.

Because the scale factor varies with the distance from the central meridian, each point on the line will have a different scale factor. There are several solutions to the problem of determining the grid scale factor of a line. The solution depends on the required accuracy of the reduction, the lengths of the lines, and to a lesser extant the size of the project and its location in the zone. To determine the appropriate method for computing the line scale factors for the lines in a project, compute the point scale factors at the worst case situations — the ends of the longest lines and at the east-west extremes of the project — and compare the magnitude of the corrections to the required accuracy of the project. The simplest solution is to use a single factor for an entire project. If this solution is applicable it may also be possible to combine the grid scale factor with a single elevation factor thus allowing the reduction of surface distances to the grid in a single step.

When it is determined that a line grid scale factor for each line is required there are three commonly used methods. Each approach involves computing one or more point grid scale factor along the line to be reduced. Approximately the same results can be obtained using the mid-point scale factor (k_m) or the mean of the end point scale factors ($k_1 + k_2$)÷2. The third method requires computing the point scale factors for the end points and the mid-point of the line and is computed as follows:

$$k_{12}=(k_1+4k_m+k_2)\div 6$$

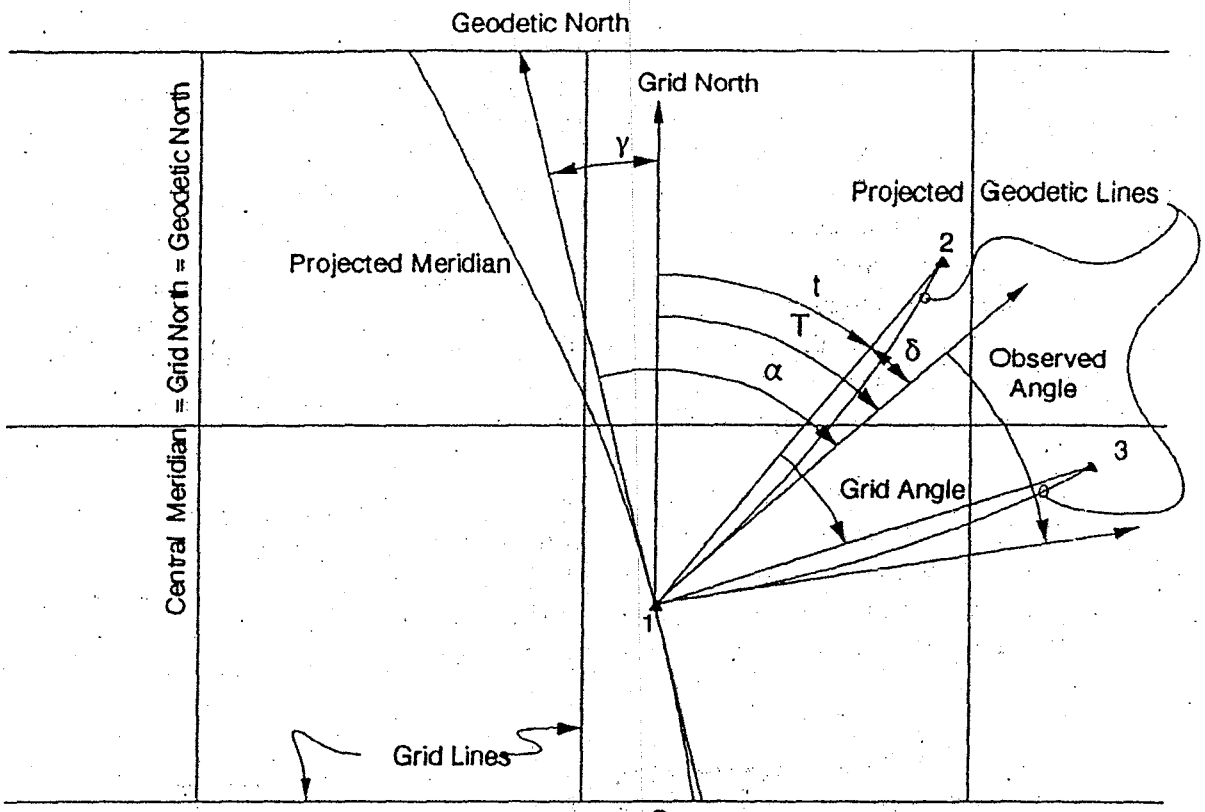
Convergence and the Arc to Chord (t-T) Corrections

Due to the fact that the state plane grid is constructed from orthographic lines on a plane and geodetic meridians are converging lines on the globe, geodetic north and grid north do not coincide anywhere except along a zone's central meridian. This difference between geodetic north on the grid and grid north is the "convergence angle", sometimes referred to as the "mapping angle". The convergence angle is symbolized as "γ". Note the change from SPCS 27 where convergence was symbolized "$\Delta\alpha$". Put another way, the convergence angle is the difference between the geodetic azimuth of a line "α" and the projection of that azimuth on the grid "T". Hence by definition:

$$\gamma=\alpha-T$$

Recalling that one of the properties of a conformal map projection is that angles on the surface are represented by the same angles on the grid. This is true however, only for lines of infinitesimal length. From a purely academic perspective, grid azimuths should be used with grid angles, while geodetic azimuth should be used with observed angles. The projected geodetic azimuth (T) is <u>not</u> the grid azimuth. The grid azimuth is the azimuth computed from two grid coordinate pairs and is symbolized "t". The relationship between the projected geodetic azimuth (T) and the grid azimuth (t) is a subtle, but computable quantity symbolized as "δ" or as "(t-T)". As can be seen in figure 4, the term (t-T) is often referred to as the "arc to chord" correction. "t-T" is the angular correction to the line of sight between two points and can be viewed as the difference in "pointing" observed on a curved surface like the earth surface or the ellipsoid and the pointing on the grid. To perform survey computations on the grid the observed angles need to be corrected for the arc-to-chord correction. The relationship among grid azimuth (t), geodetic azimuth (α), convergence (γ), and the arc-to-chord correction (δ) is shown in figure 4 and at any point is

$$t=\alpha-\gamma+\delta \quad (\text{where } \delta=(t-T))$$



$$t=\alpha-\gamma+\delta$$

where:

α=Geodetic Azimuth Reckoned From North
T=Projected Geodetic Azimuth
t=Grid Azimuth Reckoned From North
γ=Convergence Angle=Mapping Angle
δ=(t-T)=SecondTerm Correction=Arc to Chord Correction

Figure 4

In order to apply the arc-to-chord correction to an observed angle the correction needs to be computed for both the backsight and foresight. As can be seen in figure 4, the correction to the backsight will increase the observed angle while the correction to the foresight will decrease it. The arc-to-chord correction is technically a function of the latitude of the point, the difference in longitude of the point and the central meridian, the azimuth of the line, and the length of the line. The following equation will compute an approximate value for δ_{12}, the d from point (N_1,E_1) to point (N_2,E_2), that will meet the requirements of most surveys.

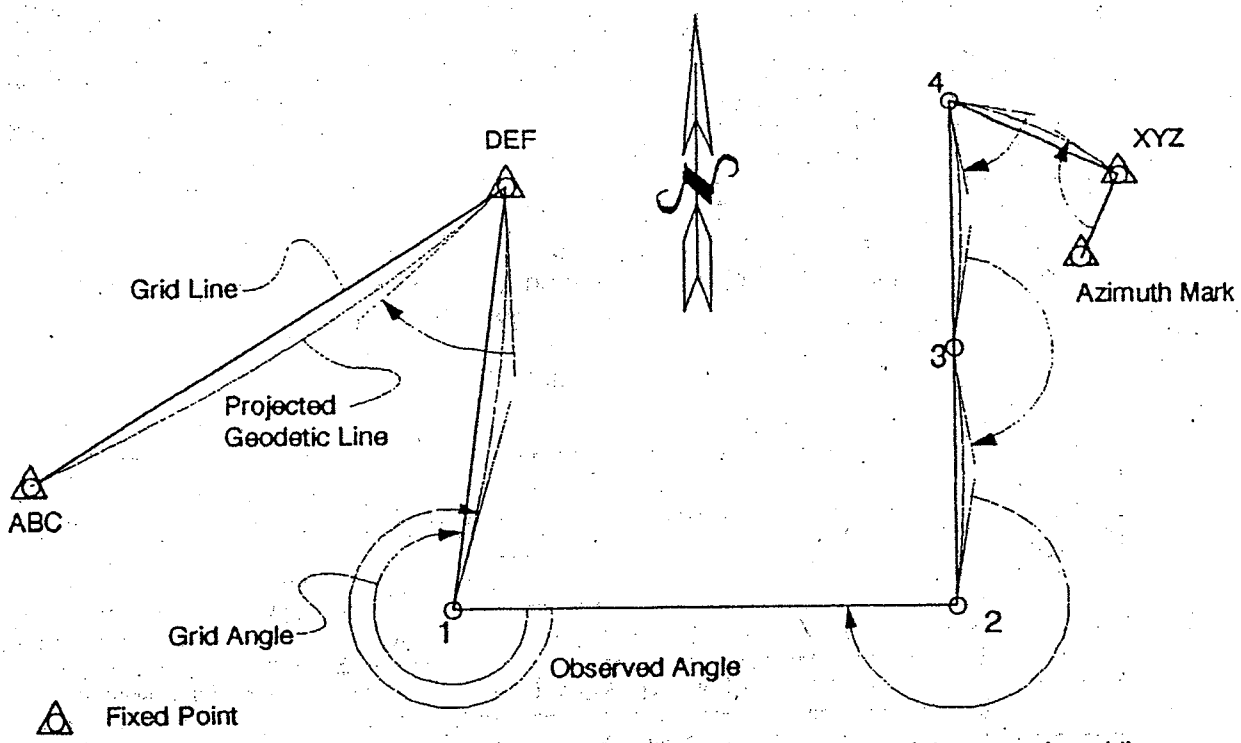
$$\delta_{12}= - 25.4 \bullet \Delta N \bullet E_3 \bullet 10^{-10} \text{ seconds}$$

where the coordinates are in meters and

$$\Delta N=N_2-N_1$$
$$E'_1=E_1-213360$$
$$E'_2=E_2-213360$$
$$E_3=(2E'_1+E'_2)\div 3$$

An Example Traverse

Figure 5 is a sketch of an example traverse. (This is an example and is not intended to demonstrate good traverse practices.) The traverse was run to establish coordinates on points 1,2,3, and 4. The starting azimuth was derived from the published data for station XYZ. The closing azimuth was computed from the coordinates of stations DEF and ABC. In the sketch, the heavy lines represent the final grid lines. The fine curved lines depict the projected geodetic lines, and the short lines the observed line of sight. With today's calculators and desktop computers it is almost as easy to calculate elevation and grid scale factors for each line as it is to calculate mean values to be used in the reduction of field data. That is the approach that will be illustrated here. Note that in this example the linear units are meters.



NOTE: The projected geodetic lines are concave toward the central meridian.

Figure 5

The following steps are required to compute any traverse:

1. Obtain the starting and closing coordinates and azimuths.
2. Compute the elevation factors for the traverse legs. Barring better information a mean geoid height (N) from the published data will generally be adequate for computing the elevation factors.
3. Using preliminary azimuths calculated from the observed angles and ellipsoid distances, compute approximate coordinates for each point.
4. From the approximate coordinates compute the (t-T) corrections for the observed angles and the grid scale factors of the lines.
5. Apply the scale factors to the ellipsoid distances to compute the grid distances.

Station	Obs. Angle Rt.	Preliminary Azimuths	Horizontal Distance H	Mean Elev of the Line (H)	Ellipsoid Distance S	Preliminary Coordinates North	Preliminary Coordinates East	Station
Az Mark								Az Mark
		*025°09'32.4"						
XYZ	088°11'50.1"					*147,271.976	306,474.391	XYZ
		293°21'22.5"	3447.119	827.083	3446.688			
4	066°03'50.1"					148,638.405	303,310.133	4
		179°25'12.6"	4788.087	758.346	4787.540			
3	179°54'26.6"					143,851.110	303,358.582	3
		179°19'39.2"	4832.376	720.770	4831.852			
2	270°28'07.6"					139,019.591	303,415.289	2
		269°47'46.8"	9596.632	755.780	9595.539			
1	277°31'09.5"					138,985.482	293,819.811	1
		007°18'56.3"	8163.292	726.463	8162.400			
DEF	050°34'43.3"					*147,081.777	294,859.224	DEF
		†237°53'39.6"						
ABC						*141,427.408	285,846.563	ABC

* These values from step 1 are fixed

† Note the difference between this value and the one from step 1 is the total angular error and includes errors in observation and the (t–T) corections.

$$S = D \times R \div (R + H + N) = D \times 6{,}370{,}000 \div (6{,}370{,}000 + H - 30)$$

Table 1

Station	E'	ΔN Ahead	ΔN Back	E_3 Ahead	E_3 Back	(t-T)" Ahead	(t-T)" Back	k_p	k_{12}	Grid Distance	Station
AzMark											AzMark
						0.00	0.00				
XYZ	93,114							1.0000069			XYZ
		1,366	-1,366	92,059	91,005	-0.32	0.32		1.0000033	3,446.699	
4	89,950							0.9999997			4
		-4,787	4,787	89,983	89,966	1.09	-1.09		0.9999998	4,787.539	
3	89,999							0.9999998			3
		-4,831	4,831	90,036	90,018	1.10	-1.10		0.9999999	4,831.852	
2	90,055							1.0000000			2
		-35	35	83,658	86,857	0.00	0.00		0.9999899	9,595.442	
1	80,460							0.9999798			1
		8,097	-8,097	81,153	80,806	-1.67	1.66		0.9999808	8,162.243	
DEF	81,499							0.9999819			DEF
		-5,655	5,655	75,491	78,495	1.08	-1.13				
ABC	72,487										ABC

Note: The ahead and back in the table refer to the direction of the line between the points. For example, in the columns labeled ΔN Ahead refers to the line from XYZ to 4 while Back refers the line from 4 to XYZ.

$$E_3 = (2E'_1 + E'_2) \div 3 \qquad k_p = 0.9999 + E'^2 \times 1.23244 \times 10^{-14}$$

$$(t-T) = -25.4 \times \Delta N \times E_3 \times 10^{-10} \qquad k_{12} = (k_1 + k_2) \div 2$$

Table 2

Station / Az Mark	Obs. ANgle Rt.	(t - T)" Backsight Correction	(t - T)" Foresight Correction	Angle Correction	Grid Angle	Intermediate Azimuths	Correction for Closure	Corrected Grid Azimuths	Station / Az Mark
						*025°09'32.4"		*025°09'32.4"	
XYZ	088°11'50.1"	0.00	-0.32	-0.32	088°11'49.8"		† +01.2"		XYZ
						293°21'22.2"		293°21'23.4"	
4	066°03'50.1"	0.32	1.09	0.77	066°03'50.9"		† +01.2"		4
						179°25'13.1"		179°25'15.5"	
3	179°54'26.6"	-1.09	1.10	2.19	179°54'28.8"		+01.1"		3
						179°19'41.9"		179°19'45.5"	
2	270°28'07.6"	-1.10	0.00	1.10	270°28'08.7"		+01.1"		2
						269°47'50.6"		269°47'55.2"	
1	277°31'09.5"	0.00	-1.67	-1.67	277°31'07.8		+01.1"		1
						007°18'58.4"		007°19'04.1"	
DEF	050°34'43.3"	1.66	1.08	-0.58	050°34'42.7"		+01.1"		DEF
						237°53'41.1"		*237°53'47.9"	
ABC		-1.13							ABC

* These are fixed grid values

† The extra 0.2" was apportioned to these angles because they have the shortest lines of sight.

Grid Angle = Obs. Angle - Backslight Correction + Foresight Correction

Table 3

Station	Grid Azimuth	Grid Distance	Unadjusted North	Unadjusted East	Accumulated Correction North	Accumulated Correction East	Adjusted Coordinates North	Adjusted Coordinates East	Station
XYZ			*147,271.976	*306,474.391	0.000	0.000	*147,271.976	*306,474.391	XYZ
	293°21'23.4"	3,446.699							
4			148,638.423	303,310.129	0.014	-0.012	148,638.437	303,310.117	4
	179°25'15.5"	4,787.539							
3			143,851.129	303,358.510	0.034	-0.030	143,851.163	303,358.480	3
	179°19'45.5"	4,831.852							
2			139,019.608	303,415.070	0.053	-0.047	139,019.661	303,415.023	2
	269°47'55.2"	9,595.442							
1			138,985.890	293,819.687	0.093	-0.082	138,985.983	293,819.605	1
	007°19'04.1"	8,162.243							
DEF			147,081.651	294,859.335	0.126	-0.111	*147,081.777	*294,859.224	DEF

Compass Rule Adjustment

Table 4

Station	North	East	Grid Azimuth	Grid Distance	Elevation Factor	Scale Factor	Surface Distance	Station
XYZ	147,271.976	306,474.391						XYZ
			293°21'23.9"	3,446.715	0.9998749	1.0000033	3,447.135	
4	148,638.437	303,310.117						4
			179°25'16.3"	4,787.518	0.9998857	0.9999998	4,788.067	
3	143,851.163	303,358.480						3
			179°19'46.2"	4,831.833	0.9998916	0.9999999	4,832.357	
2	139,019.661	303,415.023						2
			269°47'56.1"	9,595.477	0.9998907	0.9999899	9,596.667	
1	138,985.983	293,819.605						1
			007°19'03.3"	8,162.272	0.9999808	0.9999808	8,163.321	
ABC	147,081.777	294,859.224						ABC

Final Adjusted Results

Table 5

6. Apply the (t-T) corrections to the observed angle to obtain the grid angles and compute the balanced azimuths .
7. Adjust the traverse and compute the final SPCS 83 coordinates of the points.

Step 1

Obtain the state plane coordinates for points XYZ, DEF, and ABC. Compute the starting grid azimuth from station XYZ to the azimuth mark from the published geodetic azimuth and convergence angle.

$$t = \alpha - \gamma = 205°41'16.3" - 000°31'43.9" = 205°09'32.4"$$

The ending azimuth is computed by inversing the published coordinates of points DEF and ABC.

$$t = \arctan((285846.563 - 294859.224) \div (147427.408 - 147081.777)) = 237°53'47.9"$$

Step's 2 &3

Table 1 shows the results of steps 2 & 3. For this example an approximate value for N of 30 meters was used the ellipsoid distances.

Step's 4 & 5

Table 2 shows the various intermediate values needed to compute the arc-to-chord, (t-T), corrections and the line grid scale factors for each traverse leg. Because the (t-T) correction corrects for the difference between sighting on a curved surface and on a plane, it is necessary compute a correction for both backsights and foresights.

In this example a mean of the point grid scale factors was used to compute the line grid scale factors.

Step 6

Table 3 illustrates the application of the (t-T) correction. It is helpful to refer to a sketch when applying the arc-to-chord correction. After the grid angles have been computed it is usually necessary to correct them for closure. In this example the error in closure amounts to 1.1" per angle with 0.2" left. In this case the 0.2" is divided between the angles over station XYZ and point 4 on the assumption that all else being equal the angles with the shortest sights have the greatest potential for error.

Step 7

Table 4 shows a compass rule adjustment and table 5 the final adjusted results and surface distances.

Some Final Comments

The Arizona Plane Coordinate System, 83 provided a convenient method for permanently recording the positions of survey points to the accuracy of the original work and ties all surveys to single reference framework. If at least two known points are used to control a survey most blunders will be detected.

Throughout this paper the linear units used have been meters. There are several reasons for this:

1) A meter is meter whereas a foot isn't always a foot.
2) The coordinates published by the NOAA are in meters. (The country has been on the metric system legally since 1866 and practically since 1893.)
3) Using meters on the SPCS,83 grid helps to distinguish between SPCS,27 and SPCS,83. State law currently requires that coordinates shown on plats be in International feet.

Appendix

Transverse Mercator Mapping Equations

Definitions, Notations, Arizona, and GRS Constants

Note: Linear units are in meters and angular units are in radians unless otherwise indicated.

GRS-80 Ellipsoid constants:

a	=6,378,137 = semi-major axis
b	=6,356,752.3141403 = semi-minor axis
f	=0.0033528106811911 = flattening = $(a-b)/a$
e^2	=0.00669438002290 34 = eccentricity squared = $2f-f^2$
e'^2	=0.00673949677549 66 = second eccentricity squared = $e^2/(1-e^2) = (a^2-b^2)/b^2$
r	=6367449.14577 = radius of the rectifying sphere
U_0	=–0.005048250776
U_2	= 0.000021259204
U_4	=–0.000000111423
U_6	= 0.000000000626
V_0	= 0.005022893948
V_2	= 0.000029370625
V_4	= 0.000000235059
V_6	= 0.000000002181
S_0	= Meridional distance from the equator to the defining latitude of the zone multiplied by the central meridian scale factor of the zone and is constant for the zone.

SPCS 83 Notations

ϕ	Geodetic latitude positive north
λ	Geodetic longitude positive west
ω	Rectifying latitude
N	Northing on the grid (formerly y)
E	Easting on the grid (formerly x)
λ_0	Central meridian (110°10'00", 111°55'00", or 113°45'00" for Arizona)
E_0	False easting of λ_0 (213,360 for all three Arizona Zones)
S	Meridional distance
ϕ_0	Latitude of grid origin (31°00'00" for all three Arizona Zones)
N_0	False northing of ϕ_0 (0 for all three Arizona Zones)
k_0	Central meridian grid scale factor = 0.9999 for the central and east zones =0.999933333... (14999 +15000) for the west zone.
ω_0	$=\phi_0 + \sin\phi_0 \cos\phi_0 (U_0 + U_2 \cos^2\phi_0 + U_4 \cos^4\phi_0 + U_6 \cos^6\phi_0)$
S_0	$=k_0 \omega_0 r$ = 3,430,631.2260 for the central and east zones and 3,430,745.5918 for the west
k	Grid scale factor of a point
k_{12}	Grid scale factor of a line
ΔN	$N_2 - N_1$
ΔE	$E_2 - E_1$
E'	$E - E_0$
γ	Meridian convergence

δ_{12} Arc-to-chord correction (t–T) from point 1 to point 2

R Radius of curvature in the prime vertical = $a/(1 - e^2 \sin^2 \phi)^{1/2}$ (note: Latitude dependent)

r_0 Geometric mean radius of curvature scaled to the grid = 6,369450m for central and east zones and = 6,369,660m for the west zone. $r_0 = k_0(1 - e^2)^{1/2}/(1 - e^2 \sin^2 \phi)$ where $\phi = 34°$

t $\tan \phi$ or grid azimuth depending on the context.

η^2 $e'^2 \cos^2 \phi$

Direct Conversion

(Geodetic to State Plane)

The following computations start with geodetic coordinates of a point (ϕ, λ) and yield the Transverse Mercator grid coordinates (N , E), the convergence angle (γ), and the grid scale factor (k) of the point. All angles are in radians (one radian=180÷π degrees). All linear units are in meters (one international foot = 0.3048 meters exactly).

$L = (\lambda - \lambda_0) \cos \phi$ (Note: the sign convention for SPCS 27 was $(\lambda_0 - \lambda)$)

$\omega = \phi + (\sin \phi \cos \phi)[U_0 + \cos^2 \phi \{U_2 + \cos^2 \phi (U_4 + U_6 \cos^2 \phi)\}]$

$S = k_0 \omega r$

$R = k_0 a / (1 - e^2 \sin^2 \phi)^{1/2}$

$A_2 = (R\, t)/2$

$A_4 = [5 - t^2 + \eta^2 (9 + 4\eta^2)] / 12$

$A_6 = [61 - 58 t^2 + t^4 + \eta^2 (270 - 330 t^2)]/360$

$N = S - S_0 + N_0 + A_2 L^2 [1 + L^2 (A_4 + A_6 L^2)]$

$A_1 = -R$

$A_3 = (1 - t^2 + \eta^2)/6$

$A_5 = [5 - 18 t^2 + t^4 + \eta^2 (14 - 58 t^2)]/120$

$A_7 = (61 - 479 t^2 + 179 t^4 - t^6)/5040$

$E = E_0 + A_1 L [1 + L^2 (A_3 + L^2 (A_5 + A_7 L^2))]$

$C_1 = -t$

$C_3 = (1 + 3\eta^2 + 2\eta^4)/3$

$C_5 = (2 - t^2)/15$

$\gamma = C_1 L [1 + L^2 (C_3 + C_5 L^2)]$

$F_2 = (1 + \eta^2)/2$

$F_4 = [5 - 4 t^2 + \eta^2 (9 - 24 t^2)]/12$

$k = k_0 [1 + F_2 L^2 (1 + F_4 L^2)]$

The terms A_6, A_7, C_5, and F_4 may be omitted when computed within zone boundaries.

Inverse Conversions

(State Plane to Geodetic)

These computation start with the SPCS 83 grid coordinates (N,E) and yield the geodetic coordinates (ϕ, λ), the convergence angle (γ), and the grid scale factor (k) of a point.

$\omega = (N - N_0 + S_0)/(k_0 r)$

$\phi_f = \omega + (\sin \omega \cos \omega)[V_0 + \cos^2 \omega \{V_2 + \cos^2 \omega (V_4 + V_6 \cos^2 \omega)\}]$

$R_f = k_0 a/(1 - e^2 \sin^2 \phi_f)^{1/2}$

$Q = E'/R_f$

$$B_2 = -t_f(1+\eta_f^2)/2$$
$$B_4 = -[5+3t_f^2+\eta_f^2(1-9t_f^2)-4\eta_f^4]/12$$
$$B_6 = [61+90t_f^2+45t_f^4+\eta_f^2(46-252t_f^2-90t_f^4)]/360$$
$$\phi = \phi_f + B_2 Q^2 [1+Q^2(B_4+B_6 Q^2)]$$

$$B_3 = -(1+2t_f^2+\eta_f^2)/6$$
$$B_5 = [5+28t_f^2+24t_f^4+\eta_f^2(6+8t_f^2)]/120$$
$$B_7 = (61+662t_f^2+1320t_f^4+720t_f^6)/5040$$
$$L = Q[1+Q^2\{B_3+Q^2(B_5+B_7 Q^2)\}]$$
$$\lambda = \lambda_0 - L/\cos\phi_f$$

$$D_1 = t_f$$
$$D_3 = -(1+t_f^2-\eta_f^2-2\eta_f^4)/3$$
$$D_5 = (2+5t_f^2+3t_f^4)/15$$
$$\gamma = D_1 Q[1+Q^2(D_3+D_5 Q^2)]$$
$$G_2 = (1+\eta_f^2)/2$$
$$G_4 = (1+5\eta_f^2)/12$$
$$k = k_0[1+G_2 Q^2(1+G_4 Q^2)]$$

The Arc-to-Chord Correction

The following equations produce and use approximate values for ω and ϕ_f. ϕ_f is accurate to within 3 seconds. The formula given earlier under the section on convergence yields results that will agree very closely with the ones generated by these equations when ΔN is less than 30 km.

$$\Delta N = N_2 - N_1$$
$$N_m = (N_1+N_2)/2$$
$$\omega = (N_m - N_0 + S_0)/(k_0 r)$$
$$\phi_f = \omega + V_0 \sin\omega\cos\omega$$
$$F = (1-e^2\sin^2\phi_f)(1+\eta_f^2)/(k_0 a)^2$$
$$E_3 = 2E_1 + E_2$$
$$\delta_{12} = -\{\Delta N E_3 F[1-(E_3^2 F)/27]\}/6$$

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Grid and Ground Coordinates

A simplified explanation with step-by-step instructions.

This paper originally appeared in Professional Surveyor Magazine as a 3 part series of articles.

EDM & GPS Measurements—Why Don't They Agree? Part One
Jul-Aug 1999, Volume 19 No 6

EDM & GPS Measurements: Why Don't They Agree? Part Two
October 1999, Volume 19 No 8

EDM & GPS Measurements: Why Don't They Agree? Part Three
April 2000, Volume 20 No 4

This version is a compilation of the three installments along with some additional diagrams and tables. It was used as the workshop notes for a class held at the New Jersey Society of Professional Land Surveyors Conference, SurvCon 2000 - Sighting the Future, February 2, 3, 4, 2000 Atlantic City, New Jersey

GPS & EDM Measurements – Why Don't They Match?

Have you ever measured the distance between 2 points that were established by GPS with your EDM? Have you been surprised or alarmed? If so, this workshop is for you.

This will be a "hands on" class to teach you why this happens and how to solve the problem, so bring your calculators. The theory and equations will be kept to a minimum, but the topics will include GPS vectors as measurements, State Plane verses "Ground" Coordinates and grid lengths verses ground surface distances.

New Jersey Society of Professional Land Surveyors
SurvCon 2000 - Sighting the Future
February 2, 3, 4, 2000
Atlantic City, New Jersey

Instructor:
Jesse Kozlowski, PLS
GPS Project Manager
Taylor Wiseman & Taylor
www.taylorwiseman.com

EDM & GPS Measurements – Why Don't They Agree? Part One.

A question put to me often by surveyors all over the country is "Why do my EDM measurements disagree with the State Plane Coordinates I was given by the GPS surveyor?" In the majority of cases, the source of this problem has very little to do with GPS. It has everything to do with how the surveyor is using the coordinates that were produced using GPS.

In the series of articles to follow, we will explore the proper use of the State Plane Coordinate System. The whole process will be covered step by step by the end of the series. The purpose of this first article will be to dispel a common myth. Have you ever heard someone say, "The traverse that we ran was close to the shore, so we didn't have to bother with any of that 'reduction stuff' since we were so close to sea level"? What do you think? Is that an accurate statement?

Ok, before we jump to any conclusions, it will be necessary for us to be acquainted with some of the basics. I learn best by working through a problem. For this demonstration, I have selected a pair of monuments located in Absecon, New Jersey. This location was chosen because it is situated only 25 seconds east of the Central Meridian and it is very close to sea level. It will serve as a perfect illustration of the problem that many surveyors are having across the country.

The State Plane Coordinate System in New Jersey is based on the Transverse Mercator Projection. Refer to **Table 1** in the appendix for the Mapping Projection Parameters and a map of New Jersey showing Absecon in relation the Central Meridian as **Exhibit 1**. The same example could be developed for other states that use the Lambert Projection. The NGS Data sheets for the two control points used in this example can be found in the appendix as **Exhibit 2** and **Exhibit 3**.

In this example, we want to set our Total Station on **STEELMAN 2 RM 4** to begin a traverse and we want to use **STEELMAN 2 AZ MK 2** as an azimuth mark for orientation on the NJPCS. The first thing you would probably do is measure the distance with your EDM. After reducing the slope distance to a horizontal distance, you might record something very close to 354.838 meters. I said, "very close" because we are not discussing measurement error theory or instrument calibration here. As it turns out, this distance actually is the ground distance between these two marks, so we will use it in this example.

Notice that we are given the distance between the two marks on the NGS data sheet as 354.840 meters. You say, "Not bad shooting." We appear to be within a couple of millimeters of the published distance. You might think that this confirms your assumption that no "State Plane Reduction Stuff" will be necessary and proceed to traverse happily off into the Pine Barrens. Everything will be fine until you close your traverse on another control point several kilometers away.

Some of you may have also noticed something else before you picked up the instrument. When you inverse the pair of State Plane Coordinates, you get a distance that is very different from the one you measured. The inversed grid length is 354.8048 meters, which is 33 millimeters shorter than what you measured. What is going on here?

Part of the answer is that the distance shown between the two marks on the NGS Data sheet is the Geodetic Distance, which means that it is measured along the curved surface of the ellipsoid. At this location, the ellipsoid would be around 25 meters over our heads. How do I know this you ask? We are given all the information that we need right on the NGS Data sheet to determine this interesting fact and also other useful things such as reducing our measurement to the grid surface. An examination of **Figure 1** and **Figure 2** will clarify this.

Ok, now it is time to reach some conclusions. Notice that the NGS Data sheet provides us with the Scale Factor for the control stations along with the coordinates. For both points the scale factor is 0.99990000. This scale factor is especially interesting to us because it is the exact same scale factor used to define the SPCS in New Jersey. Refer to **Table 1**. This makes perfect sense because these two stations are practically on top of the Central Meridian.

Those of you that compared your measured distance to the inversed grid length may have also been interested to know what the accuracy of your measurement was. If you divided the published distance by the perceived error of 33 millimeters, you may have concluded that you achieved a dismal accuracy of around 1 part in 10,000. What you were really looking at was the affect of the scale factor along the Central Meridian, which actually is 1 part in 10,000. What do you suppose would happen when this traverse closes on another control point several kilometers away and no account is taken for the scale factor? Even worse, what if only a loop traverse was run and closed back on itself at STEELMAN 2 RM 4? The problem would go undetected as it so often does. See **Figure 3** for an example of a connecting traverse and the misclosure that results when the distances are not reduced to the grid. **Table 2** presents the differences between coordinates computed on the grid and coordinates computed on the ground.

<u>Summary of Part One</u> – The whole purpose of this entire exercise was to demonstrate that the proper use of the State Plane Coordinate System requires our distances be reduced to the grid no matter what the elevation is. It is true that elevation can be an important element in the reduction computation. We purposely chose an example at sea level so that elevation would not be a factor in order to focus your attention on another very important aspect of the State Plane Coordinate System; the Scale Factor. It has been demonstrated that ignoring the grid scale factor will introduce as much as 1 part in 10,000 distortion into any final surveying work product.

Hopefully, the myth has been dispelled and I have captured your attention. The next article will cover each step of the computation to reduce an EDM measured distance to the corresponding grid length. You will see how the elevation and the scale factor are used in the computation. You will also be introduced to another aspect that must be considered called the Geoid Height.

EDM & GPS Measurements – Why Don't They Agree? – Part Two

This is the second in a series of three articles addressing the familiar "Grid versus Ground" issue. Included is a detailed look at the distance reduction computations for the Absecon, New Jersey example from the previous article. You will need to refer back to the first article for the data and other information used in the 4 steps below. The reduction computation can be found in the appendix as **Exhibit 4.**

Step 1. Reduce The Slope Distance To The Horizontal Distance.

Everyone is familiar with this computation. The most common method used is to multiply the slope distance by the sine of the zenith angle. Another useful way employs the Pythagorean theorem to solve for the horizontal distance given the difference in height between the two marks and treating the slope distance as the hypotenuse of a right triangle. In our example, this computation was assumed to have already taken place. The reduced horizontal distance is 354.838 meters. This is labeled as the "Horizontal Distance in **Exhibit 4.**

Step 2. State Plane Coordinate Inverse Computation.

Using the Pythagorean theorem once again, compute the grid inverse of the two marks given their respective northing and easting coordinates. The grid inverse is equal to the square root of the sum of the squared delta northing and the squared delta easting coordinates. The inversed grid length is 354.8048 meters or 354.805 rounded to three decimal places. Refer to **Exhibit 4** to see the "SPCS Grid Inverse Computation".

Problem Presents Itself

Here is where the problem presents itself. Comparing the inversed grid length to our reduced horizontal measurement reveals a discrepancy of 33 millimeters. Faced with this situation, some surveyors begin to question the control rather than their procedures. For whatever the reason is many surveyors are not comfortable with the state plane coordinate system, which is evident, by how often this problem arises. Often times the choice is made to ignore the discrepancy hoping the problem will go away somehow. This course of action only makes the situation worse by allowing the problem to become more complex than it really is. Let's examine this in greater detail.

How Close Is Close Enough?

Dividing 354.805 meters by 0.033 meters indicates the accuracy of our work up to this point which equals 1 part in 10,752. Is that acceptable? How close is close enough and how is that decided? This decision will be made using objective error analysis.

Error Propagation

Making the following assumptions – that the control is accurate, that the EDM is working properly, that it is capable of measuring to precision of at least +/- 5mm + 5ppm, and that the EDM and target have been carefully centered over the marks to within 1 to 2 mm, we can also assume that 33 millimeters is too much discrepancy to be acceptable. Using well know error propagation methods, we find that over this length the difference should not exceed 12 millimeters 95 percent of the time.

The error propagation is computed as follows:

$(0.005^2 + ((5/1000000)*354.8)^2 + 0.002^2 + 0.002^2)^{1/2}$ = +/-0.006m one sigma (68% confidence)

+/-0.006m * 1.96 = +/-0.012m or +/-12mm 95% confidence

Clearly something else is involved here. The explanation is simple, but not well known or readily accepted by those having difficulty with this subject. The measurement made at the earth's surface cannot be compared directly to the corresponding length on the surface of the grid. These two surfaces are separated by hundreds of meters. See **Figure 2.**

Point Emphasized

To emphasize this point, consider a traverse located in an area where the difference in elevation ranged around 200 meters. Would you use only observed slope distances from this traverse in all the subsequent computations that you perform such as traverse closure, boundary line analysis, subdivision design, etc? You wouldn't waste your time with such nonsense, right? Every single slope measurement gets converted to a horizontal distance before the measurement is used in further computations.

Likewise, before a comparison can be made in this example, a transformation must take place, that is, the ground surface measurement must be reduced to the grid and then compared to the grid length obtained by inversing the pair of state plane coordinates. That brings us to step 3.

Step 3. Ground To Grid Reduction

There will be two parts in this reduction. The first part reduces the measurement to the surface of the ellipsoid to produce a *Geodetic Distance.* The second part reduces the geodetic distance to the surface of the state plane coordinate grid producing a grid distance. Later it will be shown how to combine these two parts into one reduction step, which will produce a grid distance directly from the ground surface horizontal measurement.

Step 3, Part 1. Horizontal Distance Reduced To The Ellipsoid

This step can be compared to the "sea-level" reduction used for NAD27. It is similar, but different in that rather than reducing the line length to sea level, the line is reduced to the surface of the ellipsoid.

The separation between the geoid *(sea level for our purposes here)* and the ellipsoid is called the *Geoid Height*, symbolized thus **"N"**. It is the distance measured from the surface of the ellipsoid to the surface of the geoid. In the contiguous USA, the geoid height ranges between minus 25 and minus 35 meters, except for places in the Rocky Mountains where the separation becomes as low as around minus 5 meters. In Alaska the Geoid Heights are positive.

The affect of ignoring this separation causes the length to be reduced to sea level rather than the surface of the ellipsoid. An error of up to 5-PPM can result because the NAD83 State Plane Grids are projected from the ellipsoid and not mean sea level. Try the following computations for yourself with and without the geoid height to see what the affect is. The error may seem small on this one line, but it will accumulate in the traverse as a systematic error.

To transform the distance to the ellipsoid, a "reduction" factor is needed. This factor is known as the *Elevation Factor* in the NOS NGS 5 manual to distinguish it from the *Grid Scale Factor*, which we will use in the step to follow. I prefer to use the term Ellipsoid Factor rather than Elevation Factor because it is more meaningful considering the surface that the measurement is actually transformed on to.

The ellipsoid factor is simply a ratio between the geodetic distance and the horizontal ground distance. Dividing the mean radius of the earth by the average ellipsoid height of the line added to the earth's mean radius will yield this ratio I call the ellipsoid factor. Multiplying the horizontal ground distance by the ellipsoid factor produces the geodetic length. Take note that the use of the terms "reduce" or "reduction" does not imply that the magnitude of the distance decreases. In the example before us, the geodetic distance is actually longer than the ground distance by a very small amount. **Figure 2** provides a visual clue why this is true at the location in our example. For any location where the terrain elevation is near sea level, the ellipsoid will be overhead. The result of this computation is labeled as the "Geodetic Distance" under the "Ground to Grid Computation" heading in **Exhibit 4.**

Step 3, Part 2. Geodetic Distance Reduced To The SPCS Grid

To make this part of the reduction, the scale factor for each of the points will be needed. Fortunately, we rarely, if ever, will be required to compute the scale factor. It will always be available to us on the NGS data sheet along with the coordinates and convergence. Most software programs that make SPCS computations also output the scale factor. If you are interested in how to actually compute the scale factor, you may refer to the NOS NGS 5 manual for the equations.

For our purposes, it is more important to know what the scale factor is and what it does. Recall from the previous step the way in which the ellipsoid factor was computed. That ratio represents the difference between the geodetic distance and the ground distance. Likewise, the scale factor is the ratio between the grid distance and the geodetic distance. To obtain the grid distance, we simply multiply the geodetic distance by the average scale factor for the line. The result of this computation is labeled as the "SPCS Grid Distance" under the "Ground to Grid Computation" heading in **Exhibit 4.** That ends the reduction computation. In Step 4, we make a comparison to check our work.

Combined Factor

As stated above, the two parts of Step 3 can be combined into one step. To do this, simply multiply the ellipsoid factor by the grid factor to produce a single factor called the *Combined Factor*. The same grid distance will result when the ground distance is multiplied by the combined factor. A single combined factor can be computed for many surveying projects and used to reduce all the distances measured in that area.

Step 4. Comparison Between Published and Measured

Now you can compare the grid distance computed from your measurement to what we may regard as the published value, that is, the inversed grid distance from the published coordinates. This step is labeled, "Comparison" in **Exhibit 4**.

Relieved or alarmed? If the discrepancy exceeds what you would expect from your EDM now, then there is cause to question the control. If the marks were established using static or fast static GPS surveying techniques, the difference should be very small. The stipulation of course is that the base line between the marks was observed in the GPS survey. In other words, if the two marks were surveyed with GPS using a radial method and the base line between them was not observed, then the discrepancy could be larger than desired since the two marks would be "side shots".

Try This Experiment

If you still aren't convinced that extreme accuracy can be obtained from GPS, try this experiment some time. Take your GPS receivers to an EDM calibration base line. Be sure to use care in setting over the marks just as you would if you were calibrating your EDM. You will find that the GPS base lines you process will match the published "mark-to-mark" distances on the NGS data sheet extremely well.

Here are the results of some comparisons made on the Burlington County EDM Calibration Base Line located near Lumberton, New Jersey.

The published Mark-to-Mark distance on this line between the 0 meter mark and the 1000 meter mark is 1000.0414m +/-0.6mm. These marks were occupied during a static GPS survey for around 50 minutes on two separate occasions. The results obtained by post processing the GPS data collected during these two occupations yielded the following base line lengths:

Results on 12/5/95 = 1000.041m +/- 0.0003
Results on 12/6/95 = 1000.043m +/- 0.0003

The length on the first day is within 0.001m agreement with the published distance at the millimeter level. Luck perhaps? The results on the second day differ from the published distance by 2 millimeters rounding up. Two millimeters in one thousand meters equates to an accuracy of 1 part in 500,000. You can see similar results using RTK with careful centering of the antenna pole.

<u>Summary of Part Two</u> - The above exercise provides evidence to support the opening statement in the first article. The grid versus ground issue has very little to do with GPS. It has everything to do with the way the surveyor is using the coordinates that were produced from GPS measurements. If you take the time to do the test on an EDM base line, the grid versus ground issue will begin to vanish before your eyes.

The EDM and GPS are capable of producing comparable measurements. This article dealt entirely with the transformation of distances from one surface to another. In the next and final article of this series, we will examine how to transform the coordinates from one surface to another. These "localized" coordinate systems can be useful in certain applications. The disadvantages of local coordinates will also be presented.

EDM & GPS Measurements – Why Don't They Agree? – Part Three

This is the final part in this three part series of articles on the subject of "Grid vs. Ground" coordinates or measurements – whichever way you see it. This article will begin with an explanation why the title, "EDM & GPS – Why Don't They Agree?" was chosen to present this topic.

Problem Discovered

Think for a moment how this grid vs. ground problem is first discovered. It isn't discovered during the GPS survey. After careful survey planning, data collection, and data processing the usual finished product is a file of coordinate values. Confidence in these values comes from the QA/QC phases of the data processing and relying on good surveying procedures during the data collection. Any problems discovered during the QA/QC, like high RMS, misnamed stations, incorrect antenna height measurements, obstructions, etc. must be remedied before producing the final coordinates. Those problems are unrelated to the "grid vs. ground" issue.

This issue only gets discovered when someone physically occupies a pair of these points to measure between them with an EDM. Without fail, this one act will always prompt the question, "Why don't they agree?" by surveyors not familiar with State Plane Coordinates. As it turns out, this problem has existed for a long time, but it took the wide spread use of GPS to force surveyors into learning how to use the SPCS properly. Prior to GPS, many surveyors would occupy geodetic control monuments to tie their traverses to the SPCS, but the coordinates that they produced were a kind of 'pseudo-state plane coordinate' because they did not make the necessary reductions to their EDM measurements. In essence, they were traversing along the surface of the ground and not on the surface of the grid. Refer to **Figure 3** and **Table 2** for a comparison of coordinates computed on the grid with coordinates computed at ground.

This went undetected for a long time, but now because so many points are being established with GPS, the reduction computations can no longer be ignored. The reason for this is that the measurements made with GPS are extremely good when made properly and with the mere push of a button, the processing software will produce real SPC values on the grid, not the ground. The surveyors that have ignored this up to now will have to catch up to learn what actually happens when that button gets pushed. It is hoped that this series of articles has been helpful in this regard.

Some Review

Before we get started with the subject of Part 3, let's look back at what has been covered so far in the first two parts.

In the first article, it was demonstrated that the distortion that is built into the projection cannot be ignored no matter what the elevation is. The example being used is at sea level and the distortion is 1 part in 10,000 – that is just over a foot in only two miles!

The second article explained how to reduce a distance measurement to the grid surface step by step. A single combined factor can be computed for the whole project area and used to reduce all the distances without the loss of accuracy required for the application.

It should also be mentioned that the subject of this series of articles has been restricted to the treatment of lengths and coordinates. It was felt that the arc to cord correction for angular measurements was small enough in most applications that it could be ignored.

Now, we will examine how to apply this knowledge to the coordinates themselves rather than the distances.

Coordinates Can Be Scaled Too

By now you are familiar with how to apply the combined scale factor to a distance to reduce it to the grid. A distance of 1000.000 meters between to points on the ground is 999.900 meters on the SPCS grid surface where the combined factor equals 0.99990000.

This same principle can be applied directly to the coordinates themselves to increase or decrease their magnitude, that is, raise them from the grid to a surface near the ground or visa versa. This method of manipulating the SPCS can give rise to heated debates between the advocates of this method and those that are opposed to this method. Obviously, there are positive and negative aspects to everything, including how coordinates are produced and what they are used for.

A Continuing Debate

This article will not promote either side of this debate, but will only describe how this is already being done and how these so called, "localized ground" coordinates are being used. Some of the issues raised by both sides will be mentioned. Various organizations, agencies, counties, and DOTs around the country have adopted methods and procedures for dealing with the grid versus ground fact. These methods can be separated basically into two categories – 1) Use the SPCS as it was designed to be used as published by NGS, and 2) Create hybrid coordinate systems that are related to SPCS in some way.

Number two can be further broken down into two additional categories – 2a) The hybrid coordinate systems are designed to best fit the average elevation for an entire county. Each county has its own system, and 2b) Use the SPCS as in number one above for the entire state, but create localized coordinate systems for various projects, like highway construction for example.

As has been already stated, this topic can be debated and good arguments presented by the parties on all sides. My personal view is that each method can be used to great advantage in those applications that are well suited for them. Each surveyor needs to be well acquainted with the SPCS to be able to recognize when to apply one of these methods. The biggest point in this regard relates to the ability to follow the footsteps of the previous surveyor. Often times, the way in which a system of localized coordinates was produced does not get passed on with the northing and easting values. That is the number one reason given to discourage this practice. Nevertheless, a surveyor faced with this problem needs to know how to unravel a tangled situation like this. Knowing how to create a localized coordinate system is the very same knowledge required to return a hybrid coordinate system back into something that begins to resemble State Plane Coordinates.

Absecon Example Revisited

Recall that the scale factor used to reduce the line to the grid was computed by averaging the scale factors for the end points of the line. The elevations of the end points were also averaged to find the average elevation of the line in order to reduce it to the ellipsoid.

What would happen to the coordinates of the end points if they were divided by the combined factor? The answer is that they would increase in magnitude in proportion to one another. In the table below, the grid coordinates of the control have been divided by a ***Combined Factor of 0.99990382*** to produce 'ground coordinates'.

Point Name	Grid North	Grid East	Ground N	Ground E	Delta N	Delta E
STEELMAN 2 AZ MK	64612.500	150488.828	64618.715	150503.304	-6.215	-14.476
STEELMAN 2 RM 4	64948.826	150601.837	64955.074	150616.324	-6.248	-14.487

Scaling the grid coordinates in this way has the desired affect of producing new coordinates that are situated "near the ground" surface. This means that when an inverse is computed, the resulting length will match the horizontal distance measured with an EDM - in this case 354.839 meters.

Something else becomes immediately apparent. The ground coordinates are shifted northward by around 6 meters and eastward by around 14.5 meters. For some surveyors, this shift isn't quite enough. They would recommend applying a greater shift by either adding or subtracting a large constant so that these scaled ground coordinates are not mistaken for State Plane Coordinates. Adding 300,000 to the northings and subtracting 100,000 from the eastings would result in the following:

Point Name	Project Datum N	Project Datum E
STEELMAN 2 AZ MK	364618.715	50503.304
STEELMAN 2 RM 4	364955.074	50616.324

The reader will discover that the inverse of the so-called "Project Coordinates" will still produce the same ground distance.

Some situations and applications require that the shift between the grid coordinates and the scaled ground coordinates be kept as small as possible. This requirement may also come from the desire on the part of some surveyors to have it "both ways" – State Plane Coordinates that are at ground and match their EDM measurements. Having reached this point in this article, it is hoped that the reader will be comfortable with the fact that it is impossible to have it both ways. The SPCS is not at ground and coordinates at ground are not State Plane Coordinates. With that said, we will now examine how to produce a set of ground coordinates that are "localized" to keep the shift from the corresponding grid coordinates as small as possible.

We will use the connecting traverse depicted in **Figure 3** as an example. The following table contains the ***Localized Ground Coordinates*** for the starting and ending control points.

Point Name	Grid N	Grid E	Elev	Comb Fact	Local N	Local E	GN - LN	GE - LE
STEELMAN 2 RM 4	64948.826	150601.837	9.317	0.999903880	64948.647	150601.938	0.179	-0.101
STEELMAN 2 AZ MK	64612.500	150488.828	2.301	0.999904980	64612.289	150488.919	0.211	-0.091
PINE 2	68739.917	148331.659	12.590	0.999903380	68740.103	148331.542	-0.186	0.117
PINE 1	68936.075	148765.899	14.730	0.999903020	68936.280	148765.824	-0.205	0.075
Centroid	66809.330	149547.056	9.735	0.999903815	66809.330	149547.056	0.000	0.000

The SPCS coordinates for the point named "Centroid" are computed by finding the average of all the Northing and Easting values in a project. This would comply with the definition for "centroid" as taken from the *Geodetic Glossary* (NGS, Sept.1986), *"The point whose coordinates are the average values of the coordinates of all the points of the figure."*

In this case, the centroid is the average of the starting and ending control points in the traverse. The centroid then becomes a "hinge" point or a "peg" to keep the shift as small as possible. Note the last two columns in the table listing the shifts. The shifts for the centroid are zero because the local ground coordinate is identical to the grid value for the centroid. The local ground coordinates are computed as follows:

Ground N = Centroid N - [(Centroid N - Grid N) / Combined Factor]
Ground E = Centroid E - [(Centroid E - Grid E) / Combined Factor]

These equations find the latitude and departure between the centroid and any other point in a project. Just as a grid length can be "raised" to the ground by dividing it by the combined factor, the latitudes and departures can also be transformed to produce "ground" latitudes and departures. Algebraically summing these "increased" latitudes and departures with the Northing and Easting values of the centroid will yield the local ground values for the point. One variation for computing local ground coordinates includes the option to "localize" on an existing control monument rather than computing a centroid. All other computations remain the same.

It is a very simple procedure and used very conveniently to deal with the grid vs. ground issue in certain applications, like construction layout for instance. Keep in mind that the very same attribute of these local coordinates considered to be wonderful and desirable by some surveyors is exactly the same attribute considered to be unacceptable by other surveyors. These local coordinates look just like State Plane Coordinates, but are not – by a very small amount, which makes them dangerous. In one respect these coordinates make certain tasks easier, while in other respects, the work load is increased because much more book keeping is required to document how the coordinates where produced. Here is an example of some notes that should accompany a set of local ground coordinates –

Project Name: Absecon Traverse
Date: 01/26/00
Reference Information: NJSPLS Example
Units: Meters
Vertical Datum: NAVD88
Horizontal Datum: Local Ground System

The horizontal coordinates are local ground coordinates that are based on the State Plane Coordinate System of ***New Jersey*** NAD83 ***(1986).***

The relationship between this local ground system and the grid system is as follows.

1) Both systems share a common centroid point with identical coordinates of
North 66809.330 **East** 149547.056

2) The projection surface used for the local ground system is based on the following:
Average Elevation = 9.735m
Combined Factor = 0.999903815

3) State Plane Coordinates can be computed from the local ground coordinates by:

$N_G = N_C - [(N_C - N_L) \times CF]$ $E_G = E_C - [(E_C - E_L) \times CF]$

Where : N_G, E_G = **Grid North , Grid East**
N_C, E_C = **Centroid North, Centroid East**
N_L, E_L = **Local North, Local East**
CF = Combined Factor

<u>Summary of Part Three</u> – The final part of this series described how surveyors in various counties, state DOTs, and other agencies or organizations are modifying the SPCS to produce 'near ground' coordinate systems. Advocates of this procedure want to use a system of coordinates that easily relates to the measurements made with their EDMs without applying scale factors. They also want the hybrid system of coordinates to have the appearance of State Plane Coordinates. Surveyors that are opposed to creating modified projections raise concerns about the confusion that results when these coordinates are mistaken for State Plane Coordinates. The additional 'bookkeeping' in the form of extra metadata about the source of these coordinates can also become a disadvantage when data is shared or exchanged in a GIS application for example.

Conclusion –

The State Plane Coordinate System was created to enable surveyors to traverse great extents across a curved earth, yet treat it as though it were a flat surface. The SPCS was designed so that "ordinary" surveying mathematics, like trigonometry would still be applicable eliminating the need for complex geodetic computations on an ellipsoidal surface. The most complex aspect of the SPCS is the need to account for the distortion caused by projecting a curved surface onto a developable "flat" surface. Since the SPCS was designed to be conformal, that is, retain the shape of areas, the distortion is forced into the lengths between points. The computations to account for the length distortion are simply applied through the use of scale factors.

Even with all of its simplicity and utility, the SPCS has never really been fully accepted by the majority of surveyors. Even where the SPCS is being used, it is being modified in various ways to accommodate the user's requirements. The time has come to accept the fact that the SPCS does not fulfill all the needs of surveyors and other mapping professionals. One of the biggest disadvantages of the SPCS is that it is only 2 dimensional. The technology existing today and that will be used by surveyors in the future will allow for 3 dimensional positioning with great ease.

Once upon a time, it was thought that the sun orbited a flat earth. Forward looking surveyors will be open to the idea that there must be a better way to position points and describe those positions. True 3-D COGO is possible today. The coordinate system already exists in the form of Earth Centered, Earth Fixed (ECEF) coordinates. Using ECEF coordinates eliminates the grid/ground distance dilemma. There are no zones or projection constants to apply using ECEF coordinates. One set of solid geometry equations is equally applicable around the world. This subject will be detailed in a future article. In the mean time, readers are encouraged to explore this issue for themselves. Additional information on 3-D COGO can be obtained on the web at http://www.zianet.com/globalcogo/index.html

Appendix - Tables, Figures & Exhibits

Table 1 - Mapping Projection Parameters for the New Jersey SPCS (NAD83)

Central Meridian	W 74°30'00.000000"
Latitude of Origin	N 38°50'00.000000"
Origin Northing	0.0000 m
Origin Easting	150000.0000 m
Scale along Central Meridian	0.999900000000

Exhibit 1 – Map of New Jersey Showing Central Meridian and SPCS Origin

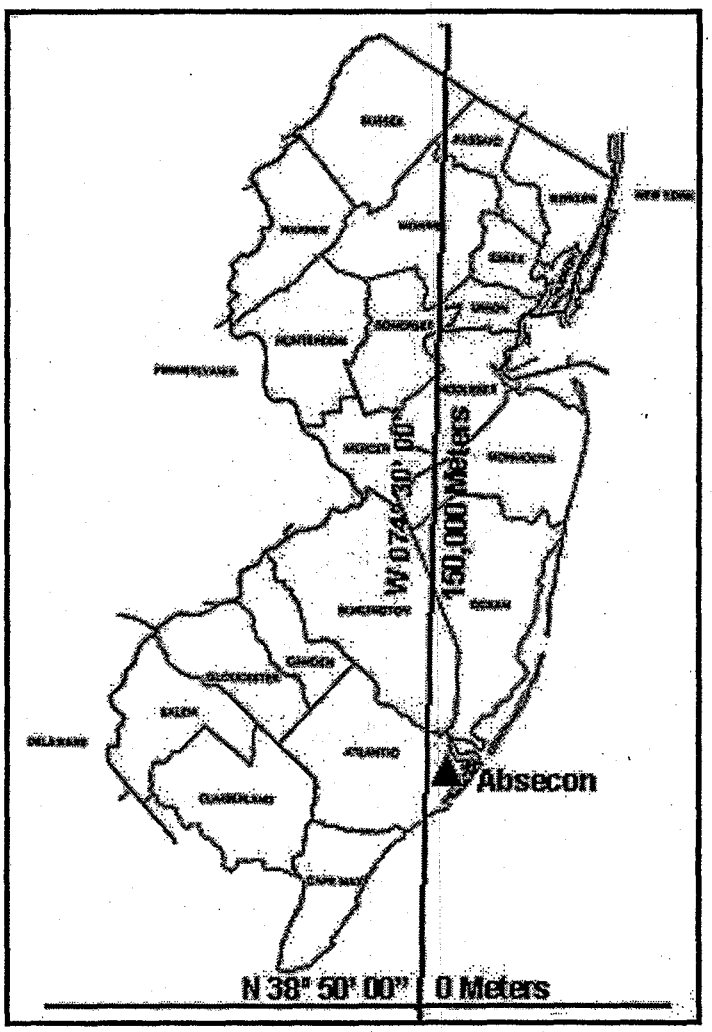


Exhibit 2 – NGS Data Sheet for STEELMAN 2 RM 4

```
JU4104 ***********************************************************************
JU4104  DESIGNATION -  STEELMAN 2 RM 4
JU4104  PID         -  JU4104
JU4104  STATE/COUNTY-  NJ/ATLANTIC
JU4104  USGS QUAD   -  OCEANVILLE (1989)
JU4104
JU4104                         *CURRENT SURVEY CONTROL
JU4104  ___________________________________________________________________
JU4104* NAD 83(1986)-  39 25 06.31893(N)    074 29 34.83764(W)    ADJUSTED
JU4104* NAVD 88     -         9.4    (meters)      31.     (feet)  GPS OBS
JU4104  ___________________________________________________________________
JU4104  X           -   1,319,118.861 (meters)                     COMP
JU4104  Y           -  -4,754,336.670 (meters)                     COMP
JU4104  Z           -   4,028,294.964 (meters)                     COMP
JU4104  LAPLACE CORR-          -1.57  (seconds)                    DEFLEC96
JU4104  ELLIP HEIGHT-         -24.79  (meters)                     GPS OBS
JU4104  GEOID HEIGHT-         -34.11  (meters)                     GEOID96
JU4104
JU4104  HORZ ORDER  -  FIRST
JU4104  ELLP ORDER  -  THIRD     CLASS II
JU4104
JU4104.The horizontal coordinates were established by GPS observations
JU4104.and adjusted by the National Geodetic Survey in March 1993.
JU4104
JU4104.The orthometric height was determined by GPS observations.
JU4104
JU4104.The X, Y, and Z were computed from the position and the ellipsoidal ht.
JU4104
JU4104.The Laplace correction was computed from DEFLEC96 derived deflections.
JU4104
JU4104.The ellipsoidal height was determined by GPS observations
JU4104.and is referenced to NAD 83.
JU4104
JU4104.The geoid height was determined by GEOID96.
JU4104
JU4104;                    North         East     Units   Scale      Converg.
JU4104;SPC NJ       -    64,948.827   150,601.837   MT  0.99990000 +0 00 16.0
JU4104;UTM  18      - 4,363,333.591   543,641.486   MT  0.99962345 +0 19 19.0
JU4104
JU4104|-------------------------------------------------------------------|
JU4104| PID    Reference Object                     Distance      Geod. Az |
JU4104|                                                           dddmmss.s |
JU4104| JU0332 STEELMAN 2                           43.496 METERS 03132    |
JU4104| JU4103 STEELMAN 2 AZ MK 2                  354.840 METERS 19835    |
JU4104|-------------------------------------------------------------------|
```

Exhibit 3 – NGS Data Sheet for STEELMAN 2 AZ MK 2

```
JU4103 ***********************************************************************
JU4103  DESIGNATION -  STEELMAN 2 AZ MK 2
JU4103  PID         -  JU4103
JU4103  STATE/COUNTY   NJ/ATLANTIC
JU4103  USGS QUAD   -  OCEANVILLE (1989)
JU4103
JU4103                        *CURRENT SURVEY CONTROL
JU4103  ___________________________________________________________________
JU4103* NAD 83(1986)-  39 24 55.41251(N)    074 29 39.56332(W)     ADJUSTED
JU4103* NAVD 88     -         2.3    (meters)       8.     (feet)  VERTCON
JU4103  ___________________________________________________________________
JU4103  LAPLACE CORR-          -1.60  (seconds)                    DEFLEC96
JU4103  GEOID HEIGHT-         -34.12  (meters)                     GEOID96
JU4103
JU4103  HORZ ORDER  -  THIRD
JU4103
JU4103.The horizontal coordinates were established by classical geodetic methods
JU4103.and adjusted by the National Geodetic Survey in June 1990.
JU4103.No horizontal observational check was made to the station.
JU4103
JU4103.The NAVD 88 height was computed by applying the VERTCON shift value to
JU4103.the NGVD 29 height (displayed under SUPERSEDED SURVEY CONTROL.)
JU4103
JU4103.The Laplace correction was computed from DEFLEC96 derived deflections.
JU4103
JU4103.The geoid height was determined by GEOID96.
JU4103
JU4103;                    North         East     Units   Scale      Converg.
JU4103;SPC NJ       -    64,612.500   150,488.828   MT  0.99990000 +0 00 13.0
JU4103;UTM  18      - 4,362,996.737   543,530.373   MT  0.99962333 +0 19 15.9
JU4103
JU4103|-------------------------------------------------------------------|
JU4103| PID    Reference Object                     Distance      Geod. Az |
JU4103|                                                           dddmmss.s |
JU4103| JU4104 STEELMAN 2 RM 4                    354.840 METERS 01835      |
JU4103| JU0332 STEELMAN 2                         397.348 METERS 01959      |
JU4103|-------------------------------------------------------------------|
```

Exhibit 4 - Distance Reduction Computations - Absecon, New Jersey Example

Solution to Absecon Example - Ground To Grid

Point Name	North	East	Elev	Geoid Ht.	Scale Factor
STEELMAN 2 RM 4	64948.827	150601.837	9.4	-34.11	0.999900000
STEELMAN 2 AZ MK 2	64612.500	150488.828	2.3	-34.12	0.999900000

SPCS Grid Inverse Computation

Delta N, Delta E	=	-336.327	-113.009
Delta N^2, Delta E^2	=	113115.6509	12771.0341
Delta N^2 + Delta E^2	=	125886.6850	
$(\text{Delta } N^2 + \text{Delta } E^2)^{1/2}$	=	354.805	

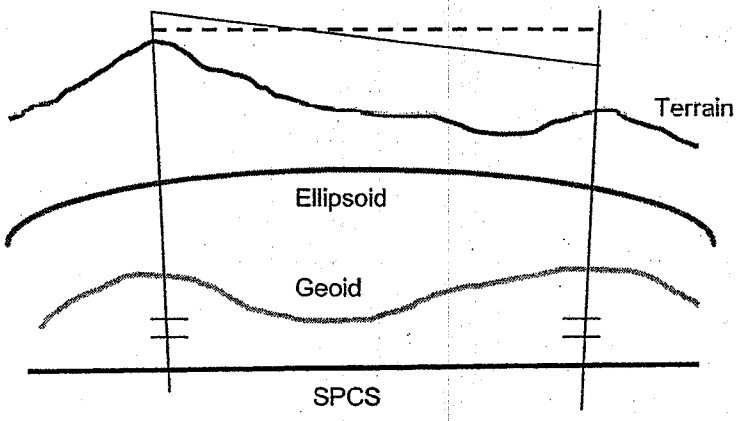
Ground To Grid Computation

Horizontal Distance	=	354.838	= (EDM Slope Measurement Reduced to Horizontal)
Ave Elev of Line	=	5.850	= (9.4 + 2.3) / 2
Ellipsoid Factor	=	1.000004436	= (R / (R + N + H)) = (6,372000 / (6,372000 + (-34.1) + 5.850))
Geodetic Distance	=	354.840	= Horizontal Distance x Ellipsoid Factor
Average Scale Factor	=	0.999900000	= Sum of Scale Factors / 2
SPCS Grid Distance	=	354.804	= Geodetic Distance x Average Scale Factor

Comparison

Published Grid Dist	=	354.805	
Reduced EDM Dist	=	354.804	
Difference	=	0.001	Difference = 1 part in 354,800

Figure 1 - Reference Surfaces



In New Jersey, the surface of the State Plane Coordinate System is approximately 640 meters below the surface of the ellipsoid along the central meridian.

Figure 2 - Reference Surfaces in Absecon, NJ

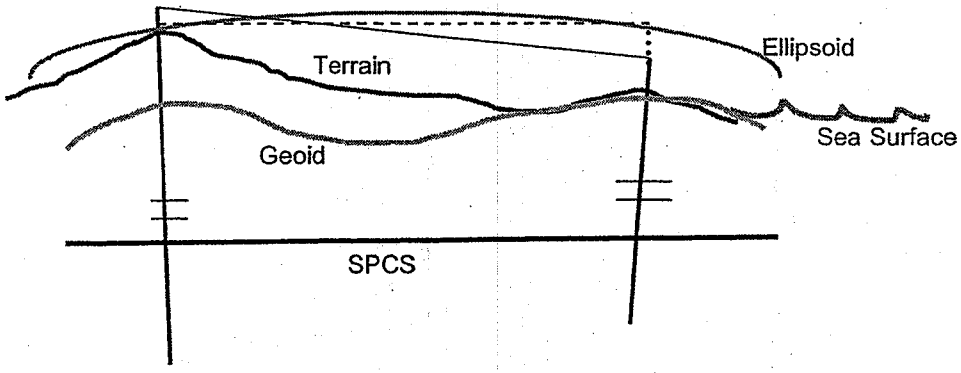
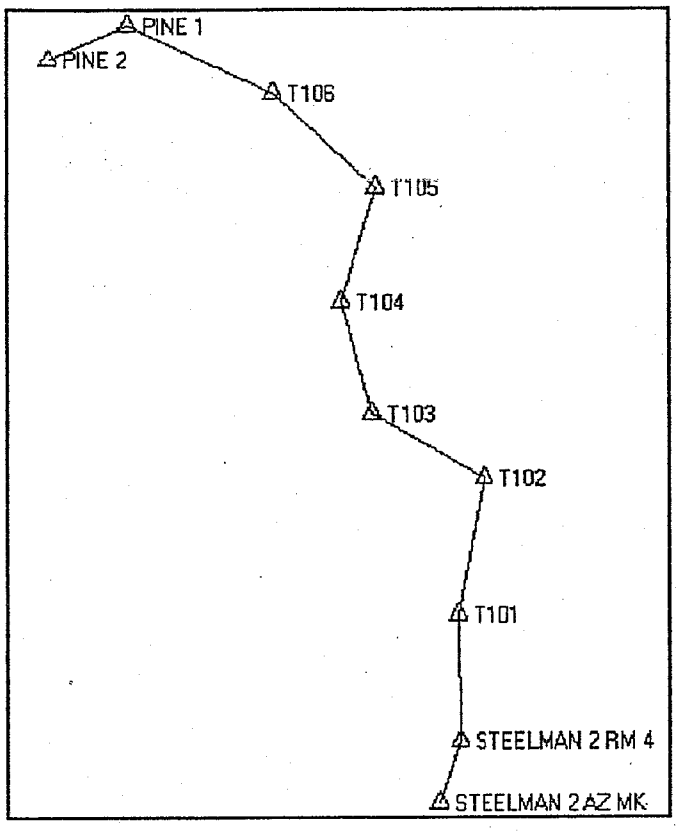
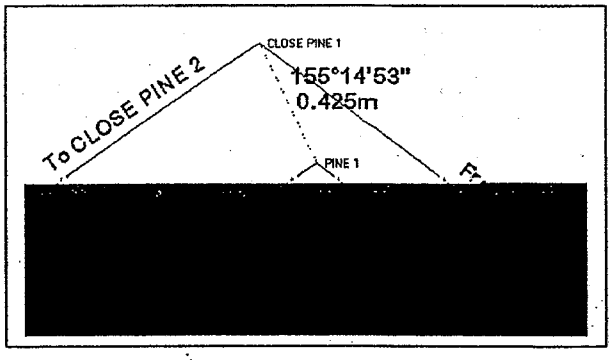


Figure 3 – Traverse Example Showing Closure





Traversing along the ground without reducing the distances to the grid causes a misclosure at PINE 1 of 0.425m. The total distance traversed was 5170.34 meters. This equates to an accuracy ratio of 1 part in 12,000 which is the direct result of not applying the grid scale factor of 1 part in 10,000.

Table 2 – SPCS Compared To 'Pseudo SPCS'

		SPCS		Pseudo SPCS			SPCS minus Pseudo			Grid-Grnd
Point Name	Grid Dist	N	E	N	E	Grnd Dist	Delta N	Delta E	Radial	Delta Dist
STEELMAN 2 AZ MK		64612.500	150488.828							
	354.805					354.839				-0.034
STEELMAN 2 RM 4		64948.826	150601.837							
	708.965					709.034				-0.069
T101		65657.760	150595.104	65657.829	150595.103		-0.069	0.001	0.069	
	773.436					773.511				-0.075
T102		66417.949	150737.639	66418.091	150737.652		-0.142	-0.013	0.143	
	713.073					713.142				-0.069
T103		66774.287	150119.985	66774.463	150119.938		-0.176	0.047	0.182	
	639.648					639.709				-0.061
T104		67391.941	149953.694	67392.177	149953.632		-0.236	0.062	0.244	
	668.972					669.037				-0.065
T105		68033.350	150143.741	68033.648	150143.697		-0.298	0.044	0.301	
	773.436					773.511				-0.075
T106		68555.980	149573.600	68556.329	149573.501		-0.349	0.099	0.363	
	892.668					892.753				-0.087
PINE 1		68936.075	148765.899	68936.461	148765.721		-0.386	0.178	0.425	
	476.49					476.536				-0.046
PINE 2		68739.917	148331.659	68740.284	148331.439		-0.367	0.220	0.428	

The U.S. Survey Foot versus the "International Foot" and their relationship to determining Arizona State Plane Grid Coordinates derived from the North American Datum 1983 (NAD-83)

Background:
Back in the late seventies, the U.S. National Geodetic Survey (NGS) encouraged all of the states to revise their respective statutes concerning the impending arrival of the North American Datum of 1983, (NAD-83). As a part of that endeavor, NGS (under direction from the Carter Administration) informed all parties concerned that the basis for the system would be meters in the International System of Units (F. Systeme International d'Unites, or simply SI). This would be in contrast to the existing NAD-27 system of state plane grid coordinates in feet, based on converting from metrics utilizing the "U.S. Survey Foot".

The surveying community in Arizona (the Arizona Professional Land Surveyors, or APLS, which was formed in 1980) took upon itself the objective of working with the Arizona state legislature to revise the state statutes to comply with the suggestions of the NGS.

The members of the legislature were very skeptical of any federal mandate and especially on any system based solely on metrics. The legislators wished to maintain a Arizona State Plane Grid System based on feet, this was even after attempts demonstrating that "the foot system" they were already employing was in reality originating in meters and then converted into feet). Further communications with NGS indicated that NGS would publish NAD-83 coordinates in BOTH meters AND feet (IF so requested and ordained by the respective state legislature), then NGS would apply a conversion from meters to feet using SI units.

With all that information, APLS and the legislators hammered out the revisions to the appropriate Arizona Revised Statutes (ARS) to convert from NAD-27 to NAD-83 coordinate systems. As a part of that change it was noted that the conversion shall be from meters to feet using the SI conversion (as versus the old conversion utilizing the U.S. Survey Foot). These revisions were passed and signed into law.

Meanwhile, the Reagan Administration had taken over, and began reviewing what the previous administration had been up to in the preceding four years. One of Carter's goals was the metrification of the United States to conform to the rest of the world's measurement system. For various reasons tied to free market driven economies, the Reagan Administration considered any mandated federal policy to be unwarranted, and began to systematically dismantle the previous efforts towards metrics.

This including, reversing the previous decisions to have all federal surveying organizations (NGS, USGS, BLM, COE, USBR, DMA, etc.) to begin using the SI conversion to obtain coordinates in feet.

Also, because of the confusion arising from the fact that several states had already acted on the previous request to use SI as the conversion of choice, (and Arizona being one of those states), the NGS decided NOT to calculate and distribute NAD-83 in feet as had been previously promised, but would henceforth and forever after, publish data in meters only!

Currently:
Well...., this has set the stage for some interesting potential problems when dealing with Arizona State Plane Grid Coordinates based on NAD-83.

Where ever and whenever necessary, the federal agencies are using NAD-83 meters and then converting to "feet" by using the U.S. Survey Foot, which is 3.2808333333333... (...3333 as far out as you wish to carry the decimal conversion.)

Meanwhile, the State of Arizona (by law, in the ARS) is mandated to use NAD-83 state plane grid coordinates in feet, (where ever necessary to do so), and they are converted from those SAME metrics coordinates, (the same as those that the feds are using), by using *the international system (SI)* conversion, which is:
1 foot = 0.3048 meter (*International Foot*), -or-
3.280839895.....(constantly changes as you increase the number of decimal places.)

In the U.S. (since 1893) the yard has been defined in terms of the meter by the relation:
1 yard = 3600/3937m or lm = 39.37 inches exactly.
This legal ratio is used to define the "U.S. Survey Foot".

In 1959, the U.S. agreed to establish a new "uniform" relationship between the yard and the meter as:
1 yard = 0.9144 meter exactly or
1 foot = 0.3048 meter (International Foot) or
1 inch = 25.4mm

So What's the problem?:
Since the new value for the yard is smaller by two parts per million than the Survey Foot system (about 1/8 inch per mile!!) one might believe than only in large-scale geodetic survey projects would this difference be important.

The potential for BIGGER problems:
However, the land surveying profession, (and anyone attempting to mix the apples and oranges of NAD-83 coordinates in feet, that are converted from both SI & U.S. Survey Foot), should be uniquely sensitive to the problems that arise in converting metric to non-metric and /or from converting record measurements (non-metric) to metric.

Example:
The origins (the zero, zero (0,0) points of the Arizona coordinate systems were located far enough south and west, so as to insure that there would be no chance of negative coordinates within the respective state plane grid zones (west, central and east).

This means that in the central area of the state near Phoenix you already have "northings" at or 1 million (1,000,000) in feet, after conversion. This is where the problems can arise.

If you were to have NAD-83 **metric** coordinates of 153,000 East, and 290,000 North, and then convert them to **feet** you would obtain:

> 501,967.50 East and 951,441.67 North (rounded to two decimal places, using eight decimals for conversion from a handheld HP-41CX calculator), using 3.280833333 for U.S. Survey Feet.

Using 3.280839895 (**SI conversion**) you would obtain:

> 501,968.50 East and 951,443.57 North.

The difference between the two is +1.00' East and +1.90' North, between the conversion from the U.S. Survey Foot and the conversion from SI.

It is very easy to discern from this quick example that it is of a paramount imperative to know precisely which conversion has been utilized when one receives information in NAD-83 "FEET"!!!

Another item to ponder:
Also -- be aware of the possibility of someone receiving coordinates in feet and "converting" them back to meters, (differently than the original meter to foot conversion), and then giving you what may appear to be "original" meters; (this becomes the "ticking time bomb" scenario waiting to explode later, when and where it can do the most damage).

What to do?

Suggestions:

"ONLY" transfer (and receive) NAD-83 coordinatcs in the "original", unadulterated meters, and if available, also transferred (received) with the geographic coordinates (latitudes and longitudes) from which the metric coordinates themselves are derived, as this is additional verification on the metrics. Then perform any necessary conversions internally within the control of one's own local surveying and mapping environment.

"NEVER" transfer NAD-83 coordinates, IN FEET, outside your own organization (without plenty of caveats).

"NEVER" accept coordinates in NAD-83, IN FEET, without the original metrics and complete confirmation as to the conversion(s) utilized by the originating/sending organization; even then, double check with some independent source as to how the "feet coordinates" were derived.

Into the Future:
Until such time (if ever) as either the federal government changes from the U.S. Survey Foot, to the SI definition; and/or, the State of Arizona revises its ARS to change from SI to the U.S. Survey Foot, -- we will just have to be aware that there are potentially, at least, two separate systems that may be operating within Arizona for state plane grid coordinates based on NAD-83.

Conclusions:
My advice is to learn to live with, (and eventually, begin to love) meters; and it would appear that NAD-83 is a good place to start.

Note:
This reply is not intended to be an exhaustive discussion on the possible, potential corruption of state plan grid coordinates. We have not even begin to discuss such diabolic strategies as the "traverse adjusted grid conversion"; or localized coordinate systems such as the ground level systems; or "regionalized" topography driven systems.

THANKS and Good Luck with any endeavors using SPCs!

AZ footsies
Posted By ***Christian Clausen*** *on 6/27/2001 at 11:07 PM*

Just a quick question...

Several weeks ago I received a glorious letter informing me that I passed the Principles and Practice (professional) NCEES exam.

Next week, I will be sitting for the Arizona state statutes exam (2 hr).

Reading through my copy of the state statutes, I have come across the following wording:

'foot value 0.3048 meter exact'

I know for a fact that 0.3048 meter is an International foot (not a US survey foot, which is 39.37 inches).

Simply put, for my own piece of mind and complete understanding of surveying practice in Arizona, my question is this: Is there an AZ RLS out there that can definitely confirm that the International foot is the 'foot' in AZ?

Any help is greatly appreciated!

Re: AZ footsies
Posted By ***JOE GLIDDEN*** *on 6/27/2001 at 11:21 PM*

Christian,
I thought we all used the U.S. foot. It is only One hundredth difference in a mile, but going out 14,000 miles it might fuss things up a little. I think we should stay in meters most of the time and convert to feet at the last minute. Conversion factors are a bummer. (I live in Washington and we are on the U.S. foot. (39.37/12)

Jobo

Re: AZ footsies
Posted By ***Christian Clausen*** *on 6/27/2001 at 11:47 PM*

I'm with you Jobo... I've always used the US Survey foot or the meter. With that exam coming up in less than a week, I sure am interested in finding out if, in fact, Arizona uses the International foot instead of the US Survey foot.

Anyone else have a clue?

Re: AZ footsies
Posted By ***Grant Bobbitt*** *on 6/27/2001 at 11:53 PM*

Christian,

The definition of the US Foot is 1200/3937 = 0.304800610 meters. (That's all I can display).

This has come up before, I suppose if you do a search up top you might find it.

There is a warning in Foote/Davis that if you want more that 6 decimal place accuracy, you should use the 1200/3937.

Also, Foote/Davis says that many hand held calculators are programmed with the International conversion.

Some of the posters on that previous post warn that if you don't use enough decimal places when you convert state plane coordinates, you can have big problems.

I guess I would get a definition in a reference like Foote/Davis - It looks like your state is rounding it off. I can't imagine they intend to go against everyone else. Same as Jobo except he is giving you the reciprocal equation or how many feet in a meter which agrees with the textbook.
Grant

Re: AZ footsies
Posted By ***Christian Clausen*** *on 6/28/2001 at 12:01 AM*

I appreciate the info Grant.

I feel completely confident in my knowledge of the definitions of the various 'feet' - 39.37"/12 for USft and .3048m for International foot.

My main concern is that the AZ Statute says '0.3048 meter exact value'. I looked up the international foot conversion in a surveying textbook and...viola! '0.3048m (International foot) =denotes exact value'.

I'm with you and Jobo... I don't see why the USft would be deviated from in AZ, but the wording of the statute seems to indicate that that is the case!

Christian (still unsure of AZ 'feet')

Re: AZ footsies
Posted By ***Philip Huntoon*** *on 6/28/2001 at 12:37 AM*

All the HP calculators I have ever used use 2.54cm = 1 inch....ie the 1 foot = 0.3048 meter value; not the survey definition.....

The difference is about 2 parts per million.....

My TI-99 goes out to 14 places; When I fire it up again; I will get a more accurate number....

This 2 ppm "error" can easily get swamped with the temperature error of an EDM....

As Grant says; "when one converts state plane coordinates, you can have big problems."....This is because the 2ppm error is large; when one is away from the reference point of grid...

JoBo; conversion factors are cool Remember that a nickel is 5 grams; a US bill is 1 gram; a penny is 0.750 diameter

Re: AZ footsies
Posted By ***JOE GLIDDEN*** *on 6/28/2001 at 12:48 AM*

Philip,
Well Philip, you are cool. What would this board do without you. I now like Engineers.

Jobo

Re: AZ footsies
Posted By ***Glen Cameron*** *on 6/28/2001 at 6:45 AM*

Using the calculator in Windows I get
1200/3937 =
0.30480060960121920243840487680 9754

Cheers,

Re: AZ footsies
Posted By ***Brian D. Ewing, PLS*** *on 6/28/2001 at 8:32 AM*

Christian,
In Arizona, some boneheads legislated the International Foot as the unit for SPCS '83. The US Survey Foot is still the basis of all PLSS records, though. It will only make a significant difference when converting coordinates, but watch yourself; not all software knows about Arizona's idiocy. BTW, I'm licensed in AZ and CA.
---Brian
Modified By ***Brian D. Ewing, PLS*** *on 6/28/2001 at 8:34 AM*

Re: AZ footsies
Posted By ***Cliff Wagner*** *on 6/28/2001 at 8:42 AM*

Christian,
The use of US vs International foot you are speaking of only relates to the State Plane Coordinate System in Arizona. While I am not in Arizona so I can't relate to you the exact place to look, I do know that this is defined in the SPCS specs, which may in fact be what your looking at...The standard varies State to State...some use international, some US...In either case most daily survey work would show negligible differences. However if you convert from metric values it will make a difference....I suggest you look at reference material for your SPCS to find the standard

Re: AZ footsies
Posted By ***Lewis Clark*** *on 6/28/2001 at 9:56 AM*

Isn't NAD '83 on the international foot? Aren't all states that have legislated NAD '83 using the international foot?

Re: AZ footsies
Posted By ***Christian Clausen*** *on 6/28/2001 at 9:56 AM*

Brian D. Ewing and Cliff Wagner:

I appreciate your sensible, to-the-point answers.

Sure enough, the Arizona coordinate system uses the International foot per my statute book. Seems odd, but at least I know now.

Thankfully for anything other than SPC's the US foot is used.

Appreciate the enlightenment, folks.
Thanks again!

Christian

Re: AZ footsies
Posted By ***Mr. Geodesist*** *on 6/28/2001 at 10:01 AM*

When NGS developed NAD 83 each state was asked to specify which definition of the foot they wanted to use. If there is a state law specifying the International foot this is what NGS will publish. You can look at an NGS data sheet and see what foot has been adopted by any state by looking at the SP coordinates given on the data sheet.

Re: AZ footsies
Posted By ***Cliff Mugnier*** *on 6/28/2001 at 10:13 AM*

For NAD27 State Plane Coordinates, Arizona uses the U.S. Survey Foot.

For NAD83 State Plane Coordinates,
Arizona uses the S.I. Foot.

Re: AZ footsies
Posted By ***angelo fiorenza*** *on 6/28/2001 at 10:17 AM*

Paraphrasing "Surveying-10th Ed., by Moffitt aand Bossler (1998):

When SPSC83 was developed, states were given the option to choose meters, US or International Foot as the preferred units. 9 states adopted the US foot and the meter; 6 states adopted the International foot and the meter; the 21 states adopted the meter exclusively; the remaining 14 states have (at that time) not yet legislated a preference. I remember seeing a table of these states somewhere...I'm guessing it was on an ACSM page somewhere.

Re: AZ footsies
Posted By ***Jeremiah Teague*** *on 6/28/2001 at 10:20 AM*

We were discussing this issue on the Arizona discussion board. Ron Platt sent this first message Many thanks Ron.

Last week I gave you (Laura) a rather abbreviated answer whether we use "International Foot" or "U.S. Survey Foot" (as I did J.O. a couple weeks ago); so for the benefit of all (meaning myself), I have researched this topic and come up with the following. Someone please straighten me out soon if I got his all screwed up ! !

Arizona Revised Statue 33-132 specifically defines the value of the foot to be used in our NAD83, Arizona Central State Plane Coordinate system. You can view this statue at http://www.azleg.state.az.us/ars/33/00132.htm.

In part, the first paragraph of the statute reads:

"33-132. Coordinates of system; zone definitions

A. The plane coordinates of a point on the earth's surface, to be used in the position or location of such point in the appropriate zone of the system, shall consist of two distances, expressed in feet and decimals of a foot (foot value 0.3048 meter exact). One of these distances, to be known as the "X-coordinate", shall give the position in an east-and-west direction, and the other, to be known as the "Y-coordinate", shall give the position in a north-and-south direction. These coordinates shall depend on and conform to the coordinates on the Arizona coordinate system, 1983, of the horizontal control stations of the national geodetic survey in this state, as these coordinates have been determined by the survey." (Underlines added for emphasis).

It is from the 0.3048 meter per foot value we get the 3.280839895 feet per meter value used in our projection files. I.e., the ratio derived is 1 foot / 0.3048 meter = x feet / 1 meter.

The question whether this value is "International Foot" or the "Survey Foot" (or "U.S. Survey Foot") is answered as follows:

Researching conversion charts one can find that the "US foot" = 0.3048006096 meters (to ten decimal places) and that the "International foot" = 0.3048 meters. See http://www.genaware.com/html/support/faqs/gmap/gmap28.htm.

The NGS has a document at http://www.ngs.noaa.gov:80/PUBS_LIB/FedRegister/FRdoc59-5442.pdf that defines the "U.S. Survey foot" as 1200/3937 meter or 0.3048006096 meters.

So, two different sources confirm the value of the "U.S.Survey Foot". It is also now obvious that the value specified by statute (0.3048 meter exact) , and the value of 3.280839895 feet per meter (as derived from 0.3048 meter exact) used locally in our projections is NOT the "U.S. Survey Foot".

The final question whether we are certain the value we use is the "International Foot" or not, is best answered through further study of the NGS document at http://www.ngs.noaa.gov:80/PUBS_LIB/FedRegister/FRdoc59-5442.pdf.

A portion of said document refers to the "International Yard" as 0.9144 meter (or one foot = 0.3040 meter exact from dividing by 3 feet), we can be fairly certain the division of the "International Yard" may in definition be

extrapolated to verbally mean "International Foot". I am sure there is a real statement to this effect out there somewhere.
Also, based on the data provided at http://www.genaware.com/html/support/faqs/gmap/gmap28.htm we may also conclude the answer is "yes we use the International Foot even though ARS 33-132 did not expressly say so".

I also found an article at http://www.wsdot.wa.gov/Metrics/ftmtr2.htm that offers a bit of history (J.O.) regarding the two (left) feet.

Lastly, one might ask why we use the "International Foot"? Especially since the NGS designed the State Plane Coordinate System as a "Geodetic" network, and since the article at http://www.wsdot.wa.gov/Metrics/ftmtr2.htm suggests geodetic applications be based on the "U.S. Survey Foot".

The NGS document at http://www.ngs.noaa.gov:80/INFO/Policy/st_plane.html is the policy by which authority is vested with the states to decide which value is used. Arizona obviously chose...well, you answer that.

Thanks for all the great info Ron! It coincides with what I've learned in the past that it was the State legislature's decision to go with the meter/foot relationship that results in us using the International Foot designation. Here's what the Arc docs say about the differences:

In 1959, the directors of the National Bureau of Standards and the United States Coast and Geodetic Survey agreed on a redefinition of the inch-centimeter relationship. This redefinition defined 1 inch as equal to 2.54 centimeters, exactly, or 1 foot as equal to 0.3048 meters, exactly. However, their agreement stipulated that the older value for 1 meter equaling 39.37 inches, exactly, be retained for identifying the 'U.S. survey foot'. One of the reasons for this retention was that the State Plane Coordinate Systems, which are derived from the national geodetic control network, are based on the relationship of 1 meter equaling 39.37 inches, exactly. The difference between these two values for the foot is very small, two parts per million, which is hardly measurable but not trivial when computational consistency is desired. Fundamental survey units, such as rods, chains, statute miles, acres, sections, and townships, all depend on the relationship of 1 meter equaling 39.37 inches, exactly.

c-ya,
Whit.

Re: AZ footsies
Posted By ***Mark Mayer*** *on 6/28/2001 at 10:37 AM*

Oregon also uses the International foot for NAD83, the US Foot for NAD27.

Re: AZ NAD83 uses International Foot
*Posted By **Earl Burkholder** on 6/28/2001 at 11:40 AM*

NGS has always used meters. When state plane coordinates were "invented" in the 1930's NGS published plane coordinates in feet using 39.37 feet = 12.00 meters. The datum was NAD27.

According to a paper "The metric system for surveyors" by Ralph Moore Berry (1973), prior to 1959 several relationships were used:

British - 1 inch = 25.39997 mm
Canada - 1 inch = 25.4 mm
US - 1 inch = 25.40005

In 1959 a compromise was reached whereby the International Foot would be 0.3048 meters (same as the Canadian 1 inch = 25.4 mm). But thousands of state plane coordinates were already published otherwise and it would not be prudent to change them. Therefore it was decided that (NAD27) state plane coordinates would continue unchanged but they would be called U.S. Survey Foot to distinguish them from other foot units defined by the International Foot relationship. A July 1, 1959 notice in the Federal Register (24 FR 5348) states that the U.S. Survey Foot shall continue to be used "until such time as it becomes desirable and expedient to readjust the geodetic survey networks in the United States, after which the ratio of a yard equal to 0.9144 meter (the international definition) shall apply."

So, once NAD83 is published, which units are to be used for state plane coordinates? Meters? The October-November 1988 issue of P.O.B. has an article (page 90 & 92) by James E. Stem, NGS Geodesist, who describes the senario and requested input from the surveying community on the foot conversion issue. He describes action of several states, Arizona being one of them, who attempted to live up to the intent of the 1959 decision. But the upshot of it all is states can adopt whatever conversion they choose and NGS publishes coordinates accordingly.

Could all this confusion be used as an argument to use meters whenever possible? The meter is defined at a very high degree of precision and used by scientists and spatial data professionals world-wide. Humm.....

*Modified By **Earl Burkholder** on 6/28/2001 at 11:44 AM*

Re: AZ footsies
*Posted By **JOE GLIDDEN** on 6/28/2001 at 11:57 AM*

Earl,
You are right on the money! If we stich to meters like NGS has since 1807 there would be no problem. How do we get the message out? They won't listen to me, I'm too old.

Jobo

Re: AZ footsies
*Posted By **Cliff Wagner** on 6/28/2001 at 1:02 PM*

Oh No I disagree with Jobo...Must be a celestrial transferance going on somewhere...First off I can work in any units you give me as long as they are defined...I often use meters in working on government jobs but I still think in feet..its hard for me to visualize meters because I grew up

with feet, All my conversions that I have stored in my head would have to be restored and there just isn't any more room up there...everyone who works with rods...chains...links..perches..etc can handle a little metric-foot conversion...I do think the metric system will eventual win out but I hope its after I'm long gone

---

Re: AZ footsies
*Posted By **James Webb** on 6/28/2001 at 1:34 PM*

Cliff,

I to have trouble visualizing meters because each of my feet is a foot !

:>}

Jimbo

Re: AZ footsies
*Posted By **JOE GLIDDEN** on 6/28/2001 at 1:38 PM*

Cliff,
I think we agree. All I am saying is to use metric when dealing with the GPS info. Wait until the last minute to convert. (Don't do it in your equipment. Your are dealing with very long distances and as Earl has pointed out, the meter is very well defined. They will also have to wait before converting to meters totally, say in 20 years.

Jobo

Re: AZ footsies
*Posted By **Bill McComber** on 6/28/2001 at 2:50 PM*

Interesting thread, Colorado started with the U.S. Survey Foot. When NAD83 rolled around, the law changed to the S.I. Foot, then at the first sunset review after that, it changed back to the U.S. Survey foot. So, depending on the era of the survey, you can run into various combinations of numbers.

Bill Mc, PLS Co

Re: AZ footsies - from one of the BONEHEADS!
*Posted By **Gregg T.** on 6/29/2001 at 2:06 PM*

Although this topic has been fairly well discussed, I would like to clarify the comment made by Brian Ewing: "In Arizona, some boneheads legislated the International Foot as the unit for SPCS '83."

In the late 1970s early 1980s the National Geodetic Survey was preparing for the transfer from the NAD-27 to NAD-83 datum.

During the initial couples of years (the end of the Carter administration and into the Reagan administration), NGS had proffered a model law, and NGS had assured everyone that they the conversion would METERS and "INTERNATIONAL FEET" - ONLY - and that the U.S. Survey Foot conversion would not be an viable option, and would not be supported.

At that time period (early to mid 1980's) the Arizona Professional Land Surveyors (APLS) Associate was actively involved in promoting the NGS proposed model law (including the International Foot) as the basis for the new Arizona State Plane Coordinate System.

Initially, APLS had supported the NGS recommended language of just METERS as the basis of the conversion from NAD-27-to-83; however, at that time many members in the AZ Legislature were "anti-metric" in orientation and natural proclivity.
So, APLS suggested the (NGS promoted) Int. Ft. conversion, as the only other alternative, which was eventually accepted by the legislative members on the various committees through which the bill needed to traverse.

Various members of APLS, (including myself), from various private, municipal, county, state and federal sector employers testified in favor of the proposed legislation. I was in the federal sector at the time and testified that, indeed, NGS has drafted the model law we were just 'copying' and following their directive; and also noted that NGS had insisted on the International Foot conversion as the only conversion that they would support. (This was from the Carter administration influence, for those of you who might remember, we were all going to be completed metricified by the early '80s. Remember the dualing/dueling speed limit signs in both MPH & KPH ? And remember that the MPH signs were going to be 'phased-out' on the federal highway system?

Anyway, The AZ legislative was approved as drafted.

THEN the Reaganites finally took hold in the Federal Government and various agencies who were hell-on-wheels for metrics began to recant and back off their "all-metrics or nothing-at-all" positions, including NGS.
The initial federal pressure to maintain and preserve the US Survey Foot came from BLM.
The Reagan revolution was rather anti-metrics (similar in philosophical tenure to the AZ legislature, past and present.)
So the next thing we know, NGS has eased it original hard line and no longer pressured the states for metrics and international feet definition in their respective state statutes.

But the damage has been done.
AZ had already redefined it as Int. Ft. versus Survey Feet.

As noted above, this only applies to the AZ SPCGS, the relevant citations are elsewhere in this thread.
And, no one from APLS, or any other survey oriented group, wanted to go right back and open up the legislative process again. If you have ever dealt with government and legislators at any level you will know why they did not return to change the definites.
And, at the time it was not viewed as a major problem, since so few people were actually using the AZSPGCS (the true realization of tremendous impact of GIS, at all levels of government was still a full decade in the future.

The big problem is that the state law did not apply to federal agencies which continued to use USSF versus I.F.

The other insight is that now, when dealing with data in the Arizona State Plane Grid Coordinate Systems, in AZ (if not everywhere) it should be each professional surveyor's due-diligence to inquire as to the meta-data behind the coordinates. Did, in fact, the creator of the coordinators use an I.F. versus USSF conversion.

The problem is so invasive that our survey organization (within a quasi-governmental utility provider) will NOT supply NAD-83/HARN-94-2001 SPGCS except in METERS. Requests and user

of the documents can then convert as they deem appropriate.

We trust no one who submits AZ SPGCS data for which we can not confirm the meta-data to a high degree of verification.

Now, 15 years after the passage of the original AZ SPGCS statute revisions, and 4-years after they became mandatory, it may be time for the state surveying society (APLS) to have a debate on whether the statutes are in need of revision (especially the antiquated section in ARS 33-133 & 137.)

In closing, I would like to relate a quick story about testifying before the AZ Senate Commerce Subcommittee, which was reviewing the proposed legislative changes. A very well know political leader, committee chair, and banker was very, very concerned that if we changed the coordinate system, it would change the location of the property he owned!!!!! At first I thought he was just 'shining' me - and when I chuckled he reacted very negatively, and verbally reprimanded me for me flippant attitude. Before I could reply, I was rescued by another senator on the committee, who was a pilot for a major(at the time Pan Am) airline, and with his background in navigation he could assure the banker (and real estate broker, I might add), that changing a coordinate system would NOT change the location of the property.
This was one of my first insights into how very, very little most people know about our field of expertise, and revealed that an even more minuscule fraction, of otherwise very intelligent people, have any insight and understanding of what it is all about.

Well, this note is another 'novel-for-television' response.

Thanks, if you made it this far, for taking the time to peruse it.

Gregg Tuttle, AZ-RLS

SRP LAND-Surveys Division

RLS member & Chair, AZ-BTR

Re: AZ footsies
*Posted By **JOE GLIDDEN** on 6/29/2001 at 2:17 PM*

Gregg,
What a excellent post! I think Brian was talking more out of frustration than anything else. The whole Metric debate has us all going. Here in Washington State they had us going Metric until last year. Too many horror stories. I believe as surveyors dealing with GPS however we should stay metric and forget the US foot against the International foot.

A US foot man,

Jobo

Re: AZ footsies
*Posted By **Philip Huntoon** on 7/1/2001 at 2:37 AM*

Gregg; great post on the history of the metric stuff....

Philip

Re: AZ footsies
Posted By ***Brian D. Ewing, PLS*** *on 7/2/2001 at 9:30 AM*

Gregg,
Good post, but it reinforces my view that the International Foot was legislated out of ignorance and lack of foresight. Admittedly, at the time no one knew that new technologies (particularly GPS) would cause widespread usage of SPCS83. In my opinion, the International foot is a seldom used unit that should not have been selected for this purpose, given that all land records and SPCS27 use the US Survey foot. If you want to go metric, use the SI Meter, not some half-assed unit like the International Foot (which is a misnomer, since the international standard is the SI Meter). I've performed surveys in 23 countries on 4 continents, and the only place I ever used International feet was in Arizona.
Thanks for the historical info. I didn't know NGS was pushing the international foot at the time.

This should be taken as an example of how not to do things the next time there is a need to legislate units for a coordinate system.
---Brian
Modified By ***Brian D. Ewing, PLS*** *on 7/2/2001 at 3:02 PM*

Post a reply to this thread

DATUMS

Joseph F. Dracup
National Geodetic Survey (ret.)

A DATUM is the foundation on which other structures and/or work are built:

- In surveying, Geographic Information Systems (GIS) and Land Information Systems (LIS) DATUMS are required to provide horizontal and vertical positioning bases for the systems.

- In the interest of uniformity, employing the latest national DATUM for each of the fundamental elements (horizontal and vertical) is best. However, this does not preclude employing several DATUMS in the preliminary compilations provided relationships exist, or can be determined, to acceptable accuracies between the specific DATUMS and the present National DATUM.

- Relationships exist between all U.S. National DATUMS and also between these DATUMS and many local systems. For a discussion on this subject see: Dracup, J.F. 1991, "Methods for Converting Earlier Datum Plane Coordinates to North American Datum of 1983", *Surveying and Land Information Systems*, Vol. 51, No.2, 1991, pp. 75-82.

- Any local DATUM (system), regardless of its internal accuracy and geographic extent, that has not been converted to a National DATUM has only a limited value for GIS and LIS identification purposes. Several possible solutions to this problem follow.

- The best way to convert local systems to a National DATUM or one for which a relationship with the National DATUM exists is to connect the two systems by surveying methods.

  - where only one point of the local system is tied to the National DATUM, all points in the local net can be assigned National DATUM coordinates, but no fully satisfactory verification of the connection is obtained regardless of the number of National DATUM points involved in positioning that one point.

  - for horizontal position conversions, at least two points and preferably three or more points in the local system should be connected to at least two and preferable three National DATUM points. This method provides verification for the differences in distances (scale) and azimuth (orientation) between the National DATUM and the local net. If a recomputation is not feasible, these differences can be incorporated in translation and rotation formulas discussed in Dracup (1991).

  - where a recomputation of the local horizontal system is feasible, but the original observations are not available, angles and distances computed from the local coordinates can be used.

  - for vertical position (elevation) conversions, at least two points (bench marks) and preferably three or more of the local net should be directly tied to at least two National DATUM bench marks. As a general rule, a mean of the differences can be used to convert all the local bench marks.

  - As a last resort, in very isolated places where connections to the National DATUM are not practical, coordinates scaled from the best available map based on a National DATUM may be used. Such coordinates should always be prominently identified as <u>SCALED</u> values.

The following National DATUMS have been or are presently used in the conterminous United States for horizontal surveys.

1 New England 1879-1901
2. U.S. Standard 1901-1913
3. North American (NAD) 1913-1927
4. North American Datum of 1927 (NAD27) 1927-
5. North American Datum of 1983 (NAD83) 1983-

Datums (1), (2) and (3) are identical for all purposes, only the names were changed.

- Prior to 1879 detached pieces of the triangulation were computed on independent astronomic DATUMS. This practice was continued in a few instances after 1879 by the U.S. Coast and Geodetic Survey (USC&GS) for special surveys in isolated regions and by the U.S. Geological Survey (USGS) for mapping control in similar locations such as mining districts, where connections to the national DATUM were not feasible. One such case is a small second-order network observed by the USC&GS for the International Boundary Commission (IBC) at Nogales, AZ in 1892-1893. Most of these independent systems, including the Nogales net, were eventually incorporated in NAD and/or NAD27.

- In 1910, the U.S. Standard Datum was extended into Arizona for the first time. The records may show horizontal control points established in Arizona on the U.S. Standard Datum, NAD, NAD27 and NAD83. As a case in point, positions for points on the Maricopa-Yavapai county line are available for NAD, NAD 27 and NAD 83.

- About 1990, the C&GS/NGS began observing High Accuracy Reference Networks (HARN) on a state-by-state basis with points spaced at 15-60 miles, using the Global Positioning System (GPS). These control points have NAD83 horizontal positions and are considered an upgrade to the existing reference network, but independent of it. The relative accuracy of the HARN positions is about one part in a million. In addition to horizontal positions, the points also have vertical coordinates referred to the reference ellipsoid and identified as ellipsoid heights. These heights can be reduced to orthometric heights, the usual form of elevations, by applying the geoid height at each point. Points in the statewide existing reference system are adjusted to the HARN, resulting in two sets of NAD83 coordinates being available for each point. While the differences in the coordinates are unlikely to be of sufficient size to cause problems when used for GIS/LIS purposes, there will always be opportunities for confusion to arise. For this reason, some sort of positive identification system should be instituted.

The following National DATUMS have been or are presently used in the conterminous United States for vertical surveys

1. Sea Level Datum of 1929 (SLD29) 1929-. In 1973, SLD29 was renamed to the National Geodetic Vertical Datum of 1929 (NGVD29) 1929-
2. North American Vertical Datum of 1988 (NAVD88) 1988-

    - Prior to 1929, there were several adjustments of the National leveling network as it existed in 1898, 1903, 1907 (partial) and 1912 (partial). Each of these computations can be considered a separate DATUM, although the differences in elevation for common bench marks are likely to be quite small.

    - In addition to the USG&GS (C&GS/NGS), geodetic quality leveling has been observed in Arizona by the USGS and other organizations including the railroads.

GEODETIC CONTROL

Joseph F. Dracup
National Geodetic Survey (ret.)

Prior to 1985, most horizontal GEODETIC CONTROL was established primarily at the federal level by the USC&GS/NGS, and mostly by triangulation methods. Other agencies, federal and otherwise including the private sector used mostly traverse methods. These methods, along with rarely used trilateration, are known as the classical methods when employed individually, and as mixed-mode methods when employed in combination.

Since 1985, the Global Positioning System (GPS), employed in the differential mode (DGPS), has dominated the horizontal GEODETIC CONTROL scene and will continue to do so in the foreseeable future. In the differential mode, 2 or more receivers are operated simultaneously observing the same satellites.

Where higher accuracies for vertical control are required, the classical method for establishing vertical GEODETIC CONTROL using levels and rods will continue to be employed. However, for many purposes, the vertical coordinates available from GPS observations, corrected for the geoid-height components, will be satisfactory. Geoid heights can be estimated presently to about 0.1m (4 in.) in many cases and rarely worse than 0.3m (1ft.)

The methods described in the discussion on DATUMs for converting local systems to a National DATUM can be modified when GPS is the modus operandi, especially in those cases where a single Base Station or Continuous Operating Reference Station is employed. Regardless of the procedures developed, the observing scheme should provide sufficient verified information to rescale and reorient the original net to the National DATUM.

GEODETIC CONTROL is expressed by positions defined by latitudes and longitudes and associated plane coordinate systems referenced generally to a National DATUM, such as NAD27 or NAD83, and/or elevations above (or below) an adopted reference surface, such as NGVD29 or NAVD88.

- Plane coordinate systems include the State Plane Coordinate System (SPCS), Universal Transverse Mercator grid (UTM), Local Mapping Systems based on conformal map projections and Tangent Plane grids, used as intended or as modified for scale and elevation, the latter identified as Project Datum Coordinates.

GEODETIC CONTROL is classified in accordance with the relative accuracy between adjacent points at the 95% confidence level, known statistically as 3 sigma.

- Accuracies are expressed in proportional parts for horizontal surveys and by a simple formula for leveling. As examples, the accuracy standard for First-order surveys made following the classical horizontal methods is 1:100,000 between points; and for First-order class I leveling, 1.5mm $\sqrt{K}$ where K is the distance between bench marks in kilometers.

- Accuracy standards for surveys established by classical practices follow:

  Horizontal Surveys

First-order	1:100,000
Second-order class I	1: 50,000
Second-order class II	1: 20,000
Third-order class I	1: 10,000
Third-order class II	1: 5,000

  Vertical Surveys

First-order class I	1.5 mm $\sqrt{K}$
First-order class II	2.0 mm $\sqrt{K}$
Second-order class I	3.0 mm $\sqrt{K}$
Second-order class II	4.0 mm $\sqrt{K}$
Third-order	6.0 mm $\sqrt{K}$

- Accuracy standards for horizontal surveys established by DGPS

AA	1:100,000,000
A	1: 10,000,000
B	1: 1,000,000

  Plus First-order to Third-order class I standards from above.

- Specifications for carrying out field operations, for all accuracy standards, are designed to produce accuracies better by a factor of about 2 than the standard itself. That is, specifications for First-order horizontal surveys will give accuracies of about 1:200,000 on the average and First-order class I leveling about 0.8 mm $\sqrt{K}$.

High-precision photogrammetry surveying (HPPS) can be used to establish densification networks to second-order class II (1:20,000) standards within a reasonable time span and at modest cost.

When favorable topography exists, GPS kinematic surveying methods can provide better results than HPPS for such networks and probably in a shorter time and at less cost, in many instances.

Large amounts of GEODETIC CONTROL data are held by numerous Federal agencies, state and local governments and the private sector. By far, the largest collection, best organized, most up-to-date and the easiest to obtain are the data files of the C&GS/NGS.

- Several Federal agencies, including the USGS, Forest Service among others, maintain large data holdings obtained from their own surveys, most accomplished to geodetic standards or close approximations to them. The one question that must always be asked, when requesting these data, are their current status with NAD83 or NAVD88.

- The same question is mandatory when requesting data from state and local governments and private holdings.

- In other times, the C&GS issued lists of field geographic positions when the adjusted data would be delayed. These computations, while reasonably consistent within themselves, were based on only two positions from the geodetic files and no effort was made to distribute the closures. No SPCS were computed by the C&GS, although it is known some users did and probably considered their surveys to be computed on NAD27. Data history should be known prior to using.

- Project Datum coordinates are coordinates that have been modified to create a grid at average project ground level. When SPCS and other mapping grids are so modified, the modification usually includes averaging out the scale reduction inherent to such systems as well. These grids have long been favored by those whose survey work is at ground level. Unfortunately, they are often placed in the files as SPCS values rather than modified coordinates. In some cases Project Datum coordinates have found their way to other users who thought they were SPCS values, with the inevitable result.

- Caution should always be exercised when using GEODETIC CONTROL data for which little information about its origin exists. Even data from the best sources should be examined for obvious mistakes.

Methods for Converting Earlier Datum Plane Coordinates to North American Datum of 1983

Joseph F. Dracup

ABSTRACT. *With the completion of the North American Datum of 1983 (NAD 83), one of the major questions remaining is, "What should be done with the thousands of points positioned only on North American Datum of 1927 (NAD 27) and earlier datums via plane coordinate systems?" In retrospect, few pre-NAD 27 systems were converted to that datum. The reasons for retaining these pre-NAD 27 systems are valid in many cases, even today. However, if the decision is to convert to the State Plane Coordinate System of 1983 (SPCS 83), recomputation is the best of several methods available. Other effective procedures include the use of translation and rotation equations for pre-NAD 27 systems and weighted mean differences for most SPCS 27.*

Introduction

The new adjustment of the North American Datum (NAD 83) resolved most of the problems inherent to NAD 27 and included almost all of the points in the national geodetic files. The total amounted to more than 250,000 stations of all orders of recognized accuracies. However, a large number of points were connected to NAD 27 and earlier datums by the public and private sector for which observations did not appear in these files. Of course, these points were not included in the NAD 83 fundamental adjustment. Furthermore, many of these points are still being used to extend and control surveys. It is to this group of points and their possible conversion to NAD 83 that this article is directed. Plane coordinates will be used as the means for making the transformations, although references to geodetic practices will be given on occasion.

Plane Coordinate Systems in Use in the United States

Plane coordinate systems are probably as old as surveying itself. In the United States, it is generally recognized that Simeon Borden set up the first comprehensive grid system when he designed a five-zone tangent plane grid for Massachusetts in the 1840s. In the intervening period, several types of grids have been employed.

Pre-NAD 27 Systems

Prior to the development of SPCS in 1933, most systems were based on tangent plane grids. These systems are necessarily restricted to areas within a 20- to 25-mile radius of the origin, limiting the maximum scale error introduced to about 1:50,000. Even today, this grid is entirely satisfactory for most purposes when kept within the aforementioned area restriction. Tangent plane systems are identical in concept to grids long employed by land surveyors, where the origin is defined by assumed coordinates or the points referenced to quadrants relative to the origin with orientation at or nearby the origin.

When converting tangent plane coordinates to any other grid, it is important to remember that many systems were referenced to some elevation surface other than sea level. This matter must be dealt with when converting such systems to other grids. In particular, one must be aware that NAD 83 is referenced to the ellipsoid, not to sea level. The difference between the two surfaces in NAD 83 amounts to about 100 feet in the conterminous United States. This difference is subtracted from sea-level elevations to obtain ellipsoidal elevations. In some instances, tangent plane grids were rotated about the origin; this rotation must be considered in the conversion process as well.

NAD 27 Systems

With the advent of SPCS, which used conformal mapping projections for the grid systems, much larger areas could be accommodated within a zone or band. Corrections could be easily made to account for the scale errors. Only the Michigan Lambert system was referenced to an elevation other than sea level. However, numerous local grids, known as project datum coordinates, were developed for special situations. These grids were derived from SPCS and referenced to local elevations. Such coordinate systems must be handled properly in any transformation procedure. Surprisingly, a few major tangent plane systems were set up after SPCS came into use, notably the one used by the Cleveland Regional Geodetic Survey.

Later, about the time of World War II, the Universal Transverse Mercator (UTM) grid was developed by the military. Still later, special grids, such as the oblique system, were used in the Northeast Corridor

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of the United States. UTM coordinates based on NAD 27 or earlier datums can be transformed to NAD 83 by the same methods described for SPCS. Converting the Northeast Corridor grid and similar oblique systems by approximate methods would require more treatment than is discussed here.

Other Grid Systems

The Military Grid System (Sharp 1943) covered the conterminous United States with seven zones, each 9 degrees wide in longitude. A polyconic projection was used within each zone. This system was developed during World War I and continued in use by the military during the early years of World War II. Coordinates were in yards. Conversion to NAD 83 would probably be done best via geodetic coordinates once the actual datum used is determined. During World War II, several harbor defense grids were established on the Lambert conformal projection. In most cases, the coordinates were defined in yards. Here, too, determining which datum was used may be difficult. In most instances NAD 27 was used for grids in the United States. The method suggested for those SPCS where the fundamental parameters were changed for NAD 83, discussed next, should work for these systems as well.

SPCS 83

With the NAD adjustment of 1983, most states retained their essential SPCS parameters, thus making the transformation process somewhat easier. However, most changed to meters as the defining unit of measure. The SPCS associated with NAD 83 positions is termed SPCS 83, while the previously existing system is distinguished by the designation SPCS 27 (Stem 1989). The National Geodetic Survey (NGS) has so far published SPCS 83 coordinates in meters only, although there are indications that this policy may be modified. Several states retained the U.S. Survey Foot when converting from meters, and about the same number indicated their preference for the International Foot for such conversions. The U.S. Survey Foot was implicit for SPCS 27. Only one state retained the SPCS 27 constant on its easting origin (but in meters), and many placed constants on their northing origins. The changes in origin constants and the unit of measurement used in the particular SPCS 83 must be taken into account when converting from SPCS 27 to SPCS 83.

The following states radically changed the basic parameters for their SPCS 83 (Stem 1989). Consequently, transformations from SPCS 27 to SPCS 83 will be complicated if the recomputation method is not used. A practical solution (Method 2) is suggested later.

1. California, zone 7 SPCS 27, to California, zone 5 SPCS 83
2. Montana, North, Central, and South zones SPCS 27, to Montana, SPCS 83
3. Nebraska, North and South zones SPCS 27, to Nebraska, SPCS 83
4. New Jersey, SPCS 27, to New Jersey, SPCS 83
5. New York, East zone SPCS 27, to New York, East zone SPCS 83
6. South Carolina, North and South zones SPCS 27, to South Carolina, SPCS 83

Changes in the easting origins for the Michigan Central and South zones and the Texas North Central zone also complicate the conversion process. This is because while the angular change in the particular convergence is a constant, it does not translate into a constant linear shift in the Eastings. Method 2 (described later) is satisfactory for use in these instances, too.

Changes in scale-factor parameters for zones in Mississippi and Wyoming can be converted to SPCS 83 by scaling SPCS 27 accordingly. Shifts in the Northing origins for the Louisiana zones, Maine East zone, Maryland zone, Mississippi zones, New York Long Island zone, Tennessee zone, and Wyoming zones for SPCS 83 can be taken into account when equating SPCS 27 to SPCS 83.

Synopsis of the Problem

The first and perhaps the most important question that needs to be answered is, "Should coordinates for points on earlier datums be converted to NAD 83?" Clearly, numerous considerations enter into this decision. The most obvious is cost, not just for the conversion process, but also for time spent documenting these changes or notations in the records and derived documents. Another is the need for users to familiarize themselves with a new system, especially users of pre-SPCS 27 grids.

Perhaps the most important consideration is, "Will the new system be better than the one it replaces?" Relative accuracy of converted points falls into this category as well. However, for cartographic and parcel identification purposes, a lesser accuracy is usually more acceptable than for control or extension of control needs. These are the more basic considerations. Other factors may carry more weight in some instances, of course. A brief discussion of the various systems follows.

Pre-NAD 27 Systems

Possibly about 100 to 200 local coordinate systems of various sizes were connected to earlier datums. Most systems were developed for cities, counties, water-

ways, and a few large land parcels such as mines, major engineering projects, and the like. Some were converted to NAD 27; a majority were not. Those systems that were transformed served mostly cartographic purposes. It can only be assumed that most of the other systems continued to be satisfactory for local survey control and mapping uses. At least one system proved unsatisfactory, but the local surveyors found ways to work around the problems. One point cannot be overemphasized: no significant improvement in relative accuracy of points in pre-NAD 27 surveys can be expected even if a completely new computation is made. Furthermore, in those cases where the conversion is made via translation and rotation formulas or similar approximations, some lessening in accuracy is to be expected.

NAD 27 Systems

Surveyors undoubtedly connected thousands of local surveys to SPCS 27. Of this number, possibly only 200 or so are in the major system category, such as those established for or by states, counties, and cities. Most of the basic points in these surveys became part of NAD 83; however, only a handful of the large number of points supplemental to these nets were included. It is here that the greatest conversion effort is expected to be found in the coming years. The basic considerations enumerated earlier apply here as well, although the matter of familiarity is not thought to be as big a problem. Unlike the pre-NAD 27 surveys, there is a possibility that the accuracy of points connected to NAD 27 might be improved if the surveys are actually recomputed rather than converted by empirical methods. These improvements could result because the constraints of NAD 27 were largely removed in NAD 83. In instances where other conversion methods are used, accuracy estimates will have to be made on a case-by-case basis.

Conversion Methods

Prior to the adoption of NADCON (Dewhurst 1990a, 1990b) by the Federal Geodetic Control Committee (FGCC) in 1990 as the national standard for converting NAD 27 geodetic coordinates to NAD 83 (American Congress on Surveying and Mapping [ACSM] 1990), NGS recommended three methods for transforming NAD 27 to NAD 83 (Wade and Doyle 1987). Vincenty (1987) examines the entire problem. Other investigations have taken different approaches (Shrestha 1987; Vincenty 1988; Shrestha and Dicks 1990), including a solution using polynomials (Moffitt and Jordan 1987). All have their merits. Some use SPCS in the process. None touches specifically on pre-NAD 27 systems, although two NGS proposals, recomputation and average latitude and longitude shifts or equivalent plane coordinate shifts, can accomplish the task. Recomputation is included again in the following discussions, but with a less rigorous emphasis than suggested by NGS. Only two examples are given here, neither being complete in a total computational sense. Regardless, both should provide sufficient details for anyone considering the use of either method. A few comments on graphic solutions are found at the end of this section.

Recomputation

Recomputation should always be considered the method of first choice when using the new coordinates for survey control or extension of control. A rigorous adjustment is best, but any recomputation method that distributes the closures resulting from substituting the SPCS 83 coordinates for the control points is far better than any empirical solution. This is true even in those cases where it is necessary to employ previously adjusted (balanced) azimuths and distances in the process.

Since many local surveys are based on a single control point and established by spur or loop traverses, recomputation seems the simplest solution. No scale correction for length changes between NAD 27 and NAD 83 come into play, only azimuth changes. Moreover, if less than 4 seconds of arc, such changes can often be ignored because the maximum error introduced in the coordinates is only about 1:50,000 of the distance between points. Standard rotation formulas can be employed to compute the shifts in the coordinates rather than a strict recomputation, if desired.

Pre-NAD 27 Systems

Except for a few pre-NAD 27 local systems that use only geodetic coordinates, the majority of systems were tangent plane grids. These will be transformed best by translation and rotation (T and R) equations (1) and (2). There may be a few instances, however, when the conversions would be better handled via geodetic coordinates. In such cases, see Dracup et al. (1973) for a method to compute geographic coordinates from tangent plane coordinates. For this computation, it is essential that the distances be at sea level (ellipsoid for NAD 83).

The basic T and R formulas follow:

$$N_P = N_0 + yM\mathrm{Cos}A - xM\mathrm{Sin}A + K_N \quad (1)$$

$$E_P = E_0 + xM\mathrm{Cos}A + yM\mathrm{Sin}A + K_E \quad (2)$$

where

(N_P, E_P) = converted SPCS 83 plane coordinates

(N_0, E_0) = SPCS 83 plane coordinates for the origin

(y, x) = $(Y - Y_0)$ and $(X - X_0)$ respectively, where (Y_0, X_0) are the local coordinates of the origin and (Y, X) are the local coordinates of the point.

M = scale change expressed as a factor, consisting of the average change in length (ΔL) due to the new adjustment and the average SPCS 83 scale factor (SF) for the included area of the SPCS, multiplied by 1.0000048 to place the local coordinates on the ellipsoid. Also, where the local system is not at sea level, the elevation factor (EF) for the elevation basis must be included.

A = orientation shift consisting of the SPCS convergence at the origin (C_0) plus the rotation angle of the local system (R_0), if any, the sum taken negatively (i.e., $-[C_0 + R_0]$), plus the average change in azimuth (ΔA) due to the adjustment. C is positive for points east of the central meridian and negative to the west.

(K_N, K_E) = smoothing out quantities that can take almost any form and, if needed, are usually applied at the extremities of the system. The need for these quantities is generally evident when checking the equations with known data.

Note that the quantities involving (x, y) must be in the same unit of measurement as (N_0, E_0). In theory, (A) should include the second term or arc-to-chord correction at the origin, but since this correction varies with length and direction of line, it is best to apply it to the converted coordinates individually should the need arise. These corrections are largest at the extremities of a zone (N-S for the Lambert system and E-W on the transverse Mercator grid). They can amount to more than 3 seconds of arc (about 1:70,000 of the distance) on 10 km lines (Adams and Claire 1935a, 1935b).

The average change in length (ΔL) and azimuth (ΔA) due to an adjustment can be determined by taking a representative sample of the data for each parameter for points on both datums. Unless one is familiar with geodetic inverses, the easiest approach is via plane coordinate differences including the origin in each computation (equations [3] and [4]). In a large sample, points either very close to the origin or far away should, as a general rule, be excluded. In many cases, only those points used as control for the scheme to be converted should be involved in these determinations.

Changes are computed as follows:

$$\Delta L = [(\Delta N^2 + \Delta E^2)^{1/2}/SF] - [(x^2 + y^2)^{1/2}\, EF(1.0000048)] \quad (3)$$

$$\Delta A = [(\mathrm{Tan}^{-1}\, \Delta E/\Delta N) + C_0] - [(\mathrm{Tan}^{-1}\, x/y) - R_0] \quad (4)$$

where (ΔL, ΔA) are the changes in length and azimuth; (ΔN, ΔE) are the differences (point minus origin) in the SPCS 83 coordinates; C_0 is the SPCS 83 convergence at the origin; R_0 is the rotation angle of the local system, if any; SF is the average SPCS 83 scale factor for the included area; EF is the elevation factor, if any, to reduce the local system to sea level length; (1.0000048) is the elevation factor to place the sea level length on the ellipsoid, and (x, y) are as before (equations [1] and [2]) in the same unit of measure as (N, E).

For those cases where a geodetic solution is considered best, results from equations (3) and (4) can be used to correct the local system distances and azimuths to NAD 83. In tangent plane systems, azimuths from the origin and distances reduced to sea level (ellipsoid for NAD 83) are considered to be geodetic values. Resulting geographic positions (Dracup et al. 1973) are considered to be on NAD 83. Where needed, some form of smoothing out corrections might be considered. See the previous statement concerning the K_N and K_E quantities.

Accuracy of Converted Coordinates

The error introduced by using a mean scale factor and ignoring the second term (arc-to-chord correction) in equations (1) and (2) would rarely amount to more than 1:100,000 of the distance between the origin and a point. The largest errors are found in those instances where the range of differences on which ΔL and ΔA are based is wide. Several such cases were noted in the past when developing (T and R) formulas for converting pre-NAD 27 coordinates to SPCS 27. For example, the basic nets for Queens and Richmond (Staten Island) counties, New York, were warped in fitting them to NAD 27. As a result, some converted coordinates for supplementary surveys were in error by as much as 0.4 m, although errors of 0.1 m to 0.2 m were the rule. Similar results for similar reasons were found in several local Maryland systems and elsewhere. Most of the principal points of these and many other local nets were included in NAD 83. The results for the converted coordinates should be improved, perhaps by as much as a factor of two.

The hypothetical example shown in Table 1 uses (T and R) equations. Real data are employed for two points in the Arizona Central zone. The excellent

Table 1. Hypothetical example using (T and R) equations.

NAD 83		LOCAL SYSTEM (1,250 ft. elev.)
AM 2	N_0 = 289,500.346	PEORIA x = +2,535.789m.
	E_0 = 192,345.905	y = -3,378.266m.
	-C = +0° 07' 31"28	ΔL = 1.0000050
	ΔA = - 02.00	SF = 0.9999048
	A = +0 07 29.28	EF = 0.9999450 (1,150ft)
		M = 0.9998548

	Converted NAD 83	NAD 83	Diff.
PEORIA	N_p = 286,117.056	286,117.065	+0.009
	E_p = 194,873.962	194,873.983	+0.021

agreement between the converted and known SPCS 83 coordinates is due to the fact that the NAD 27 Phoenix net, on which the coordinates are based, was changed only slightly by the NAD 83 adjustment.

NAD 27 SPCS

Earlier, it was mentioned that several methods were available to transform SPCS 27 to SPCS 83 values. This section discusses two additional methods. The first is used for converting those SPCS 27 coordinates where the fundamental constants remained essentially unchanged for SPCS 83, and the second applies where the constants changed significantly.

Method 1 is based on the premise that where four or more points are common to NAD 27 and NAD 83, well-known, weighted-mean practices can be used to satisfactorily convert SPCS 27 points in the same general vicinity to SPCS 83. For best results, the points common to both datums should be about equally spaced. There is one condition, however: The points known only on NAD 27 must have been positioned from national network points identical to or near the control points. While all tests of the method involved four or more known points, situations are bound to arise, especially for traverses, when only two or three points are known on both datums. In these cases, common sense dictates that one use what is available, unless this results in an extremely poor geometric relationship. Among such poor relationships are those cases where three or more control points are on line with the points to be converted, and those where one or more of the control points are very distant from the points. In these instances, the use of only the closest control points should be considered, even if this means using only a single control point for conversion purposes.

Method 2 involves selecting a point common to both datums as the origin for a modified tangent plane system, where the local coordinates are simply the differences between the SPCS 27 coordinates of the origin and the SPCS 27 coordinates of the other points (i.e., $[X - X_0]$ and $[Y - Y_0]$). The standard (T and R) equations (1) and (2) and the (ΔL) and (ΔA) equations (3) and (4) are also modified to account for differences in convergence and scale factors.

Method 1. Step 1: Compute the datum differences (ΔN, ΔE) for the points selected as control for the conversion process as follows:

$$\Delta N = N - N_c, \text{ where } N_c = YQ + N_0 \quad (5)$$

$$\Delta E = E - E_c, \text{ where } E_c = (X - X_0)Q + E_0 \quad (6)$$

(N, E) = SPCS 83; (N_c, E_c) = SPCS 27 translated to SPCS 83

(X, Y) = SPCS 27; and X_0 = SPCS 27 constant on origin

Q = SF_1/SF_2, where SF_1 is the scale factor on the central meridian for SPCS 83 and SF_2 is the SPCS 27 scale factor on the same meridian. This quantity only enters computations for Mississippi and Wyoming zones.

(N_0, E_0) = constants on origins for SPCS 83. N_0 also includes differences between SPCS 27 and SPCS 83 latitude origins where necessary, and these will need to be computed.

Note that Y and $(X - X_0)$ must be in same units as (N_0, E_0). For Michigan North zone, SPCS 27 must be divided by 1.0000382 to reduce the coordinates to sea level.

Step 2: Compute the distances between each SPCS 27 only point and the control points using SPCS 27

coordinates and standard grid computation methods for distance (i.e., $S = [\Delta X^2 + \Delta Y^2]^{1/2}$).

Step 3: Compute weights for each line as follows:

$$P = 1/S^2 \tag{7}$$

where P = weight and S^2 = distance squared. S should be divided by some number to make $1/S^2$ between 0.5 and 10 in most instances. The same divisor must be used for all distances in each set of weights. In the following example, (S) was divided by 100,000.

Step 4: Compute the weighted mean shifts (ΔN_p, ΔE_p) accordingly:

$$N_p = (P_1 \Delta N_1 + P_2 \Delta N_2 + P_3 \Delta N_3 \ldots)/(P_1 + P_2 + P_3 \ldots) \tag{8}$$

$$E_p = (P_1 \Delta E_1 + P_2 \Delta E_2 + P_3 \Delta E_3 \ldots)/(P_1 + P_2 + P_3 \ldots) \tag{9}$$

Step 5: Translate the SPCS 27 values to SPCS 83 (equations [5] and [6]). The converted coordinates (N_p, E_p) are

$$N_p = N_c + \Delta N_p \tag{10}$$

$$E_p = E_c + \Delta E_p \tag{11}$$

This method was tested in four locations of the network. Brief reports for each location follow. An example of the application of Method 1 is shown in Table 2.

1. Phoenix, Arizona: For eight points within a quadrilateral with perimeter sides averaging about 25 km, the average vector length was 0.04 m, with a maximum of 0.09 m. Other tests were also made with equally good results. The NAD 27 network is minimally constrained.
2. San Francisco, California: For 16 area points controlled by four stations with corners spaced about 27 km apart, the average vector length is about 0.11 m, with a maximum of 0.22 m at a point some distance from the area boundary. The NAD 27 network appears to be somewhat constrained.
3. Gainesville, Florida: For seven points in an arc about 60 km in length with two control points at each end, the average vector length is 0.24 m, with a maximum of 0.35 m. The NAD 27 net is moderately constrained.
4. Duluth, Minnesota: For 42 area points controlled by four stations at the corners spaced about 85 km apart, the average vector length is 0.40 m, with a maximum of 0.85 m. Additional tests with closer spaced control points showed a marked improvement in the vector lengths. Example is from these tests. The original vector length for HINCKLEY was 0.81 m. In this area, the NAD 27 network is very distorted.

On the basis of these tests, and assuming that control is available in acceptable configurations at about 15 km spacing, the accuracy of the converted coordinates using Method 1 can be expected to be on the order of 0.1 m to 0.2 m. Even in the worst case, the error should rarely exceed 1.0 m.

Method 2. Coordinates in the Michigan East, Central, and West SPCS 27 zones that were eliminated in SPCS

Table 2. SPCS 27 values transformed to SPCS 83 values for station HINCKLEY in Minnesota Central zone using Method 1.

Control Stations	ΔN	ΔE	Distance	P
POMROY	-4.655	-19.325	58,724	2.8998
SANDSTONE	-5.180	-18.534	34,271	8.5143
CROSBY	-4.927	-18.839	51,427	3.7811
BEROUN	-4.465	-18.889	51,235	3.8095

Note: ΔN and ΔE in meters. Distances to HINCKLEY are in feet.

HINCKLEY	N_c =	216,738.115	E_c =	902,236.092
	ΔN_p =	- 4.906	ΔE_p =	- 18.787
Converted SPCS 83	N_p =	216,733.209	E_p =	902,217.305
SPCS 83	N =	216,733.050	E =	902,217.308
		- 0.159		+ 0.003

83 can be converted by this method. However, when using this procedure in those states in which two or more SPCS 27 zones were consolidated into a single SPCS 83 zone (Montana, Nebraska, and South Carolina), more consideration must be given to the arc-to-chord corrections (see note accompanying equations [1] and [2]). In some cases, rapidly changing scale factors may limit the size of the area to be converted. No major problems in methodology are expected for the other SPCS suggested for this method.

After selecting some point common to both datums as the origin, the local SPCS 27 coordinates (y, x) are translated and rotated by equations (12) and (13):

$$N_p = N_0 + yMCosA - xMSinA + K_N \quad (12)$$

$$E_p = E_0 + xMCosA + yMSinA + K_E \quad (13)$$

(N_p, E_p), (N_0, E_0), (y, x), (K_N, K_E) are defined for equations (1) and (2).

$$M = (SF_1/SF_2)(\Delta L)(1.0000048)$$

where SF_1 is the average SPCS 83 scale factor for the included area, SF_2 is the equivalent SPCS 27 value, ΔL is the change in length due to the adjustment expressed as a factor, and 1.0000048 is as defined previously. An elevation factor (EF = 0.9999618) will also be needed to reduce Michigan Central and South zones (SPCS 27) to sea level.

$$A = [-(C_1 - C_2) + \Delta A]$$

where C_1 is the SPCS 83 convergence for the origin and C_2 is the SPCS 27 value. ΔA is the change in aximuth due to the adjustment. ΔL and ΔA are computed as follows:

$$\Delta L = [(\Delta N^2 + \Delta E^2)^{1/2}/SF_1] - [(x^2 + y^2)^{1/2} \quad (1.0000048)/SF_2]EF \quad (14)$$

$$\Delta A = [(Tan^{-1} \Delta E/\Delta N) + C_1] - [(Tan^{-1} x/y) + C_2] \quad (15)$$

See equations (3), (4), (12), and (13) for definitions of notations. EF is the elevation factor when Michigan Central and South zones coordinates are involved. The same general comments given with the section on converting pre-NAD 27 systems apply here.

Graphic Methods

In the early years following the completion of NAD 27, conversions were occasionally made by plotting lines of equal coordinate change and scaling the datum shifts. Such methods are probably useful even today for some purposes. Speert (1937) and Sharp (1943) give an example and explanation using geographic coordinates. SPCS can be substituted for these values.

Conclusion

Three major points are made. First, it is not mandatory to convert a survey system to NAD 83. Second, when a decision is made to convert a system, recomputation is the best solution, with the amount of rigor depending on the particular circumstances. Third, when an approximate solution is acceptable, any of several transformation methods will provide more or less the same accuracy in the converted coordinates, although some may be more appealing in a mathematical sense or more adaptable to computers.

ACKNOWLEDGMENT

An earlier version of this article was presented at the ACSM/ASPRS Annual Convention in Denver, Colorado, March 18-23, 1990.

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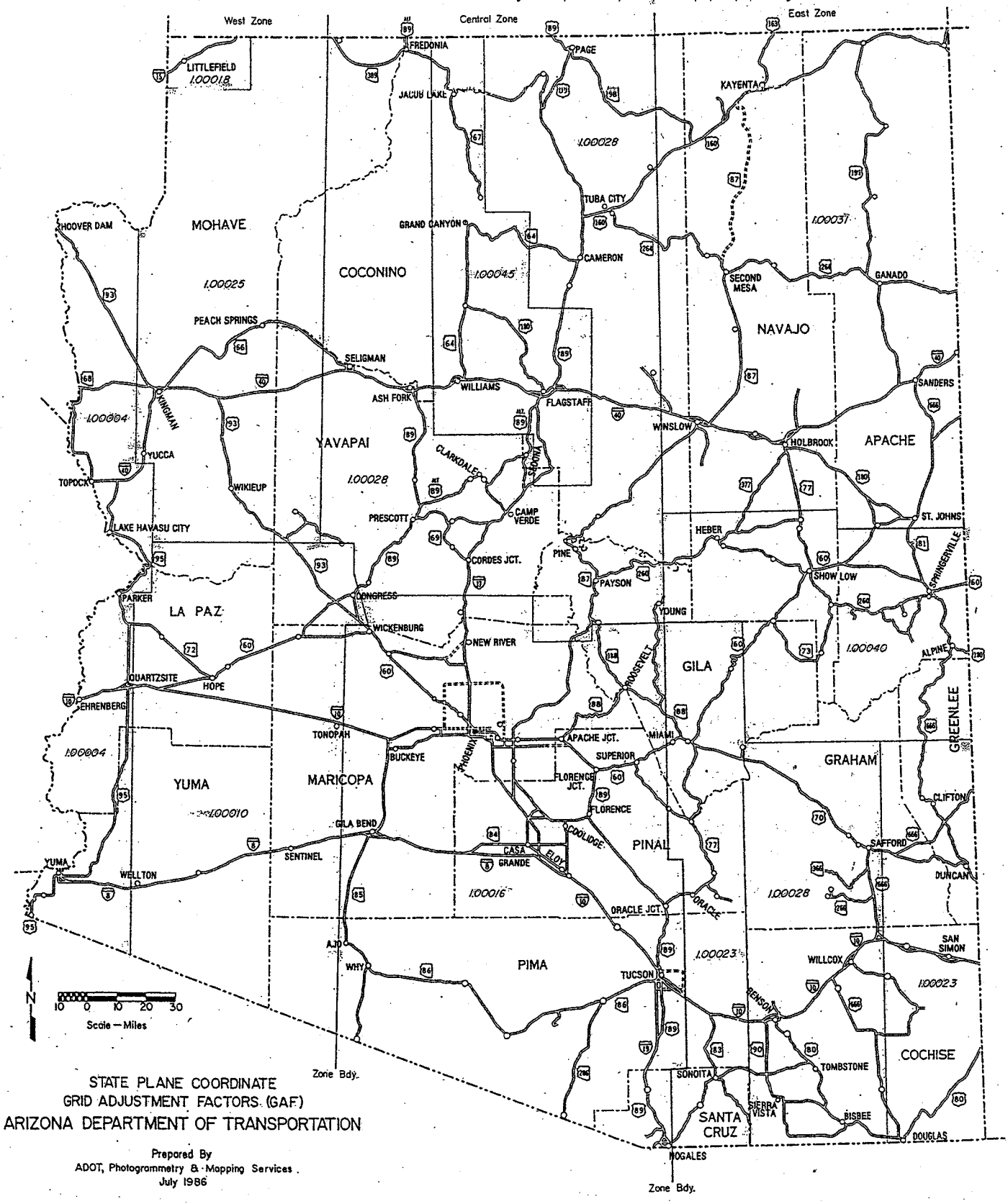

STATE PLANE COORDINATE
GRID ADJUSTMENT FACTORS (GAF)
ARIZONA DEPARTMENT OF TRANSPORTATION

Prepared By
ADOT, Photogrammetry & Mapping Services
July 1986

In the absence of a Data Quality Report, ask questions about undocumented data before you use it.

- What is the reliability of the provider?
- Is the data relevant to the project at hand?
- What projections, coordinate systems, and datums were used?
- Where did it come from?
- What is the age of the data?
- Why was the data compiled?
- How was the data checked?
- What was the density of observations used for its compilation?
- How accurate are the positional data?
- How precise are the positional data?
- Does the data seem logical and consistent?
- In what software programs was it originally produced?
- In what format is the data kept?
- Who originally performed the work? The review and checks?

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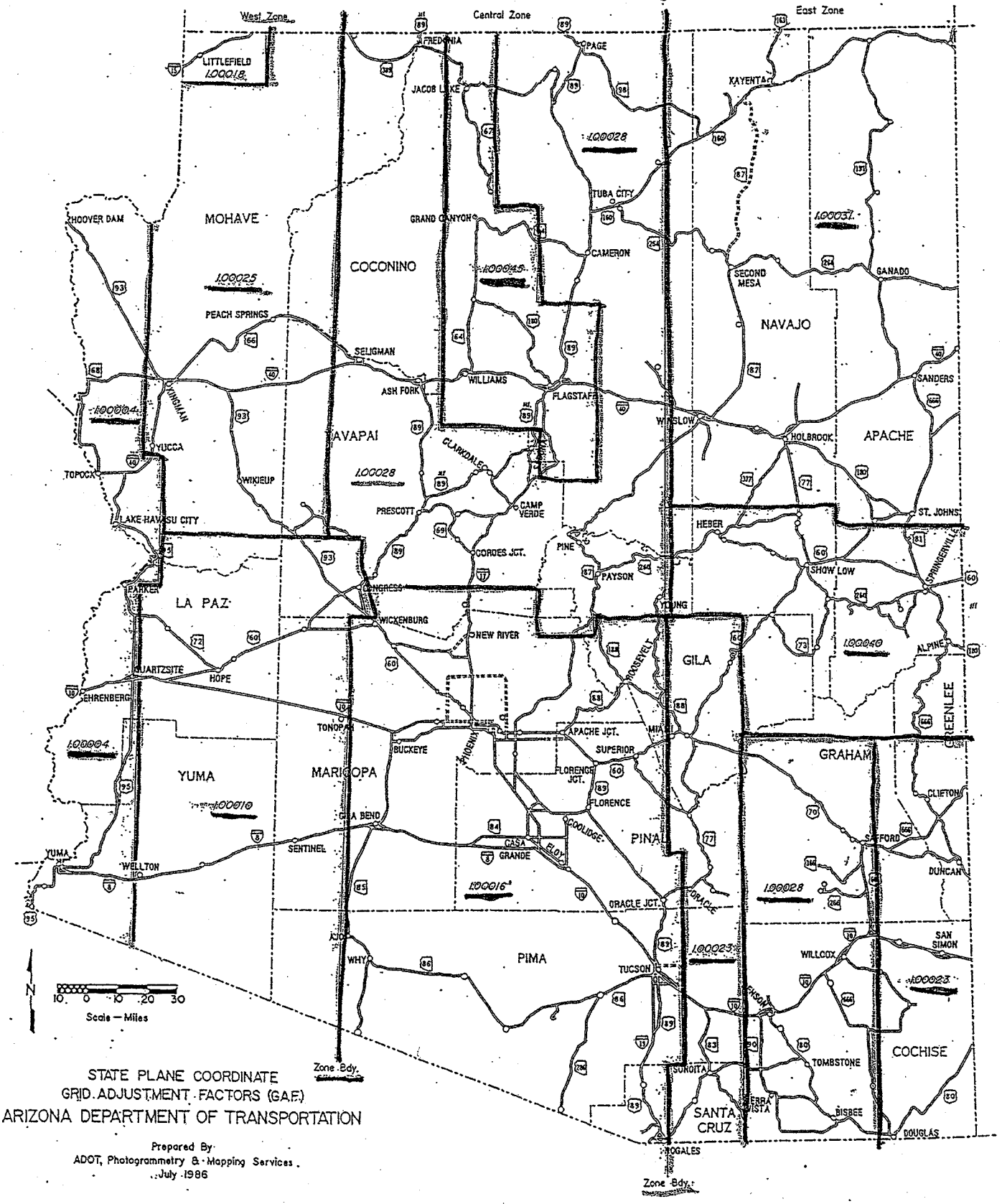

STATE PLANE COORDINATE
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THE HORIZONTAL CONTROL SYSTEM

The basis of the horizontal control system for the City of Scottsdale is the Land Survey performed by Hunsaker and Associates (H&A) as published on January 17, 1991. H&A values were achieved through the use of the Global Positioning System (GPS). H&A data in their report included latitude and longitude values along with respective state plane coordinate values.

The City's Land Survey Unit converted the H&A state plane coordinate values into a modified coordinate system for the purpose of establishing a horizontal basis for the City's Geographic Information System (GIS) . The method used in creating the modified coordinate system is unique but produced values compatible with the varying surface of the earth. The modified coordinate system will be hereinafter referred to as the "City Grid".

Station 7192 of the H&A report is the initial point of the City Grid. Station 7192 was also a primary control station of the H&A network. The state plane coordinate of this station has the same numerical value as the City Grid bearing the same station number. All other state plane values will be different when compared to City Grid values. The bearing and distance value determined by inversing between any two City Grid coordinates will be equal to actual field measurements on the surface. Station 7192 is a 1919 Government Land Office (GLO) brass cap on a 2 inch iron pipe 0.88 feet above the surface representing the Northeast corner of Section Nineteen (19), Township Four(4) North, Range Five(5) East of the Gila and Salt River Meridian, Maricopa County, Arizona, Latitude 33° 40' 01.02932" North, Longitude 111° 52' 26.82154" West and being also the intersection of Deer Valley Road and 96th Street.

A scale factor of 0.999801686 (ASF) was determined as the common scale factor. The common scale factor represents the average of all H&A station scale factors for stations within the city. The common scale factor was applied to each state plane coordinate station inversed from the initial point. This method yielded reasonable accuracy when comparing any station to station dimension for surface values.

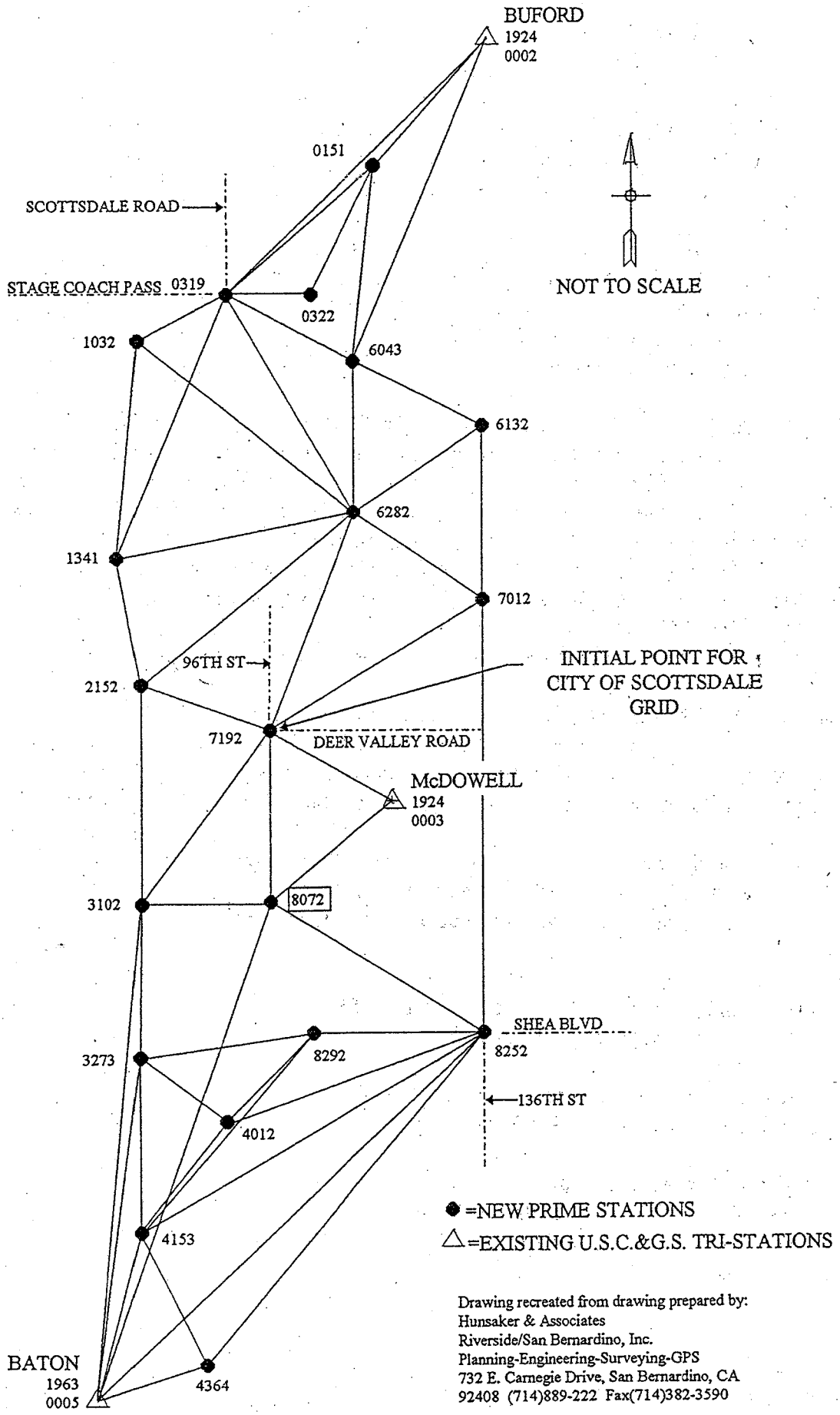
The GIS has as its origin the City Grid as established through the above described process. The City Grid coordinates are a part of this document and are to be used when doing work within the City of Scottsdale having to do with land measurement and when data is presented to the city for its consideration.

Example:

COS ground distance = ASP Grid Distance ÷ ASF
5000.992 = 5000.000 ÷ 0.999801686

To ASP Distance = COS ground distance x ASF
5000.000 = 5000.992 x 0.999801686

ASP refers to Arizona State Plane.
COS refers to City of Scottsdale.
ASF refers to Average Scale Factor for COS.



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CITY of TEMPE

All coordinates shown hereon indicate project datum ground coordinates, projected from the Arizona Coordinate System, Central Zone, by a combination scale and sea level factor of 1.0001572*, and translated by –200,000 in the X direction, and by –600,000 in the Y direction.

Coordinate values hereon are listed in Y-X, or Northing-Easting, order.

To convert coordinate values shown hereon to their corresponding Arizona Coordinate System values, utilize the following formulas.

X_{ACS} = X coordinate value on Arizona Coordinate System
Y_{ACS} = Y coordinate value on Arizona Coordinate System
X_{PD} = X project datum coordinates
Y_{PD} = Y project datum coordinates

$X_{ACS} = (X_{PD} + 200{,}000) \times 0.9998428$
$Y_{ACS} = (Y_{PD} + 600{,}000) \times 0.9998428$

Distances shown hereon indicate GROUND LEVEL (1200 feet A.M.S.L.) distances converted from sea level datum.

* Derivation of Combination Factor.

In the Central Zone of the Arizona State Plane Coordinate System, the grid distances along the Central Meridian (longitude 111• 55', approximately midway between Rural and McClintock Roads) are <u>shorter than</u> the actual *geodetic* and ground distances.

The approximate Scale Factor to apply along the Central Meridian is 1/10,000.

AVERAGE Scale Factors derived for the City of Tempe were:

0.9999002	ground to grid
1.0000998	grid to ground

The Sea Level Reduction Factor, assuming a mean radius of the earth of 20,906,000 feet and an average ground elevation of 1200 feet, is approximately:
(20,906,000/20,906,000 + 1200) = 0.9999426 ground to seal level, the inverse = 1.00000574 sea level to ground

The COMBINED FACTORS therefore become:

(0.9999002 x 0.9999426) = 0.9998428	ground to sea level grid
(1.0000998 x 1.0000574) = 1.0001572	sea level grid to ground

Greg Tuttle
SRP
From Mike Nunez

CITY OF PHOENIX

Quarter Section Corner Compilation

On

Arizona State Plane Coordinate System

With the exception of those quarter corners and section corners located by the Photogrammetry and Mapping Division which are indicated for each township; all remaining corners were coordinated using the City of Phoenix Quarter Section Survey Data.

All township boundary traverses began and ended on corners coordinated by the Photogrammetry and Mapping Division from established control points. The section and quarter section line traverses began and ended on township boundaries and were run, in most instances, from East to West using the counter clockwise angles. Identical results could not be obtained when using clockwise angles or traversing from South to North. For this reason and because many of the angles and distances indicated on the quarter section maps are computed, caution is advised when using this data even though the majority of the traverses indicate good closure.

A copy of the City of Phoenix Quarter Section Index Map and Quarter Section Survey Data Maps should be obtained to relate each point number to the proper corner. In all north townships the lower left hand corner is designated by the quarter section number. In the south townships the upper left hand corner is designated by the quarter section number without the leading 0.

Should there be any questions regarding the compilation of this data please contact the Photogrammetry and Mapping Division, Geodesy Branch, Arizona Highway Department. Any questions regarding the actual corner description and location should be referred to the City of Phoenix Quarter Section Mapping Section. In some instances more than one corner is in existance.

T1SR2E

Grid Factor: 0.9998444
Ground Factor: 1.0001556

	ADJUSTED DATA		RAW DATA		DIST.
Boundary	1.4'	1:20,240	1.2'	9"	29,068
27th Ave. N-S	0.7'	1:14,991	0.8'	5"	10,597

Boundary was run independent of T1SR3E to obtain acceptable closure. Corners are evidently not common. This resulted in duplicate point designation on the common boundary.

T1SR3E

N 6, N 5, N 4, NW 3, N 3, NW 2, N 2, NW 1, N 1 and NE 1 were located by P&M Survey Data.

	ADJUSTED DATA		RAW DATA		DIST.
Boundary	2.6'	1:25,488	3.0'	38"	65,817
S. Mountain Ave.	1.2'	1:17,295	1.2'	7"	20,965
Dobbins Road	1.5'	1: 8,719	1.1'	25"	13,159
Mineral Road	.7'	1:15,998	.6'	7"	10,540

Boundary was run independent of T1SR2E to obtain acceptable closure. Corners are evidently not common. This resulted in duplicate point designation on the common boundary.

T1SR4E

N 6 located by P&M Survey Data

	ADJUSTED DATA		RAW DATA		DIST.
NW 1/4 Sec. 6	.4'	1:25,668	.4'	0"	10,550

T1NR4E

Grid Factor: 0.999850
Ground Factor: 1.00015

West Boundary common to T1NR3E
North Boundary Roosevelt St., Van Buren St. and Broadway Road are defined by P&M Survey Data also NW 20, NW 32, and S 32.

	ADJUSTED DATA		RAW DATA		DIST.
East Boundary to Broadway Rd.	1.9'	1:11,261	3.5'	38"	21,217
East Boundary to Baseline Rd.	1.0'	1:13,703	1.0'	4"	13,223
Vineyard Rd.	.2'	1:27,913	.5'	25"	5,243
Southern Ave.	.1'	1:47,244	.1'	14"	5,250
Roser Rd.	.2'	1:24,022	.4'	11"	5,236
Elwood St.	.2'	1:33,970	.2'	8"	5,237
Transmission Rd.	.3'	1:19,767	.3'	3"	5,252
Durango St.	.2'	1:21,603	.3'	1"	5,274
Buckeye Road	.4'	1:13,215	.6'	20"	5,295
Harrison St.	.5'	1:21,096	.7'	34"	10,494

T1NR3E

Grid Factor: 0.999850
Ground Factor: 1.00015

South Boundary defined by P&M Survey Data except S 35
North Boundary defined by P&M Survey Data except N 6, N 4, NE 1, NE 36, E 36, NE 25, and NE 24 are defined by P&M Survey Data.

	ADJUSTED DATA		RAW DATA		DIST.
19th Ave.	2.8 '	1: 10,425	4.0'	17"	29,410
Roosevelt St. 19th to 7th Ave.	.04'	1:125,373	.1'	7"	5,331
Roosevelt St. and 40th St.	.3 '	1: 15,190	.4'	9"	5,249

Remaining section of Roosevelt defined by P&M Survey Data

Roeser Road and 40th St	.5'	1:11,782	.9'	39"	5,330
40th Street	.5'	1:34,388		9"	15,999
S 35	1.0'	1: 5,299	1.0'	13"	5,356
Vineyard Road	2.0'	1:15,137	2.8'	12"	31,628
Southern Ave	2.3'	1:13,882	2.8'	14"	31,622
Roeser Road	2.0'	1:15,679	2.3'	8"	31,689
Broadway Road	1.9'	1:17,022	1.7'	10"	31,756
Elwood St	1.1'	1:29,112	.8'	8"	31,794
Lower Buckeye Road	1.9'	1:16,795	2.2	18"	31,832
Durango St.	1.7'	1:18,187	1.7'	14"	31,817
Buckeye Road	1.1'	1:29,773	1.1'	3"	31,811
Harrison St.	.9'	1:33,206	1.3'	16"	31,874
Van Buren St 19th to Central	.4'	1:18,256	.5'	5"	8,016
Van Buren St Central to 16th St.	.1'	1:86,452	.1'	3"	5,338

T1NR2E

Grid Factor: 0.999857
Ground Factor: 1.000143

East Boundary common to T1NR3E
North Boundary defined by P&M Survey Data
Remaining section of Roosevelt St. defined by P&M Survey Data SE 36, SW 36, NW 21, W 16, NW 16 are defined by P&M Survey Data

	ADJUSTED DATA		RAW DATA		DIST.
Boundary to L. Buckeye Rd.	1.4'	1:33,139	3.1'	41"	47,913
Boundary Buckeye Rd. to Van Buren	1.2'	1:13,047	1.7'	17"	15,745
Vineyard Rd.	.4'	1:12,200	.4'	.3"	5,265
Southern Ave.	.6'	1: 8,134	.7'	4"	5,253
Roeser Rd. 19th to 27th Ave.	.3'	1:20,689	.3'	3"	5,208
Roeser Rd. 35th to 43rd Ave.	.4'	1:15,041	.3'	1"	5,264
Broadway Rd. 19th to 27th Ave.	.1'	1:68,882	.2	12"	5,164
Broadway Rd. 35th to 43rd Ave.	.4'	1:13,177	.4	4"	5,251
Elwood St.	1.3'	1:16,676	2.4'	24"	21,284
Lower Buckeye Rd.	1.5'	1:14,285	2.2	15"	21,230
Durango St.	.3'	1:66,851	2.0'	34"	21,174
Buckeye Rd.	1.0'	1:19,374	1.4	10"	21,119
Harrison St	2.4'	1:13,022	2.0'	6"	31,573
Van Buren St.	.6'	1:16,893	.9'	13"	10,495

Remaining section of Van Buren St. defined by P&M Survey Data

Roosevelt St. 43rd to 51st Ave.	.1'	1:97,000	.1'	3"	5,288
Roosevelt St. 51st to 59th Ave.	.1'	1:68,187	.1'	3"	5,27[illegible]
Roosevelt St. 59th to 67th Ave.	.2'	1:34,318	.2'	10"	5,16[illegible]

T2NR1E

Grid Factor: 0.9998510
Ground Factor: 1.000149

East Boundary common to T2NR2E
N 35, NW 35, W 26, NW 23, and N23 defined by P&M Survey Data

	ADJUSTED DATA		RAW DATA		DIST.
Thomas Rd.	.3'	1:23,477	.5'	7"	7,895
83rd Ave.	1.0'	1: 8,216	1.6'	8"	7,913
Camelback Rd.	1.3'	1: 5,903	1.1'	12"	7,912
Osborn Rd.	.4'	1:23,706	.7'	10"	10,531
Indian School Rd.	.3'	1:39,289	.5'	28"	10,535
Campbell Ave.	1.0'	1: 9,858	1.3'	10"	10,536

T2NR2E

Grid Factor: 0.9998510
Ground Factor: 1.000149

South Boundary defined by P&M Survey Data also N 1, NW 22, and N 21
East Boundary common to T2NR3E except E 25 which was located from
NE 36 and is defined as 2516 in Data Tables.

	ADJUSTED DATA		RAW DATA		DIST.
Boundary 67th Ave and Camelback	.5'	1:62,081	.9'	16"	28,929
Boundary 43rd Ave and Northern	1.2'	1:25,001	2.3'	24"	29,067
Encanto Blvd.	.6	1:50,208	.8'	5"	31,509
Thomas Rd.	.9	1:33,209	1.5'	10"	31,496
Osborn Rd.	1.3'	1:23,872	2.3'	16"	31,496
Indian School Rd.	.8'	1:38,025	1.5'	11"	31,498
Campbell Ave.	.8'	1:38,966	1.0'	3"	31,488
Camelback Rd.	1.3'	1:12,038	1.7'	13"	15,752
Missouri Ave.	.8'	1:20,488	.8'	8"	15,769
Bethany Home Rd.	.8'	1:19,560	.9'	4"	15,784
Maryland Ave.	.9'	1:17,516	1.0'	9"	15,790
Glendale Ave.	1.0	1:16,501	1.0'	5"	15,804
Orangewood Ave.	.6'	1:27,314	.6'	1"	15,781

T2SR3E

Grid Factor: 0.999847
Ground Factor: 1.000153

South Boundary common to T1NR3E
NW 6, NW 3, NW 19, NW 25, NW 36, W 36, CTR 35, CTR 34, and W 33, E 25, N 3
are defined by P&M Survey Data
West Boundary common to T2NR2E except W 30
North Boundary common to T3NR3E
East Boundary common to T3NR4E except E 36 which is not coordinated
and NW 36

	ADJUSTED DATA		RAW DATA		DIST
19th Ave.	.7'	1:43,688	.9'	13"	31,737
Northern	2.4'	1: 6,580	2.1'	15"	15,759
16th St.-40th St. to Osborn Rd.	2.8'	1:14,103	.7'	37"	40,135
36th St.-McDowell to Thomas	.1'	1:70,711	.1'	5"	5,366
Encanto Blvd.	2.1'	1:11,022	3.6'	32"	23,905
Thomas Rd.	1.5'	1:17,727	3.3'	28"	26,545
Osborn Rd.	2.2'	1:14,551	4.1'	25	31,837
Indian School Rd.	2.4'	1:13,022	5.0'	34"	31,854
Campbell Ave.	3.2'	1:10,111	3.9'	13"	31,869
Camelback Rd.	2.7'	1:11,814	4.0'	24	31,882
Missouri Ave.	2.2'	1:14,592	3.2'	20"	31,862
Bethany Home Rd.	2.7'	1:11,848	3.9'	26"	31,861
Maryland Ave.	3.1'	1:10,408	4.5'	28"	31,836
Glendale Ave.	2.4'	1: 8,721	3.4'	26"	21,135
Orangewood Ave.	2.2'	1: 8,399	2.3	3"	18,425

T2NR4E

Grid Factor: 0.999847
Ground Factor: 1.000153

West Boundary common to T2NR3E except NW 31 which was defined by P&M Survey Data and stored in Data Tables as 3715 and W 31 also defined by P&M Survey Data
South Boundary common to T1NR4E
NW 29, NW 33, NW 27, W 22 are defined by P&M Survey Data
NW 17 is defined by point 39 and N 17 by point 40

	ADJUSTED DATA		RAW DATA		DIST.
North and East Boundary to Campbell Ave.	1.5'	1:18,185	1.5'	1"	26,584
Remaining East Boundary	1.0'	1:18,192	1.4'	26	18,573
Oak St.	.4'	1:17,996	1.0'	38"	7,896
Thomas Rd.	1.3'	1: 8,208	.9'	15"	10,538
Osborn Rd.	1.7'	1: 9,100	1.4'	38"	15,918
Indian School Rd.	1.8'	1: 9,025	.4'	39"	15,890
Campbell Ave.-56th St.	.3'	1:29,766	.5'	18"	7,952
Campbell Ave.-44th St.	.4'	1:12,672	.4'	27"	5,268
Camelback Rd.	1.5'	1:10,639	1.6'	3"	15,867
Missouri Ave.	1.3'	1:11,967	1.5'	5"	15,880

T3NR2E

Grid Factor: 0.999845
Ground Factor: 1.000155

South Boundary common to T2NR2E
East Boundary common to T3NR3E
N 28, NW 28, W 21, NW 4, N 4, NW 2, NW 1 are defined by P&M Survey Data
N 1 located from NW 1

	ADJUSTED DATA		RAW DATA		DIST.
West Boundary to Peoria	1.9'	1: 6,963	2.1'	8"	13,116
West Boundary to Bell	2.4'	1: 7,646	2.2'	10"	18,166
Bell Road	1.~~7~~ 7'	1: ~~6,118~~ 7,7[illegible]	1.~~9~~ 5'	~~6"~~ 5"	10,574
Butler Drive	1.8'	1: 8,676	2.2'	11"	15,822
Dunlap Ave.	1.9'	1: 8,261	2.3'	14"	15,836
Mountain View Road	2.4'	1: 6,500	2.7'	8"	15,817
Peoria Ave.	2.0'	1: 7,981	2.8'	35"	15,805
Cholla St.	1.9'	1:11,127	2.5'	15"	21,050
Cactus Rd.	1.3'	1:16,313	1.8'	15"	21,023
Sweetwater Ave.	1.5'	1:13,555	3.5'	46"	20,993
Thunderbird Rd.	2.1'	1:10,022	5.2'	1' 4"	20,961
Acoma Drive	1.4'	1:15,216	3.7'	46"	20,936
Greenway Road	1.1'	1:19,580	1.9'	18"	20,911
Paradise Lane	.8'	1:~~27,380~~ 16,[illegible]	2.1'	28"	20,884

T3NR3E

Grid Factor: 0.999836
Ground Factor: 1.000164

South Boundary common to T2NR3E
West Boundary common to T3NR2E
W 33, NW 6, NW 5, NE 1, NE 25, and SE 25 are defined by P&M Survey Data
E 3 is defined by point 143; N 19 by point 313; NW 20 by point 312; N 20 by point 311; NW 20 by point 1290; W 20 by point 1280; NW 30 by point 1260; NW 29 by point 1270 and W CTR 29 by point 281

	ADJUSTED DATA		RAW DATA		DIST.
Boundary 19th Ave.	3.7'	1: 8,653	4.8'	16"	31,675
Boundary Bell-19th Ave. to 7th St	.8'	1:13,360	1.5'	29"	10,402
Boundary Bell-7th St. to 40th St.	1.3'	1:~~53,~~533	~~12~~ .6'	~~4'34"~~	18,470
Boundary 40th St. to Peoria	1.4'	1:15,456	3.6'	47"	21,086
Boundary 40th St.- Peoria to 16th St.	2.8'	1: 9,466	5.3'	1' 6"	26,530
Butler Drive	1.5'	1:10,331	2.1'	44"	15,738
Dunlap Ave.	3.1'	1: 8,641	1.8'	20"	26,425
Mountain View Road	3.3'	1: 9,470	1.6'	26"	31,671
Peoria Ave. (Shea)	2.5'	1:12,580	1.6'	18"	31,679
Cholla St.	3.6'	1: 8,709	4.6'	13"	31,684
Cactus Road	2.2'	1:14,280	3.1'	15"	31,655
Sweetwater Ave.	1.9'	1:16,306	4.6'	37"	31,588
Thunderbird Road	.7'	1:45,315	2.6'	29"	31,577
Acoma Drive	.4'	1:75,515	2.3'	32"	31,561
Greenway Rd.	1.0'	1:32,145	.7'	21"	31,546
Paradise Lane 19th Ave to 24th St	~~2.6'~~ .2	1: ~~9,010~~ 19,500	~~2.2'~~ .7	~~1'38"~~ 1'	23,560
Paradise Lane 40th St to 24th St	~~1.5'~~ .1	1: ~~8,466~~ 142,050	~~3.4'~~ .2	~~1'57"~~ 6	13,117

T3NR4E

Grid Factor: 0.999836
Ground Factor: 1.000164

West Boundary common to T3NR3E except W 19 which was located from NW 30 and defined by point 3073 in Data Tables
N 6, N 3, NE 3, E 3, E 15, SE 15, S 15, NW 29, N 30 were defined by P&M Survey Data

	ADJUSTED DATA		RAW DATA		DIST.
Boundary Bell Rd.	.8'	1:19,888	1.1'	24"	15,941
Boundary Scottsdale Rd. to Sweetwater	.4'	1:27,976	.4'	3"	10,573
Boundary 68th St. to Shea Blvd.	1.6'	1:17,688	1.6'	4"	29,029
Boundary Mt. View to 40th St.	5.5'	1: 2,418	4.6'	1' 8"	13,347
Dunlap Ave.	4.8'	1: 1,141	4.8'	1"	5,447
Mt. View	2.6'	1: 2,100	2.6'	15"	5,434
Shea Blvd.	1.0'	1:10,704	1.0'	6"	10,520
Cholla St.	2.0'	1: 6,770	1.8'	8"	13,334
Cactus Rd.	1.4'	1: 9,467	1.3	8"	13,357
Sweetwater Ave.	1.5'	1:14,159	1.3'	12"	21,314
Thunderbird Rd.	1.5'	1:14,432	1.3'	7"	21,349
Acoma Rd.	1.4'	1:15,260	1.5'	2"	21,375
Greenway Rd.	1.1'	1:20,041	1.6'	12"	13,638
Paradise Lane	1.4'	1:15,565	1.6'	7"	21,500

Analysis of The City of Chandler's Municipal Survey Grid

Performed By:
Mapping Automation, Inc.
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480.893.1700

August 1999

Chandler • Arizona
Where Values Make The Difference

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Appendix A:
Originally-supplied Datafiles Used for Analysis

Appendix B:
Arc Macro Language Script Used for Data Reprojection and Coverage Analysis

1 Background

The Greiner Surveys

In 1995, the City of Chandler contracted with Greiner Engineering (Greiner) to perform a field survey of its Municipal Survey Grid, i.e., the Public Land Survey System (PLSS) sections and quarter-section corners within the City's corporate limits.

Greiner, under the direction of Mr. Robert J. Pecha, PRLS, had previously performed a conventional field survey (date unknown) of the 149 PLSS corners north of and including Pecos Road:

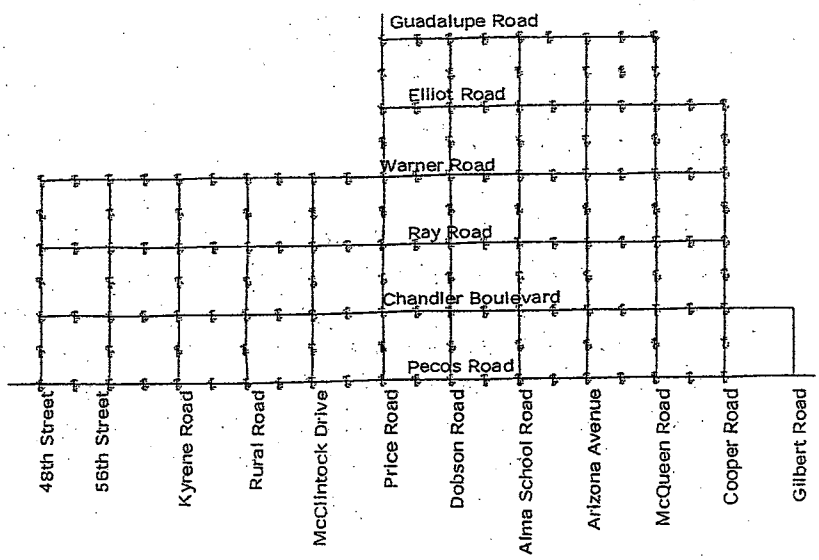


Figure 1: Greiner NAD27 Survey

This survey was referenced to the North American Datum of 1927 (NAD27), the geodetic datum in use at the time. WHAT?

In 1995, a subsequent survey using Global Positioning System (GPS) technology was performed on the 129 PLSS corners south of Pecos Road.

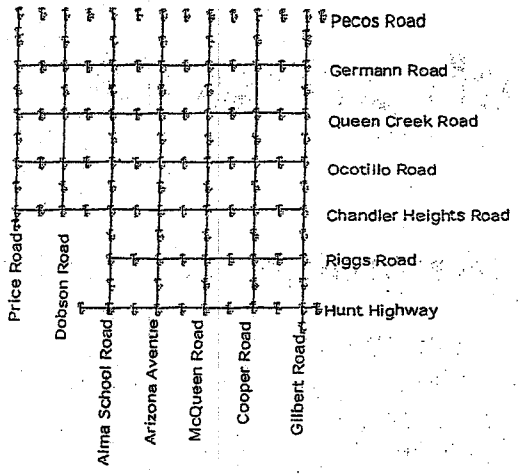


Figure 2: 1995 Greiner GPS Survey

This survey was referenced to the North American Datum of 1983 (NAD83), a closer approximation of the earth's shape over the North American continent than was obtainable in 1927.

The standardized survey coordinate system used in Maricopa County is the Arizona Coordinate System, Central Zone (ACSCZ), referenced to the NAD83 datum. Prior to enabling legislation enacted in the mid-1980s, the ACSCZ was referenced to the NAD27 datum. Accordingly, to produce a composite survey of Chandler's entire Municipal Survey Grid, Greiner mathematically reprojected the NAD27 coordinates north of Pecos Road to be on the same coordinate system as the 1995 NAD83 survey points south of Pecos Road. This composite survey will be henceforth referred to as the Greiner Ground NAD83 Survey, the 'Ground' adjective to be more fully explained later in this document.

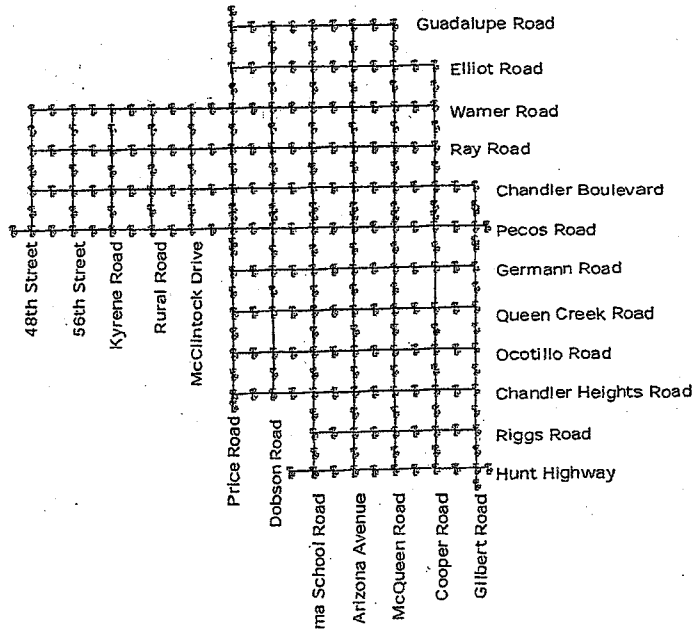


Figure 3: Greiner Ground NAD83 Survey

It should be pointed out that additional survey points beyond just the PLSS corners were included in Greiner's GPS survey of 1995: horizontal control points, vertical benchmark points, and combined horizontal/vertical reference points.

City GIS Survey Control from Salt River Project

During the last several years, the City has initiated work on a citywide Geographic Information System (GIS), based on data obtained from the Salt River Project (SRP). SRP's data was, and is, referenced to the NAD27 datum. A preliminary comparison between the original Greiner NAD27 survey north of Pecos Road and the corresponding SRP NAD27 survey points indicated that the Greiner coordinate values were offset, on average, 168.04 feet in a northeasterly direction, as shown on the following page.

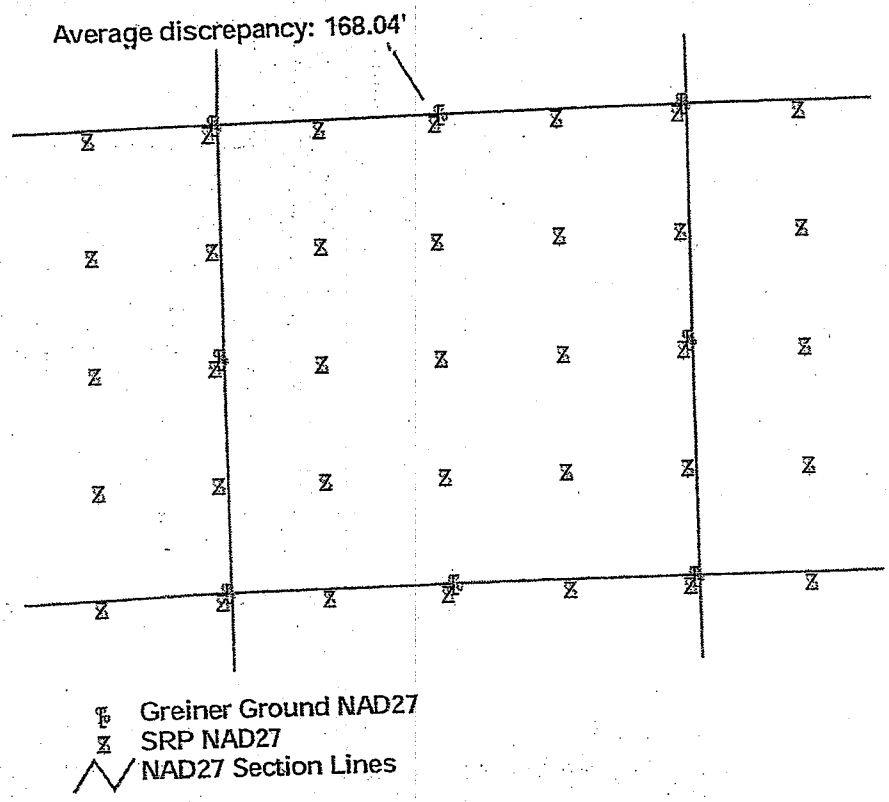


Figure 4: Average Discrepancy Between SRP NAD27 and Greiner NAD 27 Points

A comparison was then made to corresponding City of Tempe survey points along that City's coincident boundary with Chandler. The six compared points (on NAD27 datum) were within one to two inches of the SRP NAD27 coordinate values This lent independent accuracy confirmation of the SRP points, but further clouding the accuracy/reliability of the Greiner NAD 27 survey points, as well as the results of the 1995 Greiner NAD83 survey.

The next section of this report discusses the further analysis that was made on the Greiner Ground NAD83 Survey to reconcile its apparent discrepancy with the SRP survey data points.

2 Analysis

Comparison of SRP and Greiner Coordinates

The SRP NAD27 ACSCZ (Arizona Coordinate System Central Zone) data for the City of Chandler area was reprojected to the ACSCZ referenced to the NAD83 datum for comparison with the Greiner Ground NAD83 Survey. This process was performed in ESRI's ARC/INFO GIS software, using double precision numerical processing techniques. The Arc Macro Language (AML) program that performed this conversion is listed in Appendix B. The reprojected SRP coordinates will be referred to as SRP NAD83.

At this point, a comparison was made finding the nearest SRP NAD83 point to its corresponding Greiner Ground NAD83 survey point. The average discrepancy was 168.04 feet, with a standard deviation of 2.93 feet (i.e., two-thirds of all points fell within ±2.93 feet of the 168.04-foot discrepancy mean). What was especially telling, though, was the spatial distribution of the discrepancies:

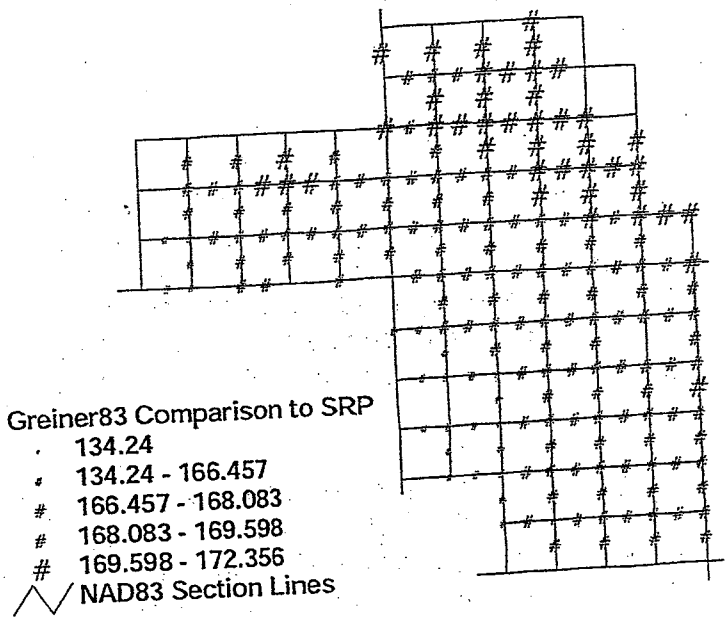
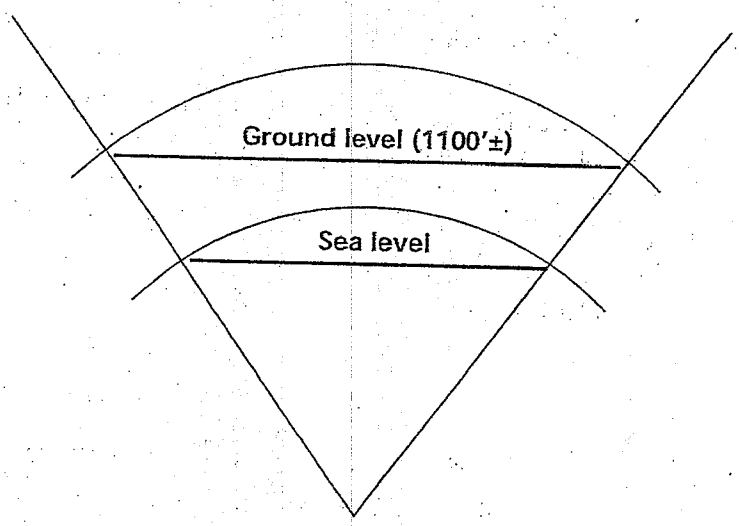


Figure 5: Spatial Distribution of Discrepancies, SRP NAD83 to Greiner Ground NAD 83 Survey

The general indication was that as the Greiner Ground NAD83 survey coordinate values increased in size (i.e., larger x and y values), the difference between the corresponding SRP NAD83 coordinates increased as well. This error as a function of coordinate size suggested that the Greiner Ground NAD83 survey coordinates were linearly scaled.

This assumption was initially proposed by Mr. Gregg Tuttle, PRLS, the Supervisor of Surveys at SRP's Land Department. He and his staff postulated that the Greiner Ground NAD83 survey coordinates had been all multiplied by a common factor to make them more closely match the at-ground surveyed distances, a common practice in local project surveying. The following diagram illustrates the difference between ACSSZ coordinate values, which are projected to a sea-level grid, and their radially-projected equivalent lengths at project ground level:



Because of the earth's basically-spherical shape, ground distances "shrink" when radially "de-projected" down to the NAD83 reference elevation of sea level. This shrinkage amount actually depends on two factors:

- the average elevation of the ground-level project (i.e., the "average elevation" for the City of Chandler)
- the "grid distortion" which comes about as a result of flattening curved lines on the earth's surface to the flat NAD83 grid

Given an average elevation of 1100'± for the Chandler area, and a grid distortion value of 1 part in 10000, the shrinkage factor (technically referred to as the *ground to grid factor*) is approximately 0.99984739. A 5,280' line surveyed at ground level will "shrink" to a 5,279.19' line at the ACSCZ sea-level reference grid. This value is in fact very close to the ground to grid factor of 0.9998472125 quoted in Greiner's "Survey Base for South Chandler Executive Summary" document supplied as part of their project deliverables.

The simple fact is that the Greiner Ground NAD83 survey coordinate values are just that: they were linearly scaled up from their computed true NAD83 values by a grid to ground factor of (1/0.9998472125=1.00015281) that, when multiplied by numbers in the six-digit range as ACSCZ NAD83 coordinates are, resulted in numeric values that were 168± feet too large when compared to the corresponding SRP NAD83 point.

When all the Greiner Ground NAD83 survey coordinate values were reduced by the ground to grid factor of 0.9998472125, the resulting comparison to corresponding SRP NAD83 points averaged 0.747 feet, with a standard deviation of 0.884 feet. The following diagram illustrates this difference distribution:

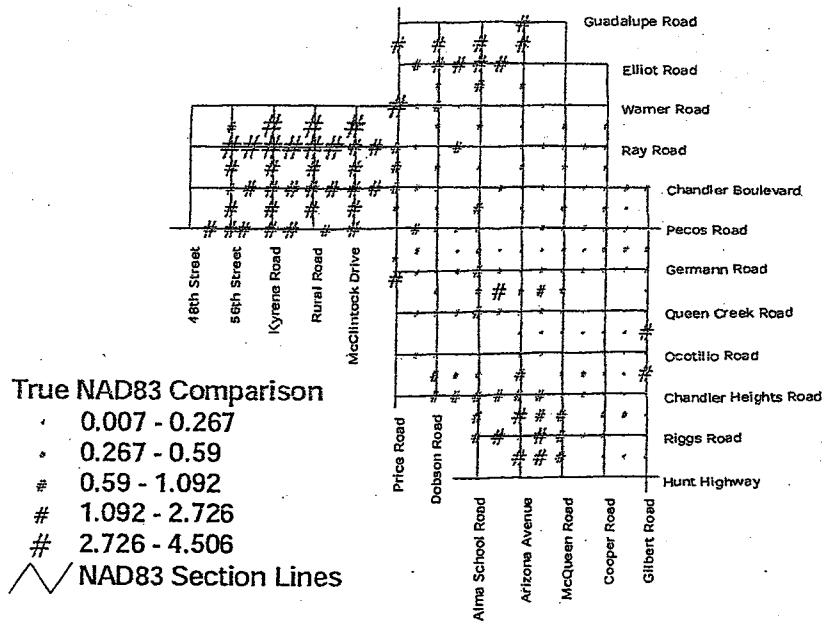


Figure 7: Spatial Distribution of Discrepancies, SRP NAD83 to True Greiner NAD 83 Survey

Somewhat troubling is the fact that the standard deviation is larger than the arithmetic mean: this indicates that there are some basic inconsistencies in the data set, most likely represented by the difference values in the 2.726 – 4.506 foot range. These points are grossly in error, occurring mostly in the old NAD27 survey area along Ray Road between 56th Street and McClintock Drive. Once these "outlier" points are removed from the matrix, a somewhat more respectable mean of 0.583 feet with a standard deviation of 0.485 feet is obtained. Further implications of these errors are more fully discussed in the following Conclusions section.

3 Conclusions & Recommendations

Somehow over the years, the Greiner Ground NAD83 survey coordinates were erroneously confused with "true" NAD83 coordinates and became accepted as such. The reason for this is fairly obvious: the coordinate range for the Greiner Ground NAD83 survey coordinates was in the same range as true NAD83 coordinates (i.e., 700,000 in the x axis, 800,000 in the y axis for the Chandler area). Recommendation 1 offers a method to distinguish such ground-level datum coordinates.

Mr. Pecha of Greiner supplied Mapping Automation with the originally-derived NAD83 coordinates for the Chandler Municipal Survey Grid project, and he stated that the same file was originally supplied to the City back in 1995. It has probably merely "fallen between the cracks", and the more "user-friendly" Greiner Ground NAD83 survey coordinates data file was used instead.

Recommendations

1. The fundamental reason for the misidentification of the at-ground Greiner coordinates as NAD83 values was that they were numerically in the same range as true NAD83 coordinates for the Chandler area. **The at-ground coordinates should be clearly distinguished from standardized coordinate systems such as State Plane by offsetting them large amounts in the x and y axes**. A suggested algorithm:

   Xground = Xnad83 * 1.00015281 – 600000
   Yground = Ynad83 * 1.00015281 – 300000

   The '1.00015281' is the grid to ground factor, or 1/0.9998472125 as stipulated by the Greiner survey report. The 600000x, 300000y offset would produce ground coordinates in the 100000x, 500000y range—not easily confused with any other standardized coordinate systems in this area.

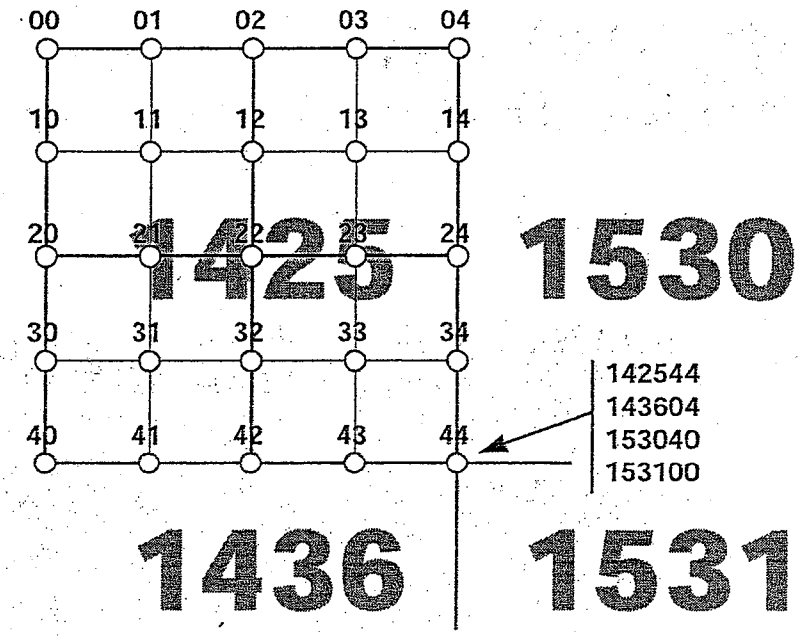
Distribution of these data files in the future should be accompanied by either a detailed "readme" file on the distribution media, or a complete Federal Geographic Data Committee (FGDC) metadata file.

2. The Notes wording on the existing hardcopy maps, although technically correct, should be clarified to read:

   "All bearings and distances shown on this map are calculated on an at-ground survey plane approximately 1,118 feet above mean sea level. This survey plane is referenced to the Arizona Coordinate System, Central Zone, NAD83 datum, by a combined grid to ground factor of 1/0.9998472125."

3. Additional analysis should be made of the survey points shown in Figure 7 that have large discrepancies. There is the possibility that the SRP points may not reference the same monuments that the Greiner survey did, for example, or are of lesser survey accuracy than the Greiner points. Since the field survey is fairly old, there is the possibility that some of the points were destroyed and re-monumented.

4. An overall survey database should be developed for integration with the City's GIS, and include (for example) coordinate values both at NAD83 sea level and at-ground. Included with this database should be a more intuitive numbering system for survey points. One possibility is to use a six digit number that would identify PLSS corners down to the quarter-quarter section level as shown on the following page.



The example section is '1425': Township **1** South, Range **4** East, section **25**. The southeast corner of this section is in matrix position 44 (row 4, column 4), and the composite number for this corner is therefore '142544'. However, this same corner could be referenced within neighboring sections as well:

- '153040' when it is referenced relative to section 15, T1SR5E (the southwest corner of that section)
- '143604' when it is referenced relative to section 36, T1SR4E (the northeast corner of that section)
- '153100' when it is referenced relative to section 31, T1SR5E (the northwest corner of that section)

Appendix A:

Originally-supplied Datafiles Used For Analysis

Four files were provided by City staff for Mapping Automation's use in this analysis:

GREINER.DWG	AutoCAD drawing files of Greiner's NAD27 survey of the City's Municipal Grid north of Pecos Road
COCS01.DWG	AutoCAD drawing file of Greiner's 1995 NAD83 survey north of Pecos Road
COCS02.DWG	AutoCAD drawing file of Greiner's 1995 NAD83 survey south of Pecos Road
GRID.SHP	ESRI shapefile of the SRP NAD27 PLSS points within Chandler

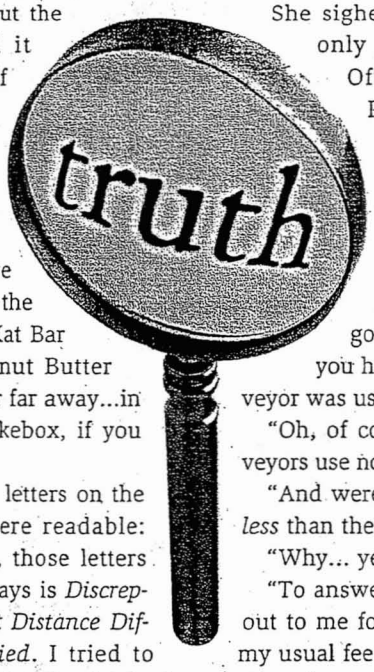
The AutoCAD drawing files were converted to Drawing exchange File (DXF) format using AutoCAD Map 2 software. All remaining data conversion was performed by ESRI ARC/INFO software, version 7.1.2, using the 'build-data' Arc Macro Language script listed in Appendix B.

by **Jerry McGray, RPLS**

Private eye-man meets plain truth.

Jerry McGray has been a managing principal of McGray & McGray Land Surveyors for 20 years, including nine years as GPS project manager. He conducts seminars on and has authored dozens of articles about GPS and geodetic surveying. He also acts as POB's editorial director and serves on POB's Editorial Advisory Board. He can be reached at mcgrayj@bnp.com.

I clawed my way out of a deep sleep, a sleep filled with dreams of renegade units of distance: meters, U.S. survey feet, international feet, varas, chains, smokes...they had all joined forces to attack me and my fragile forces for truth. The good thing about the dreams—if you could call it good—was that it was a relief to wake up. Yeah, sure...to wake up to a bleary-eyed hangover caused by way too many diet Dr. Peppers—and sure, I admit it—more than one Snickers bar. (Way more than one.) I hadn't slipped all the way to the hard stuff: the KitKat Bar followed by the Reese's Peanut Butter Cup, but temptation was never far away...in the file cabinet behind the jukebox, if you must know.



I blinked until the stenciled letters on the outside of my office door were readable: "Deifitsmy..." Wait a second, those letters are mixed up. What it really says is *Discrepancies, Inc. Devilishly Difficult Distance Difference Dilemmas De-mystified*. I tried to come up with an attention span and to think of a word to substitute for "Inc." that started with "D" as I listened to the rain pounding on the composite asphalt shingle roof. I wondered if my detective agency was about to be washed away. Nobody told me being a private eye-man was so tough. Or that I would have to sleep in the office...but that seems to be the way all us private eyes do it. How else would you start your stories? But I digress...

Right about then was when *she* walked in the door. Her gaze could melt ice cream. Actually, I guess ice cream would pretty much melt anyway, sooner or later. But this lady had the look of a professional; in fact, she might as well have carried a sign saying, "I'm an engineer." It was obvious that I was in the presence of not only an engineer but probably even a *civil* engineer! More importantly, I think this might even be...a *client*! I resolved to behave myself.

She broke the silence: "I have a problem."

"Don't we all," I smirked. But when she rolled her eyes, I remembered my resolution and asked," What seems to be the problem, Toots... I mean, Ms. Engineer?"

She sighed and plopped down into the only decent chair in the office, an Office Warehouse 31628 Deluxe Plastic w/Arms. I could see she was genuinely troubled.

"We got our surveyors to tie into a monument, and our coordinates are 2 ½ feet different from the parish's values."

I thought, "Oh boy, this is gonna be too easy." I asked, "Do you happen to know whether your surveyor was using NAD83(93) HARN Datum?"

"Oh, of course!" she answered. "Our surveyors use nothing but the latest and greatest."

"And were your coordinate values slightly *less* than the parish published values?"

"Why... yes! How did you know?"

"To answer that will require a check made out to me for $28.50 per day expenses, plus my usual fee of $12,000."

She almost knocked over the plastic bowl containing RoyBoy, my Siamese fighting fish, as she shoved the check my way. "Here, take it! Now what's the answer?"

"You and your surveyor are in the two-bit, I mean two-foot world of HARN versus non-HARN. Probably because of timing, the parish positioned its monument based upon the NAD83 values established before the High Accuracy Reference Network (HARN) was created. In this geographical area, that difference results in around 2 ½ feet, with the more recent (HARN) values being the smaller, State Plane Coordinate-wise."

She almost smiled as she got up to leave. "Thanks, private eye-man," she said. I feebly waved a goodbye as she walked out.

I flicked the last of the fish food into RoyBoy's bowl while I made a mental note to get a fresh can next trip to Safeway or Walgreen's, your—I mean my—neighborhood markets.

I had barely popped the top on a Pepsi One (by the way, if you'd like your product featured in our column...oh, never mind) when my Nokia digital phone started chirping. I answered, "Discrepancies, Inc. What's your problem?" The frantic babbling on the other end of the line could only mean one thing. Surface Adjustment Factor Syndrome! (Believe me, I've heard enough bawling and whining about SAFS to recognize the symptoms right off.) When the caller finally got coherent enough to continue, I was able to figure out that this firm had done an engineering design project tied to a Huge State Transportation Entity project. Upon project completion, they shipped a copy of their drawing file to the City Prettymuch Geographic Information System. And, of course the coordinates of the HSTE project were a long way from fitting properly into the CPGIS (I don't have to keep explaining these initials, do I?) They didn't fit by a long shot, as in 1,300 feet! A classic case of Surface Adjustment Factor Syndrome (which I once heard an elderly surveyor refer to as the 900-pound gorilla of problems. Wonder whatever happened to that old codger?).

Long story short, all I had to do was explain to the afflicted patient that the application of a surface adjustment factor not only nudges the points apart a tiny bit as was intended, but also plops them unceremoniously several hundred feet from their rightful State Plane location. After helping him figure what the factor was, and advising him how to "fix" the problem in a simple spreadsheet, there was nothing left to do but collect my exorbitant fee—plus the $28.50 per day expenses, of course.

I was about to get my sea legs and begin my usual late-morning debate with myself: Wendy's or Burger King? That's when the computer beeped with an E-mail. Yep, another distance discrepancy. This engineer had received a mapping file and loaded it into his computer –a Dail Power-Grabber running MicroSolvitKwik v.83.4. But when he tried to overlay his alignment file onto the mapping, he was off by around 20 feet! I put in a phone call to him right away.

I tried to soothe his hysteria by repeating a surveying mantra... "turn right, read left, turn right, read left." But he didn't seem to get it—probably too young. So I went right to the nasty truth: Was there ever a metric component to this project? He calmed down, sniffled and replied, "Why, yes. As a matter of fact, our design was originally done in meters. We just converted it to feet the other day."

"Umm, hmm. And do you happen to know whether you used U.S. survey feet or international feet?" I asked.

"All I know is, the difference in the two is negligible. Why do you ask?"

I said, "You're right. The difference *is* negligible... except when dealing with the massive values of State Plane Coordinates!"

You could have heard a pin drop. No wait, I think that's Sprint with the pin-dropping thing, and we were on AT&T. Anyway, he was quiet. I went through the math with him...the U.S. survey foot is officially 39.37 divided by 12 feet per meter, while the international foot is 2.54 cm per inch. We proved that the difference in the two can make roughly 20 feet of difference in the coordinates by the time you multiply them by several million.

I pulled the shade down for my afternoon nap, feeling a twinge of guilt that these clients' problems were so easily solved by simply looking at the magnitude of the discrepancies.

I pulled the shade down for my afternoon nap, feeling a twinge of guilt that these clients' problems were so easily solved by simply looking at the magnitude of the discrepancies. I wondered if my ride down Easy Street would someday find me in a cul-de-sac if these folks ever figured it out and could do their own Distance Discrepancy Detecting (DDD).

Let's see, in one short morning in private eye-land I had solved a discrepancy due to epoch differences between NAD83(86) and NAD83(9x), a discrepancy between grid coordinates and scaled "surface coordinates," and the difference between the U.S. survey foot and the international foot.

My dreams began to turn to a whole nation of surveyors who had a solid handle on the numerical conversion problems that can creep into the surveyor's work.

I mentally checked off the other common discrepancies, and found only one left: NAD83 versus NAD27. Most folks can figure that one out pretty quick. To put it on the list of magnitude = identification: around 100 feet.

As I slipped back into sleep, my dreams began to turn to a whole nation of surveyors who had a solid handle on the numerical conversion problems that can creep into the surveyor's work. A nation of surveyors who want to get it right, yet keep it plain. I have a sinking feeling that they are wising up. I began to envy RoyBoy in his isolated, watery world.

Maybe it's time for that Reese's/Reese's cocktail after all. Maybe even make it a double. ⊕

Why Use UTM Coordinates

The UTM coordinate system offers the following benefits:

A square grid

UTM Provides a constant distance relationship anywhere on the map. In angular coordinate systems like latitude and longitude, the distance covered by a degree of longitude differs as you move towards the poles and only equals the distance covered by a degree of latitude at the equator. Since land navigation is done in a very small part of the world at any one time using large scale maps. The UTM system allows the coordinate numbering system to be tied directly to a distance measuring system.

No negative numbers or East-West designators

Grid values increase from left to right and bottom to top

This is just like the X Y Cartesian coordinate system you learned high school math class. Simple Cartesian coordinate mathematics can be used. No spherical trigonometry is required!

Coordinates are decimal based

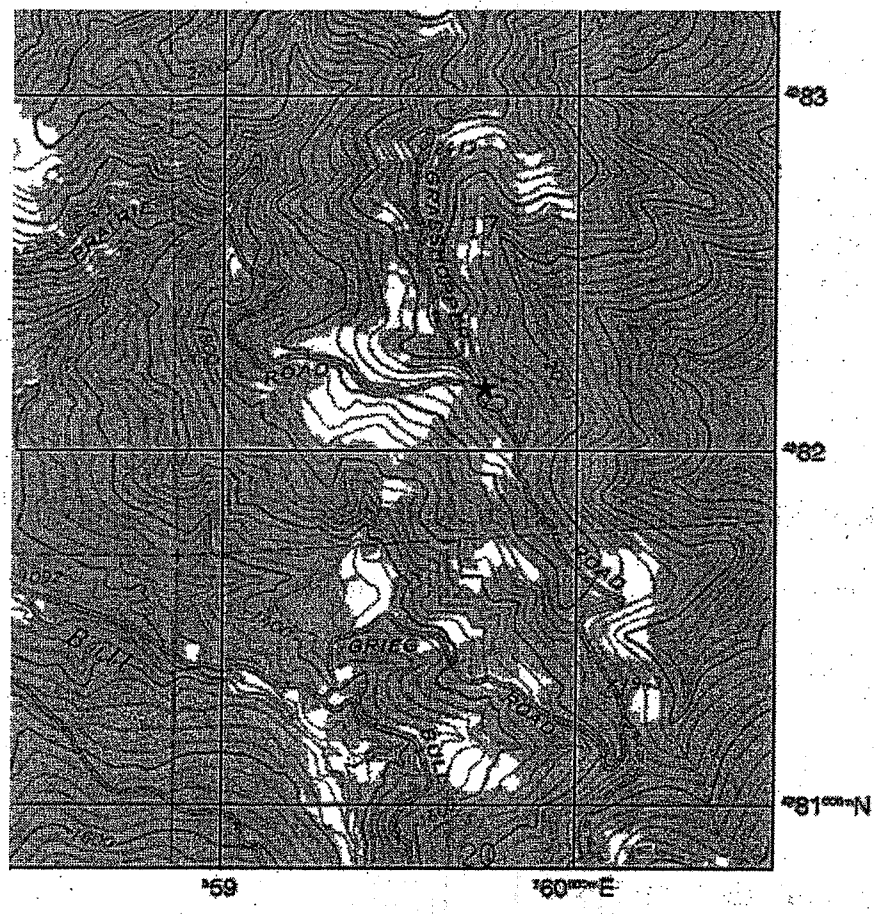
Ones, tens, hundreds and so on. No more minutes and seconds to convert.

Coordinates are measured in metric units

All UTM coordinates are measured in meters. Most of the world has already adopted the metric system. Now you won't need to remember how many feet are in a mile. And what's that in yards?

(from URL: http://www.maptools.com/Using UTM/ whyUTM.html)

A Quick Guide to Using UTM Coordinates



Standing at the road junction marked with the star on the topographic map pictured above, a GPS unit set to display position in UTM coordinates, would report a location of:

10 S 0559741
4282182

The **10 S** represents the zone you are in. The zone is necessary to make the coordinates unique over the entire globe.

The top set of numbers, **0559741**, represent a measurement of East-West position, within the zone, in meters. It's called an easting.

The bottom set of numbers, **4282182**, represent a measurement of North-South position, within the zone, in meters. It's called a northing.

The map has Universal Transverse Mercator (UTM) grid lines spaced every kilometer or 1000 meters. The vertical grid lines determine East-West position and the horizontal grid lines determine North-South position.

Look along the bottom edge of the map at the labels for the vertical grid lines.

559 and 560000 mE.

The label, 560000 mE., reads "five hundred and sixty thousand meters East." The label, 559, is an abbreviation for, 559000 mE. The two grid lines are 1000 meters apart. The horizontal grid lines are labeled in a similar manner.

(from URL: http://www.maptools.com/Using UTM/quickUTM.htlm)

Shorthand for UTM Coordinates

Most land navigation activities focus on a very small portion of the globe at any one time. Typically the area of interest to an outdoorsman is less than 20 miles on a side. This focus on a small area allows us to abbreviate UTM coordinates.

The zone information and the digits representing 1,000,000m, and 100,000m are dropped. The 1m, 10m and 100m digits are used only to the extent of accuracy desired.

A GPS unit might read

10 S 0559741
4282182

Using a notation similar to the one found on a USGS topographic map, this would be written as:

Zone 10 S 559741 mE. 4282182 mN.

An abbreviated format for the same coordinates would look like:

59 82	Describes a 1000m by 1000m square.
597 821	Describes a 100m by 100m square.
5974 8218	Describes a 10m by 10m square.
59741 82182	Describes a 1m by 1m square.

The 100m abbreviated format, **597 821**, and the 10m abbreviated format, **5974 8218**, are the most commonly used.

Notice that the easting is reported first, followed by the northing. Remember the phrase "read right up" to help you remember to read the easting from left to right, followed by the northing from the bottom up.

Also notice that when you abbreviate coordinates you should not do any rounding. **0559651** becomes **596** not **597**. This ensures that your position is still within the reported square. As accuracy decreases, the square gets bigger.

(from URL: http://www.maptools.com/Using UTM/ abbreviatedUTM.html)

More details about the UTM coordinate system

The Universal Transverse Mercator projection and grid system was adopted by the U.S. Army in 1947 for designating rectangular coordinates on large scale military maps. UTM is currently used by the United States and NATO armed forces. With the advent of inexpensive GPS receivers, many other map users are adopting the UTM grid system for coordinates that are simpler to use than latitude and longitude.

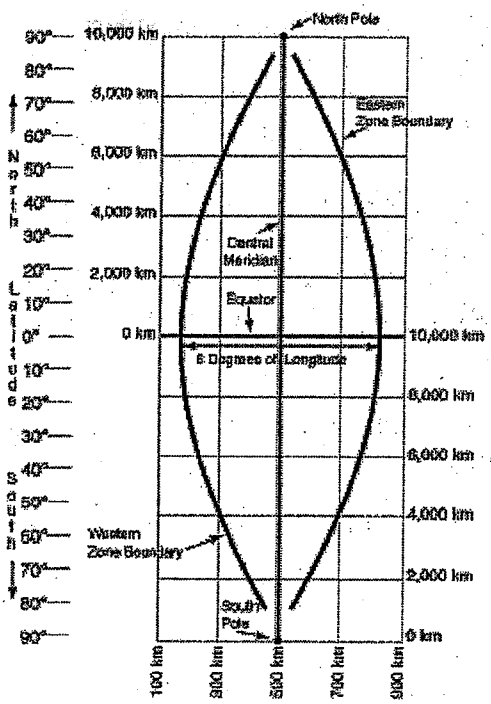
The UTM system divides the earth into 60 zones each 6 degrees of longitude wide. These zones define the reference point for UTM grid coordinates within the zone. UTM zones extend from a latitude of 80° S to 84° N. In the polar regions the Universal Polar Stereographic (UPS) grid system is used.

UTM zones are numbered 1 through 60, starting at the international date line, longitude 180°, and proceeding east. Zone 1 extends from 180° W to 174° W and is centered on 177° W.

Each zone is divided into horizontal bands spanning 8 degrees of latitude. These bands are lettered, south to north, beginning at 80° S with the letter C and ending with the letter X at 84° N. The letters I and O are skipped to avoid confusion with the numbers one and zero. The band lettered X spans 12° of latitude.

A square grid is superimposed on each zone. It's aligned so that vertical grid lines are parallel to the center of the zone, called the central meridian.

UTM grid coordinates are expressed as a distance in meters to the east, referred to as the "easting", and a distance in meters to the north, referred to as the "northing".



Eastings

UTM easting coordinates are referenced to the center line of the zone known as the central meridian. The central meridian is assigned an easting value of 500,000 meters East. Since this 500,000m value is arbitrarily assigned, eastings are sometimes referred to as "false eastings"

An easting of zero will never occur, since a 6° wide zone is never more than 674,000 meters wide.

Minimum and maximum easting values are:

160,000 mE and 834,000 mE at the equator

465,000 mE and 515,000 mE at 84° N

Northings

UTM northing coordinates are measured relative to the equator. For locations north of the equator the equator is assigned the northing value of 0 meters North. To avoid negative numbers, locations south of the equator are made with the equator assigned a value of 10,000,000 meters North.

Some UTM northing values are valid both north and south of the equator. In order to avoid confusion the full coordinate needs to specify if the location is north or south of the equator. Usually this is done by including the letter for the latitude band.

If this is your first exposure to the UTM coordinate system you may find the layout of zones to be confusing. In most land navigation situations the area of interest is much smaller than a zone. The notion of a zone falls away and we are left with a simple rectangular coordinate system to use with our large scale maps.

Frequently, in land navigation, the zone information and the digits representing 1,000,000m, and 100,000m are dropped. The 1m, 10m and 100m digits are used only to the extent of accuracy desired. Note that it's the smaller digits that are dropped in the notation used by the USGS on the edges of their maps. For example 4282000 mN. becomes 82.

Because pilots and sailors navigate over much greater distances they still favor the latitude longitude coordinate system.

(from URL: http://www.maptools.com/Using UTM/ UTMdetails.html)

Map Datums

A datum describes the model that was used to match the location of features on the ground to coordinates and locations on the map. Maps all start with some form of survey. Early maps and surveys were carried out by teams of surveyors on the ground using transited and distance measuring "chains". Surveyors start with a handful of locations in "known" positions and use them to locate other features. These methods did not span continents well. Frequently they also did not cross political borders either. The "known points" and their positions are the information that the map datum is based. As space based surveying came into use, a standardized datum based on the center of the earth was developed.

Every map that shows a geographic coordinate system such as UTM or Latitude and Longitude with any precision will also list the datum used on the map.

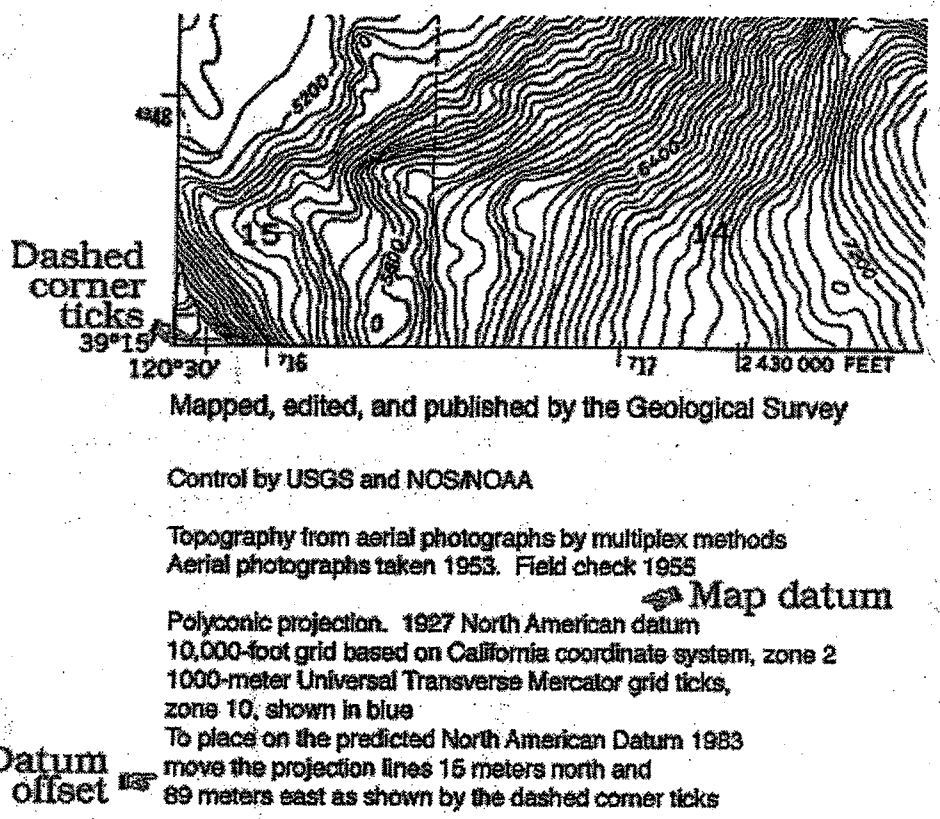
The Global Positioning System uses an earth centered datum called the World Geodetic System 1984 or WGS 84. WGS 84 was adopted as a world standard from a datum called the North American Datum of 1983 or NAD 83. For all practical purposes there is no difference between WGS 84 and NAD 83.

Most USGS topographic maps are based on an earlier datum called the North American Datum of 1927 or NAD 27. (Some GPS units subdivide this datum into several datums spread over the continent. In the Continental United States use NAD27 CONUS.)

In the Continental United States the difference between WGS 84 and NAD 27 can be as much as 200 meters.

You should always set your GPS unit's datum to match the datum of the map you are using.

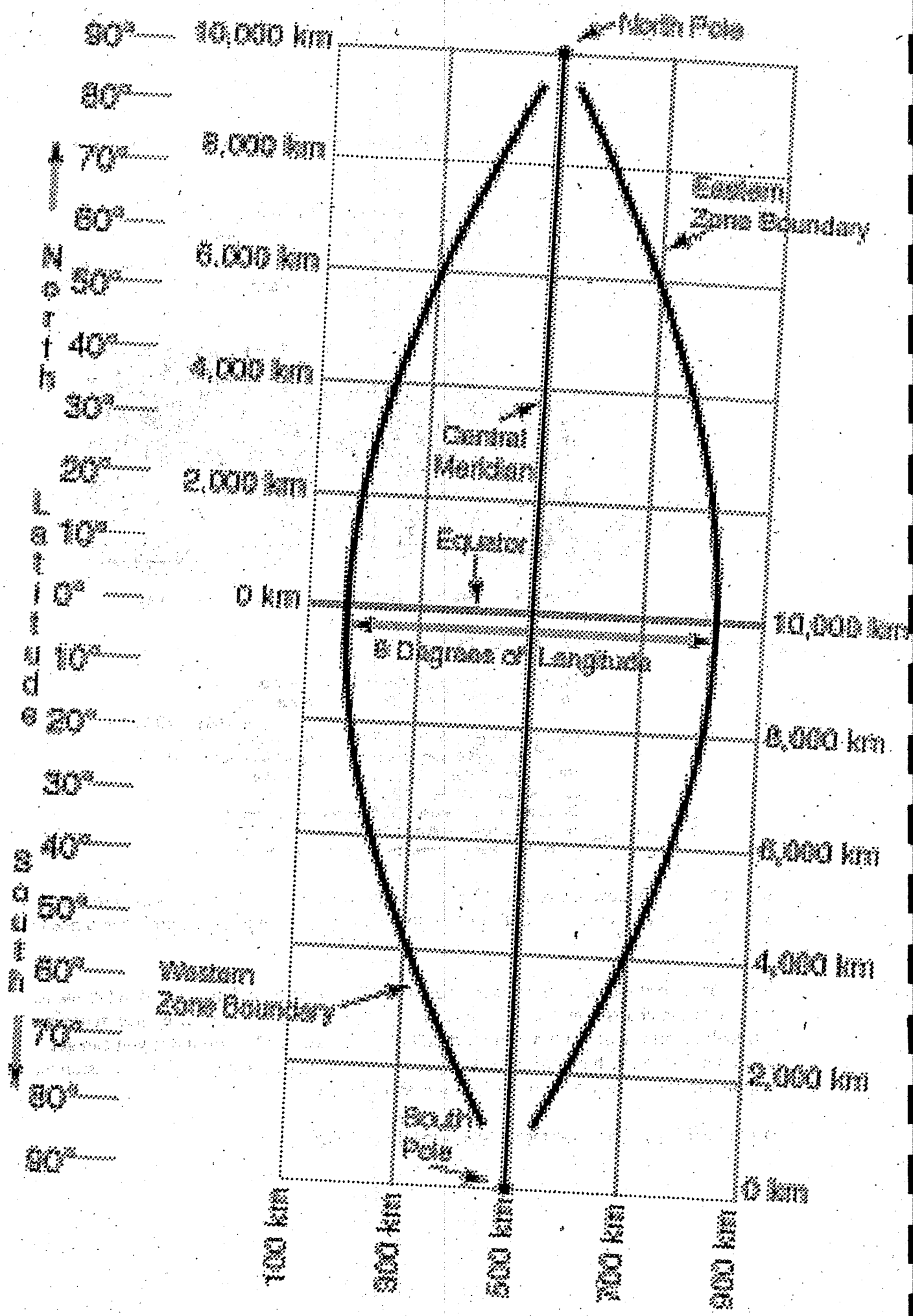
On a USGS topographic map the datum information is in the fine print at the bottom left of the map. The datum will always be NAD 27. There may be information on how many meters to shift a position to convert it to NAD 83. Think of this as the error that will be introduced if you leave your GPS unit set to WGS 84. A dashed cross in the SW and NE corners of the map gives a visual indication of the difference between the two datums.



If you have somehow set your GPS to use the Borneo Datum of 1818, it's hard to say how far off you position may be. Let's just sat that this "datum thing" is something you need to pay attention to.

If you are coordinating with aircraft, they will likely have their datum set to WGS 84, as most aviation charts now use WGS 84. Should you worry about the difference in datums? Typically a pilot will not have any difficulty locating you on the ground if you can get them within several hundred meters of your location. If you are engaged in a mission that requires more precision, then your datums should match.

(from URL: http://www.maptools.com/Using UTM/ mapdatum.html)


90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
North Latitude
South Latitude
10,000 km
8,000 km
6,000 km
4,000 km
2,000 km
0 km
North Pole
Eastern Zone Boundary
Central Meridian
Equator
18 Degrees of Longitude
Western Zone Boundary
South Pole
10,000 km
8,000 km
6,000 km
4,000 km
2,000 km
0 km
100 km
200 km
300 km
400 km
500 km

How to Read
the Universal Transverse Mercator (UTM) Grid

From GPS World,
April 1996, pp. 32, Advanstar Communications,
Eugene, OR.

N.G. Terry, Jr.

The Universal Transverse Mercator (UTM) grid is most appropriate for scales of 1:250,000 and larger. On large-scale maps such as U.S. Geological Survey (USGS) 1:24,000, 7.5 minute quadrangles, the simple numbers of the UTM grid make plotting precise locations easier than with the complex degrees, minutes, and seconds of latitude and longitude. For example, you might readily use the UTM grid while hiking to report the location of an emergency by cellular phone to 9-1-1 or to record the location of a favorite campsite or trailhead. The UTM grid has tremendous value for emergency service organizations. In Figure 1, an example of industrial use is shown.

The UTM grid is of particular interest to anyone using a GPS receiver because most models offer UTM as a coordinate system option. UTM coordinates simply measure in meters east and north from two perpendicular reference baselines. All USGS 1:250,000 and 1:100,000 topographic maps carry a full UTM grid. On those maps, grid lines are 10,000 meters (10 kilometers) apart. On 1:24,000 maps, grid lines are spaced 1,000 meters apart. Coordinates are written along the sides of a map designating specific grid lines or tic marks. The two larger numbers are known as principle digits (see Figure 2). To plot coordinate values, read right, then up. Abbreviated coordinates can be used for locally defined areas (not larger than 100 kilometers x 100 kilometers). Abbreviated coordinates are always given as an even number of digits (as in Figure1, 0491) so you know where to separate easting and northing coordinates. This allows you to abbreviate to the degree of precision you require, as will be shown later. A full UTM coordinate value defines a worldwide unique postion ("map address").

Reading UTM Coordinates. In our example, we will locate an isolated feature, a drill hole. Think of UTM grid coordinates as the drill hole's (geospatial) address. First check the bottom left corner of your map for the map's datum and for its grid. The default datum for most GPS receivers is the World Geodetic System of 1984 (WGS 84). A mismatch between datums on your map and GPS receiver can cause errors of several hundred meters. On the USGS 7.5 minute, 1980 edition Sierra Madera NE, Texas map sheet (Figure 1), the note reads like this:

> 1,000-meter Universal Transverse Mercator grid, zone 13
> 1927 North American Datum

While most USGS topographic maps use the 1927 North American Datum (NAD 27), maps are being slowly revised to NAD 83. In this example, set your GPS receiver for NAD 27. When building a waypoint or writing a grid coordinate, the first part is the grid zone. In this case, we see in the note it is Zone 13.

To find a complete UTM coordinate, read right to the grid intersection before your place of interest. In this case, it's line 704, also known by its principle digits as line 04. Then count grid lines up to the intersection (for example, 3391 or 91). The abbreviated coordinate 0491 (think 04/91) gives the location to within 1,000 meters. Measuring right in meters from line 04, we find the drill hole is another 250 meters. The complete easting component is 704250E.

Measuring up in meters from grid line 91, the drill hole is another 520 meters. The complete northing component is 3391520N. The complete grid coordinate for the drill hole is Zone 13 704250E 3391520N or abbreviated as 04259152 (think: 0425/9152).

This coordinate defines a location of the drill hole to within 10 meters, or about the size of a modest home. The abbreviated coordinates can be seen embedded in the full coordinates:

04259152 = **0425 9152**
Zone 13 7**04250**E 33**91520**N

In the field, an expedient grid reader can be made by tearing off a corner of the map and marking both edges out from the corner in 100-meter increments. Use the map's bar scale as a guide. Plastic grid readers are available from USGS and commercial vendors. With these you can easily measure coordinates to 10 meters, or near the precision of the USGS 7.5-minute map. One of the great features of UTM coordinates is the ability to provide a more precise location by simply adding a pair of digits to abbreviated coordinates. For example:

```
4 digits      0491 accurate to 1,000 meters
6 digits    042915 accurate to 100 meters, a little more than an
acre
8 digits   04259152 accurate to 10 meters, approximately the size
of a house.
10 digits 0425091520 accurate to 1 meter.
```

Another Example. Hilltop 3705 is located at grid 0577591395 or Zone 13 705775E 3391395N.

Hilltop 3774 is located at grid 04309021 or Zone 13 704300E 3390210N.

Make an expedient grid reader using the metric bar scale in Figure 1 and answer the following: What is the grid for hilltop 3726? (The answer is at the end of this sidebar.)

The Big Picture. (See Figure 3.) Let's see how our previous example fits into the world as a whole. Our example was referenced to Zone 13. The world is divided into 60 zones of latitude, each 6° wide, that extend from 84° N to 80° S. These zones begin at 180° longitude and are numbered consecutively eastward. The conterminous United States is covered by 10 UTM grid zones (see Figure 4). In the northern hemisphere each zone's northing coordinate begin at the equator as 0,000,000 and are numbered north in meters. Thus our drill hole is located 3,391,520 meters north of the equator. For the southern hemisphere, the equator is labeled 10,000,000. The easting coordinates are measured from an artificial reference line drawn parallel and 500,000 meters to the west of the zone's central meridian. (Thus each central meridian is numbered 500,000.) In the case of Zone 13, the central meridian is 105° W.

Answer: Hilltop 3726 is at grid 05589068 or Zone 13 705580E 3390680N.

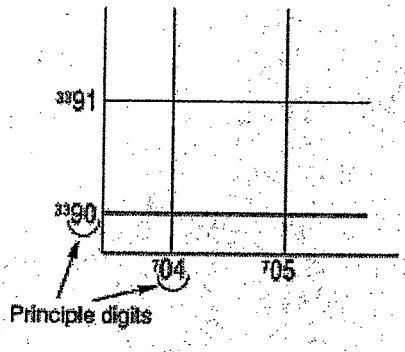


Figure 2

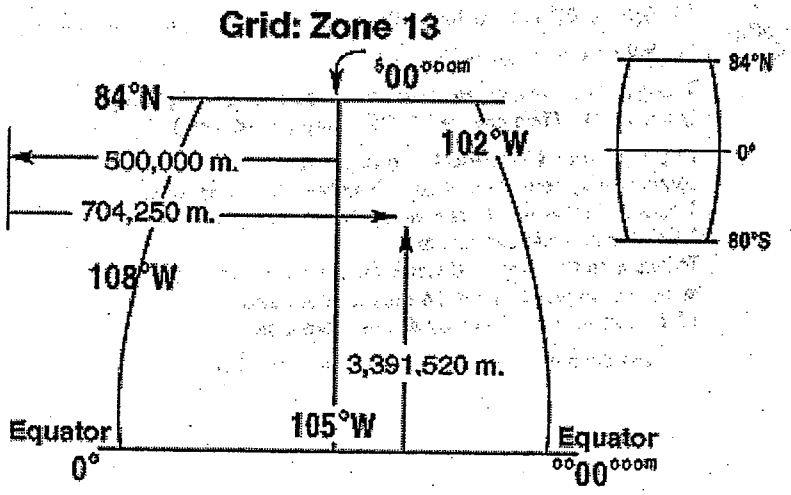
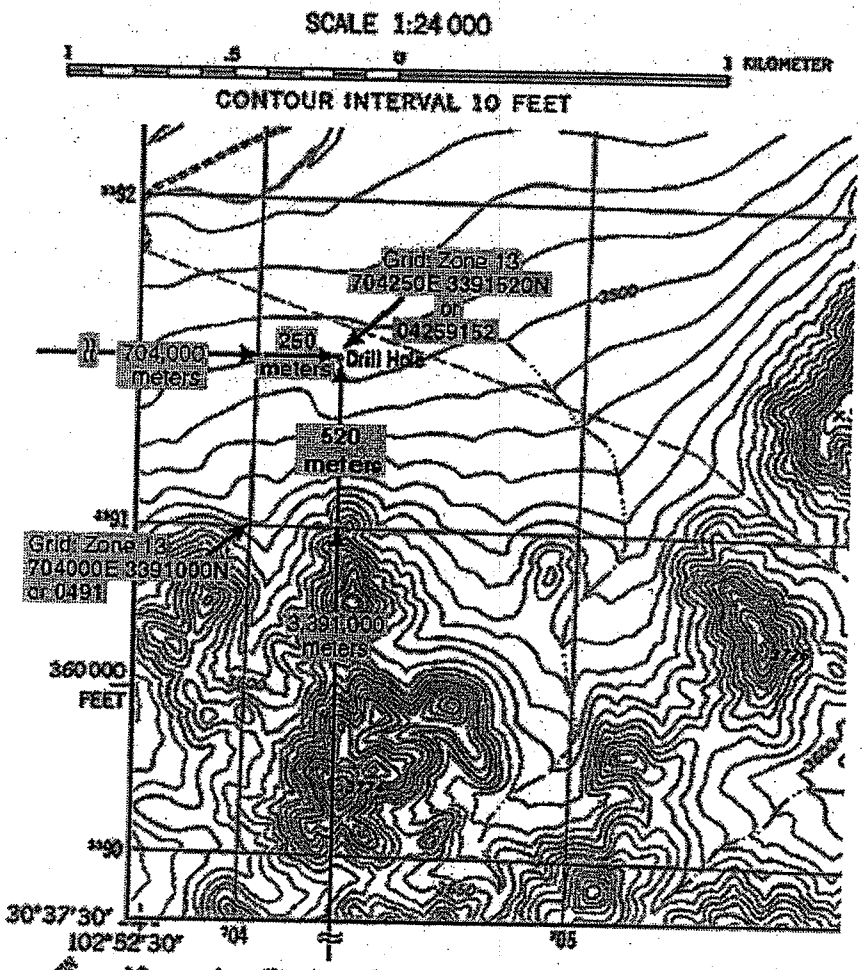


Figure 3

Figure 1



Mapped, edited, and published by the Geological Survey

Control by USGS and NOS/NOAA

Topography by photogrammetric methods from aerial photographs taken 1974. Field checked 1975. Map edited 1980

Projection and 10,000-foot grid ticks: Texas coordinate system, central zone (Lambert conformal conic)
1000-meter Universal Transverse Mercator grid, zone 13
1927 North American datum
To place on the predicted North American Datum 1983 move the projection lines 14 meters south and 41 meters east as shown by dashed corner ticks

Fine red dashed lines indicate selected fence lines

May Journal '99

Universal Transverse Mercator (UTM)

The UTM system was started in the US during WW II (1942+) in an effort to provide a world wide mapping base. Other than the ellipsoid parameters, UTM is identical to the Gauss-Kruger projection used in Canada, Great Britain and many parts of Europe, etc. With the global nature of the surveying community, this presentation is an attempt to place UTM into the computer era of the late 1990's , and to increase the understanding of the concepts and calculations. This write-up will be presented in 3 parts: part -1, the overview; part -2, conversion from NAD'27 to NAD'83 to HARN and finally part -3, the equations to calculate UTM coordinates.

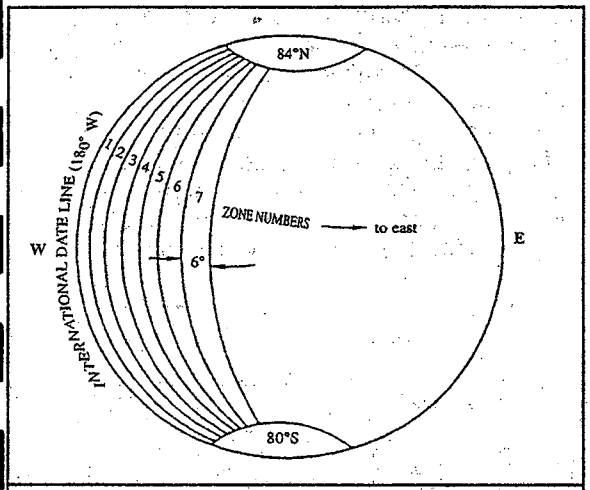


Fig. 1 - UTM ZONE NUMBERING

Conceptually, UTM is the same as the Transverse Mercator Projection used for State Plane Coordinates (Wyoming, etc.), with the exception that the zones cover the entire "earth", rather than portions of a state. As a free standing projection system UTM is excellent, but it suffers from the logical difficulty that there is no all encompassing ellipsoid which fits the entire earth. GRS'80, WGR'84, or any other ellipsoid fits well in some areas and poorly represents other areas of the world; this is in direct conflict with the modern trend of better and better fits to the ground. Based on the preceding, UTM is unquestionably the system of choice for GIS/Mapping, but it must be questioned whether UTM meets the modern GPS-surveying specifications.

As originally conceived and still implemented today, UTM divides the globe into zones 6° wide (3° wide zones are contemplated), for a total of 60 zones. Starting at the International Date Line Meridian, the zones are consecutively numbered to the east (Fig. 1 and Fig. 3) and each zone runs from 84° north to 80° south latitude (Fig.2). Both polar caps are covered by an unrelated Polar Stereographic projection. As shown in Fig.2, the central meridian of each zone has a fixed value of 500,000 m Easting (X) coordinate. For points north of the equator, the equator has a 0 m Northing (Y) coordinate and for points south of the equator, the equator has a 10,000,000 m Northing (Y) coordinate, thus the resulting coordinates are always positive (first quadrant values). As with other projections, UTM coordinates are plane coordinates near sea level or the geoid and have to be corrected for ground elevations, i.e. scale factor and elevation. By definition, the scale factor at the central meridian is 0.9996 (0.9999 for 3° zone). For a more "local flavor", Fig. 4 is included to show the UTM zones applicable for Colorado. The actual geodetic, lat/long, coordinates (HARN'92) of the 4 corner monuments are shown; thus the majority of Colorado is covered by UTM Zone 13, and only the western counties are in UTM Zone 12.

Now to the modern problem: with the advent of NAD'83 and HARN where does UTM stand?

Following below are some thoughts on applications, to be followed by the computer equations for forward and inverse calculations.

User application notes:

As stated above, UTM was started during WW II and at that time the basis of all UTM coordinates was the Clarke 1866 spheroid (NAD'27). Until recently 1995+, everything published, maps and geodetic control data, etc. was on that basis and in most cases there is no reference to the spheroid.

Now, there are at least 3 UTM coordinate datums-often again with no reference:

a) Where mandated by statutory action, the published tables for Clarke 1866 (NAD'27) values are still applicable.

b) The "original" UTM coordinates, based on Clarke 1866 with lat/long values in NAD'27 are no longer supported by NGS.

c) The "new" coordinates, based on GRS'80 with lat/long values in NAD'83 dated 1983+, are still supported by NGS.

d) The "new" UTM coordinates, based on GRS'80 with lat/long values in HARN, dated 1993+ or later. For HARN stations, NGS is now publishing these values as the primary data base.

continued on p.19

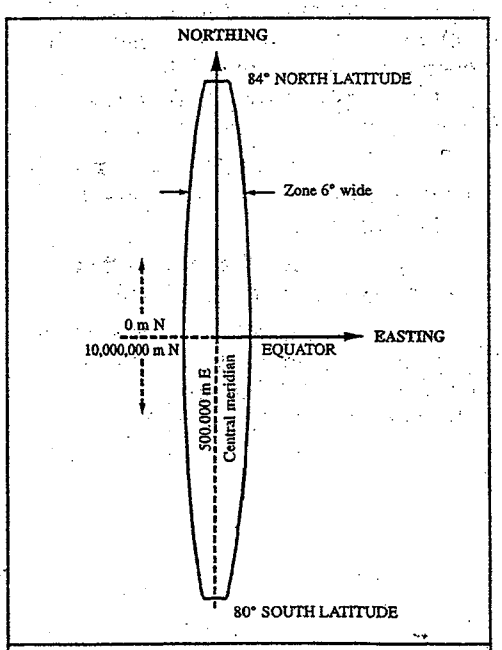


Fig. 2 - TYPICAL UTM ZONE

Universal Transverse Mercator (continued)

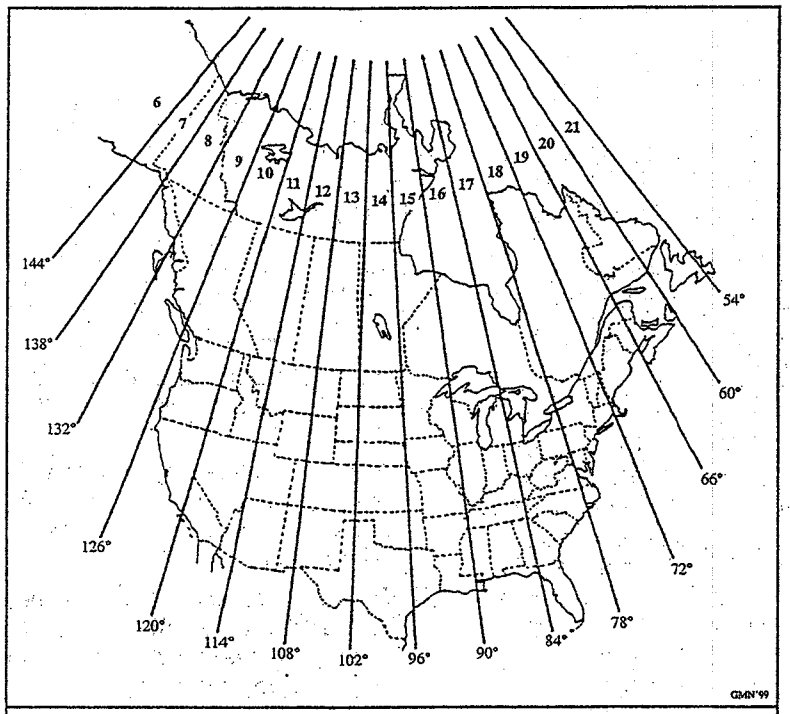


Fig. 3 - NORTH AMERICAN UTM ZONES

Surveying applications

The user should be warned that distances calculated from UTM coordinates are in the mapping plane; NAD'27 requires the scale factor, radius of curvature and elevation, NAD'83 and HARN require the 3 factors plus the geoid elevation (geoid height) in order to obtain true ground distances.

For mapping, abutting zones are ideal; surveying jobs however, may have to be extended from one zone to the next, for example a federally financed power line, etc. Despite an extensive literature search, there appear to be no published guidelines how far a UTM zone can be mathematically extended laterally (E-W) beyond the given 6° boundary. Historically the US State Plane transverse Mercator zones were limited to a 158 miles (254 km) width, ≈3° , with zone overlaps modified by political boundaries, in order of 1°. Germany uses 3° 20' wide zones, with a 20m overlap between zones. Without qualifying the value, a maximum 5m mathematical extension (≈5 miles) of the zone beyond the 6° limit should be possible without introducing major errors. Otherwise, if everything else fails, inverse (calculate the lat/long) of a point at the end of a zone and then recalculate the UTM coordinates of the same point for the next zone. The result admittedly, is a survey with 2 different UTM coordinates.

Maps, etc.

Most "old" USGS maps show the 1,000m UTM grid, based on the NAD'27 datum.

"Newer" USGS maps still show the 1,000m UTM grid, based on the NAD'27 datum but in the lower left corner of the map is the notation that coordinates have to be shifted by a certain amount to get to the NAD'83 datum. The shift is referenced to the center of the map and given in full meters, implying mapping accuracy only but not surveying accuracy.

The "newest" USGS map, starting in 1997+, now show the NAD'83 UTM coordinates!

Caution-check the lower left corner of the map for the UTM datum.

Due to the change in ellipsoid parameters, grid north, i.e. the mapping angle, is not the same for NAD'27 and NAD'83 (HARN).

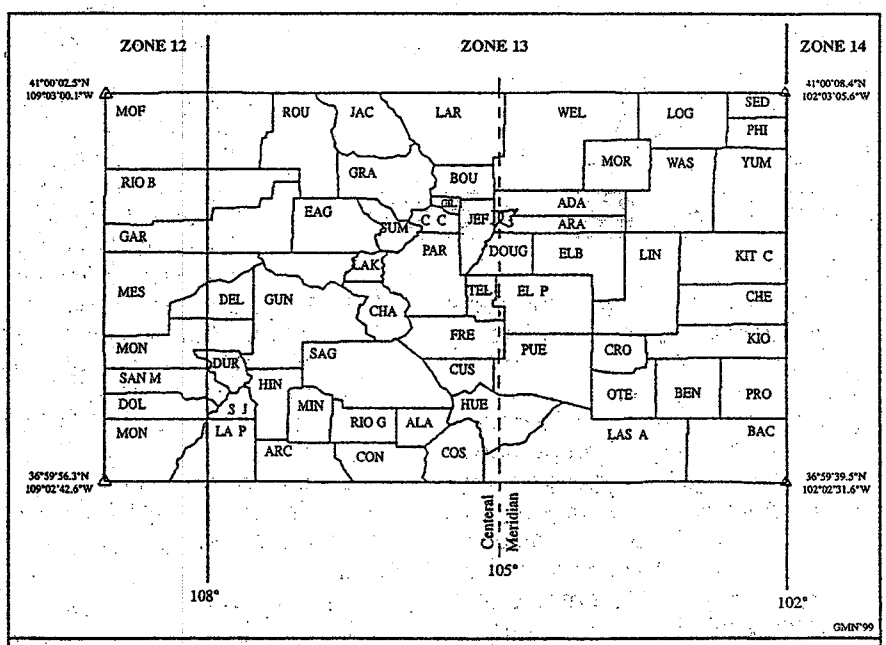


Fig. 4 - COLORADO UTM ZONES

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Universal Transverse Mercator (UTM) Part 2

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Conversion from NAD'27 to NAD'83 to HARN

This second part of the conversion between UTM data bases has to start with a disclaimer:

Dispite some commercially available computer programs, there is NO DIRECT conversion, i.e. there is NO CONSTANT MULTIPLIER, between the data bases because the published lat/long values of the control points have been adjusted (changed non-linearly) and there are 2 different ellipsoids involved.

Before dwelling on the principles, there are indeed some facts:

Facts:

1. The physical location of the control points has not changed over the years, or if so very, very little.
2. The physical distance between control points has not changed over the years, or if so very, very little.
3. The physical angle subtended between control points has not changed over the years, or if so very, very little.
4. In summary, even for surveyors, geodesists and GPS enthusiasts, the earth has changed very little over the years and yes, some of the changes can now by physically measured.
5. Once a data base has been established for a project, it can be added to either by conventional techniques or by GPS methods; the type of ellipsoid or projections does not matter, as long as everybody uses the same numbers, and in turn states the data base!

Enter mathematics! Even though written for UTM, the discussion following below can almost verbatim be applied to State Plane Coordinates (both Lambert and Mercator).

NAD'27 Data base

The NAD'27 data base represents the aggregated control station effort, covering the major part of North America, from the post colonial days to about 1970. The mathematical model is the Clarke 1866 ellipsoid.

NAD'83 Data base

Next, two major changes were made during the 1975-1980 period: first the data base was adjusted and second the ellipsoid was redefined.

The NAD'83 data base represents the mathematically derived values from the ADJUSTMENT of about 250,000 control stations in North America. The key word is ADJUSTMENT, i.e. implying that depending upon the magnitude and direction of the adjustment vector, new geodetic (lat/long) values for each station were calculated and in turn new coordinates were published; see example below.

Based in part on information from the Triangulation Network, the mathematical shape of North America was redefined as GRS'80 (WGS'84, WGS'95 etc.) ellipsoid.

Even though apparently minor in magnitude, the lat/long values changed mostly in the decimal portion of seconds or arc and the equatorial radius of the ellipsoid changed by 69 m only to 6 378 137 m; the change in coordinates is dramatic. Thus from NAD'27 to NAD'83 – depending upon the location of a point within a zone, the shift in UTM coordinates is in the range of 10-300 m (30-1,000 ft). See Fig. 5 and Fig. 6 An inspection of the UTM calculations, presented in Part 3, will show that the ellipsoid parameters have a major effect.

HARN'92

Once again the data base of the 1980's was READJUSTED nonlinearly, but with considerably smaller shifts than before; the GRS'80 (WGS'84, etc.) ellipsoid was retained. From NAD'83 to HARN – again, depending upon the location of a control station within a zone, the shift in coordinates is in the range of 1 ± m (3 ± ft).

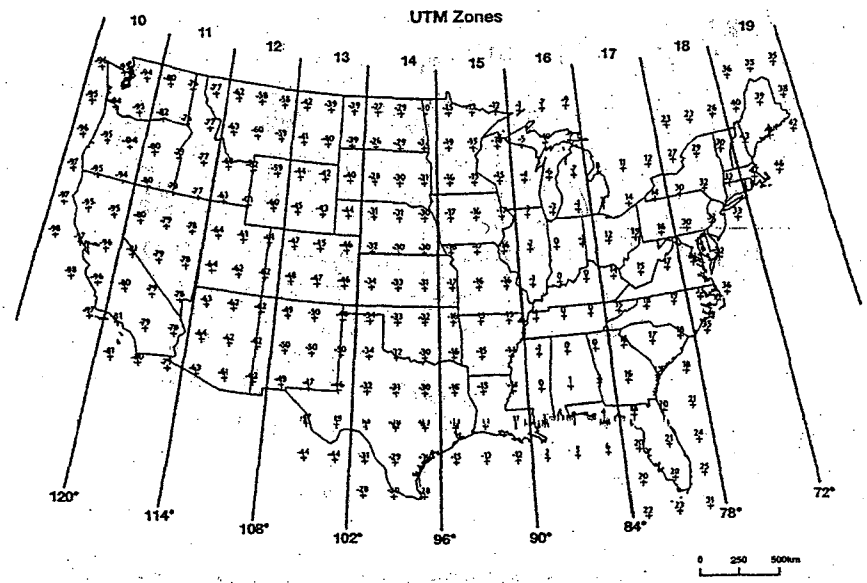

Fig. 5 **Shift in Easting (X) coordinates (m)**
NAD'27 UTM Easting + shift = NAD'83 UTM Easting

Universal Transverse Mercator (continued)

Example

A numerical example of 3 horizontal 1st order control stations in the Denver area may illustrate the concepts shown above. The 3 stations roughly form a triangle: Boulder to Morrison 32,938.16 m, Morrison to State Capitol 21,581.34 m and State Capitol to Boulder 36,143.48 m. Station Morrison is roughly due south of Boulder and Station State Capitol is the east most station.

Station	Shift from NAD'27 to NAD'83	Shift from NAD'83 to HARN'92
Boulder	213.705 m, N 12°23'34" W	0.181 m, N 33°20'27" W
Morrison	213.265 m, N 12°30'02" W	0.118 m, N 10°44'20" W
State Capitol	213.122 m, N 12°28'39" W	0.304 m, N 55°44'04" E

The meter threshold

It has been repeatedly emphasized that there is no constant conversion factor to convert from one data base to another. It also logically does NOT make sense to start with NAD'27 UTM coordinates, inverse to get the lat/long and then recalculate the NAD'83 coordinates with the "new" ellipsoid, because the geodetic values (lat/long) are NOT the same for both datums. What is a user to do?

There is about a one meter threshold, and on the risk that a large portion of the user community will be offended, there is a division between users who can accept or have unknowingly accepted an error of 1 m or larger vs. the users who need to work down to the mm range.

Users who can accept an error of one meter or larger. The answer is simple, buy or in some cases even obtain for free, any of the commercial computer programs for conversion; my bet is that most are based on the illogical assumption that the geodetic values (lat/long) are a constant and as a consequence can NOT give correct UTM coordinates below the one meter threshold, even though decimals out to the mm are shown. The proof is simple: if your program requires the input of three known shift vectors, it is OK; otherwise the answers are to the nearest meter or worse. Also, programs which either ask for or generate a single conversion factor should be viewed with **great caution**! Again, see Fig. 5 and Fig. 6.

Users who need to work down to the mm range. Working with GPS, or for that matter just working as a surveyor, requires the accurate and duplicatable generation of numbers down into the mm range. For this group of users, the only workable method for coordinate transformation is to use the least square method to calculate the most probable latitude and separately the most probably longitude shift vectors of at least 3 nearby (<5km) control stations. Now the equations together with the shift vectors, will work to calculate the correct coordinates.

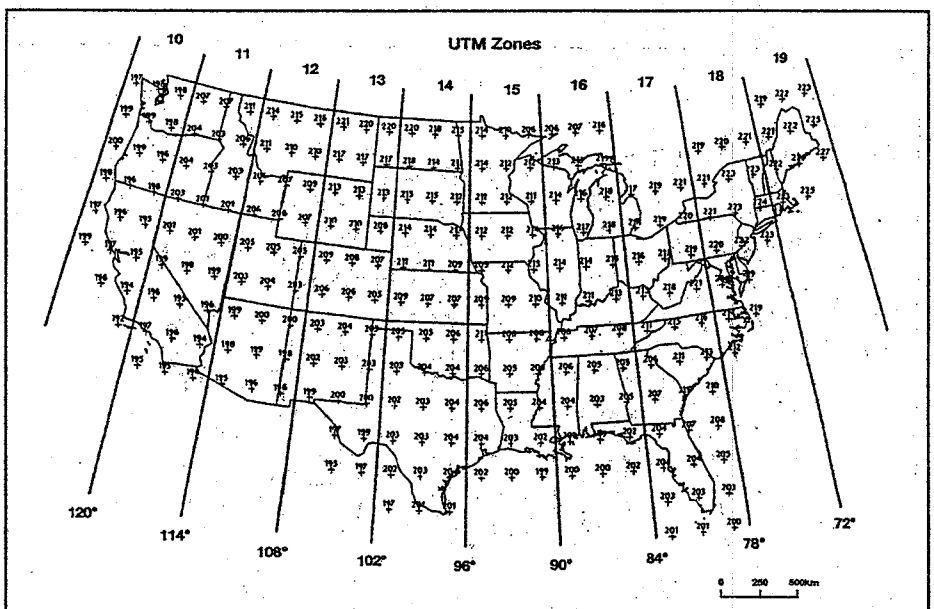

Fig. 6 **Shift in Northing (Y) coordinates (m)**
NAD'27 UTM Easting + shift = NAD'83 UTM Easting

Selected References:

DMA, 1989; The universal grids; UTM and Universal Polar Stereoscopic, DMATM 8358.2, Defense Mapping Agency (DMA) now National Imagery and Mapping Agency (NIMA)

Doyle, Frederick J. 1997; Map conversion and the UTM grid, PE & RS, Vol. 63, No. 4, April 1997.

NIMA, 1997; World Geodetic System 1984, its definition and relationships with local geodetic systems, NIMA No. DMATR83502WGS84, NSN 7643-01-402-0347

Snyder, John P., 1987; Map projections - a working manual, U.S.G.S. Professional paper 1395.

Welch, R. & Homsey, A. 1997; Datum shift for UTM Coordinates, PE & RS, Vol. 63, No. 4, April 1997.

Errata:

In the first article on UTM (p. 16, May 1999 Side Shots) the sentence in the last paragraph, containing the following phrases:

"Germany uses ... with a 20 m overlap between zones", should read: "Germany uses ... with a 20 minutes of arc overlap between zones"

"A maximum 5 m mathematical extension (≈ 5 miles)", should read: "A maximum 5 minutes of arc mathematical extension (≈ 5 miles)"

Universal Transverse Mercator (UTM) Part 3

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Division of Engineering, CSM, (303) 279-1700

Part 3 is the concluding segment of UTM coordinates and it presents the forward and inverse calculations. As shown in the references, the majority of the equations were adapted and modified from John P. Snyder's U.S.G.S. Professional Paper 1395; the only new material are the addition of the scale factor (equations F-10 to F-12), the mapping angle (equation F-13) and a numerical example based on GRS'80 (WGS'84). The equations given are exact, but the user has to be reminded that the published Lat. & Long. values of a given control point are not the same in NAD'27, NAD'83 or HARN; thus the equations can NOT be used as a direct conversion.

Even though the equations appear to be intimidating, the steps are set up to permit a cascading entry into a spreadsheet such as Excel or Mathcad, etc. As stated

Forward calculations - from Lat. & Long. to UTM — Numerical Example

Known: ϕ = Latitude, λ = Longitude — ϕ = 39°57'36.91689" N, λ = 105°17'42.59642" W

Constants - GRS'80 ellipsoid:

Equatorial radius	a = 6,378,137 m
Eccentricity squared	e^2 = 0.006 694 380 023
By definition for 6° Zone	k_o = 0.9996

Preliminary variables:

(F-1) $e'^2 = \dfrac{e^2}{(1-e^2)}$ — 0.006 739 497

(F-2) $R_N = \dfrac{a}{(1-e^2\sin^2\phi)^{1/2}}$ — 6 386 961.5 m

(F-3) Longitude for central meridian of projection zone

$$\lambda_o = 6°\left[\mathrm{INT}\left(\frac{\lambda - 180°}{6°}\right) - 1\right] + 183°$$ — 105°00'00"

(F-4) $A = (\lambda_o - \lambda)\cos\phi$ (λ in rad) — -0.003 948 66 rad

(F-5) $T = \tan^2\phi$ — 0.702 106 936

(F-6) $B = e'^2\cos^2\phi$ — 0.003 959 503

(F-7)
$$M = a\left[\left(1-\frac{e^2}{4}-\frac{3e^4}{64}-\frac{5e^6}{256}\right)\phi - \left(\frac{3e^2}{8}+\frac{3e^4}{32}+\frac{45e^6}{1024}\right)\sin 2\phi + \left(\frac{15e^4}{256}+\frac{45e^6}{1024}\right)\sin 4\phi - \left(\frac{35e^6}{3072}\right)\sin 6\phi\right]$$
= 4 425 115.9 m

(F-8) **X Coordinate of station**

$$X = k_o R_N\left[A + (1-T+B)\frac{A^3}{6} + (5-18T+T^2+72B-58e'^2)\frac{A^5}{120}\right] + 500{,}000 \text{ m}$$ — **474,790.126 m**

(F-9) **Y Coordinate of station**

$$Y = k_o\left\{M + R_N\tan\phi\left[\frac{A^2}{2} + (5-T+9B+4B^2)\frac{A^4}{24} + (61-58T+T^2+600B-330e'^2)\frac{A^6}{720}\right]\right\}$$ — **4,423,387.599 m**

(F-10) **Scale Factor**

$$k = k_o\left[1 + \frac{(1+e'^2\cos^2\phi)X^2}{2k_o^2 R_N^2}\right]$$ — **0.999 607 82**

(F-11) $\lambda' = \lambda_o - \lambda$ (in rad) — -0.005 151 613 rad

(F-12) $\eta^2 = \dfrac{e^2\cos^2\phi}{1-e^2}$ — 0.003 959 503

(F-13) **Mapping angle**

$$\Delta\alpha = \lambda'\sin\phi\left[1 + \frac{\lambda'^2\cos^2\phi(1+3\eta^2)}{3} + \frac{\lambda'^4\cos^4\phi(2-T)}{15}\right]$$ — **-0°11'22.5"**

Universal Transverse Mercator (continued)

previously, the constants which must be entered are for NAD'27 (Clarke 1966, a = 6,378,206.4 m, e^2 = 0.006 768 658 002) and NAD'83 are the same for HARM (GRS'80, a = 6,378,173 m, e^2 = 0.000 694 380 023). By definition the scale factor at the central meridian (ko) is 0.9996 for a 6° wide zone. Should the 3° wide zone be implemented, ko will have to be changed to 0.9999 and equation F-3 will have to be modified as well.

For the non-programming individuals, I can furnish a copy of my program, written in Excel'97, at the usual "you ruin it at your own risk basis"! Just call me: H-303-279-1700, e-mail: gneunzer@mines.edu, or drop me a line at: 15260 W. 48th Ave, Golden, CO 80403.

Inverse calculations - from UTM to Lat. & Long.

(R-1) $e'^2 = \dfrac{e^2}{(1-e^2)}$ same as equation F-1 above — 0.006 739 497

(R-2) $e_1 = \dfrac{1-(1-e^2)^{1/2}}{1+(1-e^2)^{1/2}}$ — 0.001 679 22

(R-3) $C = \dfrac{Y}{k_o}$ — 4 425 157.662 m

(R-4) $\mu = \dfrac{C}{a\left(1-\dfrac{e^2}{4}-\dfrac{3e^4}{64}-\dfrac{5e^6}{256}\right)}$ — 0.694 965 529

(R-5) 'Footpoint' latitude, or the latitude at the central meridian which has the same y coordinate as the station.

$$\phi_1 = \mu + \left(\frac{3e_1}{2}-\frac{27e_1^3}{32}\right)\sin 2\mu + \left(\frac{21e_1^2}{16}-\frac{55e_1^4}{32}\right)\sin 4\mu + \left(\frac{151e_1^3}{96}\right)\sin 6\mu + \left(\frac{1097e_1^4}{512}\right)\sin 8\mu$$

= 0.697 444 573

(R-6) $D = e'^2\cos^2\phi_1$ — 0.003 959 459

(R-7) $E = \tan^2\phi_1$ — 0.702 125 644

(R-8) $N = \dfrac{a}{(1-e^2\sin^2\phi_1)^{1/2}}$ — 6 386 961.661 m

(R-9) $R = \dfrac{a(1-e^2)}{(1-e^2\sin^2\phi_1)^{3/2}}$ — 6 361 772.483 m

(R-10) $F = \dfrac{X}{k_o N}$ — -0.003 948 664

(R-11) **Latitude of station**

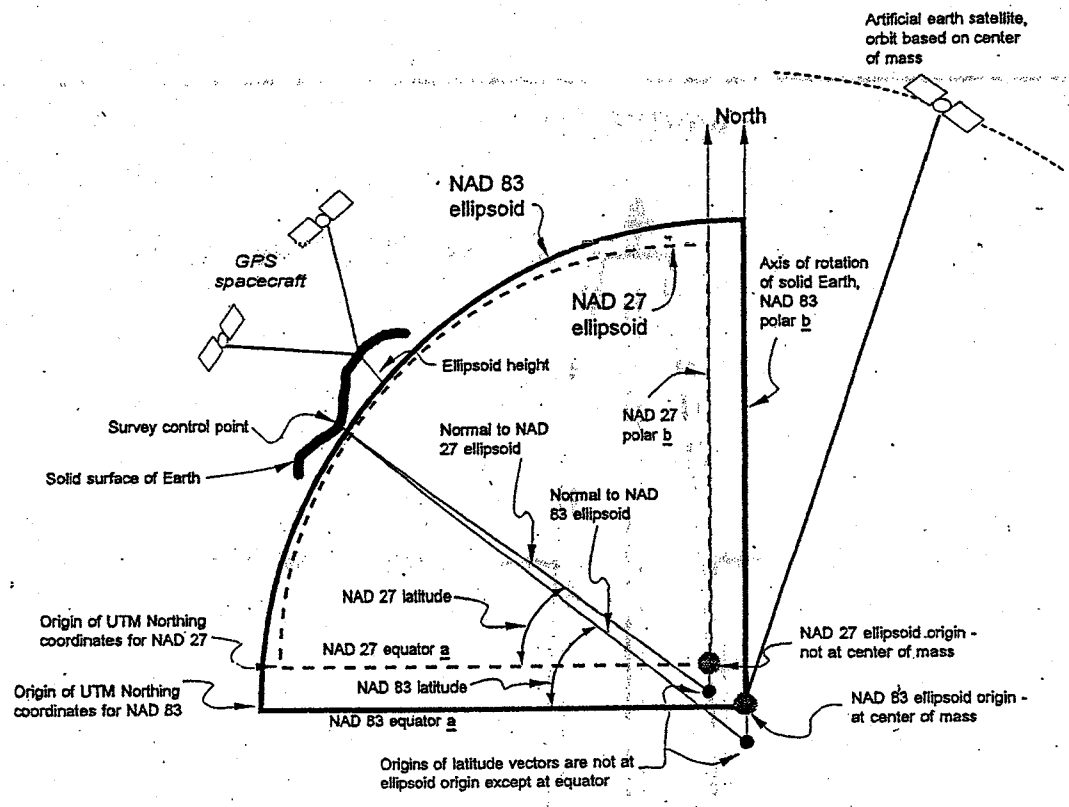
$$\phi = \phi_1 - \left(\frac{N\tan\phi_1}{R}\right)\left[\frac{F^2}{2} - \frac{(5+3E+10D-4D^2-9e'^2)F^4}{24}\right] + \left[\frac{(61+90E+298D+45E^2-252e'^2-3D^2)F^6}{720}\right]$$

39°57'36.91689"

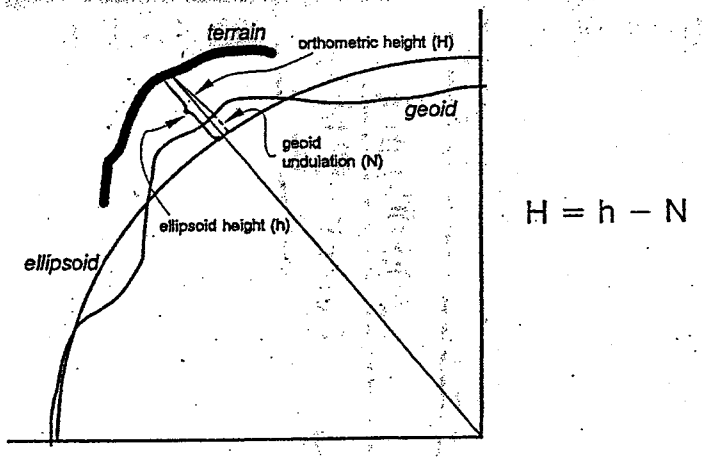
(R-12) **Longitude of station**

$$\lambda = \lambda_o + \left[F - \frac{(1+2E+D)F^3}{6} + \frac{(5-2D+28E-3D^2+8e'^2+24E^2)F^5}{120}\right]\frac{1}{\cos\phi_1}$$

105°17'42.59642"



a)



b)

Figure 1. (a) Meridianal cross-section showing geometric basis for shifts in geodetic and UTM coordinates resulting from changes in datum and ellipsoid parameters. (b) Schematic relationship of ellipsoid and geoid to terrain surface.

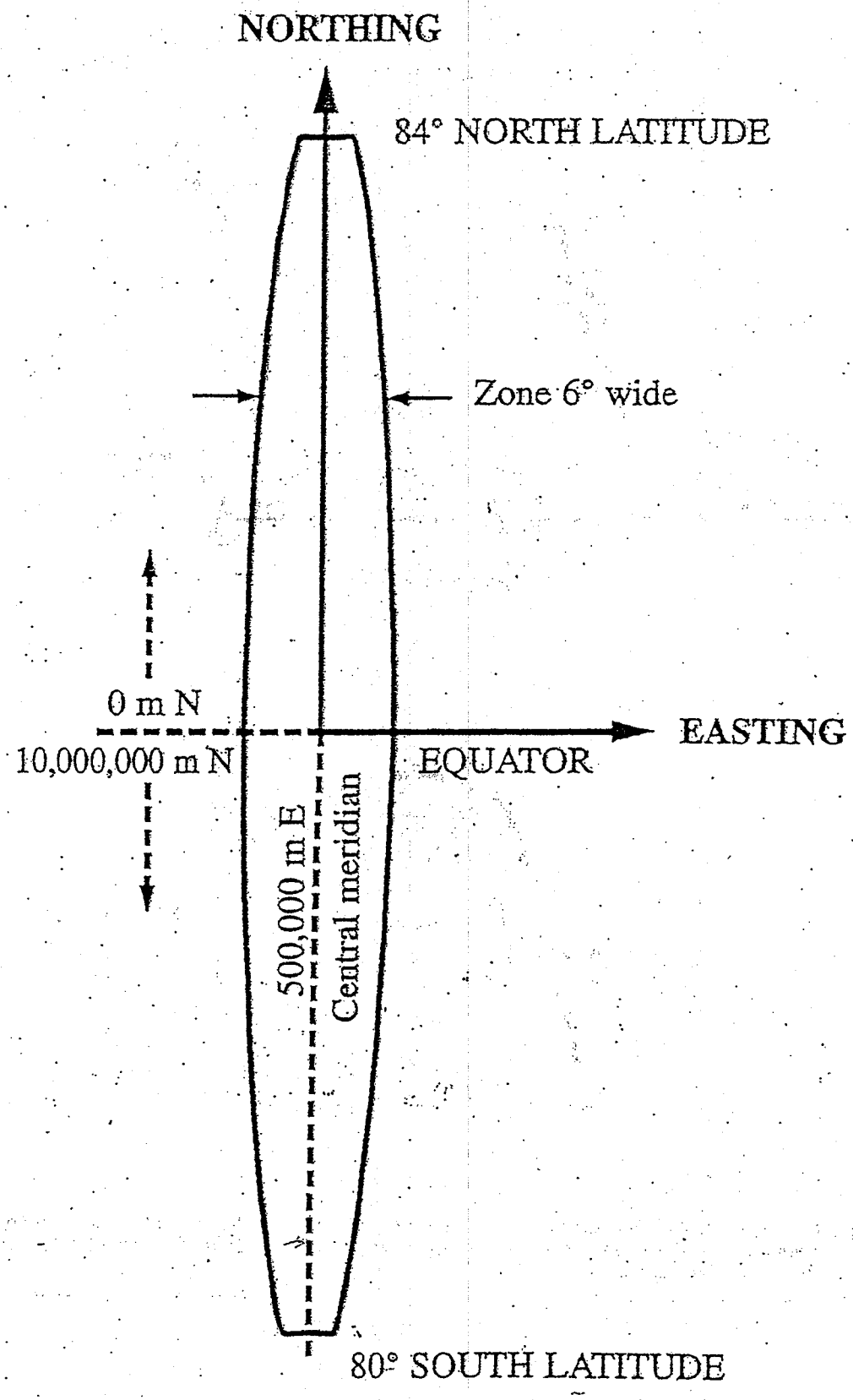
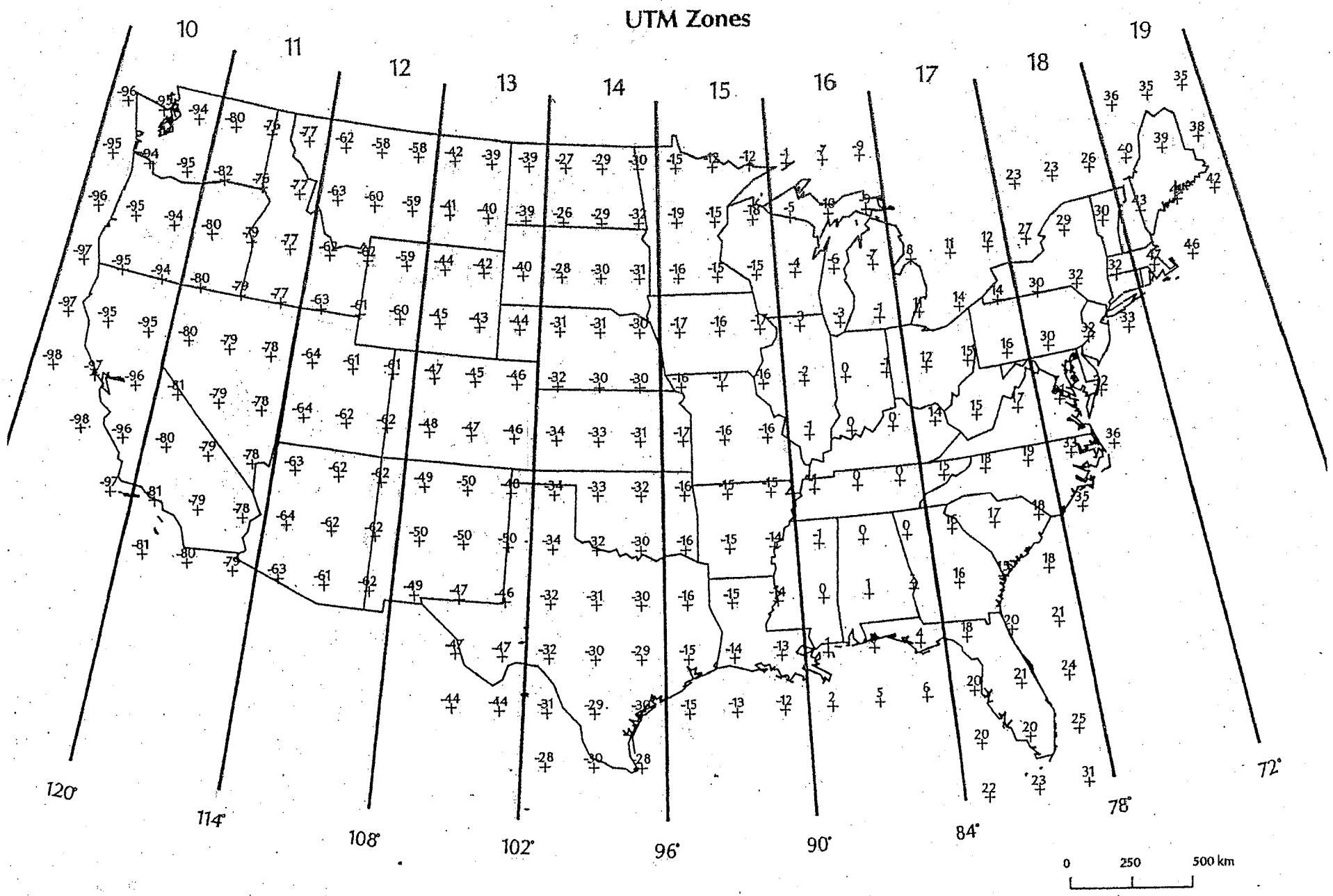


Fig. 2 - **TYPICAL UTM ZONE**



gure 4. Easting coordinate shifts in metres. NAD 27 UTM Easting + shift = NAD 83 UTM East-
ıg.

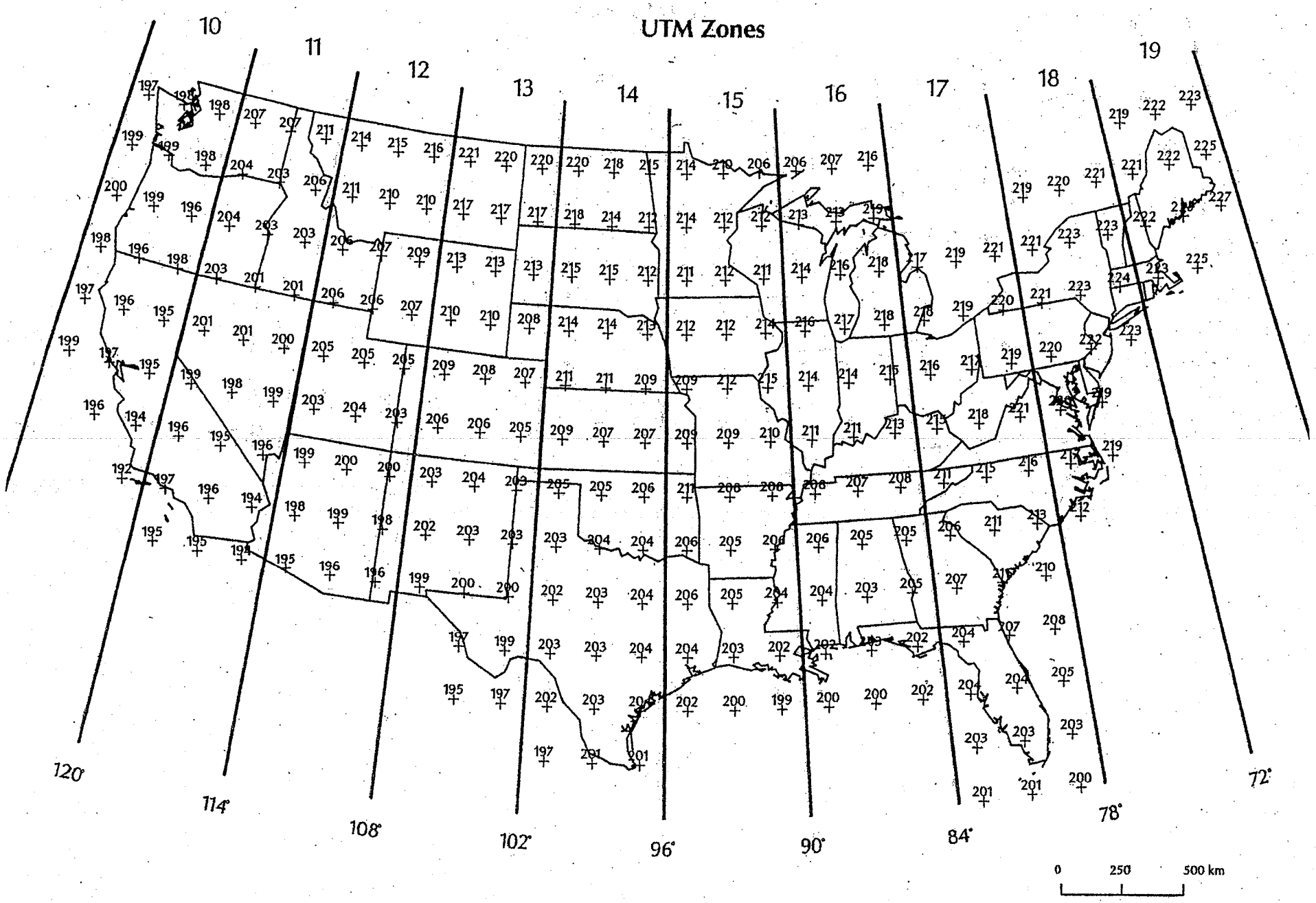


Figure 5. Northing coordinate shifts in metres. NAD 27 UTM Northing + shift = NAD 83 UTM Northing.

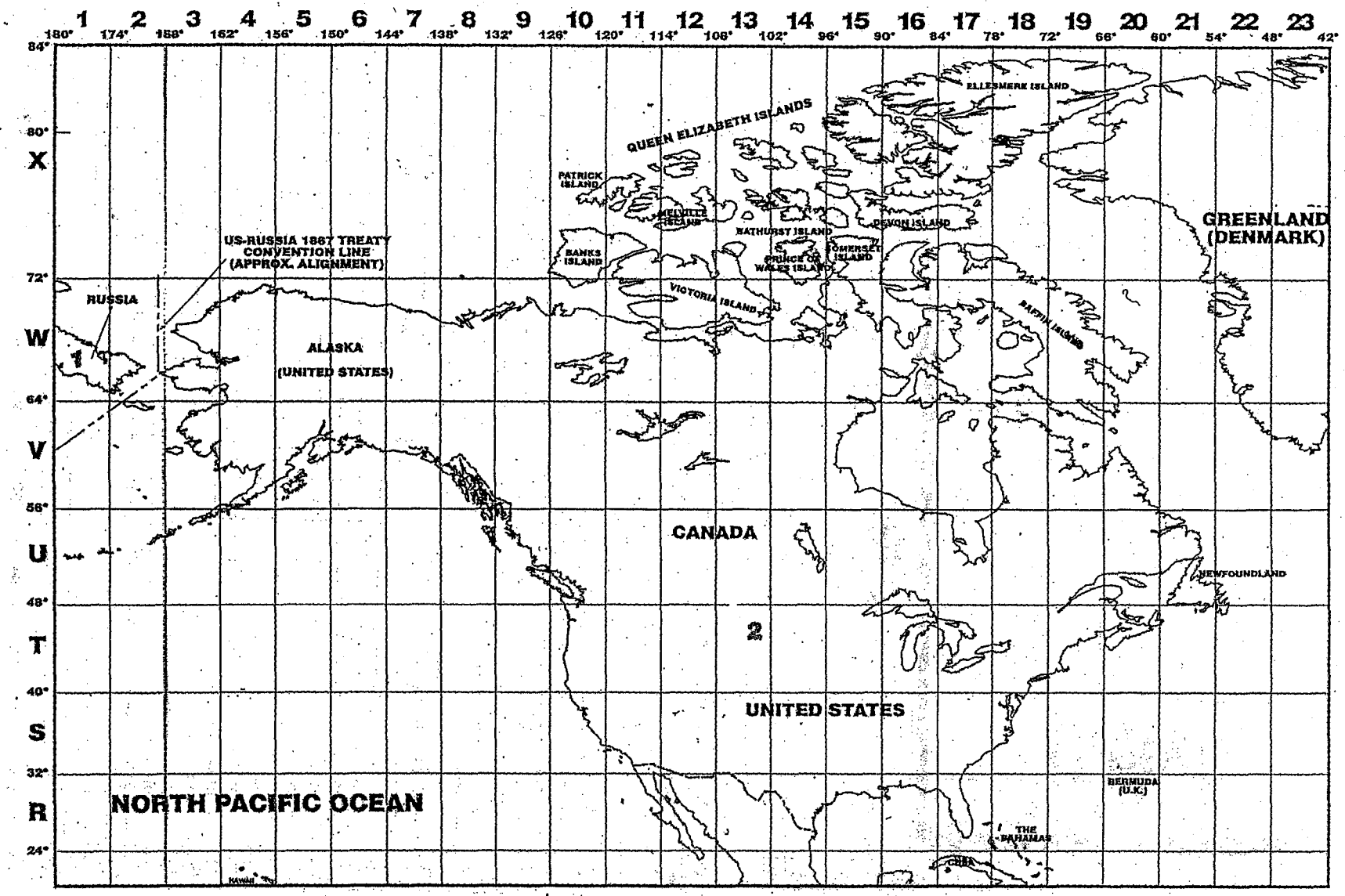


FIGURE 3-25
MGRS GRID ZONE DESIGNATIONS

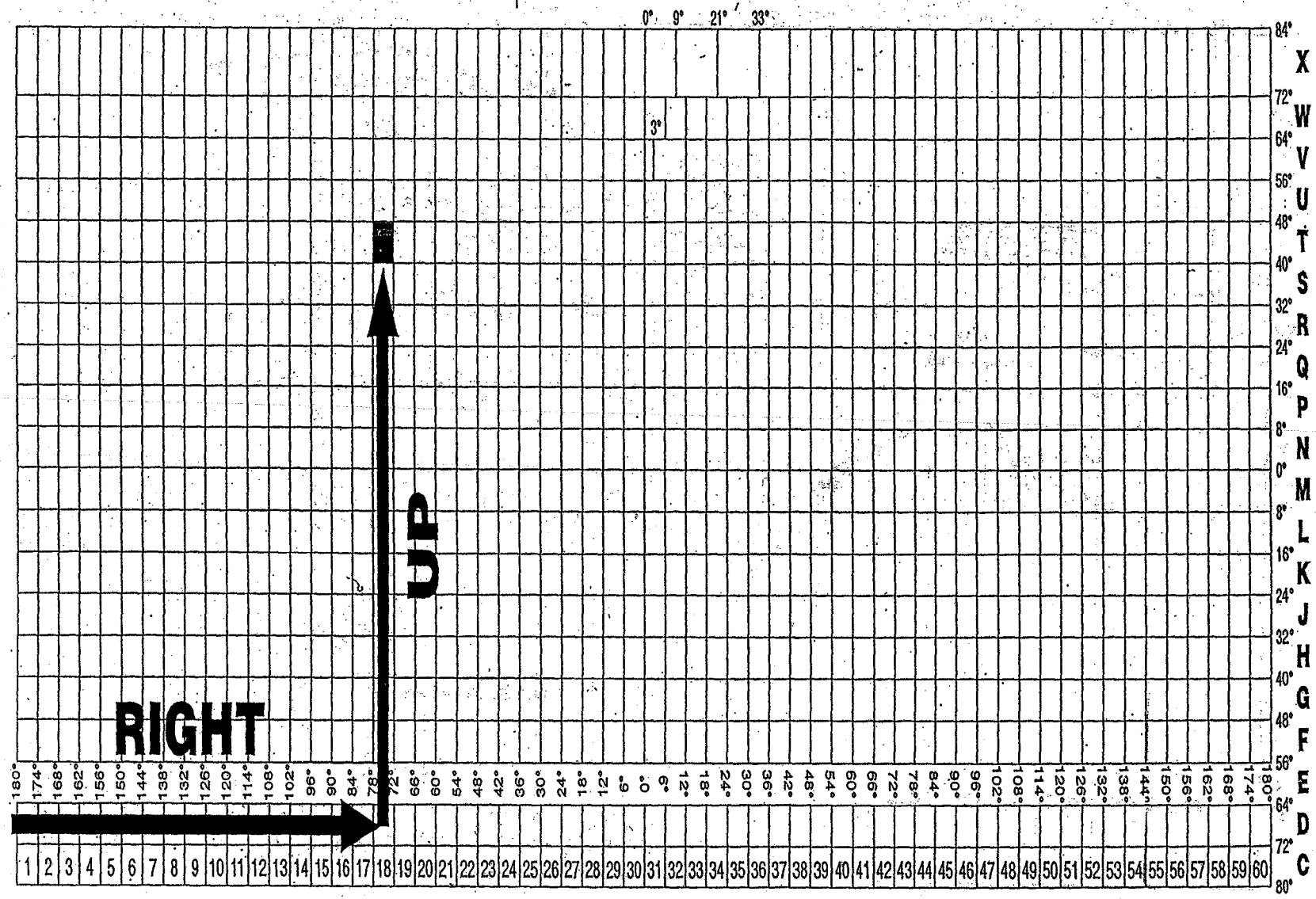


FIGURE 3-23

WORLD-WIDE MGRS GRID ZONE DESIGNATIONS

GEOSPATIAL SCIENCES DIVISION

SATELLITE GEODESY
- GPS Data
- Control Data

PHYSICAL GEODESY
- WGS 84 Earth Gravity Model
- Arctic Gravity Project
- Grids and Projections

GEODETIC SURVEYING
- Survey Operations

OTHER INFORMATION
- Publications
- Metadata
- Datum Transformation
- Scientific Applications
- DoD World Magnetic Model 2000

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Geospatial Sciences Division

Geospatial Sciences Publications

Welcome to the NIMA Geospatial Sciences Division publications listing. Many of the documents are available in Adobe's portable document format (PDF). The Adobe Acrobat viewer software is available free... Get Acrobat Reader

1. Geodesy for the Layman (HTML)
   - Geodesy for the Layman *(3,507 KB)*
2. DoD World Geodetic System 1984 - Its Definition and Relationships with Local Geodetic Systems - TR 8350.2
3. Datums, Ellipsoids, Grids and Grid Reference Systems - TM 8358.1 (HTML)
   - Datums, Ellipsoids and Grid Reference Systems - TM 8358.1
     - Chapters 1 - 5 *(6,418 KB)*
     - Chapters 6 - 7 *(7,747 KB)*
     - Chapters 8-11 *(6,824 KB)*
     - Appendix A - C *(286 KB)*
4. The Universal Grids: Universal Transverse Mercator (UTM) and Universal Polar Stereographic (UPS) - TM8358.2 *(2,883 KB)*
5. GEOTRANS v2.2.2 (GEOgraphic TRANSlator)
   - Revised version 2.2.2 posted May 16, 2002
   - Summary of Fixes from 2.0.1 to 2.2.1
   - Register - click here if you wish to be contacted with updates as they occur.
6. WGS 84 Coordinate Validation and Improvement for the NIMA and Air Force GPS Tracking Stations (NSWCDD/TR-96/201)
7. Horizontal Datums
8. Vertical Datums
9. All You Ever Wanted to Know and Couldn't Find Out About **Precise Positioning...***(In Plain English)*
10. Publications of Historic Interest
11. Ordering instructions for Geospatial Sciences Division publications

EDITION 1

September 1989

Defense Mapping Agency
Fairfax, VA

DMATM 8358.2

DMA TECHNICAL MANUAL

THE UNIVERSAL GRIDS: Universal Transverse Mercator (UTM) and Universal Polar Stereographic (UPS)

Approved for Public Release: Distribution Unlimited

DMA Stock No.
DMATM83582

FACTOIDS
About
ARIZONA
UTM Grid Coordinates
As compared to the AZ SPGCS

UTM Zones are six (06˚) degrees (longitude) in width.
STATE PLANE Zones are typical three (03˚) degrees or (*much*) less,
In Arizona on *AVERAGE* for the Central Zone is about 01˚47' 30" and about 02˚ 35' at the widest part.

UTM Zone # 12 is from 114˚W to 108˚W (NAD- 27 & 83)
UTM Zone Band "S" is from 32˚N to 40˚N (NAD-27 & 83)
UTM Zone # 12 Central Meridian = 111˚ 00' 00" W (NAD-27 & 83)
AZ State Plane Zone C.M.s

W.Z.	=	113˚ 45' 00" W (NAD-27 & 83)
C.Z.	=	111˚ 55' 00" W (NAD-27 & 83)
E.Z.	=	110˚ 10' 00" W (NAD-27 & 83)

UTM Northing origin point is 000000.000 Meters at the Equator (0˚N) (NAD-27 & 83)
AZ-SPGCS origin is 000000.00 *Feet* at 31˚N (NAD-27)
AZ-SPGCS origin is 000000.000 *Meters* at 31˚ N (NAD-83)

UTM Zone 12 is *approximately* 413 Miles (+/-) wide (in Central AZ).
AZ SPGCS Central Zone is from '*about*' 125-miles, to '*about*' 170-miles, wide.

UTM: Central Meridian false Easting = 500,000 *METERS*
STATE PLANE (AZ) Central Meridian false Easting = *500,000 FEET*

500,000-*METERS*	=	310.6-MILES
500,000-*FEET*	=	094.7-MILES

UTM: "CENTRAL Scale Factor" (at the Central Meridian) is 0.9996
That equates to a scale ratio of 1:2,500
STATE PLANE: Central scale factor (AZ E & C) is 0.9999
That equates to a scale ratio of 1:10,000

UTM NAD-27 is based on the CLARKE Spheroid of 1866
AZ SPGCS NAD-27 is based on the CLARKE Spheroid of 1866
UTM NAD-83 is based on the WGS-80 Ellipsoid
AZ SPGCS NAD-83 is based on the WGS-80Ellipsoid

NAD-27	Clarke Spheroid	Equatorial "a" 6,378,206 M	Polar "b" 6,356,584 M
NAD-83	WGS-80 Ellipsoid	Equatorial "a" 6,378,137 M	Polar "b" 6,356,753 M
GPS	WGS-84 Ellipsoid	Equatorial "a" 6,378,137 M	Polar "b" 6,356,754 M

With AZ/SPGS (NAD –27 & -83) Scale Factor and Geodetic Azimuth corrections are *somewhat simple.*
With UTM (NAD –27 & -83) both the Scale Factor & Geodetic Azimuth corrections are *rather complex.* (Use computer programs.)

NAD-83, in the conterminous U.S. the GEOID is ALWAYS BELOW the ELLIPSOID; with values, ranging from –5 Meters, to –53 Meters.

By GMT 7/8/2002

These TALES, and these DETAILS,
So many they seem,
But the details we remember...
may save the "tails" we esteem!

WATCH OUT FOR: THE LAW OF UNINTENDED CONSEQUENCES

THERE IS AN *EASY ANSWER* TO YOUR PROBLEMS,
THAT IS NEAT, PLAUSIBLE, AND WRONG!

WHAT ARE BOUNDARIES - IS A MATTER OF LAW.
WHERE THEY ARE - IS A MATTER OF FACT.
Justice Wheeler, in: Bolton v. Lann (16 TEX 96)

There are NO FACTS; only Interpretations. F. Nietzche

RTFM (Read the Freakin' Manual) Don't you just hate it when Life doesn't follow the manual?

Horatius: "EST MODUS IN REBUS" ("There is measure in (all) things", Horace, the Roman poet)

An accountant is some who knows the cost of everything and the value of nothing.
A statistician is someone who is good numbers, but lacks the personality to be an accountant.
A mathematician is a blind man in a dark room looking for a black cat which isn't there. A topologist is a man who doesn't know the difference between a coffee cup and a doughnut.

The Dopeler Effect: The tendency of stupid ideas to seem smarter when they come at you rapidly.

Make it easy on yourself, realize that: *it's not suppose to be easy!*

The chief cause of failure and unhappiness is trading what we want most for what want at the moment!

The Three (3) POTENTIAL TRUTHS IN SURVEYING:

1. *THAT WHICH WAS INTENDED* TO BE SURVEYED; (agreements, plats, maps, deeds, etc.)
2. *THAT WHICH WAS ACTUALLY SURVEYED; (physical evidence)*
3. *THAT WHICH WAS CLAIMED TO HAVE BEEN SURVEYED. (plats, descriptions, maps reports)*

In theory, there is no difference between theory and practice; In practice, there is.

The past exists only in our memories, the future only in our plans, The present is our only reality.
Robert Prisg

Familiarity is a magician that is cruel to beauty but kind to ugliness.

50% of energy extraction (oil, natural gas, coal) costs are information based.

Transfers of Knowledge (versus Information) are very difficult because the pure knowledge (versus information) is imbedded in people.

Knowledge (per se) can NOT really be measured!

As an example, Sara Lee is selling off all manufacturing and only keeping its knowledge base, branding and marketing departments.

The average Ratio of Assets associated with Intellectual Capital versus Physical Assets in the USA is estimated to be between 3:1 and 4:1.

Intellectual Capital is that intangible quantity versus the Physical - tangible assets.
The 3 categories are *Human, Structural, and Customer based* (good will.)
Human - individual -AND- Team Knowledge.
Structural - those databases, files, manuals, procedures, and guidelines, - which are transferable.
Customer Capital - those values shared w/ and between customers.

Learn the "Big Picture" – where are your competency centers of tacit value.
Look at the information/knowledge chain of flow.

Learn to Broadcast SUCCESS! Learn to also award people who successfully COPY versus just awarding the new creative-generating idea people.
Ask "who makes the office function (more/most) efficiently?"
People learn more through STORIES that any other method in the sharing of tacit knowledge – it SOCIALIZES the knowledge. It's Intellectual -AND- SOCIAL CAPITAL, that we need for the creation and formation of Knowledge.

Beware of KNOWLEDGE HORDING - the non-sharing of knowledge, especially if it reduces chances and opportunities to make new partners; sometimes knowledge is not shared if it is seen to help a co-worker who is competing for the same advancement opportunity. Understand the concept of obtaining knowledge for its own sake, -versus- knowledge that fits the corporate strategies and goals.

Employees may become drowning victims with information overflow/overload in that phenomenon known as the KNOWLEDGE EXPLOSION.
The need to have KNOWLEDGE MANAGERS - the gatekeeper who selects and edits for everyone else in the group - the point of focus of knowledge flow and transfer.

REFLECT ON WHAT IS LEARNED - WITHOUT REFLECTION THERE IS NO REAL LEARNING! (AH-HA!)

Hell If We Know What, If Anything, These Mean:

You mean drill into the ground to try and find oil? You're crazy. –
Drilling experts to Edwin Drake, 1859

Pasteur's theory of germs is ridiculous fiction. –
Pierre Pachet, Physiology Prof. at Toulouse, 1872

This 'telephone' has too many shortcomings... The device is inherently of no value. –
Western Union, 1876

Heavier-than-air flying machines are impossible. –
Lord Kelvin, Royal Society Pres., 1895

Stocks have reached what looks like a permanently high plateau. –
Irving Fisher, Prof. of Economics, Yale Univ., '29

I'm just glad it'll be Clark Gable falling on his face and not me. –
Gary Cooper rejecting lead in 'Gone With The Wind'

I think there's a world market for maybe five computers. –
Thomas Watson, IBM chairman, '53

We don't like their sound, and guitar music is on the way out. –
Decca Records rejecting the Beatles, '62

There is no reason anyone would want a computer in their home. –
Ken Olson, Pres. & founder of DEC, '77

640K ought to be enough for anybody. –
Bill Gates, '81

You might be a GPS surveyor if...

*Posted By **Trimble Man** on 5/21/2002 at 7:16 PM*

The only point you need is an iron pin under the "ONLY" tree in a field..

Before taking the family camping you look at the K-index...

You meet friends at the local watering hole and remark on how the PDOP's were pretty good today..

You go out at night and lay by the pool and know where each satellite is by memory...

People at the NGS know you by first name...

When comparing receivers with your neighbor, he shows you a Kenwood and you show him your Trimble.

Your car has GPS but you run out of gas anyway..

You have more batteries than cables in your office...

You build a deck and you use your network adjustment software to square it up...

You go golfing and rate your score at 1 sigma (instead of 2)...

You use the PPM factor when doing your checkbook... (1ppm for each dollar used)...

Your neighbor asks about flag poles and you go into a long discussion on fixed height poles....

Your eight year old knows how to run an RTK system.

You've almost been in a traffic accident while using your handheld in the car.

You know what 'AFP' means (and so does your wife).

You think 'geocaching' is a great way to spend a day off.

Your wife's eyes glaze over when she hears you talking about 'multipath'.

You refer to attractive members of the opposite sex as having good "GDOP."

You have a waypoint stored for the grocery store, the bank and the office..

You follow the waypoint route even though it means cutting a fence or two and driving through Farmer Browns prized garden...

You ask a girl for her lat/long instead of address..

GREAT surveying websites:
(w/ an AZ bias)

http://www.btr.state.az.us/ AZ/BTR

http://www.azpls.org/ APLS

http://www.igc.topica.com/lists/AZSURVEY AZSURVEY

http://www.land.state.az.us/agic/agichome.html AZ Geographic Info Council

http://www.gci-net.com/~users/S/sowens/ APLS - Saguaro Chapter

http://www.dot.co.pima.az.us/gis/maps/control/index.htm Pima County GIS

http://www.dot.co.pima.az.us/gis/maps/control/index.htm
Pima County DOT - City of Tucson DOT Geodetic Control Points

http://www.mcdot.maricopa.gov/ Maricopa County DOT (* which will have a link to the M/C Geodetic Data and Cadastral Survey * within a few weeks.)

http://azwww.az.blm.gov/plats/cadhome.html BLM - AZ Cadastral offices

http://www.acea.org/ Arizona Consulting Engineers Association (ACEA)

http://www.srpnet.com/ Salt River Project (SRP)

(w/o the AZ bias)

http://www.wfps.org/ Western Federation of Professional Surveyors

http://www.survmap.org/acsmintro.html ACSM (NSPS & others)

http://www.ncees.org/ NCEES (examinations)

http://www.ca.blm.gov/webmanual/id2.htm BLM - the "Manual"

http://www.or.blm.gov/NILS/casebook/ BLM - the "Casebook"

http://www.i-boards.com/bnp/pob/ POB (formerly RPLS) messaging

http://www.forumboard.net/1635/ the 'other' messaging board

Famous Last Words:

** I'll get a world record for this..
** It's fireproof.

** He's probably just hibernating.
** What does this button do?
** I'm making a citizen's arrest.

** So, you're a cannibal.
** It's probably just a rash.

** Are you sure the power is off?
** Yeah, I made the deciding vote on the jury, so what of it?

** The odds of that happening have to be a million to one!
** Pull the pin and count to what?

** Which wire was I supposed to cut?
** I wonder where the mother bear is.

** I've seen this done on TV.
** These are the good kind of mushrooms.

** I'll hold it and you light the fuse.
** Let it down slowly.

** Rat poison only kills rats.
** Just take whatever you want, this is a ghost town.

** It's strong enough for both of us.
** This doesn't taste right.

** I can make this light before it changes.
** Nice doggie.

** I can do that with my eyes closed.
** I've done this before.

** Well, we've made it this far.
** That's odd.

** You wouldn't hit a guy with glasses on, would you?
** Don't be so superstitious.

** Now watch this.
** What duck?

**I've learned that you cannot make someone love you.
All you can do is stalk them and hope they panic and give in.**

**I've learned that no matter how much I care,
some people are just idiots.**

**I've learned that it takes years to build up trust,
and only suspicion, not proof, to destroy it.**

**I've learned that you can get by on charm for about fifteen minutes.
After that, you'd better have a great body.**

**I've learned that you shouldn't compare yourself to others -
they are more screwed up than you think.**

**I've learned that you can keep puking long after you think you're
finished**

**I've learned that we are responsible for what we do,
unless we are celebrities.**

**I've learned that sometimes the people you expect to kick you when
you're down will be the ones who do.**

**I've learned that we don't have to ditch bad friends,
because their dysfunction makes us feel better about ourselves.**

**I've learned that no matter how you try to protect your children,
they will eventually get arrested and end up in the local paper.**

**I've learned that the people you care most about in life are taken from
you too soon and all the less important ones just never go away.**

I've learned to say "Forget 'em if they can't take a joke" in 6 languages.

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